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Research Article

A New Maintenance Optimization Model Based on Three-Stage Time Delay for Series Intelligent System with Intermediate Buffer

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For the maintenance problem of intelligent series system with buffer stock, a preventive maintenance model based on the threestage time delay theory is proposed. Firstly, the intelligent series system is decomposed into $n-1$ virtual series systems by using approximate decomposition method. The impact factor is introduced to establish the failure rate and maintenance rate model of each virtual machine. Secondly, a preventive maintenance model based on the three-stage time delay theory is proposed for each virtual series system. The machine state from normal operation to failure stage is divided into three steps: initial defect, serious defect, and fault, and different distribution functions are defined in different stages to simulate the degradation process of the machine. Based on the three-stage time delay theory, the machine cost ratio model was established by taking the machine monitoring time and buffer stock as decision variables and the minimum unit time cost rate as objective function. Finally, the rationality and validity of the model are verified by an example analysis, which provides a basis for the maintenance of the intelligent series system.

1. Introduction

Intelligent series system is an important part of modern industrial manufacturing system. Due to the variety of machines and complex layout and structure, any failure may lead to the shutdown of the entire production line and cause huge economic losses for enterprises. For the continuous series production line, the preventive maintenance can also cause downtime.

Reasonably adding buffer between two machines can improve the flexibility of the production line, reduce the production dependence between upstream and downstream machine, and reduce the impact on the stability of series system due to machine downtime. The performance of the intelligent series system is closely related to its preventive maintenance plan and buffer setting. Preventive maintenance is related to the buffer stock allocation. In order to improve the production line stability and reduce the cost, it is very necessary to jointly optimize the series system buffer stock allocation and maintenance plan of machines.

Machines in the intelligent series system are closely connected, and the failure of one machine will lead to the shutdown of the entire intelligent series system. Recently, there are many literature works on the optimal preventive maintenance strategy of the series system. Wu et al. [1] established an optimized maintenance cost model and determined the optimal condition monitoring interval and the degradation level after imperfect preventive maintenance. The authors in [2] developed a dynamic maintenance strategy joint optimization problem that integrates production and opportunity maintenance. Rooeinfar et al. [3] studied the scheduling problem of uncertain flexible pipeline with finite buffer. Zhang et al. [4] investigated the incorporation of balance and preventive maintenance in U-shaped assembly line. Two metaheuristic algorithms including elitist nondominated sorting genetic algorithm and multiobjective simulated annealing algorithm were designed to solve this problem. The authors in [5] studied the integrated control of dynamic maintenance and production in a deteriorating manufacturing system and proposed a dynamic maintenance strategy that included corrective, preventive, and opportunistic maintenance. Bouslah et al. [6] discussed the integrated production, quality, and maintenance control of the production line. Motlagh et al. [7] developed an expert system for the unreliable unbalanced production line in reality, in which all time-based parameters were random. Considering a series production system with random degradation, Wang et al. [8] proposed a predictive maintenance strategy based on the predicted failure probability of each machine and a production control strategy based on the target service level, so as to meet the dynamic stochastic demand in each period. Wang et al. [9] proposed two maintenance strategies for the series production line. The first was based on the cost rate of the machine under long-term operation, and the second was based on the single-piece machine maintenance strategy considered in the production line. The authors in [10] proposed an alternative scheduling model for railway production lines and proposed a time-based flexible displacement (FTBR) method in combination with the artificial bee colony (ABC) algorithm. Based on the concept of "energy-saving opportunity window," [11] modeled the continuous deterioration process of each machine and regarded the energy-saving opportunity window of the production system as the opportunity window of preventive maintenance. However, the above literature only proposes preventive maintenance strategies for series system not combined with buffer zones.

Buffers have been used in maintenance of machines for a long time. The authors in [12] first proposed buffer stock, considering the impact of the interstage buffer on production at different production speeds, different failure rates, and different repair rates. The method of using regeneration point for analysis and treatment was given. Currently, a large number of scholars study the optimal allocation of buffer. The authors in [13] presents a model to determine the optimal length of continuous production periods between maintenance actions and the optimal buffer inventory to satisfy demand during preventive maintenance or repair of a manufacturing facility. On the basis of this model, the authors in[14] considers that the opportunities for the fabrication of the buffer inventory and opportunities to carry out a maintenance action to the production facility are random. The authors in [15] proposed a multiobjective mathematical formula and hybrid method, which can simultaneously solve the buffer size and machine allocation problems on unreliable production lines and assembly lines with general distributed time-varying parameters. The authors in [16] proposed a tabu search algorithm to find the optimal buffer allocation plan for a serial production line composed of unreliable machines. The authors in [17] considered an imperfect production system with preventive maintenance activities in order to obtain the optimal buffer stock and minimum warranty inspection policy for sold products. The production system has the probability of changing from the normal state to the out-of-control state at any time. Under the normal state and out-of-control state, the production system will produce a certain proportion of defect items. The authors in [18] developed a method to analyze the complex tradeoff between the preventive maintenance and the buffer's contribution to system performance, considering a two-machine continuous manufacturing system with a finite capacity buffer. The authors in [19] analyzed the tradeoff between buffer capacity, spare parts inventory, and throughput for a two-stage production system with buffers and established a discrete-time Markov chain for two different situations. The numerical examples showed that the effect of a spare part on the efficiency of a transfer line was much greater than the effect of additional buffer places. All of the above research is aimed at two-stage system and does not combine buffer with intelligent series system.

There are few studies on the combination of maintenance and buffer with series system. Nahas [20] considered an unreliable serial production line. The target was to minimize the total cost of the system through finding the optimal preventive maintenance strategy and optimal buffer size at a given level of the system throughput. Extend the flood algorithm was put forward in order to solve this problem. The decomposition approximation method was used to estimate the production capacity of production line. Zandieh et al. [21] studied buffer and preventive maintenance cycle allocation issues. The model was built with three objective functions: the maximization of production rate, the minimization of buffer size, and the total number of defect units. Finally, a synthetic simulation method and a meta-heuristic algorithm were used to solve the model. Lopes [22] minimized the total cost of each product while considering product quality testing for production lines with buffers. According to the machine degradation stage and buffer level, Kang and Ju [23] used Markov decision model to obtain the optimal maintenance strategy of the machine in the production line. Alfieri et al. [24] used the approximated mathematical programming formulation of the Buffer Allocation Problem (BAP) simulation-optimization based on the time buffer concept. However, in these studies, the preventive maintenance strategy of the series system is considered as a whole, and it is not disassembled to study each machine in the production line. Preventive maintenance strategy may not be adapted to each machine in the production line.

In this paper, the buffer is added to the intelligent series system to jointly optimize the buffer stock and the optimal preventive maintenance cycle. The approximate decomposition method is used in this paper to decompose the intelligent series system into several virtual series systems with two machines and a buffer.

The approximate decomposition method is used to study the problem of production line preventive maintenance, which was first proposed by [25]. After that, it was widely used. Based on the approximate decomposition method, [26] presented an efficient method to evaluate performance of tree-structured assembly/disassembly (AD) systems with finite buffer capacity. For mixed-model flexible transfer lines, [27] proposed a general simulation model. Compared with the approximate decomposition method, the numerical results show that the proposed method is robust for predicting the throughput of transmission lines. Li et al. [28] proposed a common model that unifies several approximate methods for the analysis of tandem queueing systems with blocking. Xia et al. [29] proposed an efficient decomposition method based on a generalized exponential distribution to analyze the homogeneous transfer lines with unreliable buffers. The authors in [30] developed three heuristic approaches to solve the formulated combinatorial optimization problem. To estimate the production line throughput, an approximate decomposition method was used. Xia et al. [31] decomposed the original long line into several small decoupled subsystems and added relation condition variables between the subsystems. Bai et al. [32] proposed a new aggregation-based iterative algorithm to calculate the performance metrics of a multi-machine serial line by representing it using a group of virtual two machine lines. In this paper, based on the approximate decomposition method, the influence factors are introduced according to the importance of different machines in the series system to obtain the optimal maintenance strategy for each machine.

Another innovation of this paper is to introduce the time delay theory into the maintenance model. Many studies have presented the preventive maintenance strategies by considering buffer stock. However, the machine's degradation is not considered in this area. In this paper, the preventive maintenance strategy can be developed by introducing the time delay theory to simulate the degradation process.

The time delay theory is often used to simulate the machine degradation process. The time delay model proposed by Christer and Waller [33] is the first time to extend the time delay theory to the maintenance of industrial plants. The basic model of inspection and maintenance and some change models observed in practice are presented. Later, a large number of scholars applied the time delay theory to the field of establishing the correlation between machine maintenance cost and preventive maintenance inspection interval cycle. Wang [34] proposed such a model for a serviceable one-component system to jointly model the effect of RS and inspection with replacement on the basis of the delay-time concept. Zhao et al. [35] developed a model to evaluate the reliability and optimized the inspection schedule for a multidefect component. Gomes da Silva and Lopes [36] simulated the preventive maintenance model based on the nonhomogeneous Poisson distribution. Mahmoudi et al. [37] studied the occurrence process of machine defects presented as homogeneous Poisson distribution, and then solved the optimal maintenance cycle of preventive maintenance strategy. Based on the traditional time delay theory, Wang et al. [38] introduces a two-level inspection policy model for a single component plant system based on a three-stage failure process. They divided the machine failure into three states: original defect, serious defect, and fault, so as to simulate the fault random process. By applying the three-stage time delay theory to the simulation of machine degradation and renewal process, the clustering summary of different kinds of faults or defects that may occur in the machine can be performed in a more precise and quantitative manner.

In this paper, first, the intelligent series system is decomposed into $n-1$ virtual series systems by approximate decomposition method, and one virtual series system includes two virtual machines. The aim is to find the relationship between machines and present the cost ratio and maintenance ratio model by considering an influence factor. Then, for each two virtual machines, the buffer stock and machine monitoring time can be described as decision variables, and the total cost rate can be minimized as the objective function. A novel maintenance model based on three-stage time delay is developed to obtain the optimal preventive maintenance strategy and the buffer stock. The proposed model can be divided into four types, and it contains the whole process of machine degradation based on different machines status and monitoring time. Finally, the proposed model is compared with the maintenance model based on two-stage time delay by a case study. The overall optimal maintenance strategy and buffer stock are obtained. In the case study, this paper analyzes an intelligent series system of Shanghai Pangyuan Machinery Co.

The remainder of the paper is as follows. In Section 2, a description of general problem is presented. In Section 3, the notation and assumption are presented. The maintenance rate and cost ratio model of virtual machine and the preventive maintenance model based on the three-stage time delay theory are introduced in Section 4. In Section 5, solving method is introduced. In Section 6, numerical examples are presented and analyzed. Finally, the most important results and future work are summarized in Section 7.

2. Problem Description

The intelligent series system L includes n machines and $n-1$ buffers. It is shown in Figure 1. We assume that M_1 is never starved and the supply of raw materials needed by M_1 is continuous. M_n is never blocked, and the final product produced by M_n does not have a backlog state. The failure rate λ_i and maintenance rate μ_i of each unit are known and defined as known conditions. The approximate decomposition method is used to decompose L into $n-1$ virtual series systems, and each virtual series system includes two machines and one buffer.

After L is decomposed, $n-1$ virtual series systems are formed. M_u and M_d are virtual machines of one virtual series system after decomposition. M_u is the upstream machine, and semifinished product m_1 from M_u is input into downstream machine M_d at the production rate β through the intermediate buffer B. M_d uses m_1 as the raw material to produce product m_2 at β. B needs to be accumulated before maintenance actions adopted by M_u to ensure continuity of production process of the virtual series system.

The traditional time delay model divides the health status into defect state and fault state. The three-stage time delay model divides the health status into normal state, original defect state, serious defect state, and fault state, as shown in Figure 2. Thus, the degradation process can be divided into original defect time, serious defect time, and failure time.

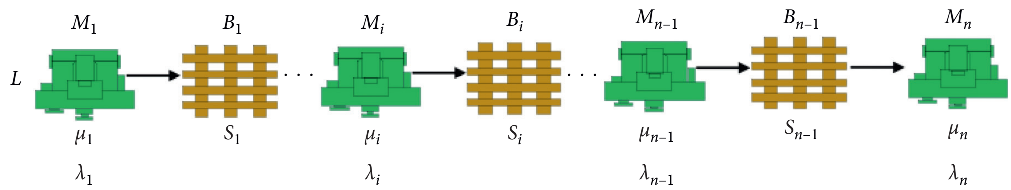


FIGURE 1: Intelligent series system L.

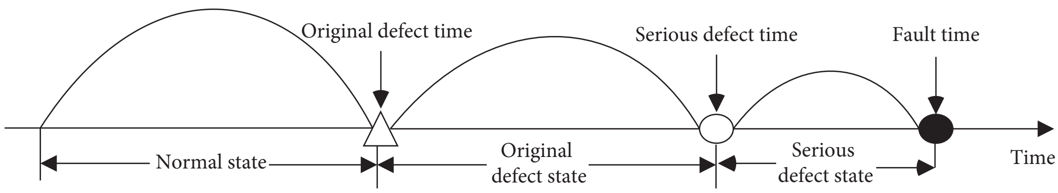


FIGURE 2: Three stages of failure.

After the upstream machine runs a certain period, B can be added by the replenishment rate α until it reaches buffer stock level S. Then, status monitoring can be executed on the upstream machine. If the monitoring result shows that it is in initial defect state or serious defect state, preventive maintenance needs to be adopted immediately. If it is stopped due to failure before monitoring, then fault repair can be performed.

3. Notation and Assumption

3.1. Notation

M_i: i^{th} machine of intelligent series system L

λ_i: failure rate of M_i

μ_i: maintenance rate of M_i

$M_u(i)$: upstream machine after decomposition

$\lambda_u(i)$: failure rate of $M_u(i)$

$\mu_u(i)$: maintenance rate of $M_u(i)$

$M_d(i)$: downstream machine after decomposition

$\lambda_d(i)$: failure rate of $M_d(i)$

$\mu_d(i)$: maintenance rate of $M_d(i)$

$t_u(i)$: maintenance time of $M_u(i)$

$r_u(i)$: remaining maintenance time of $M_u(i)$

α: replenishment rate of buffer

β: production rate of L

X: machine normal operation phase

Y: machine original defect operation phase

Z: machine serious defect operation phase

$f_x(x)$: probability density function from initial state of one machine to occurrence of original defect

$f_y(y)$: probability density function from original defect state of one machine to the occurrence of serious defect

$f_z(z)$: probability density function from serious defect state of one machine to the occurrence of failure

$F_x(x)$: distribution function from initial state of one machine to the occurrence of original defect

$F_y(y)$: distribution function from original defect state of one machine to the occurrence of serious defect

$F_z(z)$: distribution function from serious defect state of one machine to the occurrence of failure

h: buffer stock holding cost per unit

ρ: shortage cost per unit

C_r: monitoring cost of each time

C_x: maintenance cost of one time for machine in an original defect state

C_y: maintenance cost of one time for machine in a serious defect state

C_z: maintenance cost of one time for machine in a fault state

$C_m(T)$: maintenance costs in a cycle

$C_h(S)$: holding cost in a cycle

$C_s(S,T)$: shortage cost in a cycle

S: buffer stock level

T: machine monitoring time from the end of a maintenance action to the start of the status monitoring

$\text{EC}(T)$: expected length of a cycle for operational time T

$C(S,T)$: total cost in a cycle

TCR (S,T): cost rate in a cycle

W_i $(i = 1,2,3)$: random variable, maintenance time of the machine by original defect state, serious defect state, and fault state, respectively

$g_i(t)$: probability density function of W_i, $i = 1,2,3$

$G_i(t)$: distribution function of W_i, $i = 1,2,3$

3.2. Assumptions

(1) The machine status needs to be monitored after T, and monitoring time can be ignored.

(2) All the states can be accurately monitored. The corresponding maintenance actions can be adopted immediately after monitoring. Production can be resumed immediately after completing maintenance.

(3) Extra production capacity is always available in order to produce buffer stock.

(4) If there is buffer stock after completing maintenance, the next production cycle firstly consumes buffer stock.

4. The Model

4.1. Decomposing Production Line. L is decomposed into $n-1$ virtual series systems by approximate decomposition method, as shown in Figure 3. Each virtual series system has only two virtual machines and a buffer (where M_u is never starved and M_d is never blocked). Buffer in Line 1 corresponds to B_1, and buffer in Line $n-1$ corresponds to B_{n-1}. For Line i, failure rates $\lambda_u(i)$, $\lambda_d(i)$ and maintenance rates $\mu_u(i)$, $\mu_d(i)$ are unknown. Thus, the next step is to solve the failure rate and maintenance rate of virtual machine.

4.2. Failure Rate and Maintenance Rate Model of Virtual Machine. M_i in L is decomposed into virtual machine $M_d(i-1)$ and $M_u(i)$, $M_u(i)$, where $M_u(i)$ is upstream machine of Line i and $M_d(i-1)$ is downstream machine of Line $i-1$. Maintenance strategy of $M_u(i)$ is the same as M_i in L. The starvation of M_i represents starvation of $M_d(i-1)$, and failure represents failure of $M_u(i)$. Thus, λ_u and μ_u of $M_u(i)$ are required, and its maintenance strategy is formulated. $M_u(i)$ is a virtual machine decomposed from M_i. The failure of $M_u(i)$ represents failure or starvation of M_i, M_i, and the starvation of M_i is caused by failure or starvation of M_{i-1}. Failure or starvation of M_{i-1} represents failure of $M_u(i-1)$. Thus, the failure $M_u(i)$ is jointly determined by failure of M_i and failure of $M_u(i-1)$.

The failure of $M_u(i)$ is related to failure of $M_u(i-1)$ and failure of M_i. Thus, the impact factor a can be introduced. The proportion is a when the failure of $M_u(i)$ is from $M_u(i-1)$fault, and the proportion is $1-a$ when the failure of $M_u(i)$ is from M_i fault. The relationship is as follows:

$$\lambda_u(i) = a\lambda_u(i-1) + (1-a)\lambda_i, \tag{1}$$

$$t_u(i) = ar_u(i-1) + (1-a)t_i, \tag{2}$$

where $t_u(i)$ and t_i are the average maintenance time of $M_u(i)$ and M_i respectively. $r_u(i-1)$ is the average remaining maintenance time of $M_u(i-1)$ when M_i is starved. It is the maintenance time after consuming inventory in B_{i-1}.

4.3. Cost Ratio Model. For each cycle, the machine can be monitored immediately after completing buffer stock replenishment. Different maintenance strategies can be adopted based on machine monitoring status. The possible occurrence time of original defect state, serious defect state, and fault state is T_x, T_y, T_z, respectively. There are four different situations for adopting preventive maintenance based on monitoring status, T and S.

4.3.1. $0 < T < T_x$. The state monitoring time of machine occurs before the original defect time. Machine status is in a nondefective state, and it is unnecessary to execute any maintenance action. After that, in order to prevent the failure and to detect the defect in time, it is necessary to execute a state monitoring every day until the original defect is detected. Buffer stock has been replenished from first status monitoring. Buffer stock level change during a running cycle is shown in Figure 4 and preventive maintenance of machine is executed under the original defect state.

(1) The probability of Twithin $[0, T_x]$:

$$P(0 < T < T_x) = P^1 = \int_T^\infty f_x(x)\mathrm{d}x. \tag{3}$$

(2) Operation cycle of machine for $0 < T < T_x$

The operational cycle of machine includes monitoring time and maintenance time. One cycle is from the end of the above maintenance to the end of the next maintenance. For this situation, operational time is T_x, and maintenance time is W_1. Then,

$$\mathrm{EC}^1(T) = E(W_1) + E(T_X). \tag{4}$$

The total cost includes inventory holding cost, shortage cost, and maintenance cost within a cycle.

(3) Inventory holding cost for $0 < T < T_x$

The inventory holding cost can be generated from beginning to produce buffer stocks, and it can be increased with the increasing of buffer stock. From the beginning of buffer stock replenishment to the end of maintenance actions, buffer is always occupied. Thus,

$$C_h^1(S) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right) + (x-T)S\right]. \tag{5}$$

(4) Shortage cost for $0 < T < T_x$

A shortage occurs when buffer stocks are depleted and maintenance actions have not ended. Therefore, there will be a shortage of cost. The shortage time is from the end of buffer stock depletion to the end of maintenance action. Then,

$$C_s^1(S,T) = \rho\beta\int_{S/\beta}^\infty \overline{G_1}(w)\mathrm{d}w. \tag{6}$$

(5) Maintenance cost for $0 < T < T_x$

Maintenance cost includes the expected cost of preventive maintenance and all testing costs. In this case, preventive maintenance of the original defect state is carried out. $x - T + 1$ tests were conducted before the maintenance activity. Thus, maintenance cost in a cycle is

$$C_m^1(T) = C_x + (x-T+1)C_r. \tag{7}$$

The expected cost in a cycle for $0 < T < T_x$ is the summation of inventory holding cost, shortage cost, and maintenance cost. The expected cost $C^1(S,T)$is obtained:

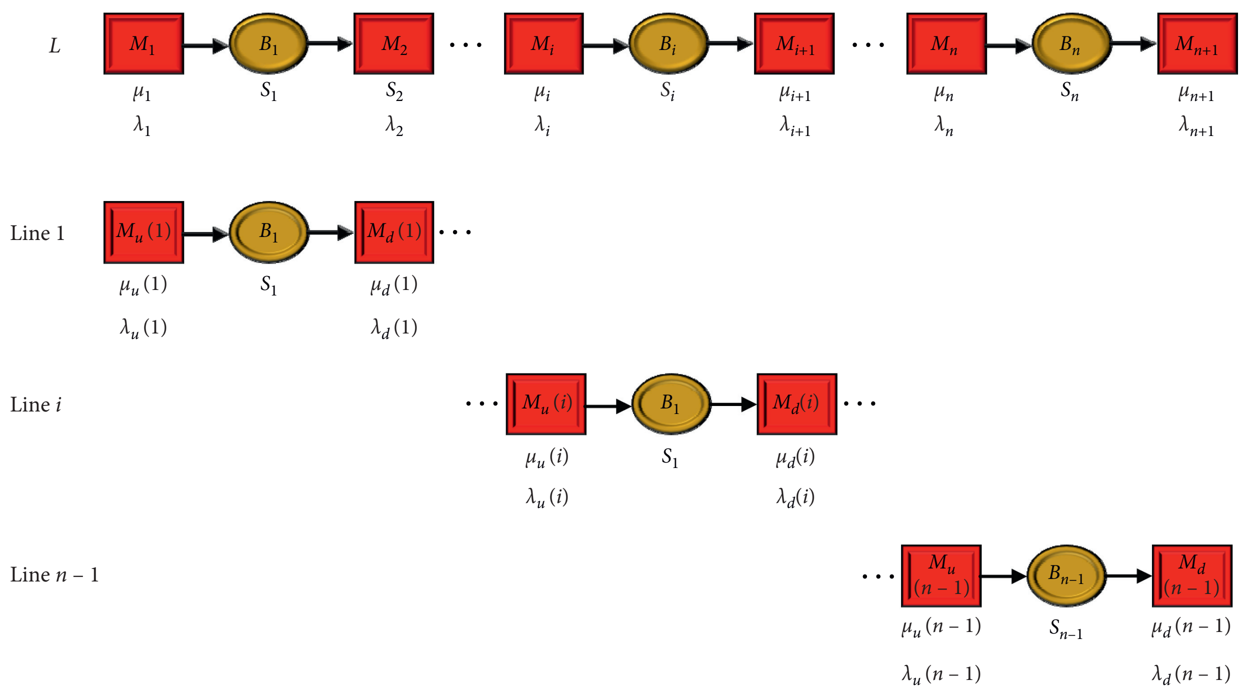


Figure 3: $n-1$ virtual series systems after decomposition.

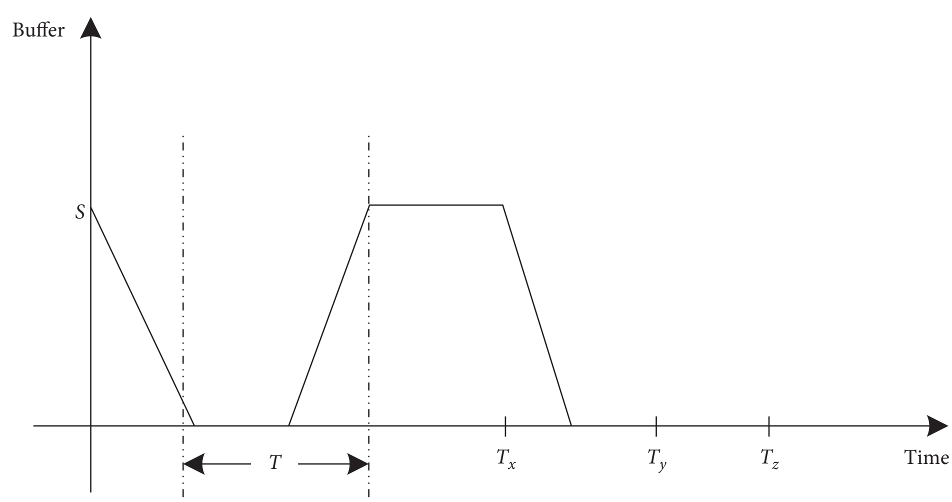


Figure 4: Buffer stock change diagram in a cycle for $0 < T < T_x$.

$$C^1(S,T) = C_h^1(S) + C_s^1(S,T) + C_m^1(T) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right) + (x-T)S\right] + \rho\beta\int_{S/\beta}^{\infty}\overline{G_1}(w)\mathrm{d}w + C_x + (x-T+1)C_r. \tag{8}$$

4.3.2. $T_x < T < T_y$. The state monitoring time of machine occurs after the original defect time and before the serious defect time. Machine status is in an original defect operation state, and it needs to execute preventive maintenance of the original defect state. After the buffer stock is replenished, machine status is monitored immediately. Buffer stock level change during a running cycle is shown in Figure 5.

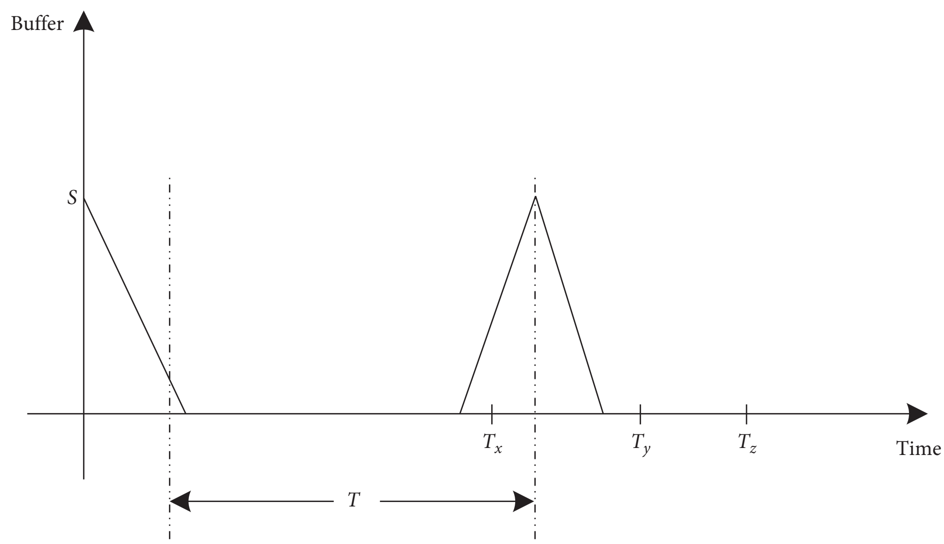


Figure 5: Buffer stock change diagram in a cycle for $T_x < T < T_y$.

(1) The probability of T within $[T_x, T_y]$

$$P\left(T_x < T < T_y\right) = P^2 = \int_0^T \int_{T-x}^{\infty} f_x(x) f_y(y) \mathrm{d}x\mathrm{d}y = \int_0^T f_x(x)\left(1 - F_y(T-x)\right)\mathrm{d}x. \quad (9)$$

(2) Operation cycle of machine for $T_x < T < T_y$

The operational cycle of machine includes operational time and maintenance time. One cycle is from the end of the above maintenance to the end of the next maintenance. For this situation, operational time of machine is T, and maintenance time is W_1. Then,

$$\mathrm{EC}^2(T) = E\left(W_1\right) + T. \quad (10)$$

(3) Inventory holding cost for $T_x < T < T_y$

The inventory holding cost can be generated from beginning to produce buffer stocks, and it can be increased with the increasing of buffer stock. From the beginning of buffer stock replenishment to the end of maintenance actions, buffer is always occupied. Thus,

$$C_h^2(S) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right]. \quad (11)$$

(4) Shortage cost for $T_x < T < T_y$

The shortage cost in one cycle is the same as in Section 4.3.1.

$$C_s^2(S,T) = \rho\beta \int_{S/\beta}^{\infty} \overline{G_1}(w)\mathrm{d}w. \quad (12)$$

(5) Maintenance cost for $T_x < T < T_y$

In this case, the machine makes one state monitoring process. The test result is the original defect state, so the preventive maintenance of the original defect state is executed. Thus, maintenance cost in a cycle is

$$C_m^2(T) = C_x + C_r. \quad (13)$$

The expected cost $C^2(S,T)$ in a cycle for $T_x < T < T_y$ is obtained as

$$C^2(S,T) = C_h^2(S) + C_s^2(S,T) + C_m^2(T) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right] + \rho\beta \int_{S/\beta}^{\infty} \overline{G_1}(w)\mathrm{d}w + C_x + C_r. \quad (14)$$

4.3.3. $T_y < T < T_z$. The state monitoring time of machine occurs after the serious defect time and before the failure time. Machine status is in a serious defect operation state, and it needs to execute preventive maintenance of the serious defect state. After the buffer stock is replenished, the machine status is monitored immediately. Buffer stock level change during a running cycle is shown in Figure 6.

(1) The probability of Twithin $[T_y, T_z]$

$$P\left(T_y < T < T_z\right) = P^3 = \int_0^T \int_0^{T-x} \int_{T-x-y}^{\infty} f_x(x) f_y(y) f_z(z)\mathrm{d}x\mathrm{d}y = \int_0^T \int_0^{T-x} f_x(x) f_y(y)\left(1 - F_z(T-x-y)\right)\mathrm{d}x\mathrm{d}y. \quad (15)$$

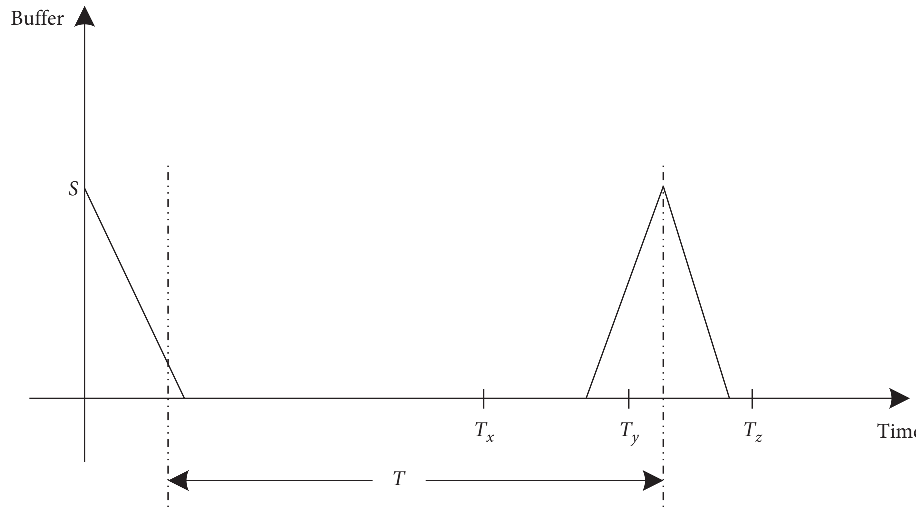


FIGURE 6: Buffer stock change diagram in a cycle for $T_y < T < T_z$.

(2) Operation cycle of machine for $T_y < T < T_z$

For this situation, operational time of machine is T, and maintenance time is W_2. Then,

$$\mathrm{EC}^3(T) = E(W_2) + T. \tag{16}$$

(3) Inventory holding cost for $T_y < T < T_z$

For this situation, from the beginning of buffer stock replenishment to the end of maintenance activities, there will be inventory occupation. Therefore, the inventory holding cost is as follows:

$$C_h^3(S) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right]. \tag{17}$$

(4) Shortage cost for $T_y < T < T_z$

In this case, preventive maintenance of machine in the serious defect state is required. Thus, the shortage cost in a cycle is

$$C_s^3(S,T) = \rho\beta\int_{S/\beta}^{\infty}\overline{G_2}(w)\mathrm{d}w. \tag{18}$$

(5) Maintenance cost for $T_y < T < T_z$

In this case, the machine makes one state monitoring process. The test result is the serious defect state, so the preventive maintenance of the serious defect state is carried out. Thus, maintenance cost in a cycle is

$$C_m^3(T) = C_y + C_r. \tag{19}$$

The expected cost $C^3(S,T)$ in a cycle for $T_y < T < T_z$ is obtained.

$$\begin{aligned} C^3(S,T) &= C_h^3(S) + C_s^3(S,T) + C_m^3(T) \\ &= h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right] + \rho\beta\int_{S/\beta}^{\infty}\overline{G_2}(w)\mathrm{d}w + C_y + C_r. \end{aligned} \tag{20}$$

4.3.4. $T > T_z$. In this case, the machine fails before the state detection is carried out. The planned state detection takes place after the failure of machine. At this time, the corrective maintenance action is executed. Buffer stock has not been replenished, or the production of buffer stock has not started before the failure shutdown. Thus, the buffer stock change during a running cycle is shown in Figure 7.

(1) The probability of T within $[T_z, \infty]$

$$\begin{aligned} P(T > T_z) = P^4 &= \int_0^T\int_0^{T-x}\int_0^{T-x-y} f_x(x)f_y(y)f_z(z)\mathrm{d}x\mathrm{d}y\mathrm{d}z \\ &= \int_0^T\int_0^{T-x} f_x(x)f_y(y)F_z(T-x-y)\mathrm{d}x\mathrm{d}y. \end{aligned} \tag{21}$$

(2) Operation cycle of machine for $T > T_z$

For this situation, before machine status is monitored, the failure has already occurred. Operational time of machine is T_z, and maintenance time is W_3. Then,

$$\mathrm{EC}^4(T) = E(W_3) + E(T_z). \tag{22}$$

(3) Inventory holding cost for $T > T_z$

In this case, from the beginning of buffer stock replenishment to the failure occurrence, there will be inventory occupation. Then,

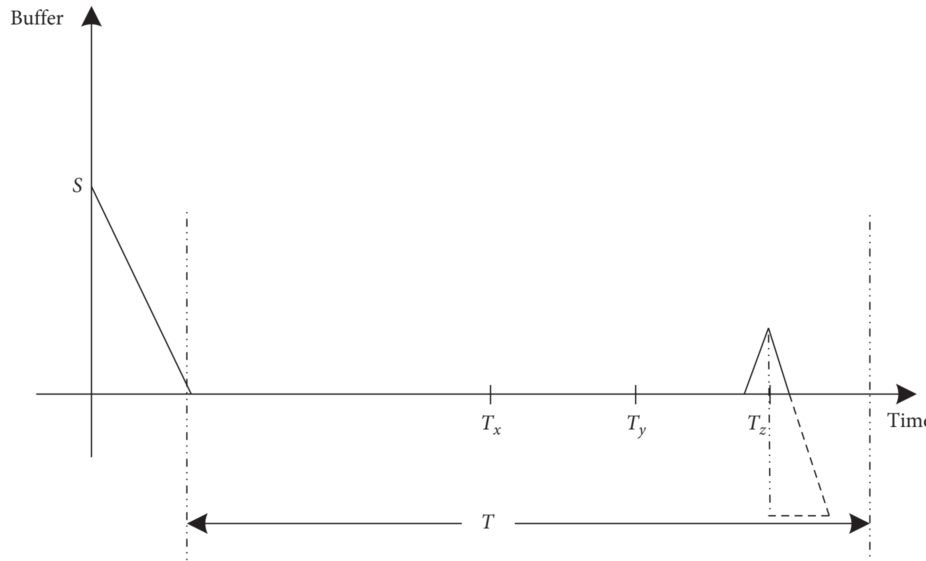


Figure 7: Buffer stock change diagram in a cycle for $T > T_z$.

$$C_h^4(S) = \frac{h}{2}\left[S - \alpha\left(T - E\left(T_z\right)\right)\right]\left[\frac{(\alpha+\beta)S}{\alpha\beta} - \left(\frac{\alpha+\beta}{\beta}\right)\left(T - E\left(T_z\right)\right)\right]. \tag{23}$$

(4) Shortage cost for $T > T_z$

In this case, the machine fails before the buffer stock is replenished. The machine breaks down, so corrective maintenance is carried out. Thus, the shortage cost in a cycle is

$$C_s^4(S,T) = \rho\beta \int_{\left(S-\alpha\left(T-E\left(T_z\right)\right)\right)/\beta}^{\infty} \overline{G_3}(w)\mathrm{d}w. \tag{24}$$

(5) Maintenance cost for $T > T_z$

In this case, the status of the machine has not been detected, and the machine breaks down. Therefore, corrective maintenance is executed. Thus, the maintenance cost of machine in a cycle is the maintenance cost under the state of failure, and there is no detection cost. Then,

$$C_m^4(T) = C_z. \tag{25}$$

The expected cost $C^4(S,T)$ in a cycle for $T > T_z$ is obtained:

$$\begin{aligned} C^4(S,T) &= C_h^4(S) + C_s^4(S,T) + C_m^4(T) \\ &= \frac{h}{2}\left[S - \alpha\left(T - E\left(T_z\right)\right)\right]\left[\frac{(\alpha+\beta)S}{\alpha\beta} - \left(\frac{\alpha+\beta}{\beta}\right)\left(T - E\left(T_z\right)\right)\right] \\ &\quad + \rho\beta \int_{\left(S-\alpha\left(T-E\left(T_z\right)\right)\right)/\beta}^{\infty} \overline{G_3}(w)\mathrm{d}w + C_z. \end{aligned} \tag{26}$$

4.3.5. Cost Ratio Model. The cost rate in a cycle is expressed as the total cost within a cycle divided by the cycle time. Then,

$$\begin{aligned} \mathrm{TCR}(S,T) &= \frac{C(S,T)}{\mathrm{EC}(T)} = \frac{P^1C^1(S,T) + P^2C^2(S,T) + P^3C^3(S,T) + P^4C^4(S,T)}{P^1\mathrm{EC}^1(T) + P^2\mathrm{EC}^2(T) + P^3\mathrm{EC}^3(T) + P^4\mathrm{EC}^4(T)} \\ &= \frac{\begin{array}{c} P^1\left(C_h^1(S) + C_s^1(S,T) + C_m^1(T)\right) + P^2\left(C_h^2(S) + C_s^2(S,T) + C_m^2(T)\right) \\ + P^3\left(C_h^3(S) + C_s^3(S,T) + C_m^3(T)\right) + P^4\left(C_h^4(S) + C_s^4(S,T) + C_m^4(T)\right) \end{array}}{P^1\mathrm{EC}^1(T) + P^2\mathrm{EC}^2(T) + P^3\mathrm{EC}^3(T) + P^4\mathrm{EC}^4(T)}. \end{aligned} \tag{27}$$

Thus, the maintenance cost ratio model is

$$\begin{cases} \min\{\mathrm{TCR}(S,T)\}, \\ S,T \in N^*; \quad S,T > 0. \end{cases} \tag{28}$$

5. Model Solving

5.1. Calculate the Failure Rate λ_u and Maintenance Rate μ_u of Virtual Machine. Equation (1) is solved by mathematical induction:

$$\begin{aligned}\lambda_u(i) &= a\lambda_u(i-1) + (1-a)\lambda_i\\ \lambda_u(i-1) &= a\lambda_u(i-2) + (1-a)\lambda_{i-1}\\ &\vdots\\ \lambda_u(2) &= a\lambda_u(1) + (1-a)\lambda_2.\end{aligned} \quad (29)$$

$\lambda_u(1)$ is equal to λ_1.

Similarly, (2) is solved by mathematical induction:

$$\begin{aligned}t_u(i) &= ar_u(i-1) + (1-a)t_i\\ t_u(i-1) &= ar_u(i-2) + (1-a)t_{i-1}\\ &\vdots\\ t_u(2) &= ar_u(1) + (1-a)t_2.\end{aligned} \quad (30)$$

Get the average maintenance time of virtual machine $M_u(i)$, and then get its maintenance rate $\mu_u(i)$.

5.2. Solving the Optimal Maintenance Cycle and Buffer Stock. In this paper, discrete iterative algorithm is used to solve the optimal solution. The specific steps are as follows:

Step 1. To assign $S = S_{\min}$

Step 1.1. $T = T_{\min}$.
Step 1.2. To solve TCR (T, S), assign TCR (T^*, S) = TCR (T, S).
Step 1.3. $T = T + \Delta T$, to solve TCR (T, S).
Step 1.4. To judge if $T < T_{\max}$. If so, it goes to Step 1.5; otherwise, go to Step 1.6.
Step 1.5. To judge if TCR $(T^*, S) >$ TCR (T, S). If so, assign TCR (T^*, S) = TCR (T, S), $T^* = T$, $T^* = T$; it goes to Step 1.3; otherwise, to record TCR (T^*, S), T^*, go to Step 1.5.
Step 1.6. To assign $S = S + \Delta S$, to judge if $S < S_{\max}$. If so, it goes to Step 1.1; otherwise, the program ends.

Step 2. Through Step 1, we can obtain the optimal operating cycle T^* under different stock allocation amounts S, as well as all the cost rates TCR (T^*, S). Record all the TCR (T^*, S) that we get. After sorting, it is easy to find the system's minimum average cost rateTCR $(T^*, S^*) = \min_{S_{\min} \le S \le S_{\max}} \{\text{TCR}(T^*, S)\}$ and the most joint strategy (T^*, S^*).

The flow chart of discrete iteration algorithm is shown in Figure 8.

6. Case Study

In this numerical example, the specific parameters and data of the intelligent series system were obtained from Shanghai Pangyuan Machinery Co.. The workshop has a lathe production line consisting of four machines and three buffers. By monitoring the equipment history fault record, the equipment fault parameters are summarized as follows. The original defect stage, serious defect stage, and failure stage of machine M_i are subject to exponential distribution independently. $f_x(x), f_y(y), f_z(z)$ are used to represent the probability density functions of machine deterioration in each stage, respectively. The definition of the exponential distribution function is given as follows:

$$f(x) = \lambda e^{-\lambda x}. \quad (31)$$

$\lambda_{i1}, \lambda_{i2}, \lambda_{i3}$ are used to represent the parameters in the exponential distribution of the $f_x(x), f_y(y), f_z(z)$ which are shown in Table 1.

The productivity of production line β is 30000 units per year. The buffer replenishment rate α is 6000 units per year. Shortage cost ρ = \$200 per unit. The cost of each machine monitoring process is \$800. The unit cost of corrective repair is \$15000, the unit cost of serious defect repair is \$7000, and the unit cost of original defect repair is \$4000. The maintenance time of each machine in the original defect state is supposed to be uniformly distributed between 0.5 and 1 day. The maintenance time of each machine in the serious defect state is supposed to be uniformly distributed between 2 and 5 days. The maintenance time of corrective maintenance is supposed to be uniformly distributed from 3 to 7 days. S varies from 0 to 211 units. T ranges from 0 to 105 days.

Using approximate decomposition method, the original production line is decomposed into three virtual series systems with two machines and one buffer. In the specific solution, $a = 0.2, a = 0.5, a = 0.8$ are, respectively, taken into the solution. Since the maintenance rate of each machine is the same, the maintenance rate of the decomposed virtual machine is the same as that of the original machine, so only the failure rate of the decomposed virtual machine needs to be solved.

In the case of $a = 0.2$, the failure rate $\lambda_u(i)$ of the virtual machine $M_u(i)$ solved is shown in Table 2.

In order to simplify the difficulty of solving and relate to the actual situation, only the case where period T and buffer stock S are integers is considered in this paper. The cost ratio model is a double integer parameter nonlinear programming problem. One discrete iteration algorithm is used to solve the model. The optimal monitoring time T_1, T_2, T_3 of the machines $M_u(1), M_u(2), M_u(3)$ is 29, 27, 27 days. The optimal stock allocation amounts S_1, S_2, S_3 of buffers B_1, B_2, B_3 are 79, 80, 79 units. Figure 9 shows the change of the cost rate of machines $M_u(1)$, $M_u(2)$, $M_u(3)$ with S, T.

When $a = 0.2$, the operating cycle of each machine in the intelligent series system, the buffer stock allocation amount, and the corresponding lowest cost rate are shown in Table 3.

Similarly, in the case of $a = 0.5$, the failure rate $\lambda_u(i)$ of the virtual machine $M_u(i)$ solved is shown in Table 4.When $a = 0.5$, the operating cycle of each machine in the production line, the buffer stock allocation amount, and the corresponding lowest cost rate are shown in Table 5.

Similarly, in the case of $a = 0.5$, the failure rate $\lambda_u(i)$ of the virtual machine $M_u(i)$ solved is shown in Table 6.When $a = 0.8$, the monitoring time of each machine in the intelligent series system, the buffer stock allocation amount, and the corresponding lowest cost rate are shown in Table 7.

6.1. Result Analysis. For different influence factors a, the monitoring time and buffer stock allocation are obtained. Table 8 is a comparison of the optimal monitoring time for each machine under different a. Table 9 is a comparison of the best stocks for each buffer under different a.

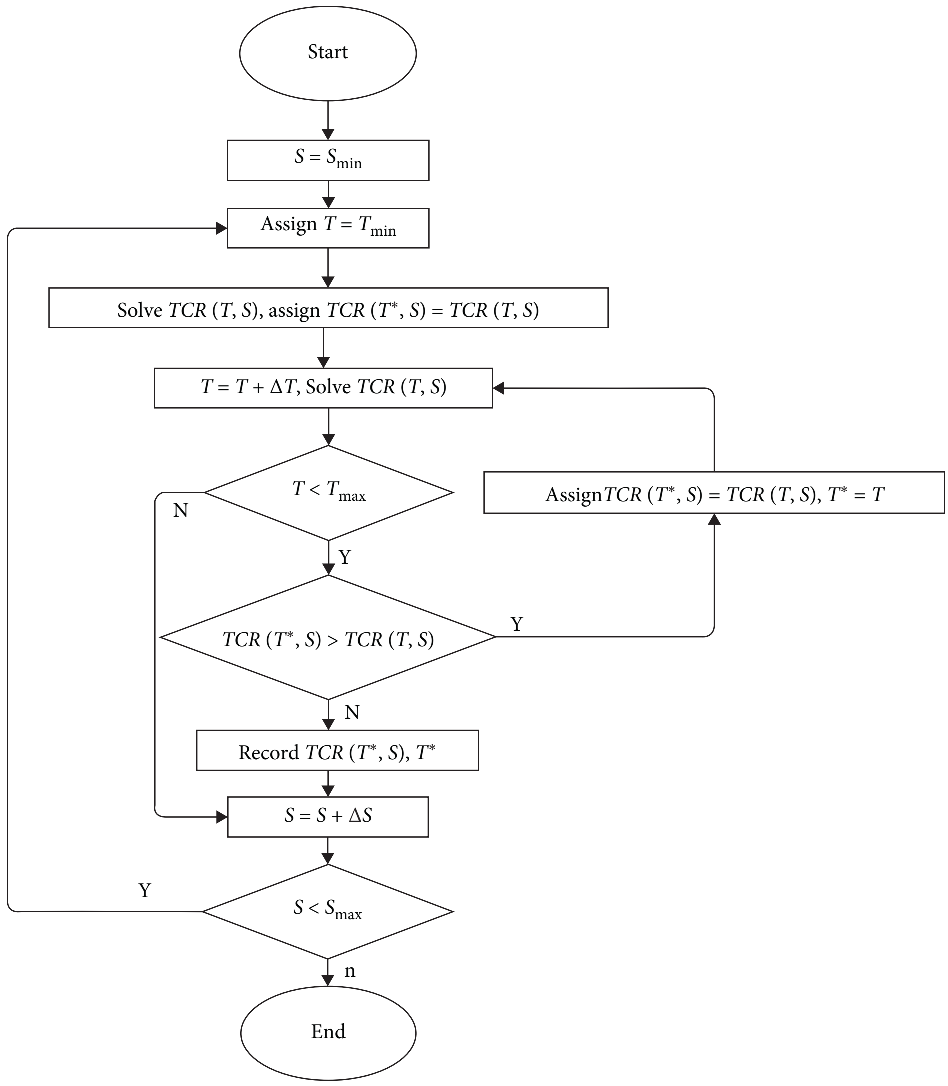


FIGURE 8: Flow chart of discrete iteration algorithm.

TABLE 1: Related parameters of failure rate distribution, unit: 1/year.

	M_1	M_2	M_3
λ_{i1}	$\lambda_{11} = 1.0$	$\lambda_{21} = 1.4$	$\lambda_{31} = 1.2$
λ_{i2}	$\lambda_{12} = 1.2$	$\lambda_{22} = 1.6$	$\lambda_{32} = 1.5$
λ_{i3}	$\lambda_{13} = 1.5$	$\lambda_{23} = 1.8$	$\lambda_{33} = 1.8$

TABLE 2: Failure rate of virtual machine $M_u(i)$ for $a = 0.2$, unit: 1/year.

$a = 0.2$	$M_u(1)$	$M_u(2)$	$M_u(3)$
$\lambda_u(i1)$	$\lambda_u(11) = 1.0$	$\lambda_u(21) = 1.32$	$\lambda_u(31) = 1.224$
$\lambda_u(i2)$	$\lambda_u(12) = 1.2$	$\lambda_u(22) = 1.52$	$\lambda_u(32) = 1.504$
$\lambda_u(i3)$	$\lambda_u(13) = 1.5$	$\lambda_u(23) = 1.74$	$\lambda_u(33) = 1.788$

As can be seen from Table 8, with the increase of a, the operational cycle of the same machine gradually increases, and the inventory of the same buffer gradually decreases. Considering the actual situation, the smaller the a is, the higher the importance of machine M_i will be. Therefore, the shorter the operation cycle is, the shorter the monitoring time is, the higher the maintenance frequency is, and the higher the inventory allocated by the corresponding buffer will be. Therefore, enterprises can choose the value of impact factor a according to the importance of the machine in the production line, so as to obtain more accurate preventive maintenance strategy and buffer stock allocation strategy.

$a = 0.2$ is fixed. For machine M_1, the optimal inventory and minimum cost rate under different monitoring time T and the optimal monitoring time and minimum cost rate under different inventory S were obtained by solving the problem, as shown in Table 9. As can be seen from Table 9, the increase or decrease of T and the increase or decrease of S will lead to the increase of the cost rate. If T is too small, the number of monitoring and maintenance processes will increase, which will lead to the increase of maintenance cost and the frequent shutdown of the machine. On the contrary, if T is too large, the possibility of machine failure shutdown will be greater, and the shortage cost will also increase. If S is

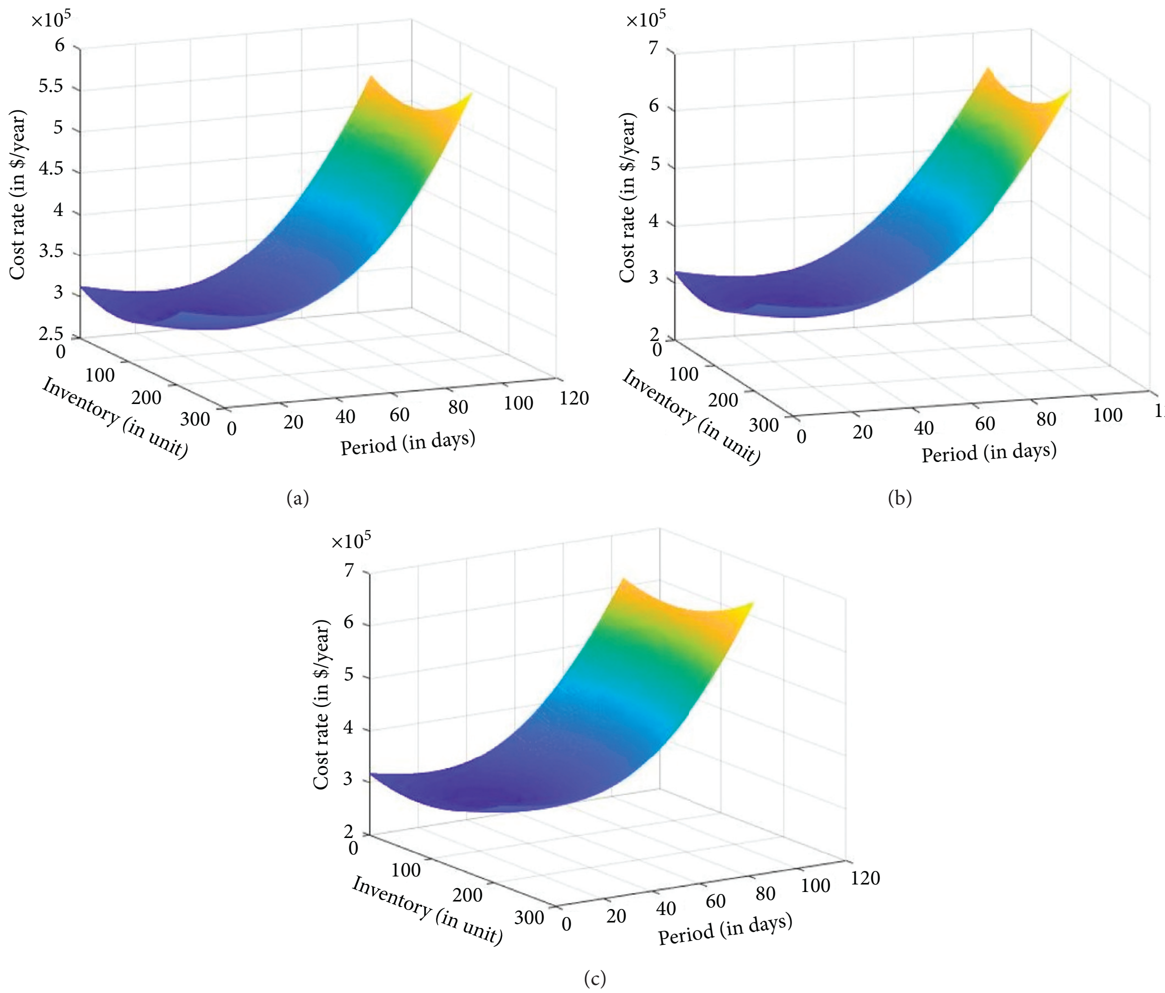


Figure 9: Diagram of the cost rate (in \$/year) of machines $M_u(1)$, $M_u(2)$, $M_u(3)$ with the monitoring time T(in days) and buffer stock S (in units).

Table 3: The lowest cost rate with the optimal monitoring time of the machine of the intelligent series system and the optimal buffer stock for $a = 0.2$

$a = 0.2$	$i = 1$	$i = 2$	$i = 3$
T_i of machine M_i (in days)	29	27	27
S_i of buffer B_i (in units)	79	80	79
Cost rateTCR (S, T)(in \$/year)	282137.8	280065.2	281066.7

Table 4: Failure rate of virtual machine$M_u(i)$ for $a = 0.5$, unit: 1/year.

$a = 0.5$	$M_u(1)$	$M_u(2)$	$M_u(3)$
$\lambda_u(i1)$	$\lambda_u(11) = 1.0$	$\lambda_u(21) = 1.2$	$\lambda_u(31) = 1.2$
$\lambda_u(i2)$	$\lambda_u(12) = 1.2$	$\lambda_u(22) = 1.4$	$\lambda_u(32) = 1.45$
$\lambda_u(i3)$	$\lambda_u(13) = 1.5$	$\lambda_u(23) = 1.65$	$\lambda_u(33) = 1.725$

Table 5: The lowest cost rate with the optimal monitoring time of the machine of the intelligent series system and the optimal buffer stock for $a = 0.5$.

$a = 0.5$	$i = 1$	$i = 2$	$i = 3$
T_i of machine M_i (in days)	29	28	27
S_i of buffer B_i (in units)	79	79	79
Cost rateTCR (S, T)(in \$/year)	282137.8	280804.5	281065.1

Table 6: Failure rate of virtual machine $M_u(i)$ for $a = 0.8$, unit: 1/year.

$a = 0.8$	$M_u(1)$	$M_u(2)$	$M_u(3)$
$\lambda_u(i1)$	$\lambda_u(11) = 1.0$	$\lambda_u(21) = 1.08$	$\lambda_u(31) = 1.104$
$\lambda_u(i2)$	$\lambda_u(12) = 1.2$	$\lambda_u(22) = 1.28$	$\lambda_u(32) = 1.324$
$\lambda_u(i3)$	$\lambda_u(13) = 1.5$	$\lambda_u(23) = 1.56$	$\lambda_u(33) = 1.608$

Table 7: The lowest cost rate with the optimal monitoring time of the machine of the intelligent series system and the optimal buffer stock with different a.

$a = 0.8$	$i = 1$	$i = 2$	$i = 3$
T_i of machine M_i (in days)	29	28	28
S_i of buffer B_i (in units)	79	79	79
Cost rateTCR (S, T)(in \$/year)	282137.8	281588.8	281531.0

Table 8: The optimal monitoring time T and the optimal stock S of each machine in the intelligent series system with different a.

	$a = 0.2$	$a = 0.5$	$a = 0.8$
T_1/S_1 of machine M_1 (in days/units)	29/79	29/79	29/79
T_2/S_2 of machine M_2 (in days/units)	27/80	28/79	28/79
T_3/S_3 of machine M_3 (in days/units)	27/79	27/79	28/79

Table 9: The optimal inventory S and the minimum cost ratio under different T and the optimal monitoring time T and the minimum cost rate under different S.

Machine M_1					Machine M_1
T	S^*	$\text{TCR}^*(S,T)$	S	T^*	$\text{TCR}^*(S,T)$
9	78	290802.0	19	29	291379.6
19	78	284631.7	39	29	286215.1
29	79	282137.8	59	29	283134.5
39	79	285132.4	79	29	283137.8
49	79	295151.5	99	29	283225.1
59	78	313425.6	119	29	286396.3
69	77	340867.0	139	29	291651.4
79	76	778048.1	159	28	298989.5
89	75	425204.7	179	28	308394.0
99	74	482245.5	199	28	319878.2

too small, it will be more likely to be out of stock, which will lead to the increase of shortage cost. If S is too large, it will inevitably lead to an increase in inventory cost.

From a practical point of view, the model results are consistent with the reality. If the monitoring time is 9 days, this means that if the detection is carried out every 9 days, the maintenance cost will be too high. On the other hand, if the buffer stock is replenished to 200 pieces, the inventory cost is high. The cost rates are highest in these extremes.

6.2. Result Comparison. The maintenance cost rate model established in this paper is combined with the three-stage time delay theory. According to the concept of three-stage fault process, the states of the system include normal, original defect, serious defect, and fault state. Compared with the traditional two-stage time delay theory, if the machine failure can be detected in the original defect state, not only the money cost but also the time cost can be saved. In this section, $a = 0.2$ is fixed. For machine M_1, the maintenance strategy proposed in this paper is compared with the maintenance strategy without buffer stock and the maintenance strategy based on two-stage time delay.

6.2.1. Comparison with a Maintenance Strategy without Buffer Stock. Buffer was added to the maintenance system in this paper. In order to illustrate the effectiveness of the model, and Table 10 compares the optimal monitoring time and the minimum cost rate of machines M_1, M_2, M_3 with and without buffer stock in the case of $a = 0.2, a = 0.5, a = 0.8$. Not taking buffer stock into account means that the buffer stock is 0. Table 10 shows that the cost ratio is smaller when buffer stock is taken into account than when buffer stock is not taken into account. It shows that the preventive maintenance strategy considering buffer stock is optimal, feasible, and effective.

6.2.2. Comparison with the Maintenance Strategy Based on the Two-Stage Time Delay Theory. According to the traditional time delay theory, there are three states of a machine: normal, defect, and failure. The defect state and fault state

Table 10: Comparison of two maintenance strategies for $a = 0.2$, $a = 0.5$, and $a = 0.8$.

$a = 0.2$	S_1	T_1^*	$\text{TCR}^*(S,T)$
Machine M_1	79	29	282137.8
	0	28	298210.2
Machine M_2	80	27	280065.2
	0	26	302058.2
Machine M_3	79	27	281066.7
	0	26	301232.9
$a = 0.5$	S_1	T_1^*	$\text{TCR}^*(S,T)$
Machine M_1	79	29	282137.8
	0	28	298210.2
Machine M_2	79	28	280804.5
	0	27	300807.4
Machine M_3	79	27	281066.7
	0	27	300559.5
$a = 0.8$	S_1	T_1^*	$\text{TCR}^*(S,T)$
Machine M_1	79	29	282137.8
	0	28	298210.2
Machine M_2	79	28	281588.8
	0	28	299125.3
Machine M_3	79	28	281531.0
	0	27	299209.3

occurred in T_y and T_z. There are three different situations for adopting preventive maintenance based on monitoring status, machine monitoring time T, and buffer stock S.

(1) $0 < T < T_y$. The state monitoring time of the machine occurs before the defect time. Machine status is in a non-defective state, and it is unnecessary to execute any maintenance action. After that, in order to prevent the failure to detect the defect in time, it is necessary to execute a state monitoring process on the machine every day until the original defect is detected. Buffer stock level change during a running cycle is shown in Figure 10. Preventive maintenance of machine is executed under the defect state.

The probability of Twithin $[10, T_y]$:

$$P(0 < T < T_y) = P_1 = \int_T^\infty f_y(y)\mathrm{d}y. \tag{32}$$

Operation cycle of machine for $0 < T < T_y$:

$$\mathrm{EC}_1(T) = E(W_2) + E(T_y). \tag{33}$$

Inventory holding cost in a cycle:

$$C_{h1}(S) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right) + (y-T)S\right]. \tag{34}$$

Shortage cost in a cycle:

$$C_{s1}(S,T) = \rho\beta\int_{S/\beta}^\infty \overline{G_2}(w)\mathrm{d}w. \tag{35}$$

Maintenance cost in a cycle:

$$C_{m1}(T) = C_y + (y - T + 1)C_r. \tag{36}$$

The expected cost in a cycle for $0 < T < T_y$ is the sum of inventory holding cost, shortage cost, and maintenance cost. The expected cost $C_1(S,T)$ is obtained as

$$\begin{aligned} C_1(S,T) &= C_{h1}(S) + C_{s1}(S,T) + C_{m1}(T) \\ &= h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right) + (y-T)S\right] + \rho\beta\int_{S/\beta}^\infty \overline{G_2}(w)\mathrm{d}w + C_y + (y-T+1)C_r. \end{aligned} \tag{37}$$

(2) $Ty < T < T_z$. The state monitoring time of the machine occurs after the defect time and before the breakdown time. Machine status is in a defect operation state, and it is necessary to execute preventive maintenance of the defect state. After the buffer stock is replenished, the machine status is monitored immediately. Buffer stock level change during a running cycle is shown in Figure 11.

The probability of T within $[T_y, T_z]$:

$$P(T_y < T < T_z) = P_2 = \int_0^T\int_{T-y}^\infty f_y(y)f_z(z)\mathrm{d}x\mathrm{d}y = \int_0^T f_y(y)(1 - F_z(T-y))\mathrm{d}y. \tag{38}$$

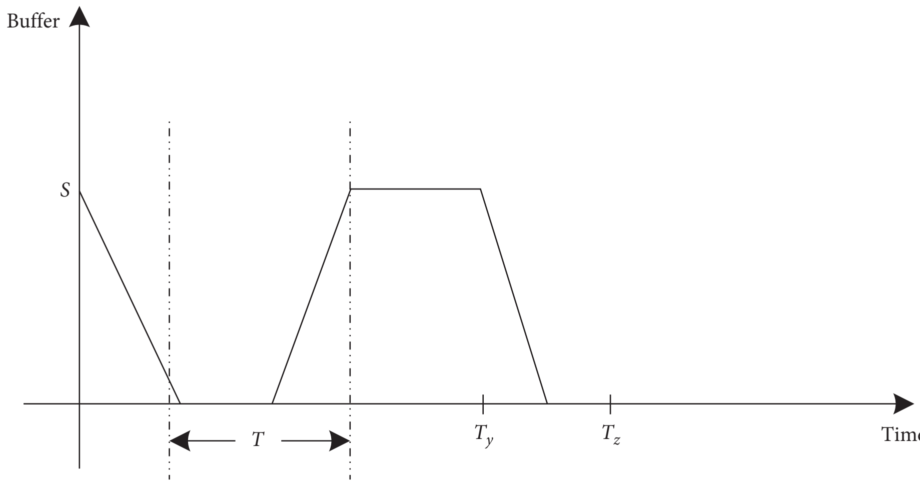


FIGURE 10: Buffer stock change diagram in a cycle for $0 < T < T_y$.

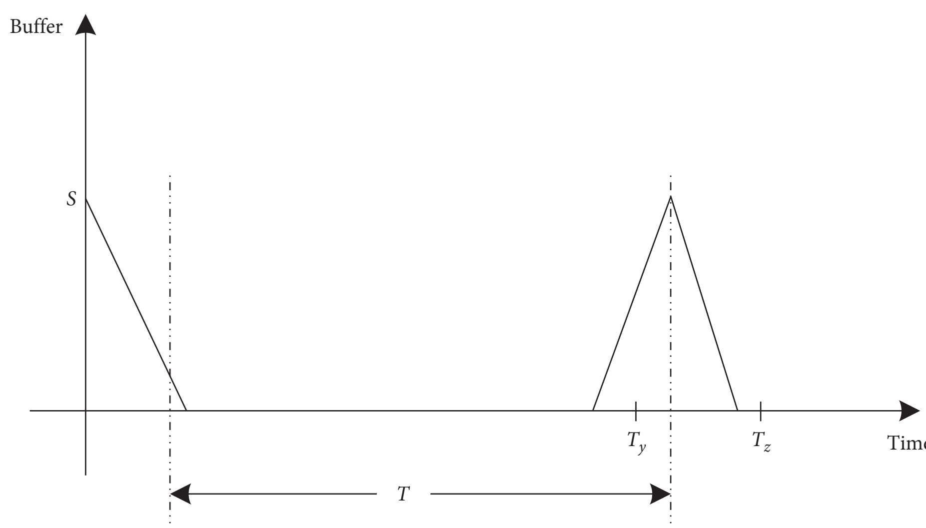


FIGURE 11: Buffer stock change diagram in a cycle for $T_y < T < T_z$.

Operating cycle of the machine for $T_y < T < T_z$:

$$EC_2(T) = E(W_2) + T. \quad (39)$$

Inventory holding cost in a cycle:

$$C_{h2}(S) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right]. \quad (40)$$

Shortage cost in a cycle:

$$C_{s2}(S,T) = \rho\beta\int_{S/\beta}^{\infty}\overline{G_2}(w)\mathrm{d}w. \quad (41)$$

Maintenance cost in a cycle:

$$C_{m2}(T) = C_y + C_r. \quad (42)$$

The expected cost in a cycle for $T_y < T < T_z$ is the sum of inventory holding cost, shortage cost, and maintenance cost. The expected cost $C_2(S,T)$ is obtained as

$$C_2(S,T) = C_{h2}(S) + C_{s2}(S,T) + C_{m2}(T) = h\left[\frac{S^2}{2}\left(\frac{\alpha+\beta}{\alpha\beta}\right)\right] + \rho\beta\int_{S/\beta}^{\infty}\overline{G_2}(w)\mathrm{d}w + C_y + C_r. \quad (43)$$

(3) $T > T_z$. In this case, the machine fails before the state detection is carried out. The planned state detection takes place after the failure of the machine. At this time, the corrective maintenance action is executed. The planned state detection takes place after the failure of the machine. Buffer stock has not been replenished, or the production of buffer stock has not started before the failure shutdown. Thus, the buffer stock change during a running cycle is shown in Figure 12.

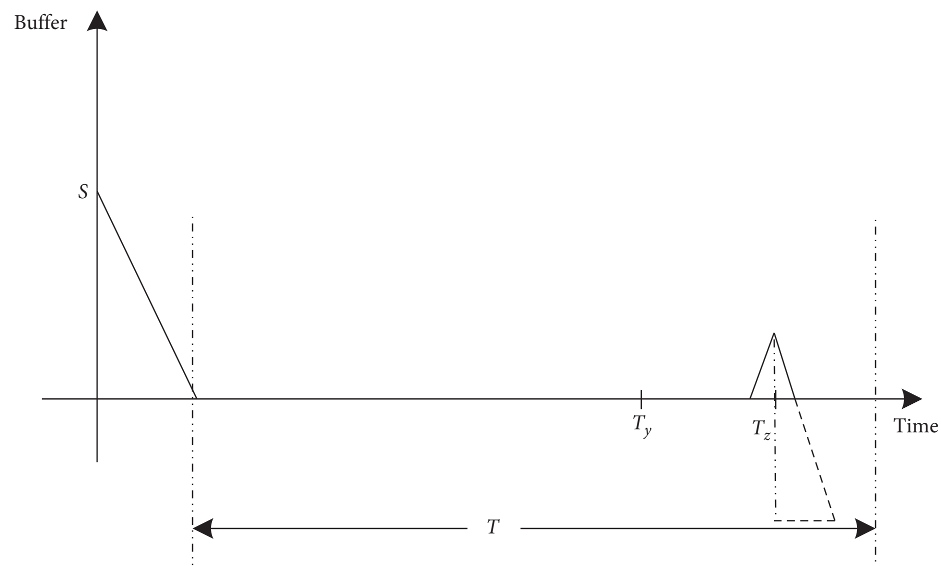


Figure 12: Buffer stock change diagram in a cycle for $T > T_z$.

The probability of T within $[T_z, \infty]$:

$$P(T > T_z) = P_3 = \int_0^T \int_0^{T-Y} f_y(y) f_z(z) \mathrm{d}y \mathrm{d}z = \int_0^T f_y(y) F_z(T - y) \mathrm{d}y. \quad (44)$$

Operating cycle of the machine for $T > T_z$:

$$\mathrm{EC}_3(T) = E(W_3) + E(T_z). \quad (45)$$

Inventory holding cost in a cycle:

$$C_{h3}(S) = \frac{h}{2}[S - \alpha(T - E(T_z))]\left[\frac{(\alpha+\beta)S}{\alpha\beta} - \left(\frac{\alpha+\beta}{\beta}\right)(T - E(T_z))\right]. \quad (46)$$

Shortage cost in a cycle:

$$C_{s3}(S,T) = \rho\beta \int_{(S-\alpha(T-E(T_z)))/\beta}^{\infty} \overline{G_3}(w) \mathrm{d}w. \quad (47)$$

Maintenance cost in a cycle:

$$C_{m3}(T) = C_z. \quad (48)$$

The expected cost in a cycle for $T > T_z$ is the sum of inventory holding cost, shortage cost, and maintenance cost. The expected cost $C_3(S,T)$ is obtained as

$$\begin{aligned} C_3(S,T) &= C_{h3}(S) + C_{s3}(S,T) + C_{m3}(T) \\ &= \frac{h}{2}[S - \alpha(T - E(T_z))]\left[\frac{(\alpha+\beta)S}{\alpha\beta} - \left(\frac{\alpha+\beta}{\beta}\right)(T - E(T_z))\right] \\ &\quad + \rho\beta \int_{S-\alpha(T-E(T_z))/\beta}^{\infty} \overline{G_3}(w) \mathrm{d}w + C_z. \end{aligned} \quad (49)$$

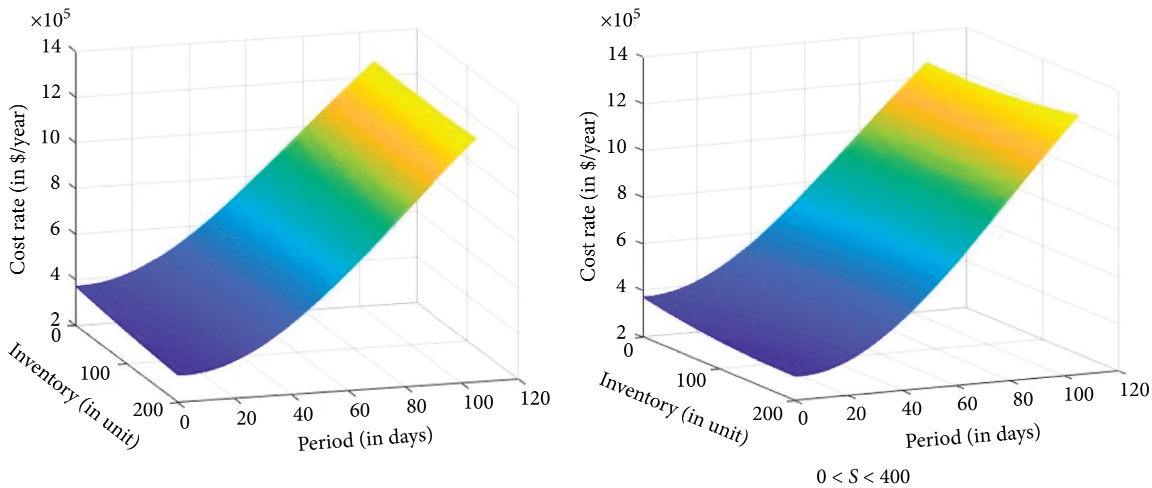
The cost rate in a cycle is expressed as follows:

$$\mathrm{TCR}(S,T) = \frac{C(S,T)}{\mathrm{EC}(T)} = \frac{P_1 C_1(S,T) + P_2 C_2(S,T) + P_3 C_3(S,T)}{P_1 \mathrm{EC}_1(T) + P_2 \mathrm{EC}_2(T) + P_3 \mathrm{EC}_3(T)}. \quad (50)$$

Based on the traditional two-stage time delay theory, the machine preventive maintenance model considering buffer stock is established as follows:

$$\begin{cases} \min\{\mathrm{TCR}(S,T),\} \\ S, T \in N^*; \quad S, T > 0. \end{cases} \quad (51)$$

The $a = 0.2$ is fixed. For M_1, all parameters in the solution remained unchanged, and the model was solved. The optimal monitoring time of machine M_1 is 3 days, the optimal stock of buffer is 211 units, and the minimum maintenance cost of machine in one year is \$318378.7, as shown on the left of Figure 12. Considering that the buffer stock has reached the upper limit previously given, the value of S is adjusted, and $0 < S < 400$ is set to solve the problem. The optimal monitoring time T of machine M_1 is 3 days, the optimal stock S of buffer is 387 units, and the minimum maintenance cost of machine in one year is \$304570.2, as shown on the right of Figure 13. As can be seen from the results, based on the traditional two-stage time delay model, the monitoring time is short and the buffer stock is high. This is because there is no distinction between the original defects and the serious defects of the machine, and the machine status cannot be accurately detected. In order to prevent the machine from being shut down, it is necessary to carry out regular monitoring, which is consistent with the actual situation. The maintenance strategy based on three-stage time delay theory can save \$22432.4 per year compared with the maintenance strategy based on two-stage time delay theory. The specific comparison is shown in Table 11.



(a) (b)

FIGURE 13: Diagram of the cost rate (in \$/year) of machine M_1 with T (in days) and S (in units).

TABLE 11: Comparison of two maintenance strategies.

M_1	S_1	T_1^*	$\mathrm{TCR}^*\,(S,T)$
Maintenance strategy based on three-stage time delay theory	79	29	282137.8
Maintenance strategy based on two-stage time delay theory	387	3	304570.2
Maintenance strategy based on two-stage time delay theory	79	29	445867.4

7. Conclusion

In this paper, a new method is proposed to solve the preventive maintenance problem of intelligent series system with buffer stock. For the intelligent series system with inventory buffer, the series system is decomposed into several virtual series systems with two machines and one buffer by approximate decomposition method. The failure rate and maintenance rate of the decomposed virtual machine are calculated by mathematical induction. The influence factor is introduced here, and the enterprise can determine the value of the influence factor according to the importance of different machines in the series system. For each virtual series system, a preventive maintenance model was built with the lowest cost rate as the objective function and the monitoring time and buffer stock as independent variables. The preventive maintenance model is combined with the three-stage time delay theory to better simulate the equipment degradation process. Finally, a case is used to verify the validity of the model.

The maintenance strategy in this paper is compared with the maintenance strategy without buffer stock and the maintenance strategy based on the two-stage time delay. It is proved that the proposed maintenance strategy based on the three-stage time delay theory is optimal. Taking the impact factor $a = 0.2$ as an example, compared with no buffer stock maintenance strategy, the annual maintenance cost of machine M_1 can save \$16072.4. Compared with the traditional two-stage time delay maintenance strategy, the annual maintenance cost of machine M_1 can save \$22432.4. Therefore, the maintenance strategy proposed in this paper can be well used in the maintenance of the series system, which can save a lot of money for enterprises

The value of impact factor a should be determined according to the importance of the machine on the production line. On the basis of the research in this study, the effective method can be adopted in following research, and the most accurate influence factor can be obtained for different machines in the series system.

Data Availability

The underlying data supporting the results of our study can be found in the article, including, where applicable, hyperlinks to publicly archived datasets analyzed or generated during the study

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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