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Multicriteria optimization of injury prevention systems to impact

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Abstract. A sensitivity analysis and multicriteria control optimization formulation is derived for mechanical systems. This formulation is implemented into an interactive optimum design code and it is applied to optimize protection systems for the prevention of injuries. The limiting isolation capabilities of the systems are determined. The effect of pre-acting control is investigated. Control forces as well as the time at which the control should act before the instant of impact are considered as design variables. The same idea used by the authors in previous articles for minimum time control problems is applied here to find the preview time. Dynamic response index, maximum acceleration, rattlespace, or maximum power of the resisting force among others can be used as performance criteria. In order to handle the multicriteria problem, both the reduced feasible region method and a min-max upper bound method are utilized. The adjoint system approach is used to calculate the sensitivities. The dynamic response of the systems and its sensitivity are discretized via space-time finite elements. The equations of motion and the sensitivity equations are integrated at-once as it is typical for the static response. This way, displacement, velocity or acceleration control conditions can be imposed easily at any point in time. Also, adjoint system response is obtained without needing primary response memorization. Mathematical programming is used for the optimal control process.

Keywords: Optimum control, preview control, multicriteria, space-time finite elements

1. Introduction

Structures and mechanical systems are frequently subject to impact. An important problem in crashworthy design is the prevention of impact-induced injuries. Therefore, crashworthy design should provide efficient impact isolation. An impact absorber should be located between the body to be protected and the structure subject to impact in order to reduce the intensity of the impact transmitted to the body as compared with the case where the body is rigidly attached to the base. This intensity can be judged by various performance criteria as the dynamic response index, the maximum acceleration, the peak displacement of the body relatively to the base (rattlespace), or the maximum power of the resisting force, among others. A limiting isolation capacity or limiting performance analysis is used to find the best possible optimal performance, independently of any specific engineering design related to the absorber device. The specific parts to be designed are replaced by a generic control force the absorber generates between the base and the body to be protected (Fig. 1). The limiting isolation capacity analysis finds the time-history of this force in order to get the best possible optimal performance. Since the control force is generic and does not represent any predetermined design, the limiting isolation capacity measures the limits on the improvements of the absorbers with respect to the prescribed performance criteria. A survey of problems related to the limiting performance analysis of injury prevention systems is given in [1]. The problem may be formulated as a control optimization problem. If we have more than one performance objective, then the problem is a multicriteria optimization problem.

If the impact absorber starts generating the control force before the instant of impact, we say the control is a preview control. It is shown that the employment of preview control leads to improvement of the impact isolation in

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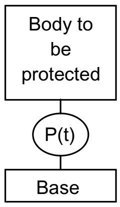

Fig. 1. Conceptual system for limiting isolation capacity analysis.

comparison with control engaging at the instant of impact [2]. However, only one-degree-of-freedom systems are considered in [2]. In the case of preview control, we add to the set of control variables the time interval that the control force initiates before the impact.

Structures and flexible mechanical systems, on one hand, as well as optimal design and optimal control, on the other hand, have been traditionally treated with separated formulations. As the theory and methods of nonlinear structural analysis have progressed, there is no distinction between flexible mechanical systems and structures. Also, in the last decades there has been the integration of optimal design and optimal control problems [3–7].

This paper presents an integrated methodology for optimal design and control of nonlinear flexible mechanical systems and applies it to injury prevention systems subjected to impact. In order to implement the methodology, one uses: (i) a nonlinear structural finite element technique to model large displacements, referring all the quantities to an inertial frame and using stress and strain measures that are invariant with the rigid body motion; (ii) a conceptual separation between time variant and time invariant design parameters, this way including the design space into the control space and considering the design variables as control variables not depending on time.

By using time integrals through all the derivations, the design and control problems are unified. In the optimization process we can use both types of variables simultaneously or by interdependent levels. Both types are designated here as design variables. The systems are discretized by space-time finite elements and the dynamic response is obtained by at-once integration as it is typical for the static response [3]. As it is known from system analysis, the boundary conditions may be imposed easily at any point of the system. Since we used at-once integration for the dynamic problem, it does not exist the difference between a point in time and a point in space, they are simply nodes of the system. This way, displacement, velocity or acceleration control conditions can be imposed easily at any point in time.

To treat the preview control problem we consider it as a time variant problem, i.e., a problem which time domain is not fixed. In this case a unit time interval is mapped onto the original total time interval, then treating equally time variant and time invariant problems. Control forces start actuating at time zero. The time interval from zero until the instant of impact is a fraction of the total time interval. This fraction is determined by imposing that, at the instant of impact, the velocity jumping condition is kept constant with design variation. Basically, it is used the same idea as before for minimum time control problems [4].

The aim of the Design Sensitivity Analysis (DSA) is the gradients calculation of the performance measures with respect to the design variables. It represents an important tool for design improvement and it is a necessary stage within the optimization process. A general overview of the DSA problems and methods of nonlinear structural mechanics is given elsewhere [8]. Basically, the methods may be classified as direct differential methods and adjoint system methods. The adjoint method is adopted in the present work [7,8].

The response analysis and corresponding DSA are implemented in the interactive optimal design code OPTIMISE in order to use optimality criteria or nonlinear programming optimization runs.

A numerical example is performed on the injury prevention system of a helicopter seat for a vertical crash landing. However, together with the reduced feasible region strategy as performed in [1], a min-max upper bound strategy is also formulated in the present work to handle the multicriteria problem [3,4].

2. Response analysis

The virtual work dynamic equilibrium equation of the system at the time t is given as

$$\delta W = \int (\rho^t \ddot{\boldsymbol{u}} \bullet \delta^t \boldsymbol{u} + {}^t\boldsymbol{S} \bullet \delta^t \boldsymbol{\varepsilon} - {}^t\boldsymbol{f} \bullet \delta^t \boldsymbol{u})^0 dV - \int {}^t\boldsymbol{T} \bullet \delta^t \boldsymbol{u}^0 d\Gamma \tag{1}$$

where all the quantities are referred to the undeformed configuration, δ represents variation of the state fields, '•' refers to the standard tensor product, upper dot '.' refers to the material time derivative, ρ is the mass density at time $t = 0$, ^{t}u is the displacement, ${}^t\boldsymbol{S}$ is the second-Piola stress measure, ${}^t\boldsymbol{\varepsilon}$ is the Green strain tensor, ${}^t\boldsymbol{f}$ is the body force per unit volume, ${}^t\boldsymbol{T}$ is the surface traction, ${}^0V$ is the undeformed volume of the body, and ${}^0\Gamma$ is the surface of the undeformed body.

A space-time finite element model is used to discretize the dynamic analysis response. After space discretization we have the governing matrix equation as

$$\boldsymbol{M}^t\ddot{\boldsymbol{U}} + {}^t\boldsymbol{C}_S{}^t\dot{\boldsymbol{U}} + {}^t\boldsymbol{K}_S{}^t\boldsymbol{U} = {}^t\boldsymbol{P} \tag{2}$$

where $\boldsymbol{M}$ is the mass matrix, ${}^t\boldsymbol{C}_S \equiv {}^t\boldsymbol{C}_S({}^t\dot{\boldsymbol{U}}), {}^t\boldsymbol{K}_S \equiv {}^t\boldsymbol{K}_S({}^t\boldsymbol{U})$ are respectively the damping and stiffness matrices, ${}^t\boldsymbol{P}$ is the loading vector and ${}^t\boldsymbol{U}, {}^t\dot{\boldsymbol{U}}, {}^t\ddot{\boldsymbol{U}}$ are respectively the displacement, velocity and acceleration vectors, all the quantities defined at time t.

For temporal modeling, finite elements of dimension Δt were considered, selecting hermitean cubic elements to model the displacements and quadratic lagrangean elements to model the loading. By taking the time derivative of the Eq. (2) on one hand, and on the other hand its integration once and then twice with average values of stiffness and damping in Δt given as

$$\bar{\boldsymbol{K}} \equiv {}^{t+\Delta t/2}\boldsymbol{K}_S({}^t\boldsymbol{U} + {}^t\dot{\boldsymbol{U}}\Delta t/2), \bar{\boldsymbol{C}} \equiv {}^{t+\Delta t/2}\boldsymbol{C}_S({}^t\dot{\boldsymbol{U}} + {}^t\ddot{\boldsymbol{U}}\Delta t/2) \tag{3}$$

one obtains four equations that combine to give the dynamic finite time-element equation as

$$\boldsymbol{D}^e_S \boldsymbol{z}^e = \boldsymbol{P}^e \tag{4}$$

where

$$\boldsymbol{D}^e_S = \left[\begin{array}{ccc|ccc} {}^t\boldsymbol{K}_S & {}^t\boldsymbol{C}_S & \boldsymbol{M} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} \\ \hline {}^t\boldsymbol{D}_{S_{11}} & {}^t\boldsymbol{D}_{S_{12}} & \boldsymbol{0}_{nxn} & {}^{t+\Delta t}\boldsymbol{D}_{S_{11}} & {}^{t+\Delta t}\boldsymbol{D}_{S_{12}} & \boldsymbol{0}_{nxn} \\ {}^t\boldsymbol{D}_{S_{21}} & {}^t\boldsymbol{D}_{S_{22}} & \boldsymbol{0}_{nxn} & {}^{t+\Delta t}\boldsymbol{D}_{S_{21}} & {}^{t+\Delta t}\boldsymbol{D}_{S_{22}} & \boldsymbol{0}_{nxn} \end{array}\right], \quad \boldsymbol{P}^e = \begin{Bmatrix} {}^t\boldsymbol{P} \\ \boldsymbol{P}_1 \\ \boldsymbol{P}_2 \end{Bmatrix} \tag{5}$$

and

$$\boldsymbol{z}^e = ({}^t\boldsymbol{z}, {}^{t+\Delta t}\boldsymbol{z}), \qquad {}^t\boldsymbol{z} = ({}^t\boldsymbol{U}, {}^t\dot{\boldsymbol{U}}, {}^t\ddot{\boldsymbol{U}}) \tag{6}$$

In Eq. (5) n stands for number of space degrees-of-freedom and ${}^t\boldsymbol{D}_{S_{jk}}$ are functions of $\bar{\boldsymbol{K}}, \bar{\boldsymbol{C}}, \boldsymbol{M}$. The Eq. (4) may be solved step-by-step, i.e., element-by-element in time, or assembled to be solved at-once. In this case, we have to assemble for a total time interval T discretized in N time nodes, resulting in the dynamic equation

$$\boldsymbol{D}_S \boldsymbol{z} = \boldsymbol{P} \tag{7}$$

where $2n$ time boundary conditions are imposed by transferring the corresponding columns of the assembled matrix $\boldsymbol{D}_S$ to the right-hand side of Eq. (7) after multiplying by the vector $\boldsymbol{U}_c$ of those conditions, resulting the equation

$$\hat{\boldsymbol{K}}_S \hat{\boldsymbol{U}} = \hat{\boldsymbol{P}}, \qquad \hat{\boldsymbol{P}} = \boldsymbol{P} - \boldsymbol{D}_c \boldsymbol{U}_c \tag{8}$$

The Eq. (8) is a nonlinear equation where $\hat{\boldsymbol{K}}_S$ is a nonsymmetrical matrix dependent on the response. Therefore, the Eq. (8) has to be solved iteratively.

3. Design sensitivity analysis

Consider now a general performance measure defined in the time interval [0,T] as

$$\Psi = \int G({}^t\boldsymbol{z}, {}^t\boldsymbol{b}, t)\, dt \tag{9}$$

where ${}^{t}\boldsymbol{b}$ is the vector of design and control variables and the other quantities were defined in Eqs (1) and (6). It contains the external forces ${}^{t}\boldsymbol{P}$. For preview control problems, consider also the impact condition

$$ {}^{t_s}\Phi({}^{t_s}\boldsymbol{z}, {}^{t_s}\boldsymbol{b}, t_s) \equiv {}^{t_s^+}\dot{u}_k - {}^{t_s^-}\dot{u}_k = v_0 \tag{10} $$

stating the velocity jump v_0 at the instant of impact t_s for the spatial degree-of-freedom k. The design sensitivity analysis problem is to derive the total design variation of the measure in Eq. (9) with respect to the design ${}^{t}\boldsymbol{b}$, for the system represented by the equation of motion, Eq. (8), and subject to the impact condition of the Eq. (10).

3.1. The adjoint method of design sensitivity analysis

The total design variation for the performance measure of Eq. (9), for preview control problems, is

$$ \bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \tilde{\delta}\Psi + \Psi_{,t}\ \bar{\delta}t_s \tag{11} $$

where $\bar{\bar{\delta}}\Psi$ and $\tilde{\delta}\Psi$ represent respectively the explicit and implicit (state dependent) design variations. For problems without preview control t_s is zero and the last parcel of the Eq. (11) vanishes. In order to formulate the adjoint method of design sensitivity analysis, replace the arbitrary variation of state fields by adjoint fields into the virtual work equation as

$$ W^a = (\hat{\boldsymbol{K}}_S\hat{\boldsymbol{U}} - \hat{\boldsymbol{P}}) \cdot \hat{\boldsymbol{U}}^a = 0 \tag{12} $$

and define an extended 'action' function

$$ A = \Psi - W^a \tag{13} $$

The basic idea of introducing an adjoint system is to replace the implicit design variations of the state fields by explicit design variations and adjoint fields, then determining these adjoint fields by vanishing the implicit design variation of the 'action' function A [7] as

$$ \tilde{\delta}A = 0 \tag{14} $$

then the total design variation of Ψ can be written as

$$ \bar{\delta}\Psi = \bar{\bar{\delta}}A + \Psi_{,t}\,\bar{\delta}t_s \tag{15} $$

The variation of t_s may be now determined by imposing the condition of the Eq. (10) after any design variation,

$$ \bar{\delta}^{t_s}\Phi = \tilde{\delta}^{t_s}\Phi + {}^{t_s}\Phi_{,t}\,\bar{\delta}t_s = 0 \tag{16} $$

or

$$ \bar{\delta}t_s = -\tilde{\delta}^{t_s}\Phi \Big/ {}^{t_s}\Phi_{,t} = -(\tilde{\delta}^{t_s^+}\dot{u}_k - \tilde{\delta}^{t_s^-}\dot{u}_k) \Big/ ({}^{t_s^+}\ddot{u}_k - {}^{t_s^-}\ddot{u}_k) \tag{17} $$

to substitute into the Eq. (15) and get the total design variation of the measure of interest.

3.2. Design sensitivity analysis modeling

In order to solve the design sensitivity analysis problem, the sensitivities are firstly performed at the element level and then the sensitivity equations are assembled. The explicit design variation of the vector of element forces of Eq. (4) is

$$ \bar{\bar{\delta}}\boldsymbol{R}^e = \bar{\bar{\delta}}\boldsymbol{P}^e - \bar{\bar{\delta}}\boldsymbol{F}^e, \qquad \boldsymbol{F}^e = \boldsymbol{D}_S^e\,\boldsymbol{z}^e \tag{18} $$

and the implicit design variation of the internal forces gives

$$ \tilde{\delta}\boldsymbol{F}^e = \tilde{\delta}\boldsymbol{D}_S^e\,\boldsymbol{z}^e + \boldsymbol{D}_S^e\,\tilde{\delta}\boldsymbol{z}^e = \boldsymbol{D}^e\,\tilde{\delta}\boldsymbol{z}^e \tag{19} $$

where

$$\boldsymbol{D}^e = \left[\begin{array}{ccc|ccc} {}^t\boldsymbol{K} & {}^t\boldsymbol{C} & \boldsymbol{M} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} & \boldsymbol{0}_{nxn} \\ \hline {}^t\boldsymbol{D}_{11} & {}^t\boldsymbol{D}_{12} & {}^t\boldsymbol{D}_{13} & {}^{t+\Delta t}\boldsymbol{D}_{11} & {}^{t+\Delta t}\boldsymbol{D}_{12} & \boldsymbol{0}_{nxn} \\ {}^t\boldsymbol{D}_{21} & {}^t\boldsymbol{D}_{22} & {}^t\boldsymbol{D}_{23} & {}^{t+\Delta t}\boldsymbol{D}_{21} & {}^{t+\Delta t}\boldsymbol{D}_{22} & \boldsymbol{0}_{nxn} \end{array}\right] \tag{20}$$

Sensitivities of Eq. (18) and the element dynamic matrix of Eq. (20) are assembled and again imposed the time boundary conditions resulting respectively $\bar{\bar{\delta}}\hat{\boldsymbol{R}}$ and $\hat{\boldsymbol{K}}$.

Now, the application of the Eq. (14) to the Eq. (13) gives the adjoint system equilibrium

$$\hat{\boldsymbol{K}}^T \hat{\boldsymbol{U}}^a = (\Psi_{,\hat{\boldsymbol{U}}})^T \tag{21}$$

Thus, the total design variation of Eq. (15) is

$$\bar{\delta}\Psi = \bar{\bar{\delta}}\Psi + \hat{\boldsymbol{U}}^a \cdot \bar{\bar{\delta}}\hat{\boldsymbol{R}} \tag{22}$$

4. Optimum design problem

The multicriteria optimal control problem to solve is generally expressed as

$$\begin{aligned} &\min\,\{\Psi_{01},\,\Psi_{02},\,\ldots,\Psi_{0m_0}\}\,, \quad \text{s.t.} \\ &\Psi_j \leqslant 0,\ (j = 1,2,\ldots,m); \quad b_{il} \leqslant b_i \leqslant b_{iu};\ \ i = 0.1,2,\ldots,n \\ &{}^{t_s}\Phi \ = v_0\,, \quad \text{if the problem is a preview control problem} \end{aligned} \tag{23}$$

where $\Psi_{0k},\,(k = 1,2,\ldots,m_0)$ are the original objectives and Ψ_j are the constraints.

In order to solve the multicriteria problems, both the reduced feasible region strategy and a min-max upper bound strategy are used. The former one consists of keeping only one of the original objectives as the only new objective, and reducing the feasible space by adding to the original constraints all the remaining original objectives:

$$\begin{aligned} &\min\ \Psi_{0k}, \text{s.t.} \\ &\Psi_{0l} \leqslant 0, (l \neq k), (l = 1,2,\ldots,m_0); \Psi_j \leqslant 0, (j = 1,2,\ldots,m); b_{il} \leqslant b_i \leqslant b_{iu}; i = 0.1,2,\ldots,n \\ &{}^{t_s}\Phi \ = v_0, \ \text{if the problem is a preview control problem} \end{aligned} \tag{24}$$

To formulate the min-max upper bound strategy, a new fictitious design variable b_0 is introduced, which is also a new objective and an upper bound for the original objectives. This way, the optimal control problem is formulated as

$$\begin{aligned} &\min\ b_0\,, \quad \text{s.t.} \\ &\Psi_{0k} \leqslant b_0,\ \ (k = 1,2,\ldots,m_0); \quad \Psi_j \leqslant 0,\ (j = 1,2,\ldots,m); \quad b_{il} \leqslant b_i \leqslant b_{iu};\ \ i = 0.1,2,\ldots,n \\ &{}^{t_s}\Phi \ = v_0\,, \quad \text{if the problem is a preview control problem} \end{aligned} \tag{25}$$

In order to handle objectives with different orders of magnitude, these are in the Eq. (25) normalized accordingly to

$$\Psi_{0k} = (\overline{\Psi}_{0k} - \overline{\Psi}_{0k}^{min})w_k \Big/ (\overline{\Psi}_{0k}^{max} - \overline{\Psi}_{0k}^{min}), \qquad \sum_{k=1}^{m_0} w_k = 1 \tag{26}$$

In the Eq. (26), $\overline{\Psi}_{0k}$ is the k-th original objective, w_k represents a weight given by the designer accordingly to his or her preference, $\overline{\Psi}_{0k}^{min}$ is its single objective ($w_k = 1$) attainable minimum and $\overline{\Psi}_{0k}^{max}$ is its maximum value among the values obtained for $\overline{\Psi}_{0k}$when all the other single objectives minimizations are performed.

The problem of Eq. (24) can also be expressed as an upper bound problem for the single formulated objective.

For preview control problems, since the instant of impact is an unknown quantity, the difficulty arises for the time discretization. Since the problem domain varies as the optimization iterates progress, a fixed unit time interval is mapped onto the original time interval T, and the original time t is transformed to the non-dimensional time $\tau = t/T$ such that we have

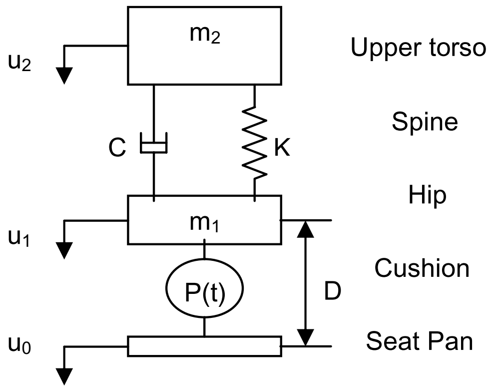

Fig. 2. Mass-model of spinal injury prevention.

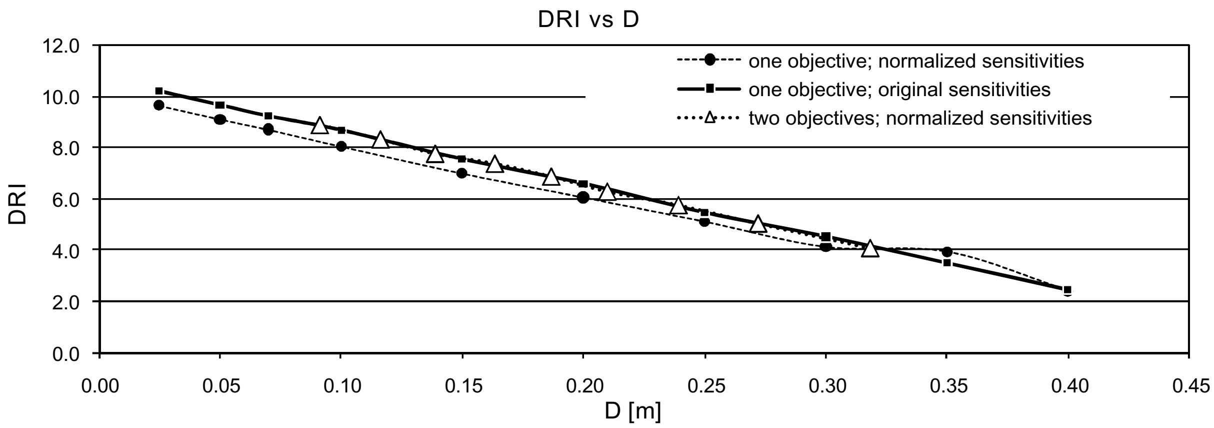

Fig. 3. Criteria space.

$$d/dt = T^{-1}d/d\tau, \qquad d^2\big/dt^2 = T^{-2}d^2\big/d\tau^2, \qquad \tau_s = t_s/T \tag{27}$$

All the performance measures are transformed accordingly, that is

$$\overline{\Psi} = \int \bar{G}({}^{\tau}\bar{\boldsymbol{z}},{}^{\tau}\bar{\boldsymbol{b}},\tau)\,T\,d\tau, \qquad {}^{\tau_s}\overline{\Phi}({}^{\tau_s}\bar{\boldsymbol{z}},{}^{\tau_s}\bar{\boldsymbol{b}},\tau_s) = 0 \tag{28}$$

Unifying problems with and without preview control, analysis and design sensitivity analysis may be ran for a fixed unit time interval with mass, damping, velocities and accelerations transformed as

$$\boldsymbol{m} = T^{-2}\boldsymbol{M}, \quad \boldsymbol{c} = T^{-1}\boldsymbol{C}, \quad \boldsymbol{U}' = T\dot{\boldsymbol{U}}, \quad \boldsymbol{U}'' = T^2\ddot{\boldsymbol{U}}, \quad ' \equiv d/d\tau \tag{29}$$

5. Spinal injury prevention in a helicopter vertical crash

Consider the problem of limiting performance of a helicopter seat cushion for the reduction of spinal injuries in a vertical crash. The system is described as the 2-mass model represented in Fig. 2 [1]. The masses $m_1 = 20.18$ and $m_2 = 34.52$ Kg of the occupant lower and upper torso, respectively, are coupled by the linear spring with stiffness $k = 96600$ N/m and the linear dashpot with damping $c = 818.1$ N.s/m, representing the spinal properties. The vertical displacement of the seat pan and of the masses m_1 and m_2 with respect to a inertial frame are respectively u_0, u_1 and u_2. Between the seat pan and the lower torso there is a cushion that generates a force $P(t)$. Assume the seat pan undergoes a half-sine wave deceleration pulse $-A\sin(\pi t/T_a)$ of duration $T_a = 0.09\,s$ due to the helicopter vertical impact at a velocity $v_0 = 9.4$ m/s. The amplitude A is calculated as $A = 0.5\pi v_0/T_a$ such that the seat has

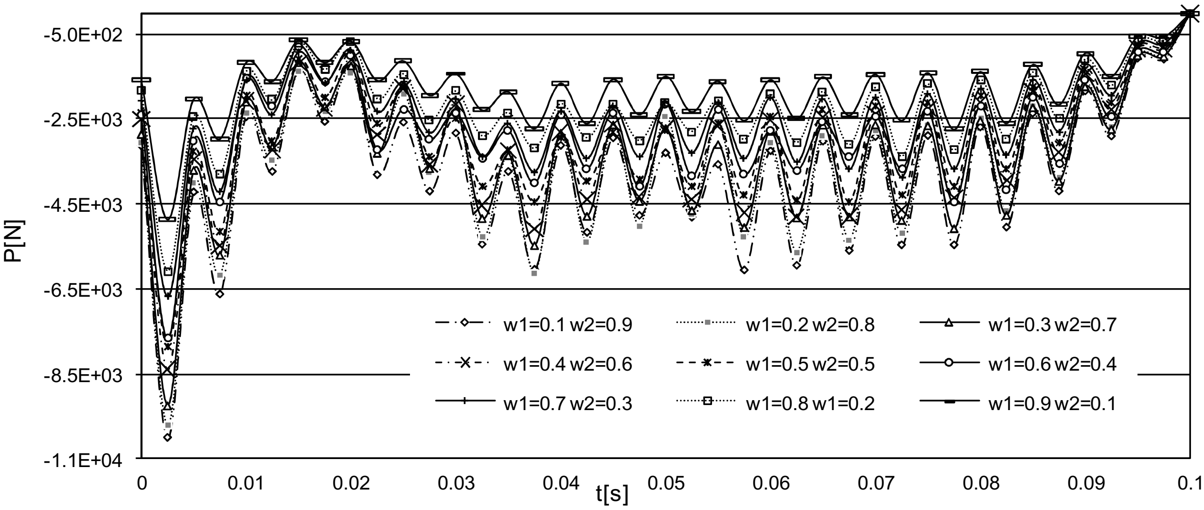

Fig. 4. Optimum control force without preview control with the min-max upper bound strategy.

a velocity jump $\dot{u}_0(t_s^+) - \dot{u}_0(t_s^-) = \dot{u}_1(t_s^+) - \dot{u}_1(t_s^-) = \dot{u}_2(t_s^+) - \dot{u}_2(t_s^-) = v_0$ and decelerates to a full stop after the time interval T_a.

To determine the limiting isolation capacity of the system one considers two performance criteria for the optimal control problem of the Eq. (25): the dynamic response index (DRI), which is used in biomechanics to predict the probability of spine fracture injuries, and the peak displacement D of the lower torso relatively to the seat pan

$$\Psi_{01} \equiv DRI = \frac{k}{m_2 g} \max_t |u_2 - u_1| ; \qquad \Psi_{02} \equiv D\,(Rattlespace) = \max_t |u_1 - u_0|$$

AGARD sets up the threshold values 15, 18, and 22 of the ***DRI*** respectively as the low, medium and high levels of spinal injury risk [1].

Firstly, the optimal control problem is performed without preview control, i.e., the impact isolator starts generating the control force at the instant of impact ($t_s = 0$). The problem is to find the control force $[P(t),\ 0 \leqslant t \leqslant 0.1s;\ -10^4\,N \leqslant P(t) \leqslant 10^4\,N]$ that minimizes the criteria objectives Ψ_{01} and Ψ_{02}. The problem has been performed by applying either a min-max upper bound strategy as in Eqs (25) and (26) or passing the second objective as a constraint (reduced feasible region strategy) which limit is relaxed from $D = 70\,mm$ to$D = 400\,mm$ by solving the problem in the form of the Eq. (24). The optimal objectives are shown in the Fig. 3 for different combinations of criteria weights or preferences w_k(min-max upper bound strategy) or for the different values of the rattlespace limit D (reduced feasible region strategy-one objective). Because the artificial variable b_0 and the control force variables have a very different order of magnitude, a scale factor was introduced such that the sensitivities are normalized to have the same order of magnitude for both types of variables. The optimal control force distribution is presented in the Figs 4 and 5, respectively for solving the optimization problem by the min-max upper bound method or by the reduced feasible region method.

In Fig. 6 is represented the pilot's spinal compression during the impact, showing the compression reaches rapidly its maximum value, then stabilizing around this value.

Secondly, the optimal control problem is performed with preview control. In this case one has performed the problem of finding the control force $[P(t),\ 0 \leqslant t \leqslant 0.1s;\ -10^4\,N \leqslant P(t) \leqslant 10^4\,N]$ plus the interval t_s that minimize the criteria Ψ_{01} such that $\Psi_{02} \leqslant 70mm$. The optimal value of the objective is $\Psi_{01} \equiv DRI = 2.4601$for an optimum pre-acting time interval $t_s = 0.20132$. Comparing with the value $DRI = 8.79765$ obtained without preview control, one gets a risk of injury about 3.5 times smaller by using a preview control than without preview control. The optimal control force is given in the Fig. 7.

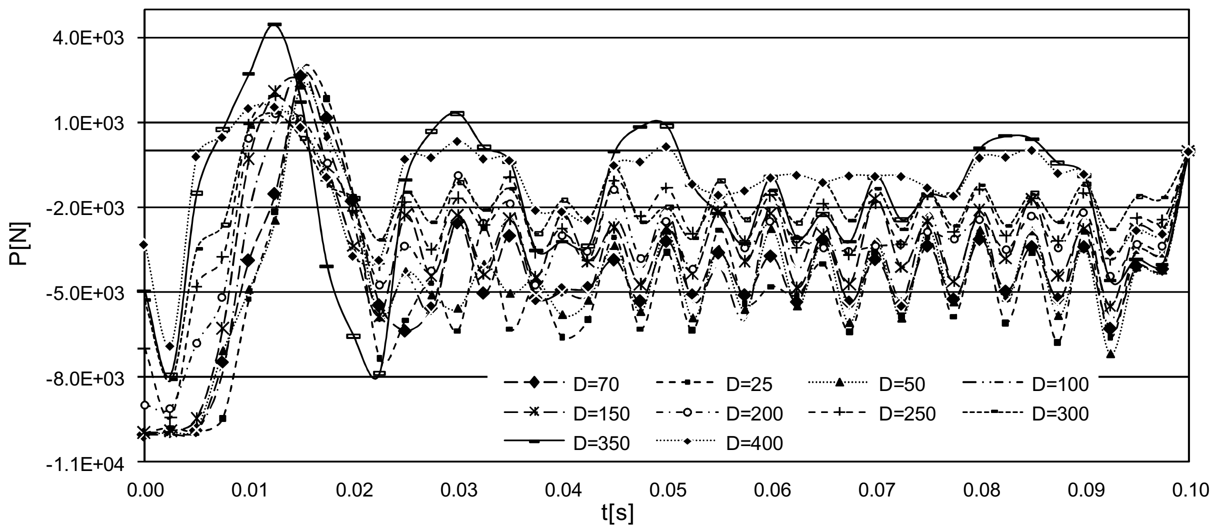

Fig. 5. Optimum control force without preview control with the reduced feasible region strategy.

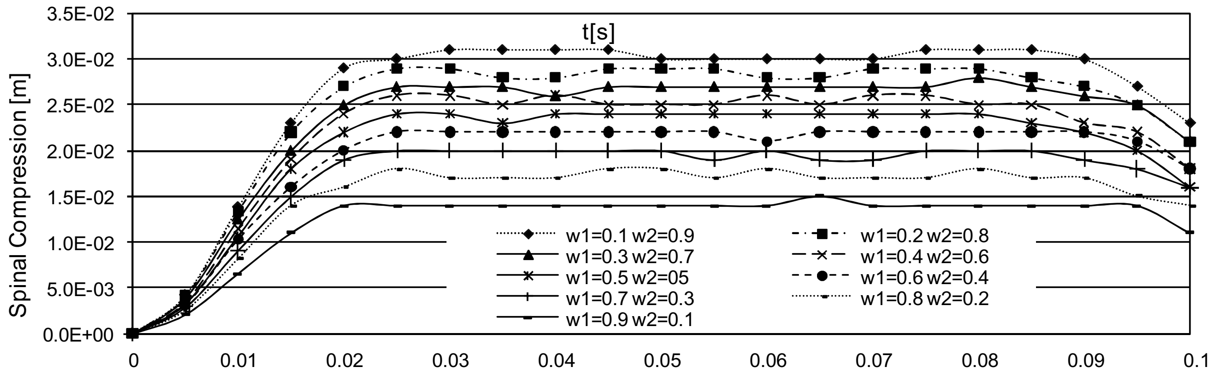

Fig. 6. History of the pilot's spinal compression during the impact.

6. Concluding remarks

An integrated methodology for optimal design and control of mechanical systems is presented. The formulation is implemented into an interactive optimum design code and it is applied to optimize protection systems for the prevention of injuries. Space-time finite elements are used to model the systems. The adjoint method is utilized to perform the design sensitivity analysis. Using at-once integration of the equations of motion and its sensitivities, the adjoint method keeps all of its advantages without its main disadvantage in dynamics: the necessity of backwards integration of the adjoint system. So, the needing of backwards integration with its major drawback – the memorization of the response history – is not associated with path-dependent problems, as it is frequently assumed, but it is associated with the fact of using step-by-step integration. This way, it is also possible and easy to apply temporal boundary conditions at any point in time.

The limiting isolation capacity of the systems to impact is determined throughout an optimal control problem. The effect of preview control is investigated. For preview control problems, since the time the isolator engages before the impact varies as the optimization proceeds, a fixed reference unit time interval is mapped onto the original time interval. The 'preview time' is a variable fraction of the total time. After the transformation of all the responses and optimization measures to the new unit time domain, the time variant domain problem may be treated as a time invariant domain problem.

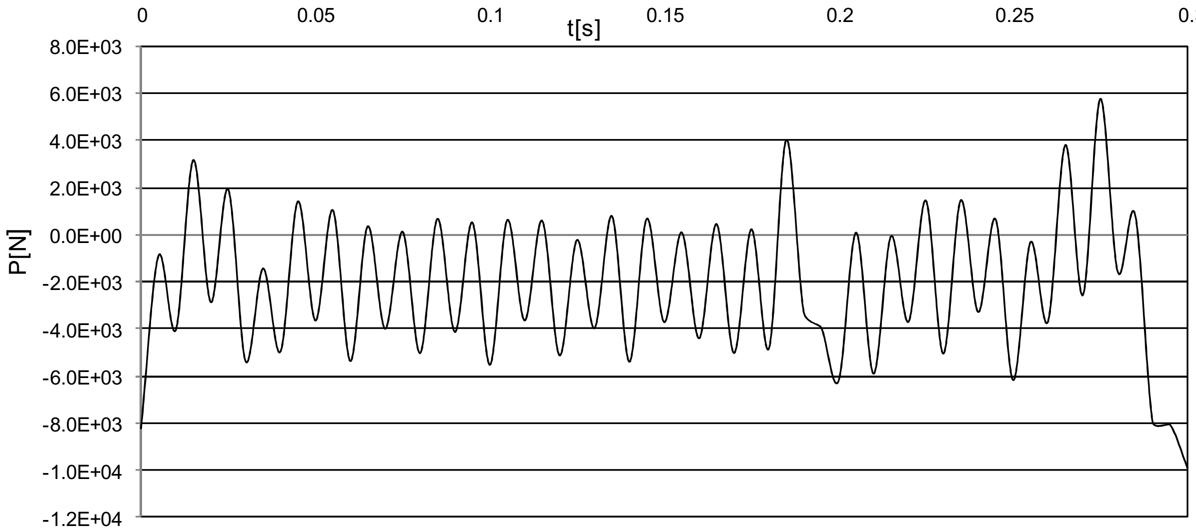

Fig. 7. Optimum control force for the limiting isolation capacity with preview control.

Numerical applications show that without cushion there is a high risk of spinal injury, which is reduced with the appropriate cushion control and characteristics of the optimal control are similar for a large range of cushion thicknesses. Also it is shown that under the optimal control, spinal compression quickly reaches its maximum value and then remains around this value. Numerical applications also show the preview control can reduce significantly the risk of injuries in comparison to a control only activated at the instant of impact.

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