Limiting performance of helmets for the prevention of head injury

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This is a study of the theoretical optimal (limiting) performance of helmets for the prevention of head injury. A rigid head injury model and a two-mass translational head injury model are employed. Several head injury criteria are utilized, including head acceleration, the head injury criterion (HIC), the energy imparted to the brain which is related to brain injury, and the power developed in the skull that is associated with skull fracture. A helmeted head hitting a rigid surface and a helmeted head hit by a moving object such as a ball are considered. The optimal characteristics of helmets and the impact responses of the helmeted head are investigated computationally. An experiment is conducted on an ensemble of bicycle helmets. Computational results are compared with the experimental results.

Keywords: Helmet, head injury, impact biomechanics, limiting performance, optimization

1. Introduction

The limiting performance of a system is found by replacing parts of the system, in particular, the part of the system such as a helmet liner that is intended to provide protection, with generic control forces, and then determining the optimal control forces that generate the best possible performance of this substitute system [1,20]. The system performance is measured by performance criteria which are usually functions of the system response and control forces. The optimal control forces are determined via optimization such that one or more performance criteria are minimized while the other criteria are constrained to remain within prescribed bounds. The resulting system performance is the theoretically optimal or best possible over all design configurations because the control forces, rather than being restricted to represent any predetermined design elements, may be passive or active, and linear or nonlinear during the optimization procedure. The limiting performance characteristics are of considerable value to mechanical system designs. First, they indicate from the design specifications alone whether a proposed design is feasible. Second, during the design cycle, they provide a measure of success of the design configurations under consideration. Third, the system characteristics for the limiting performance can be used to compare and evaluate different design criteria. Fourth, the characteristics of the optimal control forces can be used as guidelines for the improvement and optimization of a system design.

Helmets are an important safety device for head injury reduction. Because bicycle, motorcycle, and sports accidents often result in serious brain injuries, the improvement of the effectiveness of helmets as safety devices is an important injury prevention issue. The references listed in the bibliography at the end of this paper represent a small selection from the research literature on helmet safety [3,4,6–8,12,13,15,16,24,31].

In this paper, the limiting performance analysis is used to study the theoretically optimal performance of helmets for the prevention of head injuries. The objective is to show how to provide guidelines for the evaluation and improvement of helmet performance in head injury prevention. Two head injury models are used. One is a rigid head injury model, a single-degree-of-freedom (SDOF) model. The other is a two-mass head injury model, a translational head injury model (THIM).

Two impact modes are considered. In one mode, the helmeted head hits a rigid surface at a certain speed. In the other mode, the helmeted head is hit by a moving nonpenetrating object, such as a baseball. The limiting performance analysis is conducted computation-

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ally. The performance of an ensemble of currently available bicycle helmets is experimentally tested.

The main body of the paper consists of three parts. The first part deals with bicycle helmets hitting a rigid surface using a rigid head injury model. In the second part, the problem of the helmeted head hitting a rigid surface is investigated based on the THIM, a two-mass head injury model. The THIM is also employed in the third part where the problem of helmets hit by a moving ball is studied.

2. Modeling of the helmeted head under impact

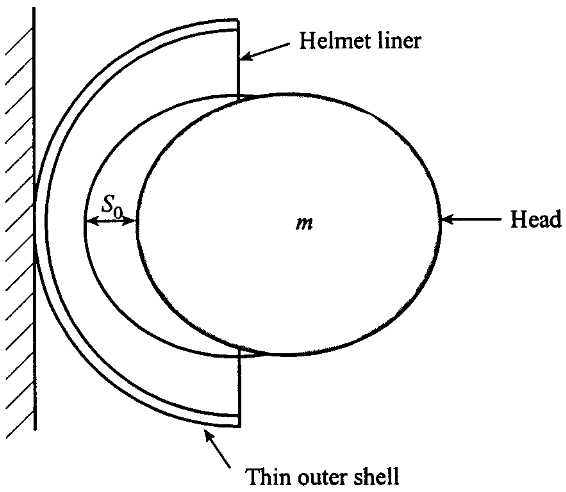
2.1. Head injury model and criteria

Over the past five decades, many models of the biomechanical dynamics of the human head have been developed [10,11,18]. These models can be categorized into direct and indirect loading models; one-, two-, and three-dimensional models; translation, rotation and combined translation and rotation models; and, recently, finite element models.

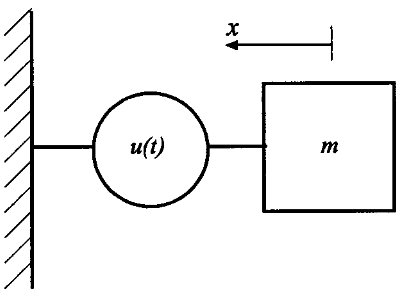
The design of helmets is often based on an analysis of a SDOF or rigid head injury model [9]. The current testing standards for helmets utilize rigid headforms [17]. The injury criteria usually employed with the SDOF model and testing standards are the severity index(SI), the head injury criterion(HIC), and the maximum head acceleration [16]. In this paper, the limiting performance of bicycle helmets is investigated using a SDOF head injury model, with the HIC and the maximum head acceleration as head injury criteria. The impact of the helmeted head of a bicyclist is depicted in Fig. 1(a).

However, questions are frequently raised as to the biofidelity of the performance of a helmet determined from experimental tests using a rigid headform which represents the rigid head injury model and the ability of helmets to mitigate actual head injuries. Therefore, a two-mass head injury model and associated head injury criteria are introduced to investigate the limiting performance of sports helmets.

Among multi-degree-of-freedom head injury models, THIM, the translational head injury model, developed by R.L. Stalnaker et al. [21,22,26,27], is chosen for the present study. The model is shown in Fig. 2 where m_1 is the mass of the impacted skull bone, and m_2 is the mass of the brain and the rest of the bones of the head [26]. The sum of m_1 and m_2 is the total mass of the head. The stiffness k and damper c_1 represent the dynamics of the skull. The damper c_2 represents primarily the damping of the brain. This one-dimensional



(a)



(b)

Fig. 1. Helmeted head under impact using SDOF model: (a) Illustration of the problem, (b) Physical model for the limiting performance analysis.

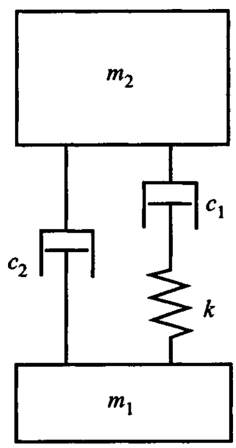


Fig. 2. Translation head injury model.

model accounts only for the translational motion of the head. Its parameters are adjusted to correspond to the impacts at four directions of the head [21,26]: (a) A–P: Anterior to Posterior (front impact); (b) P–A: Posterior to Anterior (rear impact); (c) L–R: Left to Right (side impact); and (d) S–I: Superior to Inferior (top impact).

Based on the translational head injury model, translational energy criteria (TEC) were developed [27] which include

1. The amount of energy released by the second damper c_2, which represents the energy imparted to the brain and was found to be directly proportional to the severity of brain injury. A regression equation has been developed between the abbreviated injury scale (AIS) and the energy dissipated by c_2 [28].
2. The power developed in the spring k, which represents the power developed in the skull and is related to the level of load-deformation of the skull and the rate at which this load-deformation occurs. There exists a relationship between the power level and the probability of skull fracture [28].
3. The acceleration of $m_{2,}$ which can be considered as the actual head acceleration since m_2 is about 90% of the total mass of the head.

Basically, there are two major impact modes of the helmeted head in sports. One is the helmeted head hitting a hard surface, which can happen to football and hockey helmets as well as bicycle helmets. This impact mode is described by Fig. 3(a), where the THIM is used to describe the impact dynamics of the head. The other mode is the helmeted head hit by a moving ball, which occurs in the helmets used for baseball or softball. Figure 4(a) is an illustration of this scenario where the THIM is also employed. The difference between these two impact modes may lead to different requirements of the characteristics of these two sorts of helmets. Therefore, we will treat them separately.

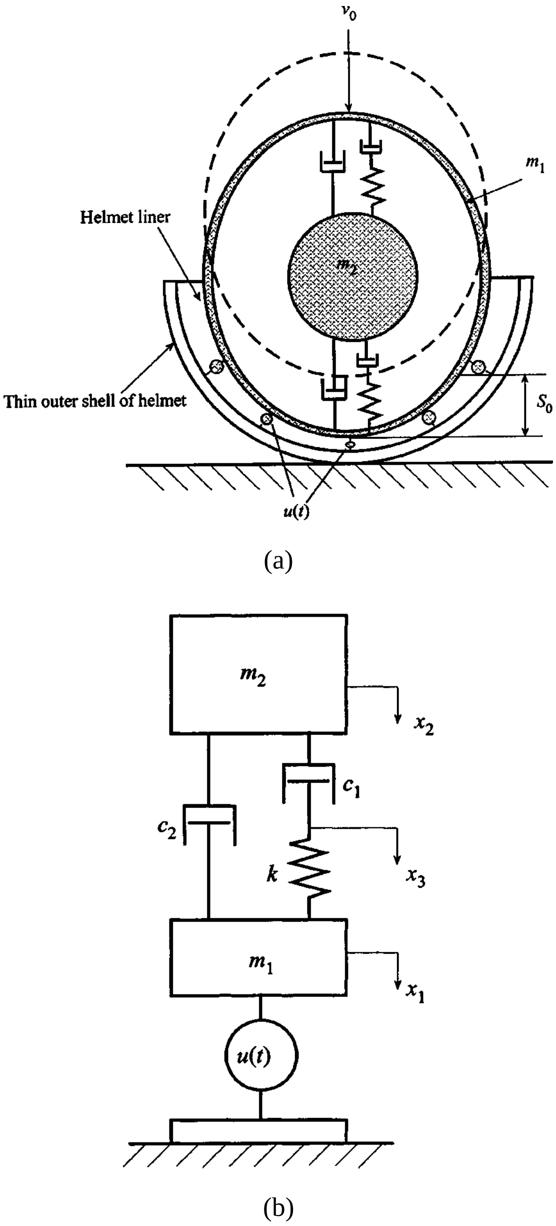


Fig. 3. Helmeted head impacting on a surface using THIM. (a) Illustration of the problem. The springs and dashpots model the dynamics of the skull and the damping of the brain. m_1: mass of impacted skull bone; m_2: mass of brain and the rest of skull bones; S_0: rattlespace of helmet, permissiable displacement of the head; $u(t)$: control force. (b) Physical model for the limiting performance analysis.

3. Analysis of bicycle helmets using a rigid head injury model

3.1. Problem statement

The helmeted head of a bicyclist under impact loading is modeled as a single degree of freedom system, as shown in Fig. 1(b). By exerting a force opposing the motion of the head, the helmet liner causes the head to decelerate during the impact. In Fig. 1(b), m denotes the mass of the head and S_0 is the *rattlespace*. The rattlespace of a helmet is defined as the maximum allowable space for the relative motion between the head and helmet in the direction of an impact. It is related to the helmet padding liner thickness and depends on

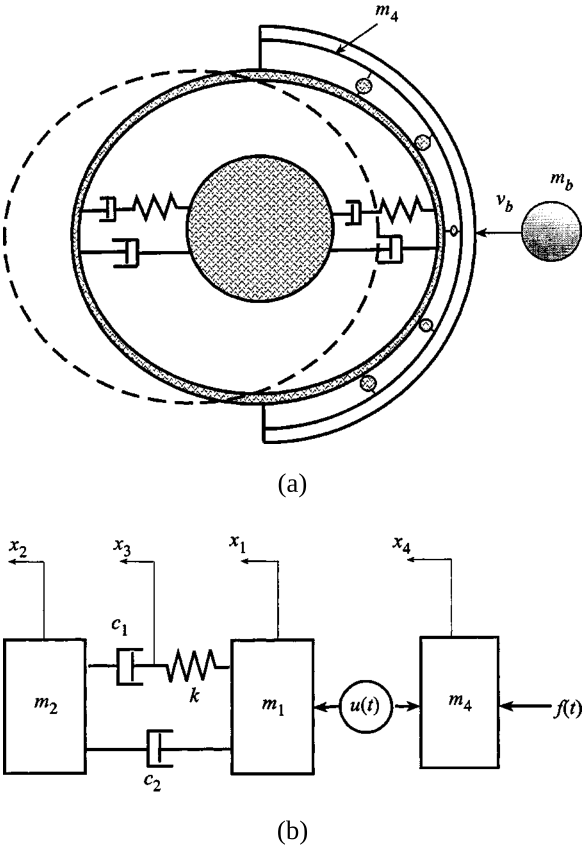


Fig. 4. Helmeted head impacted by a moving object with the head described by THIM. (a) Illustration of the problem; m_4: effective mass of the helmet; m_b: mass of the projectile, e.g., a ball. (b) Physical model for the limiting performance analysis.

the specific helmet structure. For the limiting performance analysis, the action of the helmet on the head is represented by the control force $u(t)$, which, defined in the time domain, may be linear or nonlinear, passive or active, and may be not realizable in practice. Since the protective function of a helmet is provided by the shell and, primarily, the padding liner, the control force represents the characteristics of both, but, primarily, of the liner.

One criterion used to measure the performance of the helmet for head injury prevention is the HIC. While the HIC has been susceptible to criticism [16], it is still being used as a head injury criterion associated with the rigid head injury model. The maximum acceleration of the head is commonly used for bicycle helmet standards to measure the energy attenuation function of helmets [2,25]. In this paper, in order to quantitatively evaluate the effectiveness of current bicycle helmets in head injury prevention or reduction and to qualitatively compare the differences between the two head injury criteria, both criteria will be used as system performance indices.

The problem of the limiting performance of bicycle helmets under impact loads can be described as: For a prescribed helmet rattlespace, find the optimal characteristics for the helmet liner such that the chosen head injury criterion, the helmet performance index, is minimized.

3.2. Limiting performance solution

The equation of motion for the substitute system of Fig. 1(b) is

$$m\ddot{x} = u(t), \tag{1}$$

where x is the displacement of the head. The initial conditions of the system are

$$x(0) = 0, \quad \dot{x}(0) = v_0, \tag{2}$$

where v_0 is the impact velocity. The solution of Eq. (1) is

$$x(t) = v_0 t + \frac{1}{m}\int_0^t (t-\tau)u(\tau)\,\mathrm{d}\tau. \tag{3}$$

As a performance index in the limiting performance analysis we choose the peak head acceleration,

$$J_1 = \max_{t\in[0,T]} \left|\ddot{x}(t)\right|, \tag{4}$$

where T is the duration of the control force. As another performance index we introduce the HIC,

$$J_2 = \mathrm{HIC} = (t_2 - t_1)\left[\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left|\ddot{x}_\mathrm{g}(\tau)\right| \mathrm{d}\tau\right]^{2.5} \tag{5}$$

where $\ddot{x}_\mathrm{g} = \ddot{x}/\mathrm{g}$, g is the acceleration of gravity, t_1 and t_2 (units of s), are two time instants within the impact duration. The time interval defined by t_1 and t_2 is chosen such that the value of HIC is maximized subject to the condition

$$0 < t_2 - t_1 \leqslant t_\mathrm{w}. \tag{6}$$

The time window t_w is usually taken as 0.036 s.

The problem of the limiting performance of bicycle helmets can be formulated as the following optimization problem:

$$\begin{array}{ll} \text{Find} & u_0(t) \\ \text{Minimize} & J_1 \text{ or } J_2 \\ \text{Subject to} & \max\limits_{t\in[0,T]} \{x(t)\} = S_0 \end{array} \tag{7}$$

If J_1 is to be minimized, for this case of initial impact, the solution of the optimization problem of Eq. (7) is already available [23]:

$$\ddot{x}(t) = -\frac{v_0^2}{2S_0} = \text{constant}, \tag{8}$$

and

$$u(t) = m\ddot{x}(t) = -\frac{mv_0^2}{2S_0} = \text{constant}, \tag{9}$$

so that

$$a_\mathrm{m} = \max_{t\in[0,T]} \left|\ddot{x}(t)\right| = \frac{v_0^2}{2S_0}. \tag{10}$$

For the constant acceleration of the head, the HIC becomes

$$\begin{aligned} \mathrm{HIC} &= (t_2 - t_1)\left[\frac{1}{t_2 - t_1}\int_{t_1}^{t_2} \left|\ddot{x}_\mathrm{g}(\tau)\right| \mathrm{d}\tau\right]^{2.5} \\ &= (t_2 - t_1)\left(\frac{a_\mathrm{m}}{\mathrm{g}}\right)^{2.5} \\ &= T_\mathrm{s} \cdot \left(\frac{a_\mathrm{m}}{\mathrm{g}}\right)^{2.5}, \end{aligned} \tag{11}$$

where the time interval $T_\mathrm{s} = t_2 - t_1$ is chosen to maximize HIC. The proper time interval is from the beginning of impact to the instant the helmeted head comes to a halt for the first time, that is,

$$T_\mathrm{s} = \frac{2S_0}{v_0}. \tag{12}$$

Therefore,

$$\mathrm{HIC} = \frac{2S_0}{v_0}\left(\frac{a_\mathrm{m}}{\mathrm{g}}\right)^{2.5} = \frac{v_0^4}{(2S_0)^{1.5}\mathrm{g}^{2.5}}. \tag{13}$$

If J_2 is to be minimized, the optimization problem is complicated because the time window $t_2 - t_1$ in Eq. (5) is not known at the outset and because the absolute value of the acceleration appears under the integral sign. In order to convert this problem into a mathematical programming problem and then use numerical optimization methods for the solution, we discretize the time interval $[0, T]$, where T is the duration of the control force, into identical subintervals. In addition, we approximate the control force $u(t)$ on each of the subintervals by a constant value, that is

$$\begin{aligned} &u(t) = u_k \quad \text{for } (k-1)\Delta t \leqslant t < k\Delta t, \\ &\quad k = 1, \ldots, N, \end{aligned} \tag{14}$$

where N is the number of subintervals, Δt is the length of each subinterval, $\Delta t = T/N$. Denote

$$\mathbf{U} = [u_1\, u_2 \cdots u_N]^\mathrm{T} \tag{15}$$

as the vector of design variables. The discrete form of the expression given in Eq. (3) for head displacement is

$$x_k = v_0 k\,\Delta t + \sum_{i=1}^{k} \beta_i u_i, \quad k = 1, \ldots, N, \tag{16}$$

where β_i, whose explicit form depends on the numerical integration rule, is a function of Δt. Then, the problem is formulated as

$$\begin{array}{ll} \text{Find} & \mathbf{U} \\ \text{Minimize} & J_2 \\ \text{Subject to} & \max\limits_{k\in[1,N]} \{x_k\} \leqslant S_0 \end{array} \tag{17}$$

To use the linear programming method, we convert the problem of Eq. (17) to its dual problem [1]

$$\begin{array}{ll} \text{Find} & \mathbf{U} \\ \text{Minimize} & \max\limits_{k\in[1,N]} \{x_k\} \\ \text{Subject to} & J_2 \leqslant H_\mathrm{max} \end{array} \tag{18}$$

where H_max is the prescribed limit on the HIC. The problems of Eqs (17) and (18) are equivalent.

The inequality $J_2 \leqslant H_\mathrm{max}$ is not a single condition, and must be written for every possible time window $0 < t_2 - t_1 \leqslant t_\mathrm{w}$ with t_2, t_1 assuming values in the set of discretized times $k\,\Delta t$, $0 \leqslant k \leqslant N$. It can be rewritten as

$$\int_{t_1}^{t_2} \left|u(\tau)\right| \mathrm{d}\tau \leqslant H_\mathrm{max}^{2/5}(t_2 - t_1)^{3/5} m. \tag{19}$$

Any real number α can be expressed as the difference between two non-negative numbers

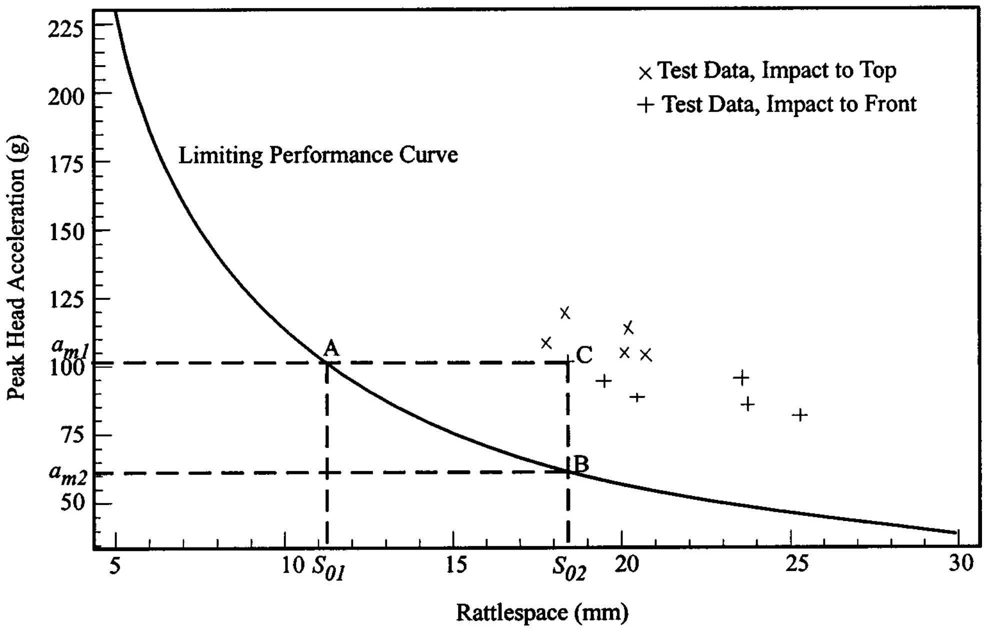


Fig. 5. Performance of bicycle helmets in tests based on peak head acceleration.

$$\alpha = \alpha^+ - \alpha^-, \quad \alpha^+ \geqslant 0, \quad \alpha^- \geqslant 0, \tag{20}$$

where α^+, α^- are defined by

$$\alpha^+ = \max(\alpha, 0), \quad \alpha^- = \max(-\alpha, 0). \tag{21}$$

The absolute value of α is given by

$$|\alpha| = \alpha^+ + \alpha^-. \tag{22}$$

Thus, if variables α and $|\alpha|$ both appear as linear terms in the formulation of an optimization problem, a change of variables to α^+, α^- eliminates the absolute value operator and yields a linear programming problem. The discretized form of the inequality (19) may therefore be stated as

$$\sum_{i=r}^{s} \gamma_i \left(u_i^+ + u_i^-\right) \leqslant H_{\max}^{2/5}(t_2 - t_1)^{3/5} m, \tag{23}$$

where r, s depend on the choice of t_1, t_2 in the computation of HIC and γ_i is determined by the numerical integration rule used in calculating the integral on the left side of Eq. (19). The linear programming formulation of the problem of Eq. (18) is

$$\begin{array}{ll} \text{Find} & u_k^+ \text{ and } u_k^-, \ k = 1, \ldots, N \\ \text{Minimize} & \max\limits_{k \in [1,N]} \{x_k\} \end{array}$$

$$\text{Subject to} \quad \begin{cases} \sum_{i=r}^{s} \gamma_i \left(u_i^+ + u_i^-\right) \\ \quad \leqslant H_{\max}^{2/5}(t_2 - t_1)^{3/5} m \\ u_k^+ \geqslant 0, \quad k = 1, \ldots, N \\ u_k^- \geqslant 0 \end{cases} \tag{24}$$

The limiting performance analysis was carried out for a range of rattlespaces with impact velocity $v_0 = 4.7$ m/s. The plots of $\min(\max_t\{\ddot{x}(t)\})$ or min(HIC) versus the prescribed rattlespace S_0, referred to as *trade-off curves*, are obtained, as shown in Figs 5 and 6, respectively.

3.3. *Experimental results and comparison with limiting performance*

Experiments for evaluating the performance of 24 bicycle helmets were conducted using a head-drop device, which consists of a head-form mounted on a support frame that slides along a vertical track (Fig. 7). Impact occurred at either the top or the frontal region of the helmet. The following measurements were made in each test:

(a) Impact velocity;
(b) Head acceleration as a function of time;
(c) Peak head acceleration;
(d) Rattlespace.

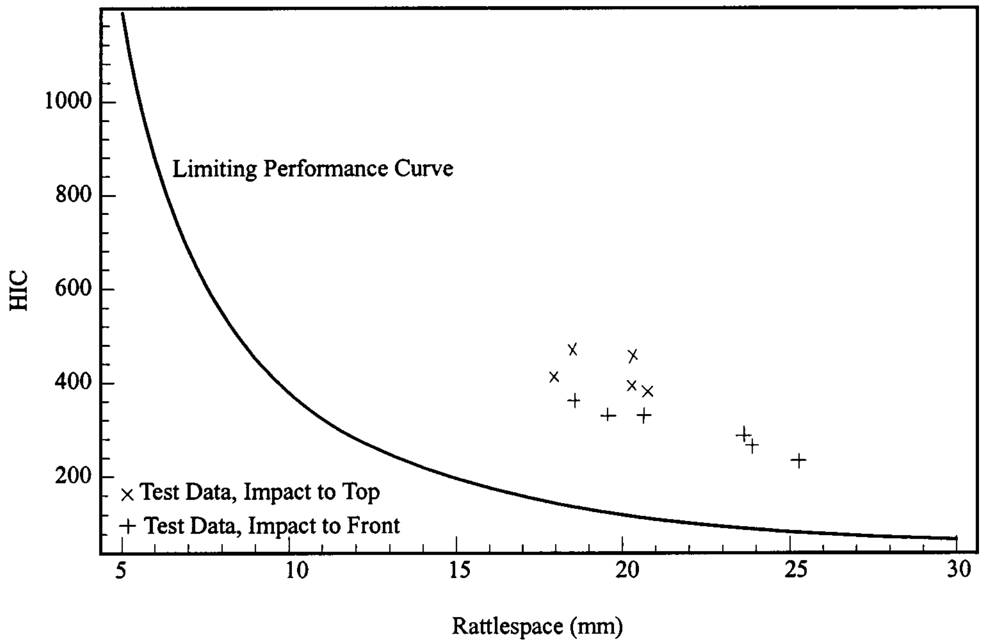


Fig. 6. Performance of bicycle helmets in tests based on HIC.

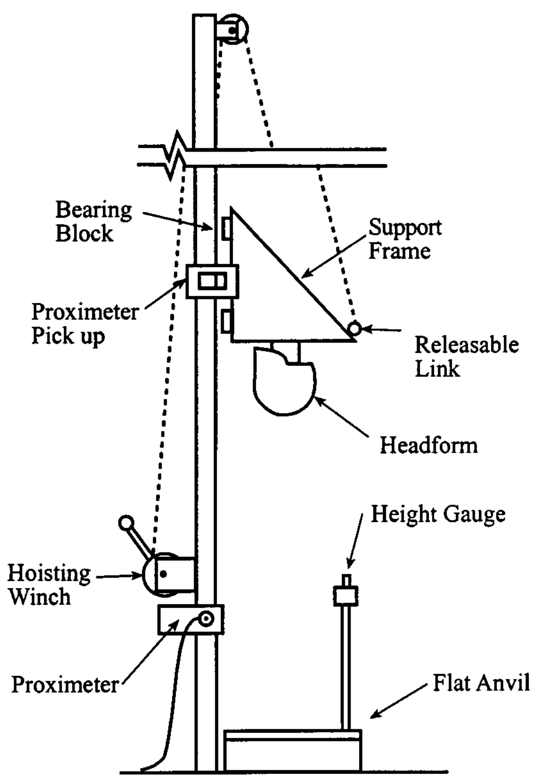


Fig. 7. Primary components of the head-drop device.

Due to the friction in the linear bearings supporting the headform, the impact velocity was measured directly using a speed sensor, rather than calculated from the drop height. The mean measured impact velocity was $v_0 = 4.7$ m/s.

To measure the rattlespace properly, some adjustments to the helmet are sometimes needed. Any soft foam lining was removed from the inner helmet prior to testing, since the soft liner offers negligible head protection and is intended primarily to make the helmet fit comfortably. The helmet was fitted snugly to the headform. Prior to testing, the pre-impact thickness of the helmet was recorded by lowering the headform and helmet onto the impact anvil and measuring the resting height of the headform. This ensured that no air gap remained between helmet and headform, as the helmet and shell were compressed under the free weight of the headform and attached assembly. During testing, the crush depth was determined by measuring the minimum height that the headform achieved during impact and subtracting this value from the original helmet thickness. This crash depth was taken as the distance corresponding to the rattlespace.

For the tested helmets, the relationship between peak acceleration and rattlespace is shown in Fig. 5, and the relationship between the HIC and rattlespace in Fig. 6.

The limiting performance curve for peak acceleration versus rattlespace in Fig. 5 shows the effectiveness

Table 1
Potential for improving helmet designs based on peak head acceleration

Impact region	Potential improvement (%)		
	Lower	Upper	Average
Top	71	110	91.1
Front	62	100	75.4

Table 2
Potential for improving helmet designs based on HIC

Impact region	Potential improvement (%)		
	Lower	Upper	Average
Top	197	304	248
Front	160	228	193

of the tested helmets in minimizing peak head acceleration. From this plot it is determined that, on average, the helmet performance could be improved by as much as 83% in minimizing peak acceleration. These results are summarized in Table 1 according to impact location.

The trade-off curve between the HIC and rattlespace of Fig. 6 shows that, on average, the helmet performance could be improved by as much as 221% in minimizing the HIC. The results are summarized in Table 2.

The average time histories of the measured head acceleration are shown in Fig. 8. For comparison, the time histories of the head acceleration by computation are also illustrated in Fig. 8. The range of rattlespace thicknesses for the 24 helmets tested is from 17.5 to 26 mm. It is evident from Fig. 8 that the experimental pulses differ significantly from both the HIC optimal pulse and the peak acceleration optimal pulse. This is especially true of the HIC optimal pulse. It is also evident from this figure that there is little agreement between the two injury criteria regarding what constitutes the ideal pulse.

3.4. Discussion

1. Some insight into the usefulness of a limiting performance analysis can be gained using a trade-off curve. For instance, with reference to the trade-off curve shown in Fig. 5, if a helmet with a rattlespace S_{02} is tested in a head-drop experiment for superior to inferior impact, the measured peak head acceleration a_{m1} of the tested helmet must be greater than or equal to a_{m2}, the theoretical minimum value of the peak head acceleration for the rattlespace S_{02}. On the other hand, if the peak head acceleration is allowed to be a_{m1}, then the corresponding theoretical minimum required rattlespace is S_{01} which is smaller than or equal to S_{02}. It is impossible for a real world helmet to have its performance point (S_0, a_{m}) lie below the trade-off curve. The closeness of the performance point to the trade-off curve indicates the effectiveness of a helmet. The closer the better. A measure of potential improvement can be defined as

$$\eta = \frac{a_{\mathrm{m1}} - a_{\mathrm{m2}}}{a_{\mathrm{m2}}}. \tag{25}$$

   If $\eta = 0$ there is no room for improvement. For actual helmets $\eta > 0$. The larger the value of η, the more the design can be improved. These interpretations can be used to quantify and evaluate the effectiveness of a helmet against head injury for the particular optimization criteria utilized.
2. When the peak head acceleration is the performance index and minimized, the head acceleration remains constant under the action of the optimal control force, as illustrated by curves 5 and 6 in Fig. 8. Its value depends on the impact velocity and rattlespace but is independent of the head mass, as indicated by Eq. (10). The optimal control force which an ideal helmet liner would deliver is constant throughout the duration of the impact. This suggests the use of a material capable of producing a constant force as it crushes. Most helmets are constructed from expanded polystyrene, with the intent of delivering a constant force upon impact.
3. When the HIC is minimized, the head acceleration is very large initially and decreases rapidly afterwards, as shown by curves 3 and 4 in Fig. 8. The ideal helmet liner should deliver an extremely large force at the onset of impact. This force should decrease rapidly to a nearly constant value for the remainder of the pulse duration. This suggests the use of a material that is extremely stiff at the onset of impact, but becomes progressively less stiff as the pulse develops. One possibility for a helmet liner to reduce the HIC is to use a breakaway material. Such a material could deliver a large force at the onset of collision.
4. Other factors indicate that the optimal pulses supported by the head injury criterion and the peak head acceleration criterion are in fact harmful. There are indications that the rate of onset of acceleration (jerk) plays an important role in de-

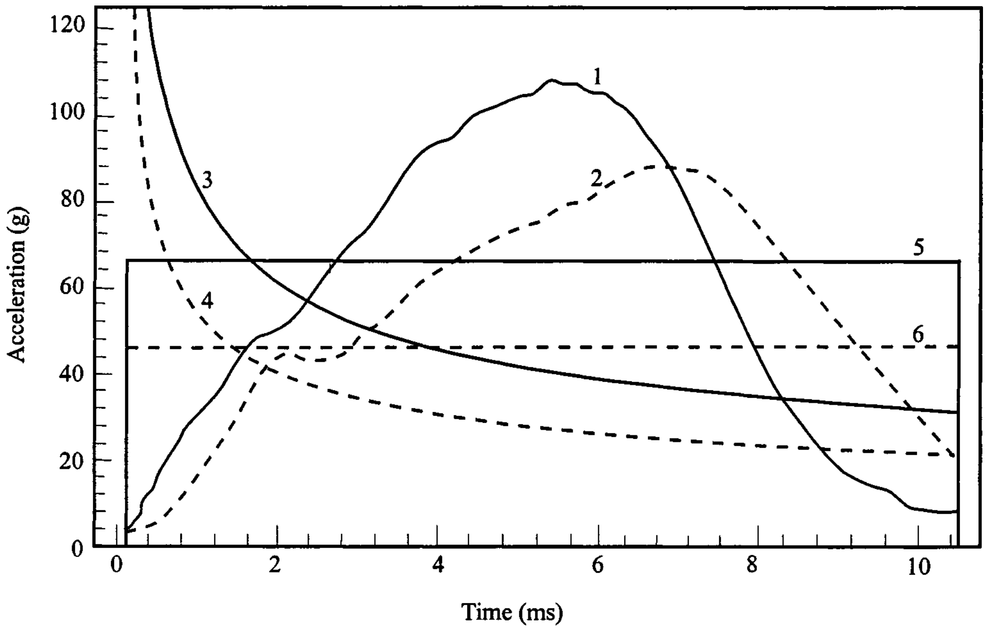


Fig. 8. Comparison of experimental and optimal acceleration. Curve 1: Test data, top impact; Curve 2: Test data, front impact; Curve 3: Minimization of HIC, $S_0 = 17.5$ mm; Curve 4: Minimization of HIC, $S_0 = 26.0$ mm; Curve 5: Minimization of peak head acceleration, $S_0 = 17.5$ mm; Curve 6: Minimization of peak head acceleration, $S_0 = 26$ mm.

termining the severity of head injury. A jerk of 2×10^5 g/s has been suggested as a reasonable upper limit [16]. Referring again to Fig. 8, it is evident that this safe limit is violated by both optimal pulses, where jerk equals infinity at the onset of impact. A reasonable jerk rate could be incorporated into the limiting performance models by imposing a secondary constraint on the slope of the acceleration pulse.

4. Helmeted head hitting a rigid surface using THIM

We will continue the study of the previous section, replacing the rigid head model with the two-mass translational head injury model (THIM). In this case, the helmeted head hitting a rigid surface is depicted in Fig. 3(a). This model can be used to describe the impact of the helmeted head occurring in sports like football and hockey as well as bicycling. As the protection function of the helmet is theoretically replaced with a control force $u(t)$, the physical model for the limiting performance analysis of the problem of Fig. 3(a) is shown in Fig. 3(b). In the following analysis, the performance of helmets for the prevention of head injury is measured by the translational energy criteria (TEC), the head injury criteria defined previously.

4.1. Problem formulation

4.1.1. Governing equations of the system

The motion of the system of Fig. 3(b) is described by the equations

$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{x}_3 \end{Bmatrix} + \begin{bmatrix} c_2 & -c_2 & 0 \\ -c_2 & c_1 + c_2 & -c_1 \\ 0 & -c_1 & c_1 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{Bmatrix} + \begin{bmatrix} k & 0 & -k \\ 0 & 0 & 0 \\ -k & 0 & k \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \end{Bmatrix} = \begin{Bmatrix} u(t) \\ 0 \\ 0 \end{Bmatrix} \tag{26}$$

with the initial conditions

$$\begin{Bmatrix} x_1(0) \\ x_2(0) \\ x_3(0) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}, \quad \begin{Bmatrix} \dot{x}_1(0) \\ \dot{x}_2(0) \\ \dot{x}_3(0) \end{Bmatrix} = \begin{Bmatrix} v_0 \\ v_0 \\ v_0 \end{Bmatrix}, \tag{27}$$

where v_0 is the initial impact velocity.

4.1.2. Performance criteria

The head injury criteria TEC, which are now used to measure the performance of helmets for the prevention of head injury, are expressed as the following system performance criteria:

$$J_1(u) = \max_t |\ddot{x}_2|,$$
$$J_2(u) = E_{c_2} = \int_0^T c_2 \big[\dot{x}_2(t) - \dot{x}_1(t)\big]^2 \,\mathrm{d}t,$$
$$J_3(u) = \max_t |P| = \max_t \big\{ \big| k\big[x_3(t) - x_1(t)\big] \times \big[\dot{x}_3(t) - \dot{x}_1(t)\big] \big| \big\}, \tag{28}$$

where J_1 is the peak head acceleration, J_2 is the energy dissipated by damper c_2 and represents the energy imparted to the brain during the duration T of the control force, and J_3 is the peak power developed in the skull. As the performance criterion of the helmet we adopt the quantity

$$J_4(u) = \max_t (x_1), \tag{29}$$

which defines the maximum displacement of the skull with respect to the helmet.

4.1.3. Problem statement

The limiting performance of the system is investigated for two cases. In Case 1, the peak head acceleration J_1 is chosen as the performance index and minimized while the energy imparted to the brain J_2 and the peak power developed in the skull J_3 are constrained to remain below prescribed threshold values. In Case 2, the energy imparted to the brain J_2 is chosen as the performance index and minimized while the peak head acceleration J_1 and the peak power developed in the skull J_3 are constrained. In both cases, the maximum displacement of the skull with respect to the helmet J_4 is bounded by the prescribed rattlespace of a helmet S_0.

These problem statements can be formulated as follows. Find the optimal control force $u_0(t)$ such that

$$J_1(u_0) = \min_u J_1(u) \quad \text{(performance index)}, \tag{30}$$

subject to

$$J_i(u_0) \leqslant D_i, \quad i = 2,3,4 \quad \text{(constraints)} \tag{31}$$

for Case 1, and

$$J_2(u_0) = \min_u J_2(u) \quad \text{(performance index)}, \tag{32}$$

subject to

$$J_i(u_0) \leqslant D_i, \quad i = 1,3,4 \quad \text{(constraints)} \tag{33}$$

for Case 2, where D_i ($i = 1, \ldots, 4$) are the maximum allowable values (threshold values) of the corresponding system performance criteria.

4.2. Computational investigation

4.2.1. Conversion to mathematical programming problems

In order to use numerical optimization methods for the solutions, we convert the open-loop control problems of Eqs (30)–(33) into mathematical programming problems. The control force $u(t)$ is discretized using Eqs (14) and (15). The system responses are discretized accordingly such that

$$x_{jk} = x_j(k\,\Delta t), \quad j = 1,2,3, \quad k = 1,\ldots,N. \tag{34}$$

Then, the system performance criteria of Eqs (28) and (29) become

$$\widehat{J}_1(\mathbf{U}) = \max_{1\leqslant k\leqslant N} |\ddot{x}_{2k}|,$$
$$\widehat{J}_2(\mathbf{U}) = \Delta t \sum_{k=1}^{N} c_2 \big(\dot{x}_{2k} - \dot{x}_{1k}\big)^2,$$
$$\widehat{J}_3(\mathbf{U}) = \max_{1\leqslant k\leqslant N} \big\{ \big| k\big(x_{3k} - x_{1k}\big)\big(\dot{x}_{3k} - \dot{x}_{1k}\big) \big| \big\},$$
$$\widehat{J}_4(\mathbf{U}) = \max_{1\leqslant k\leqslant N} (x_{1k}), \tag{35}$$

where $\dot{x}_{1k}$, $\dot{x}_{3k}$, and $\ddot{x}_{2k}$ are discretized velocities and accelerations. Thus, the optimization problems described by Eqs (30)–(33) are reformulated as follows. For Case 1, the objective is to find the optimal control force $\mathbf{U}_0$ such that

$$J_1(\mathbf{U}_0) = \min_{\mathbf{U}} J_1(\mathbf{U}) \quad \text{(performance index)}, \tag{36}$$

subject to

$$J_i(\mathbf{U}_0) \leqslant D_i, \quad i = 2,3,4 \quad \text{(constraints)}. \tag{37}$$

For Case 2, the goal is to find the optimal control force $\mathbf{U}_0$ such that

$$J_2(\mathbf{U}_0) = \min_{\mathbf{U}} J_2(\mathbf{U}) \quad \text{(performance index)}, \tag{38}$$

subject to

$$J_i(\mathbf{U}_0) \leqslant D_i, \quad i = 1, 3, 4 \quad \text{(constraints)}. \tag{39}$$

The problem of Eqs (38) and (39) is a mathematical programming problem. The problem of Eqs (36) and (37) is a minmax problem, which can be converted to a mathematical programming problem [1]. Then, the problems of Case 1 and Case 2 can be solved using nonlinear optimization methods.

4.2.2. Selection of the threshold values of system performance criteria

The threshold value for the peak head acceleration D_1 is chosen to be 200 g for both impact energy levels. The use of this seemingly low value is based on a statistical analysis of head impact data [14], which showed that levels of acceleration of 300 g to 400 g currently allowed by helmet standards were too high from the standpoint of injury protection.

The relationship between the energy imparted to the brain and the injuries of the brain is described by Stalnaker and Rojanavanich [28]

$$\text{EAIS} = 4.14\sqrt{E_\text{b}}, \tag{40}$$

where EAIS is the equivalent abbreviated injury scale, and E_b is the corresponding value of the energy imparted to the brain. If EAIS $= 2$, then $E_\text{b} = 0.2334$ J, which corresponds to the state of unconsciousness for less than one hour, and if EAIS $= 3$, then $E_\text{b} = 0.5251$ J, which corresponds to the state of unconsciousness for one to six hours [19]. The relationship between the probability of skull fracture and the peak power P_m can be approximately described using the normal distribution function [28], that is

$$p(P_\text{m}) = \int_0^{P_\text{m}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(\xi - P_0)^2}{2\sigma^2}\right) \mathrm{d}\xi, \tag{41}$$

where $P_0 = 7365$ W and $\sigma = 700$ W. Let P_u be the threshold value of P_m for a prescribed probability of skull fracture. If the probability of skull fracture is set to be 5%, then $P_\text{u} \approx 6210$ W. Also, for a probability of 15%, $P_\text{u} \approx 6640$ W.

It can be anticipated that the limiting performance of a helmet depends on the initial impact velocity v_0 or the initial impact energy E_0. For the initial conditions prescribed by Eq. (27), the relationship between v_0 and E_0 is

$$E_0 = \frac{1}{2}(m_1 + m_2)v_0^2. \tag{42}$$

Two levels of E_0 are considered here. One is that $E_0 = 53.4$ J, corresponding to the initial velocity of 4.85 m/s or the drop height of 1.2 m prescribed by a standard [17], and the other is that $E_0 = 100$ J, for an initial velocity of 6.64 m/s. The threshold values of the energy imparted to the brain and the maximum power developed in the skull are chosen according to the level of initial impact energy, respectively,

$$\begin{aligned} &D_2 = E_\text{b} = 0.2334 \text{ J} \quad \text{and} \\ &\qquad D_3 = P_\text{u} = 6210 \text{ W}, \quad \text{for } E_0 = 53.4 \text{ J}, \\ &D_2 = E_\text{b} = 0.5251 \text{ J} \quad \text{and} \\ &\qquad D_3 = P_\text{u} = 6640 \text{ W}, \quad \text{for } E_0 = 100 \text{ J}. \end{aligned} \tag{43}$$

The maximum allowable displacement of the skull with respect to the helmet D_4 is equal to the prescribed rattlespace of the helmet S_0.

4.2.3. Computational results

A reasonable range of the helmet liner thickness is from 15 mm to 30 mm. The limiting performance of the system is computed for a number of prescribed rattlespaces S_0 within these ranges for both cases. For Case 1, the relationship of the peak acceleration of m_2 versus the helmet rattlespace, the trade-off curve, is shown in Figs 9(a) and (b) for two energy levels $E_0 = 53.4$ J and $E_0 = 100$ J, respectively. For Case 2, the trade-off curves between the energy imparted to the brain and the helmet rattlespace are illustrated in Figs 10(a) and (b) for $E_0 = 53.4$ J and $E_0 = 100$ J. The time histories of the optimal control forces and the system responses are given in Figs 11 and 12.

4.3. Discussion

For Case 1 where the maximum head acceleration is minimized, the following observations can be made.

- The minimization of the peak head acceleration with prescribed bounds on the energy imparted to the brain and on the power level developed in the skull leads to a head acceleration $\ddot{x}_2(t)$ that remains almost constant for the duration of the control force, as shown in Fig. 11(b). The optimal control force which generates constant head acceleration has impulses at the onset of the impact and remains essentially constant for the rest of the time, as shown in Fig. 11(a). In Section 3 in which

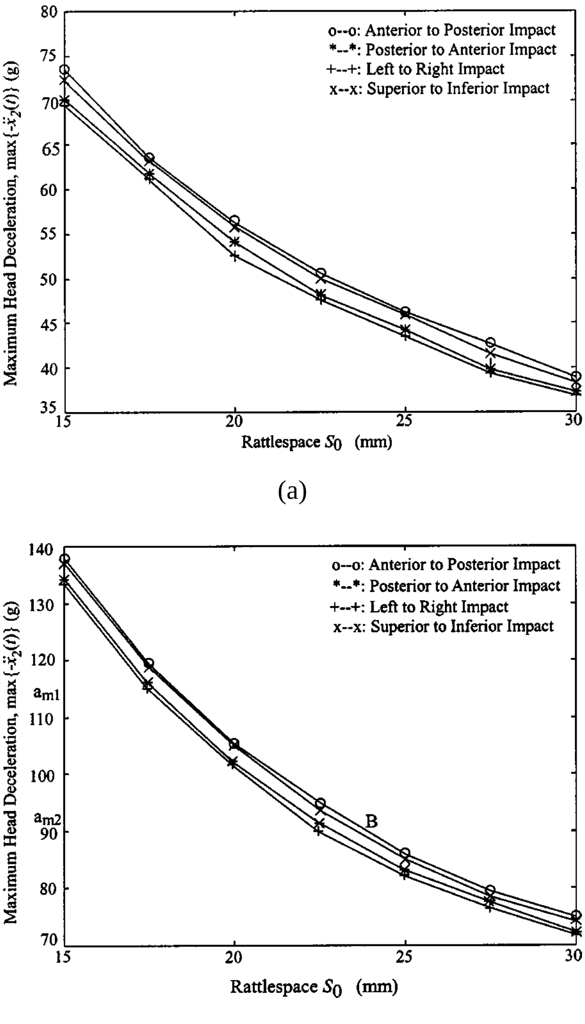


Fig. 9. Trade-off curves for Case 1 (minimize the peak head acceleration): (a) $E_0 = 53.4$ J, (b) $E_0 = 100$ J.

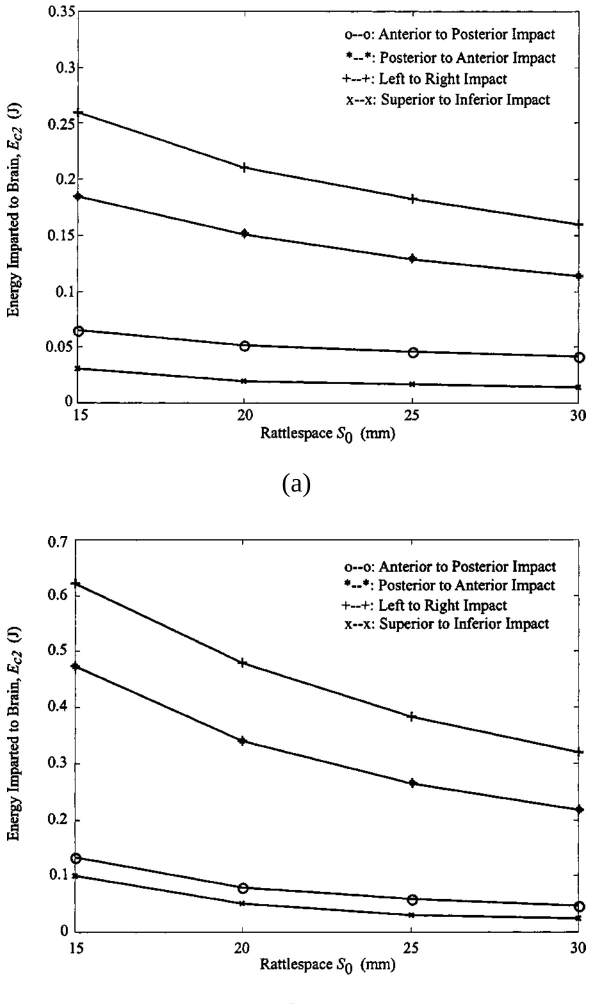


Fig. 10. Trade-off curves for Case 2 (minimize the energy imparted to the brain): (a) $E_0 = 53.4$ J, (b) $E_0 = 100$ J.

the limiting performance of the helmet using a SDOF head injury model was studied by minimizing the maximum head acceleration, a constant head acceleration during impact was also obtained, and the corresponding optimal control force is constant too. For the SDOF model, for a given initial impact velocity v_0 and a prescribed rattlespace S_0, the maximum head acceleration a_{m} is given by $a_{\mathrm{m}} = v_0^2/S_0$, that is, the maximum head acceleration is proportional to the square of the initial impact velocity or to the initial impact energy. Also, for Case 1, the peak head acceleration is given by $\max_t |\ddot{x}_2(t)| \approx v_0^2/S_0$. The peak head accelerations for the various impact directions are very close, as shown in Figs 9(a) and (b).

- A sudden jump (the initial impulse) of the control force at the onset of the impact would produce a very large rate of change of the head acceleration, the jerk, which could result in head injury too [16]. This is evident in Fig. 11(c), where the power developed in the skull has a sharp peak initially.
- If the impact is in the L–R direction and the rattlespace is small (say, $S_0 = 15$ mm), the constraint on the energy imparted to the brain ($E_{c_2} \leqslant 0.2334$ J for $E_0 = 53.4$ J and $E_{c_2} \leqslant 0.5251$ J for $E_0 = 100$ J) is an inconsistent constraint, that is, it cannot be satisfied in the optimization computation. When this happens, the constraint can be released.

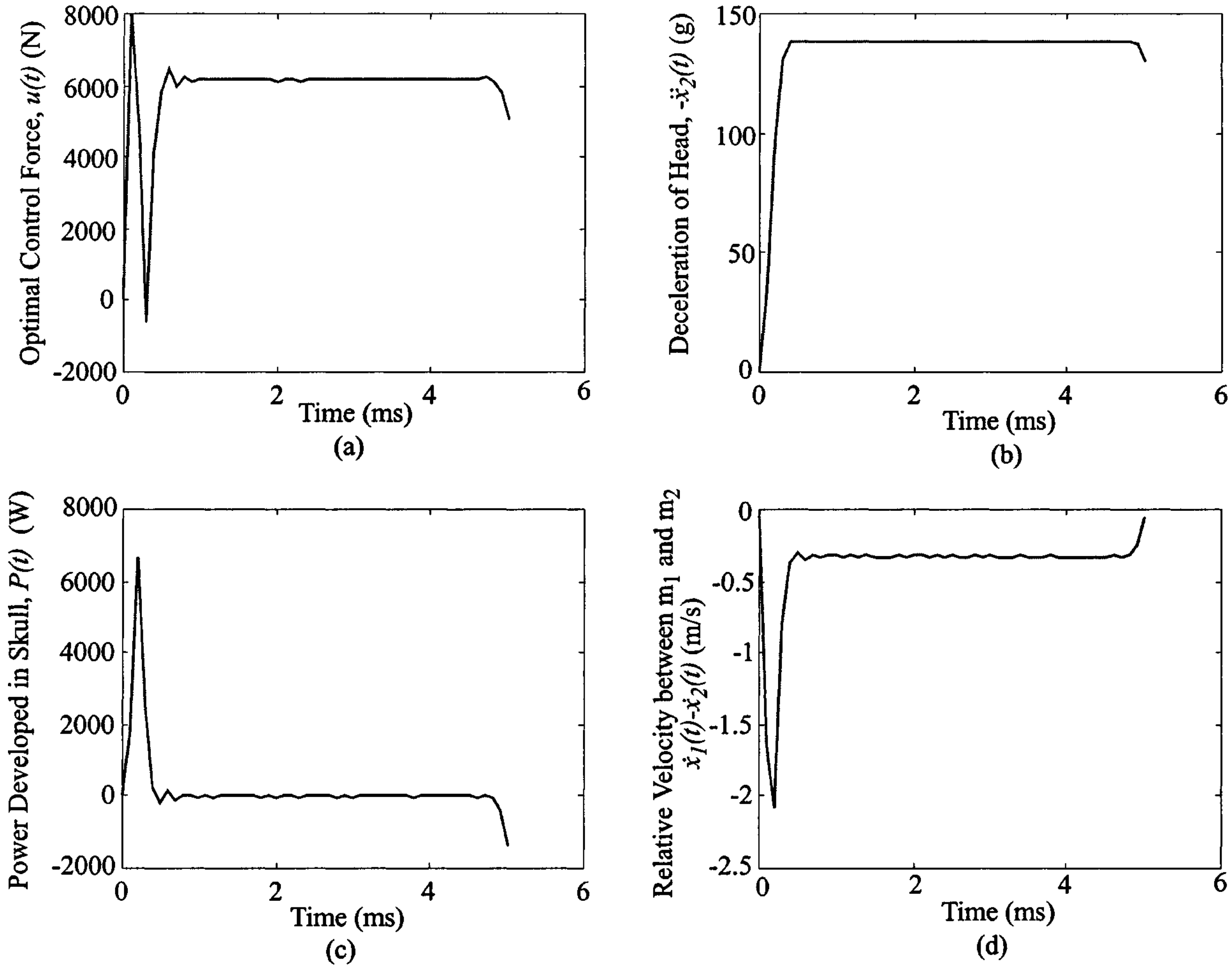


Fig. 11. Optimal control force and system responses. Case 1: Minimize the peak head acceleration, $S_0 = 15$ mm, $E_0 = 100$ J, A–P direction.

For Case 2 where the energy imparted to the brain is minimized, the following comments are in order.

- As illustrated by Fig. 12, the optimal control force and the corresponding head acceleration vary smoothly. As a result, the time history of the power developed in the skull, as shown in Fig. 12(c), varies smoothly too, with the maximum power level much lower than that for the constant head acceleration. For instance, for an impact in the A–P direction with $S_0 = 15$ mm and $E_0 = 100$ J, $\max_t |P(t)| = 1573$ W when the energy imparted to brain is minimized and $\max_t |P(t)| = 6640$ W when the peak head acceleration is minimized.
- The constraints on the peak head acceleration and the peak power developed in the skull are satisfied for the impact conditions investigated here.
- In the case of a small rattlespace or high initial impact energy, the level of minimum possible energy imparted to the brain is rather high for impacts in the directions of left to right and posterior to anterior of the head, as shown in Figs 10(a) and (b). For instance, if the impact is in the L–R direction and $S_0 = 15$ mm, $E_{c_2} = 0.62$ J for $E_0 = 100$ J and $E_{c_2} = 0.26$ J for $E_0 = 53.4$ J. Note that it is impossible for a helmet to produce a level of energy imparted to the brain less than that obtained in Case 2. Therefore, any constraints that require the energy imparted to the brain to be less than the minimum values of E_{c_2} obtained in Case 2 are inconsistent and cannot be satisfied in optimization computations.

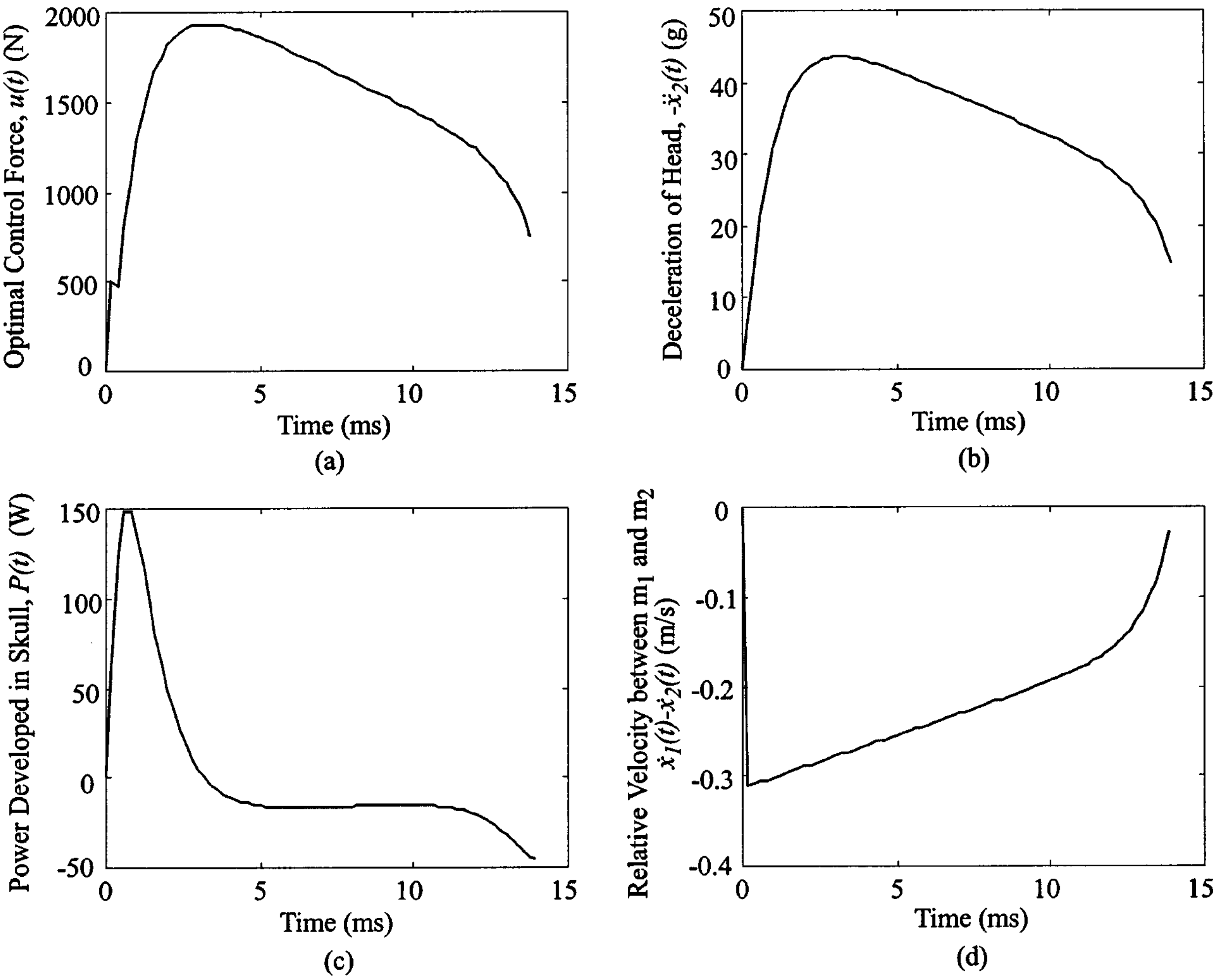


Fig. 12. Optimal control force and system responses. Case 2: Minimize the energy imparted to the brain, $S_0 = 30$ mm, $E_0 = 53.4$ J, L–R direction.

- There are significant differences among the energies imparted to the brain, especially when the rattlespace is small, for different impact directions. According to Eq. (40), the sequence of directions for which the EAIS score becomes more severe is A–P, S–I, P–A and L–R.

For each case (Case 1 or Case 2), the shape of the time histories of the optimal control forces, as illustrated by Fig. 11(a) for Case 1 and Fig. 12(a) for Case 2, is similar for the chosen rattlespace range and impact energy levels. This means that for a chosen performance index, the same optimal control can be employed for a reasonably wide range of rattlespaces and initial impact energies. However, the shapes of the optimal control forces for the minimization of the peak head acceleration and the minimization of the energy imparted to the brain are different, as indicated by Figs 11(a) and 12(a).

5. Helmeted head hit by an object using THIM

The impact to helmets used for baseball or softball is primarily from a moving ball, which is different from the impact to football and hockey helmets where the helmeted head hitting a hard surface is the primary impact mode. The difference between these two impact modes may lead to different requirements of the characteristics of these two sorts of helmets. An experimental investigation of softball/baseball helmets showed that very little benefit is gained from the use of a helmet, for the prevention of brain injuries and skull fracture under direct ball impact in which a ball hits the helmeted head in the direction normal to the helmet surface [28]. This suggests that there may be room for improvement in the performance of baseball and softball helmets.

The THIM is a one-dimensional model that accounts for the translational motion of the head. It focuses on the second resonant frequency of the head, about 700 Hz, which can be attained only for very short impacts lasting less than 3 to 4 ms [30]. Note that head impacts with small projectiles, such as softballs and baseballs, constitute one area of study in which the rotational input is very small as compared with the translational input [28], and that head impacts with balls are of short duration. Therefore, the THIM is chosen as the head injury model for the limiting performance analysis of softball/baseball helmets. Figure 4(a) illustrates the impact of the helmeted head hit by a moving ball. The limiting performance analysis of the system is based on the model shown in Fig. 4(b) where $f(t)$ represents the impact force acting on the helmet generated from a moving ball.

5.1. *Problem formulation*

5.1.1. *Governing equations of the system*

The governing equations of this system are

$$\begin{aligned} &m_1\ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k(x_1 - x_3) = u(t),\\ &m_2\ddot{x}_2 + c_1(\dot{x}_2 - \dot{x}_3) + c_2(\dot{x}_2 - \dot{x}_1) = 0,\\ &c_1(\dot{x}_3 - \dot{x}_2) + k(x_3 - x_1) = 0,\\ &m_4\ddot{x}_4 = f(t) - u(t), \end{aligned} \tag{44}$$

with all-zero initial conditions.

5.1.2. *Impact force description*

An experiment, in which a softball was propelled to impact on a Hybrid III dummy head, showed that the head acceleration was very close to a half-sine pulse [28]. Therefore, we assume that the impact force $f(t)$ applied to the helmet shell by a moving ball can be reasonably represented as a half-sine pulse

$$f(t) = F_0 \sin\frac{\pi}{T_0}t \quad \text{for } 0 \leqslant t \leqslant T_0, \tag{45}$$

where F_0 is the amplitude of the force, T_0 is the duration of the ball impact which depends on the hardness and deformation of the ball and the helmet shell. Suppose momentums are conserved during an impact, and the change of the momentum of a helmet shell is negligible as compared to that of a ball. Then, according to Newton's percussion law [5],

$$F_0 = \frac{\pi m_\mathrm{b}(1+\mu_\mathrm{b})v_\mathrm{b}}{2T_0}, \tag{46}$$

where m_b is the ball mass, μ_b is the ball restitution coefficient, and v_b is the ball impact speed.

5.1.3. *Performance criteria*

The TEC, which is defined by Eq. (28), is chosen as the head injury criteria for the limiting performance of softball/baseball helmets. As a performance criterion of the helmet we adopt the quantity

$$J_4(u) = \max_t \bigl\{ \bigl| x_4(t) - x_1(t) \bigr| \bigr\} \tag{47}$$

which defines the maximum displacement of the skull with respect to the helmet.

5.1.4. *Problem statement*

The limiting performance of the helmet for head injury prevention will be investigated in three cases where the system performance index is (a) the peak head acceleration, (b) the energy imparted to the brain, and (c) the peak power developed in the skull. Since the impact duration of a ball is usually very short, an abrupt force with large amplitude is generated and applied to the helmet shell, which could result in skull fracture. Therefore, the peak power developed in the skull will be taken as the objective function to be minimized in Case 3. These problems can be formulated as follows.

Case 1: Peak head acceleration is minimized. Find the optimal control force $u_0(t)$ such that

$$J_1(u_0) = \min_u J_1(u) \quad \text{(performance index)}, \tag{48}$$

subject to

$$J_i(u_0) \leqslant D_i, \quad i = 2, 3, 4 \quad \text{(constraints)}. \tag{49}$$

Case 2: Energy imparted to the brain is minimized. Find the optimal control force $u_0(t)$ such that

$$J_2(u_0) = \min_u J_2(u) \quad \text{(performance index)}, \tag{50}$$

subject to

$$J_i(u_0) \leqslant D_i, \quad i = 1, 3, 4 \quad \text{(constraints)}. \tag{51}$$

Table 3
Parameters of THIM

	m_1 (kg)	m_2 (kg)	c_1 (N · s/m)	c_2 (N · s/m)	k (N/m)
A–P	0.45	4.09	17000	157.6	13500000
L–R	0.25	4.29	6500	157.6	6500000

Table 4
Parameters of impact conditions

	Mass (g)	Restitution coef.	Impact speed (m/s)		Impact duration (ms)
Softball	185	0.5	36.4	27.3	1.0, 2.0
Baseball	150	0.7	39.6	29.7	1.0, 2.0

Case 3: Peak power stored in the skull is minimized. Find the optimal control force $u_0(t)$ such that

$$J_3(u_0) = \min_u J_3(u) \quad \text{(performance index)}, \tag{52}$$

subject to

$$J_i(u_0) \leqslant D_i, \quad i = 1, 2, 4 \quad \text{(constraints)}. \tag{53}$$

In Eqs (49), (51), and (53), D_i $(i = 1, 2, 3, 4)$ are the maximum allowable values of the corresponding system performance criteria.

5.2. Computational investigation

The method of converting limiting performance problems into mathematical programming problems, which is described in Section 4.2.1, is used to solve the problems of Case 1, Case 2, and Case 3.

It is indicated in a softball/baseball testing standard [17] and an experiment of softball/baseball head impact injury potentials [28] that the cases of the ball hitting the front and the side of the helmeted head are important. These cases will be investigated here. The system parameters for these two situations, as drawn from [21], are given in Table 3.

Several impact conditions are chosen for the computational investigation, as listed in Table 4. According to Eq. (46), the impact forces generated by a softball hitting at the speeds of 36.4 m/s and 27.3 m/s are the same as those generated by a baseball hitting at the speeds 39.6 of m/s and 29.7 m/s, respectively, provided that impact durations of both balls are identical. Therefore, the following computation treats the softball impact only, except when otherwise specified.

As discussed in Section 4.2.2, the selection of the maximum allowable values of system performance criteria, D_i $(i = 1, \ldots, 4)$, is based on the threshold values of corresponding injury criteria. They are assigned as follows.

$$\begin{aligned}
&D_1 = 200\ \text{g}, \\
&D_2 = E_\text{b} = 0.2334\ \text{J} \quad \text{and} \\
&\quad D_3 = P_\text{u} = 6210\ \text{W} \quad \text{for } T_0 = 2.0\ \text{ms}, \\
&D_2 = E_\text{b} = 0.5251\ \text{J} \quad \text{and} \\
&\quad D_3 = P_\text{u} = 6640\ \text{W} \quad \text{for } T_0 = 1.0\ \text{ms}, \\
&D_4 = S_0,
\end{aligned} \tag{54}$$

where S_0 is the rattlespace of helmets, which we will let vary from 6 to 22 mm.

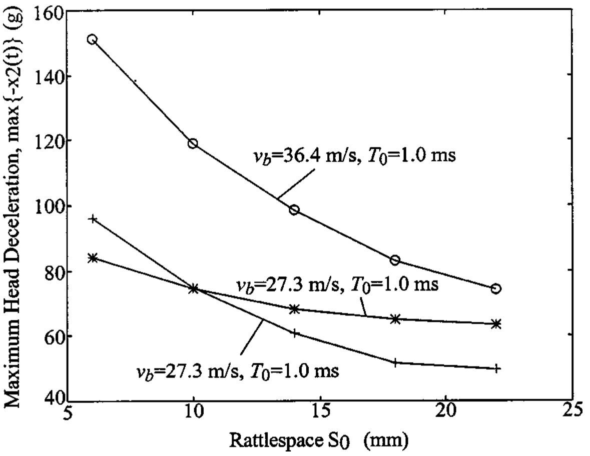
For Case 1, the trade-off curves between the peak head acceleration and the rattlespace are shown in Figs 13(a) and (b) for the front (A–P) and side (L–R) impacts, respectively. Typical time histories of the optimal control force and system responses are shown in Fig. 14. For Case 2, Figs 15(a) and (b) represent the relationship between the minimum energy imparted to the brain and the rattlespace, and Fig. 16 illustrates a sample of time histories of the optimal control force and system responses. For Case 3, the trade-off curves between the peak power stored in the skull and the rattlespace, and the trajectories of the control force and the system responses are shown in Figs 17(a) and (b), and Fig. 18, respectively.

5.3. Discussion

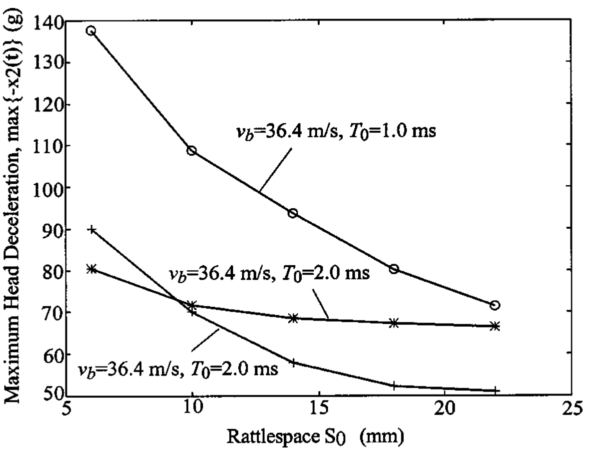
5.3.1. Comparison with experimental results

Some characteristics of the computational results can be compared with experimental data [28].

1. The performance of some current real world helmets has been tested [28]. For softball helmets subject to side impact, the computational results



(a)



(b)

Fig. 13. The trade-off curves for Case 1 (Minimization of peak head acceleration): (a) Front impact: A–P direction, (b) Side impact: L–R direction.

and experimental results are given in Table 5 for comparison. The limiting performance analysis indicates that there is considerable room for improvement of the performance of currently available softball/baseball helmets, relative to their theoretically best possible performances.

2. In the three cases investigated computationally, the injury criteria resulting from the side impact are significantly greater than those from the front impact under the same impact conditions, as indicated by a set of computational results given in Table 6. This means that, in terms of head injuries, the side impact is more dangerous than the front impact. Tests of baseball impacts to the front and side of the head without helmets also displayed the same trend [28]. The test data are also given in Table 6.
3. As expected, both computational and experimental results indicate that the reduction of the head injury criteria is significant with a decrease of the impact speed of the ball, as shown in Figs 13, 15, and 17 as well as Table 5.

5.3.2. Interpretation of the computational results

The computational solutions provide some information that is not available from the experiments.

1. According to Eq. (46), the amplitude of the impact force is inversely proportional to the impact duration. The influence of the impact duration on the injury criteria is significant. In each of the three cases, the longer the impact duration of the ball helmet contact, the lower the head injury criteria. The smaller the rattlespace, the greater the variation of the injury criterion being minimized with respect to impact duration. The impact duration of a moving ball can be correlated with the hardness and deformation of the ball and the helmet shell [17].
2. For Case 1 where the peak head acceleration is minimized, the optimal control force has large impulses initially and then remains constant, as shown in Fig. 14(a), and the response of the head acceleration basically remains constant, as shown in Fig. 14(b). When the energy imparted to the brain is minimized in Case 2, the optimal control force needs an initial impulse, and the rest part of the control force looks like a half-sine pulse, as shown in Fig. 16(a), which results in a gradual change of the head acceleration, as shown in Fig. 16(b). In terms of the system responses, there is a close resemblance between the problems of the helmeted head hitting a rigid surface and the helmeted head hit by a moving object for these two cases, except that the impact duration for the helmeted head hit by a moving ball is much shorter. For the minimization of the peak power that is Case 3, the optimal control force has a large initial pulse and then varies gradually, as illustrated by Fig. 18(a), and the time history of the power developed in the skull tends to be somewhat "bang-bang" in shape, as shown in Fig. 18(b).
3. The dependence of the head injury criteria on the helmet rattlespace is illustrated by trade-off curves. It seems that for a small rattlespace, say $S_0 \leqslant 10$ mm, the peak power stored in the skull

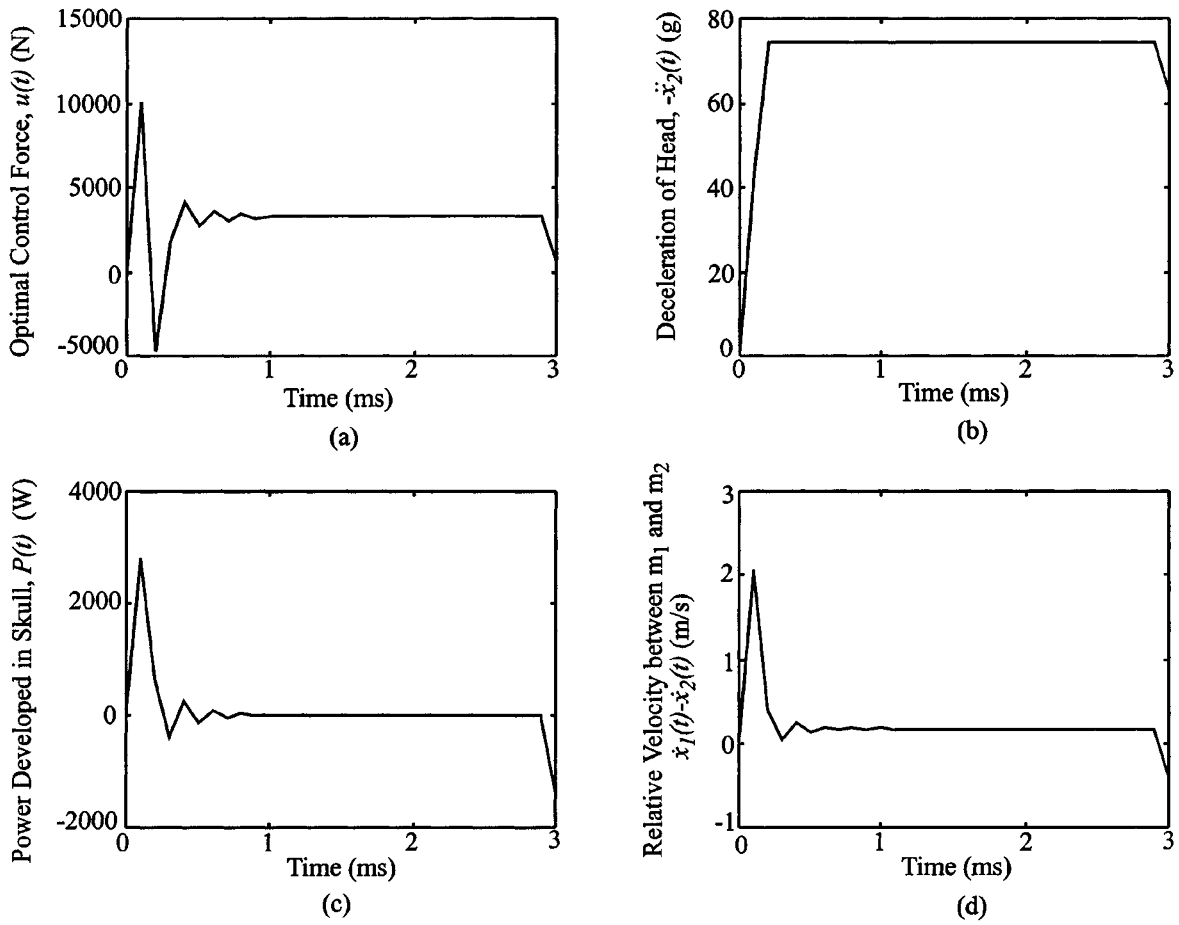


Fig. 14. Time histories of the optimal control force and the system responses. Case 1: Minimization of the peak head acceleration, A–P direction, $S_0 = 10$ mm, $T_0 = 200$ ms, $v_0 = 36.4$ m/s.

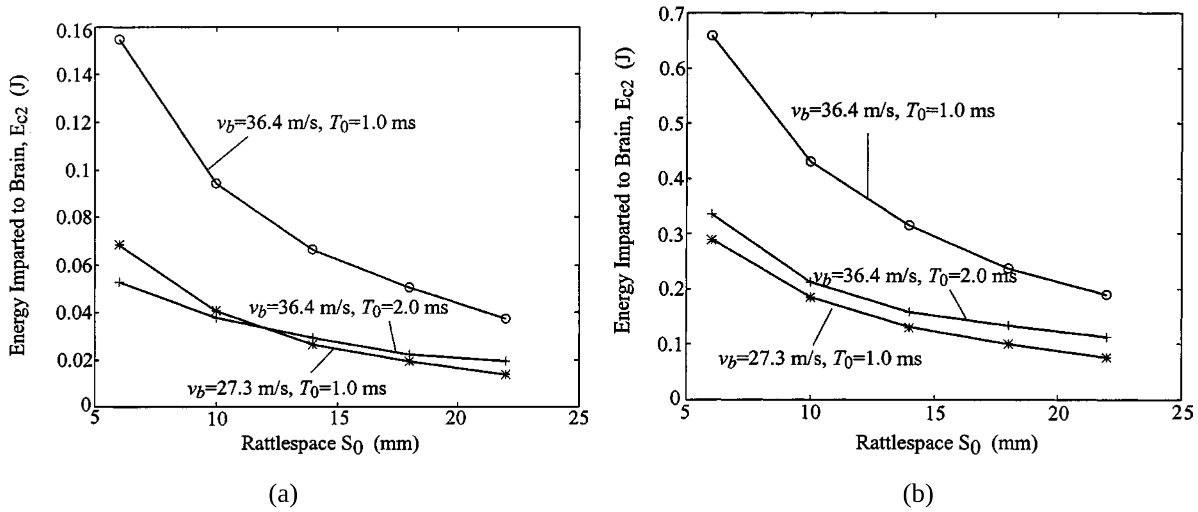


Fig. 15. The trade-off curves for Case 2 (minimization of the energy imparted to the brain): (a) Front impact: A–P direction, (b) Side impact: L–R direction.

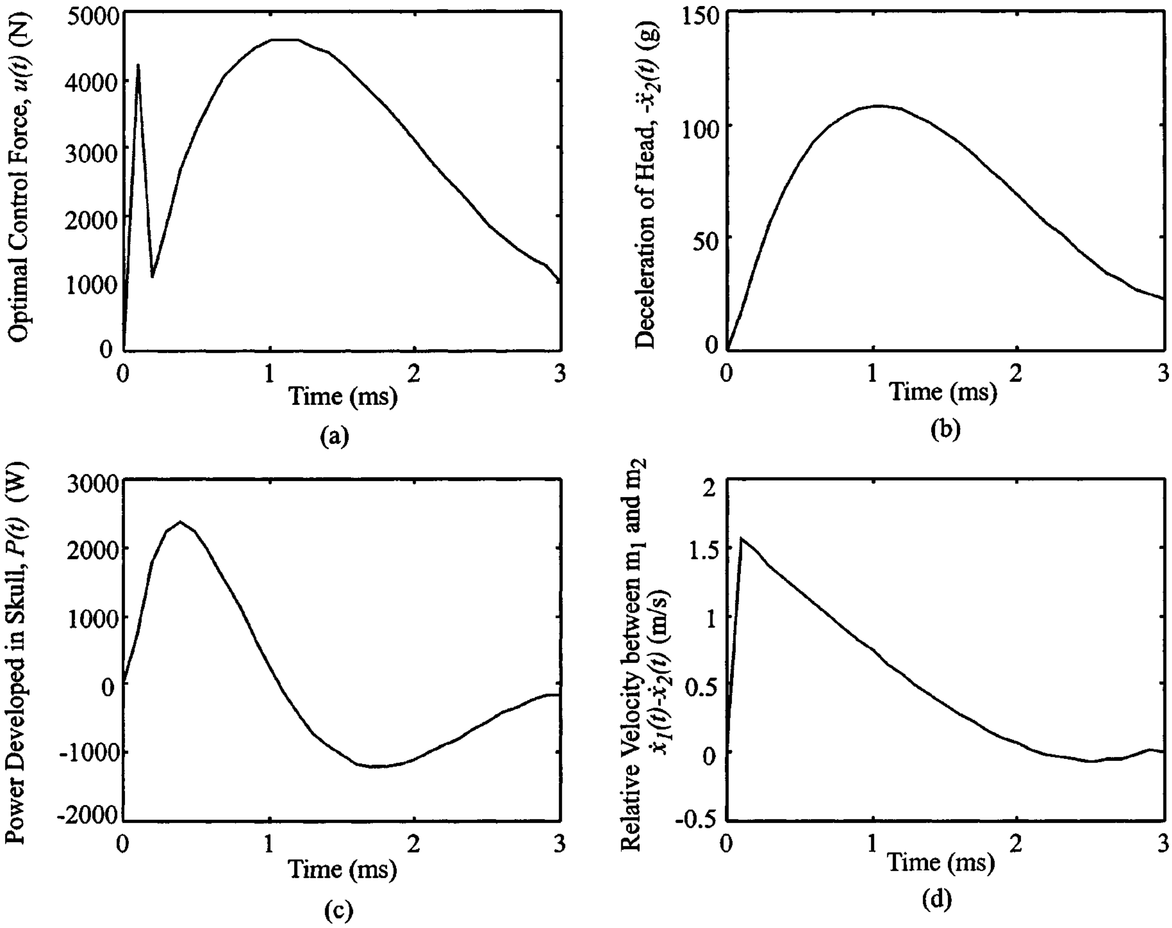


Fig. 16. Time histories of the optimal control force and the system responses. Case 2: Minimization of the energy imparted to the brain, L–R direction, $S_0 = 18$ mm, $T_0 = 1$ ms, $v_0 = 36.4$ m/s.

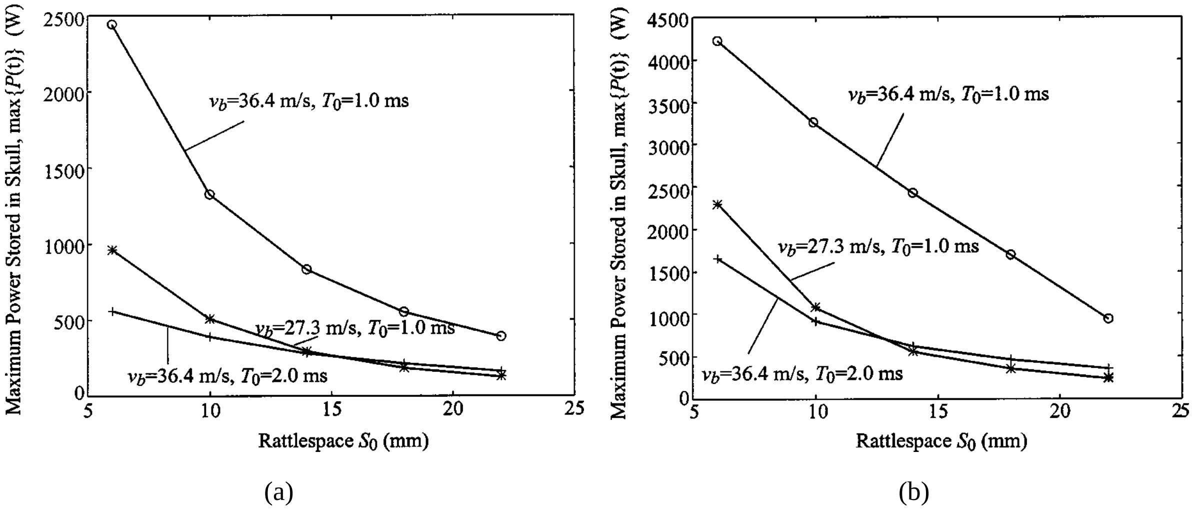


Fig. 17. The trade-off curves for Case 3 (minimization of the peak power developed in the skull): (a) Front impact: A–P direction, (b) Side impact: L–R direction.

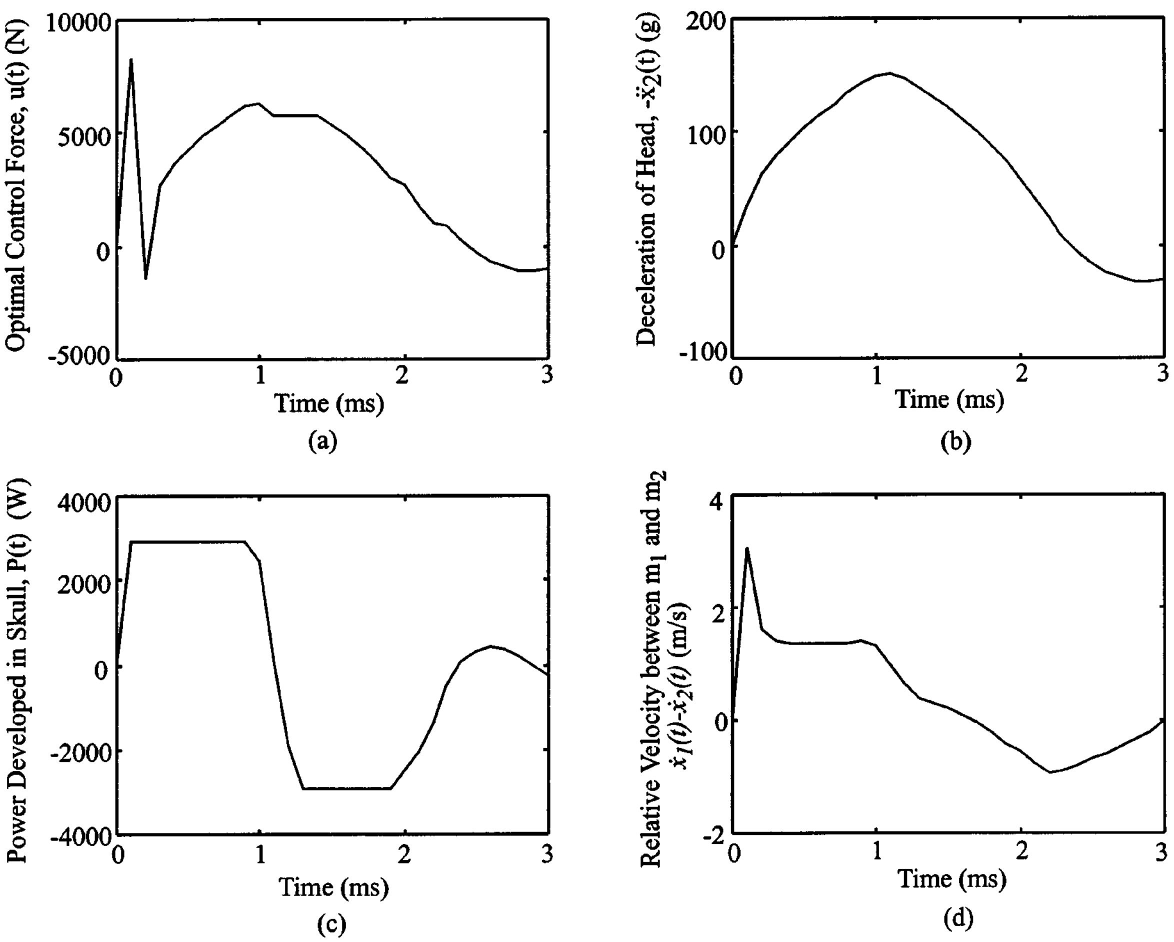


Fig. 18. Time histories of the optimal control force and the system responses. Case 3: Minimization of the peak power developed in the skull, L–R direction, $S_0 = 10$ mm, $T_0 = 1$ ms, $v_0 = 36.4$ m/s.

Table 5

Values of injury criteria for softball impact to the side of the head

	S_0 (mm)	v_b (m/s)	T_0 (ms)	E_{c_2}(J)	$\max_t \lvert P(t)\rvert$ (W)
Experiment	∼15	35.8	–	1.888	27745
Computation	14	36.4	1.0	0.393	5060
Experiment	∼15	26.9	–	0.999	13359
Computation	14	27.3	1.0	0.150	1779

Table 6

Values of injury criteria for baseball impact to the front and side of the head

	Helmet	S_0 (mm)	v_b (m/s)	T_0 (ms)	E_{c_2} (J)	$\max_t \lvert P(t)\rvert$ (W)
Experiment						
Front impact	w/o	–	37.4	–	0.608	19918
Side impact	w/o	–	37.6	–	1.611	26221
Computation						
Front impact	w/o	6	39.6	1.0	0.430	11431
Side impact	w/o	6	39.6	1.0	1.107	19434

should be the objective function to be minimized, since very high power is likely to be generated, which would result in skull fracture. For an intermediate rattlespace, say 10 mm $< S_0 \leqslant$ 18 mm, the energy imparted to the brain and the peak power stored in the skull should be minimized simultaneously, so as to achieve a balance of the helmet performance for head injury prevention. For a large rattlespace, say 18 mm $< S_0$, the energy imparted to the brain should be minimized, since the power stored in the skull is very low then.

6. Concluding remarks

This study has illustrated the applicability of the limiting performance concept to the design and evaluation of helmet performance under impact conditions. The limiting performance analysis provides a means for finding the theoretical optimal performance of the helmet for the prevention of head injuries. In addition to illustrating general trends in the relationship between the theoretical optimal performance and the helmet rattlespace, a trade-off curve can be used to quantify and evaluate the effectiveness of a helmet against head injury for the particular optimization criterion utilized. The optimal control force found from a limiting performance analysis illustrates the best possible characteristics of helmets, which can be used as a guideline for helmet design and optimization.

Two head injury models have been used in this paper. Although the SDOF or rigid head injury model is often used for the helmet tests and evaluation, there are some deficiencies. It can be questioned if a single-mass system can adequately model the rather complicated response of an actual head and helmet system. The THIM, a translational head injury model, has been proposed as a replacement for the rigid head injury model. While the THIM exhibits behavior similar to the SDOF model in terms of head acceleration, it takes into account the motion of the skull and the brain, which enables us to investigate brain and skull injuries.

The theoretical optimal performance of a helmet that is investigated in this paper is not necessarily readily realizable in practice. There are potentially significant limitations that should be recognized. For example, the head is free to rotate and translate during impact, and normally undergoes three-dimensional motion, but the THIM utilized here does not account for the possible occurrence of three-dimensional and rotational motion of a helmeted head. The neck is not represented in the study of this paper. However, the neck may play an important role in the performance of a helmeted head during an impact. Also, the THIM corresponds to a limited frequency band of the dynamics of the head. To design a helmet that is effective in head injury prevention, a head injury model with better biofidelity and other head injury criteria may be necessary. A finite element method to model the helmet may be helpful. The methods of limiting performance analysis are, of course, still applicable when more detailed modeling techniques are adopted.

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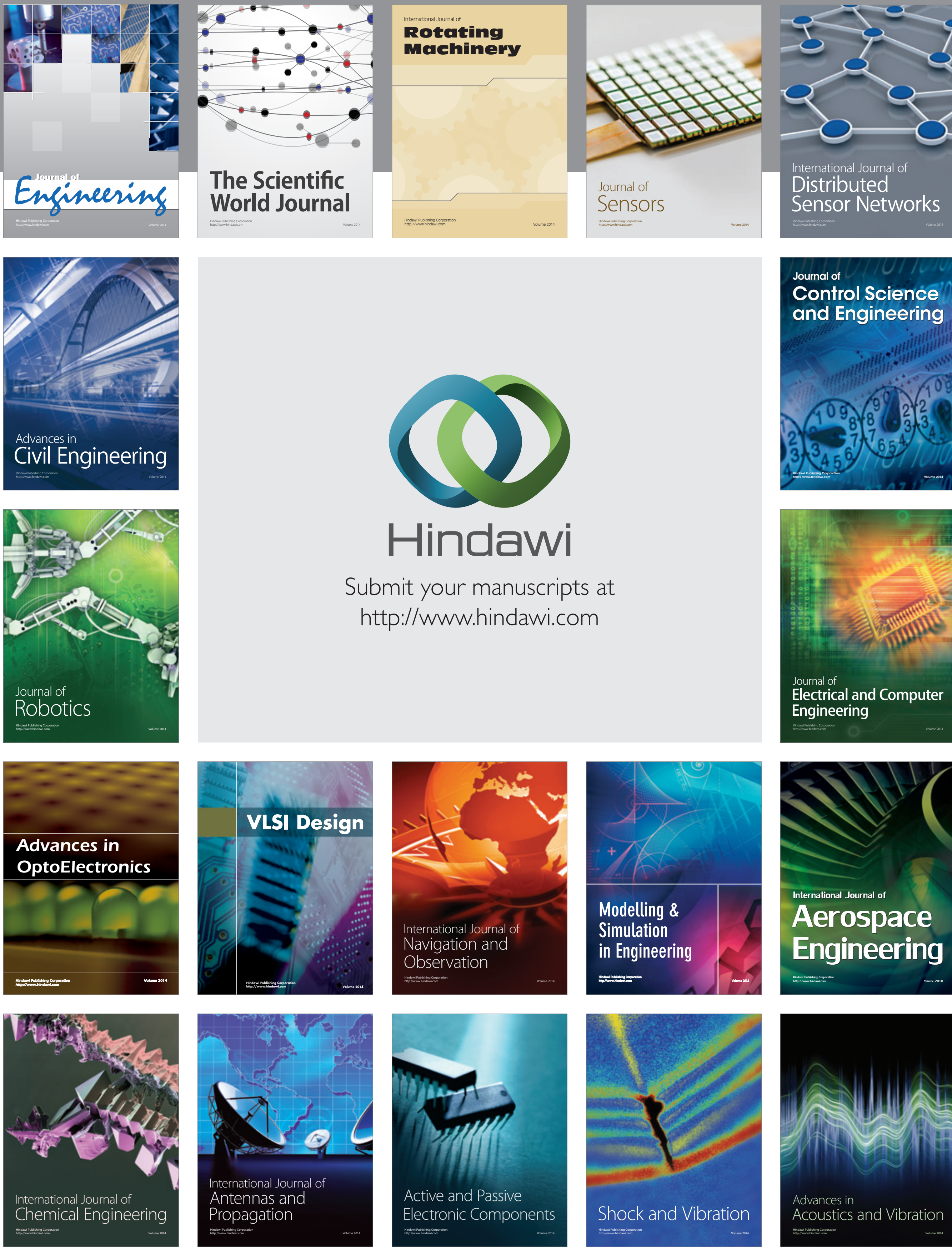
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