T. V. R. Chowdary
P. K. Sinha
S. Parthan
Department of Aerospace Engineering
Indian Institute of Technology
Kharagpur 721 302, India
E-mail: pksinha@aero.iitkgp.ernet.in

Environmental Effects on Flutter Characteristics of Laminated Composite Rectangular and Skew Panels

A finite element method is presented for predicting the flutter response of laminated composite panels subjected to moisture concentration and temperature. The analysis accounts for material properties at elevated temperature and moisture concentration. The analysis is based on the first-order approximation to the linear piston theory and laminated plate theory that includes shear deformation. Both rectangular and skew panels are considered. Stability boundaries at moisture concentrations and temperatures for various lamination schemes and boundary conditions are discussed. © *1996 John Wiley & Sons, Inc.*

INTRODUCTION

The external skin panels of modern high speed flight vehicles traveling through the atmosphere at supersonic speed are susceptible to flutter (Bisplinghoff and Ashely, 1962). The panel flutter leads to fatigue failure and has become a significant structural design problem. Panel flutter usually occurs at Mach numbers greater than one; therefore, the quasisteady theory is adequate for analyzing such problems. When panels are subjected to aerodynamic heating, thermal stresses are developed due to the restraint of thermal expansions in view of laminations and boundary constraints. Thermal stresses strongly influence the flutter behavior, because the panel is subjected to in-plane compressive stresses and may buckle in the thermal environment.

Considerable research on the flutter of isotropic panels was done by Dowell (1970) and Dugundji (1966). Olson (1967) first applied the finite element method (FEM) to study panel flutter. Recently, Bismarck–Nasr (1992) reviewed the existing literaure on the finite element flutter analysis of plates and shells. Thermal effects on flutter of isotropic panels were first studied by Schaeffer et al. (1965) who considered parabolic distribution. Yang (1976) studied the effects of temperature rise on stability boundaries using FEM. The nonlinear flutter of isotropic panels was studied by Xue and Mei (1993) who treated temperature changes as an equivalent system of mechanical loads.

The flutter response of composite panels was investigated in the past by Rosettos and Tong (1974), Gray and Mei (1993), and Liaw and Yang (1993) with the objective of assessing the influence of fiber orientation and lamination schemes on the critical flutter parameter of thin laminates. It was observed that fiber orientations and anisotropic ratios considerably influence the flutter instability of such panels. Composite materials are

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exposed to moisture and temperature during their service life. Moisture diffuses through polymer composite panels and thereby increases their moisture content levels. Moisture and temperature affect not only the matrix-dominated stiffness properties of the composite, but also have significant effects on the structural behavior of laminated plates (Pipes et al., 1976; Whitney and Ashton, 1971; Sai Ram and Sinha, 1992). The emphasis in this study is on the investigation of the effect of moisture concentration and temperature on flutter characteristics of laminated thin rectangular and skew panels. The material and geometrical linearity as well as the first-order approximation to the linear piston theory (Ashley and Zartarian, 1956) formed the basis of the present investigation. The finite element flutter analysis procedure (Chowdary et al., 1994) was accordingly modified. The reduced lamina material properties at elevated moisture concentration and temperature were used in the computation of numerical results.

GOVERNING EQUATIONS

Consider a laminated plate of length a, width b, and uniform thickness t, consisting of a number of thin laminae, each of which may be oriented at an angle θ with reference to the x axis and subjected to a supersonic air flow on the upper surface as shown in Fig. 1.

The constitutive equations for the plate when it is subjected to moisture and temperature is given by

$$\{F\} = [D]\{\varepsilon\} - \{F^N\}, \tag{1}$$

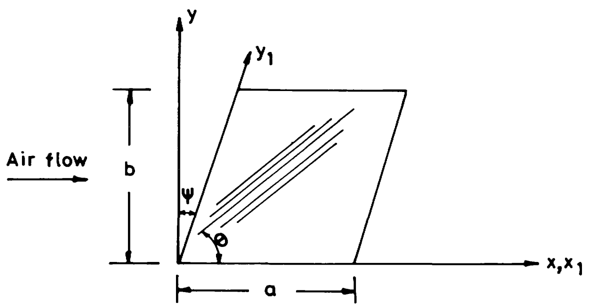

FIGURE 1 A skew composite panel.

where

$$\{F\} = \{N_x, N_y, N_{xy}, M_x, M_y, M_{xy}, Q_x, Q_y\}^T$$

are the internal force and moment resultants.

$$\{F^N\} = \{N_x^N, N_y^N, N_{xy}^N, M_x^N, M_y^N, M_{xy}^N, 0, 0\}^T$$

are the hygrothermal force and moment resultants.

$$\{\varepsilon\} = \{\bar{\varepsilon}_x, \bar{\varepsilon}_y, \bar{\gamma}_{xy}, K_x, K_y, K_{xy}, \phi_x, \phi_y\}^T$$

is the strain vector and

$$[D] = \begin{bmatrix} A_{11} & A_{12} & A_{16} & B_{11} & B_{12} & B_{16} & 0 & 0 \\ A_{12} & A_{22} & A_{26} & B_{12} & B_{22} & B_{26} & 0 & 0 \\ A_{16} & A_{26} & A_{66} & B_{16} & B_{26} & B_{66} & 0 & 0 \\ B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} & 0 & 0 \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} & 0 & 0 \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{44} & A_{45} \\ 0 & 0 & 0 & 0 & 0 & 0 & A_{45} & A_{55} \end{bmatrix}$$

is the elasticity matrix for the laminate.

Nonmechanical force and moment resultants are expressed as

$$\{N_x^N, N_y^N, N_{xy}^N\}^T = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\bar{Q}_{ij}]_k \{e\}_k \, dz$$

$$(i, j = 1, 2, 6). \tag{2}$$

$$\{M_x^N, M_y^N, M_{xy}^N\}^T = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\bar{Q}_{ij}]_k \{e\}_k \, z \, dz.$$

where $\{e\}_k$ are the hygrothermal strains of the kth lamina and are given by

$$\{e\}_k = \{e_x, e_y, e_{xy}\}^T$$
$$= [T]\{\beta_1, \beta_2\}_k^T(\Delta C) + [T]\{\alpha_1, \alpha_2\}_k^T(\Delta T),$$

in which ΔC and ΔT are the changes in the moisture concentration and temperature, respectively. The α and β are thermal and moisture expansion coefficients of the lamina. Note that subscripts 1 and 2 refer to the values along the fiber axis and transverse to the fiber axis, respectively.

$$[T] = \begin{bmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \\ \sin 2\theta & -\sin 2\theta \end{bmatrix}.$$

The stiffness coefficients of $[D]$ are defined as

$$(A_{ij}, B_{ij}, D_{ij}) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}]_k (1, z, z^2)\, dz \qquad (i, j = 1, 2, 6),$$

$$(A_{ij}) = \alpha \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} [\overline{Q}_{ij}]_k\, dz \qquad (i, j = 4, 5). \tag{3}$$

$[Q_{ij}]_k$ in Eqs. (2) and (3) is defined as

$$\begin{aligned} [\overline{Q}_{ij}]_k &= [T_1]^{-1}[Q_{ij}]_k[T_1]^{-T} \quad (i,\ j = 1, 2, 6), \\ [\overline{Q}_{ij}]_k &= [T_2]^{-1}[Q_{ij}]_k[T_2] \quad (i,\ j = 4, 5), \end{aligned} \tag{4}$$

where

$$[T_1] = \begin{bmatrix} \cos^2\theta & \sin^2\theta & 2\sin\theta\cos\theta \\ \sin^2\theta & \cos^2\theta & -2\sin\theta\cos\theta \\ -\sin\theta\cos\theta & \sin\theta\cos\theta & \cos^2\theta - \sin^2\theta \end{bmatrix},$$

$$[T_2] = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix},$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \quad (i,\ j = 1, 2, 6),$$

$$[Q_{ij}]_k = \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \quad (i,\ j = 4, 5),$$

in which

$$\begin{aligned} Q_{11} &= E_1/(1 - \nu_{12}\nu_{21}), \\ Q_{12} &= \nu_{12}E_2/(1 - \nu_{12}\nu_{21}), \\ Q_{22} &= E_2/(1 - \nu_{12}\nu_{21}), \\ Q_{44} &= G_{13}, \\ Q_{55} &= G_{23}. \end{aligned}$$

The linear strains are defined as

$$\begin{aligned} &\bar{\varepsilon}_x = \bar{u},_x, \quad \bar{\varepsilon}_y = \bar{v},_y, \quad \bar{\gamma}_{xy} = \bar{u},_y + \bar{v},_x, \\ &\overline{K}_x = \theta_{y,x}, \quad \overline{K}_y = -\theta_{x,y}, \quad \overline{K}_{xy} = \theta_{y,y} - \theta_{x,x}, \\ &\phi_x = \theta_y + w,_x, \quad \phi_y = -\theta_x + w,_y, \end{aligned} \tag{5}$$

where $\bar{u}$ and $\bar{v}$ are displacements of the midsurface; $\overline{K}_x$, $\overline{K}_y$, and $\overline{K}_{xy}$ are curvatures; and ϕ_x and ϕ_y are shear rotations of the panel.

Assuming that w does not vary with z, the nonlinear strains of the plate can be expressed as

$$\begin{aligned} \varepsilon_{xnl} &= (u_{,x}^2 + v_{,x}^2 + w_{,x}^2)/2, \\ \varepsilon_{ynl} &= (u_{,y}^2 + v_{,y}^2 + w_{,y}^2)/2, \\ \gamma_{xynl} &= (u,_x u,_y + v,_x v,_y + w,_x w,_y), \\ \gamma_{xznl} &= (u,_x u,_z + v,_x v,_z), \\ \gamma_{yznl} &= (u,_y u,_z + v,_y v,_z). \end{aligned} \tag{6}$$

Because $u = \bar{u} + z\theta_y$ and $v = \bar{v} - z\theta_x$, Eq. (6) may be written as

$$\begin{aligned} \varepsilon_{xnl} &= [\bar{u}_{,x}^{+2} + \bar{v}_{,x}^{+2} + w_{,x}^2 + 2z(\bar{u},_x\theta_{y,x} - \bar{v},_x\theta_{x,x}) \\ &\quad + z^2(\theta_{y,x}^2 + \theta_{x,x}^2)]/2, \\ \varepsilon_{ynl} &= [\bar{u}_{,y}^{-2} + \bar{v}_{,y}^{-2} + w_{,y}^2 + 2z(\bar{u},_y\theta_{y,y} - \bar{v},_y\theta_{x,y}) \\ &\quad + z^2(\theta_{x,y}^2 + \theta_{y,y}^2)]/2, \\ \gamma_{xnyl} &= [\bar{u},_x\bar{u},_y + \bar{v},_x\bar{v},_y + w,_x w,_y \\ &\quad + z(\bar{u},_y\theta_{y,x} + \bar{u},_x\theta_{y,y}) - z(\bar{v},_y\theta_{x,x} + \bar{v},_x\theta_{x,y}) \\ &\quad + z^2(\theta_{y,x}\theta_{y,y} + \theta_{x,x}\theta_{x,y})], \\ \gamma_{xznl} &= [\bar{u},_x\theta_y - \bar{v},_x\theta_x + z(\theta_y\theta_{y,x} + \theta_x\theta_{x,x})], \\ \gamma_{yznl} &= [\bar{u},_y\theta_y - \bar{v},_y\theta_x + z(\theta_u\theta_{y,y} + \theta_x\theta_{x,y})]. \end{aligned} \tag{7}$$

The application of piston theory greatly simplified the study of high-speed unsteady aeroelastic problems. The theory is strictly valid only for Mach numbers greater than 2.5. The aerodynamic pressure intensity at any point is represented as

$$p(x, y, t) = \frac{-2Q}{\sqrt{(M^2 - 1)}} \left[\frac{dw}{dx} + \frac{1}{v_a} \left(\frac{M^2 - 2}{M^2 - 1} \right) \frac{dw}{dt} \right], \tag{8}$$

where w, M, and v_a are the displacement, Mach number, and velocity of air, respectively, and $Q = \frac{1}{2}\rho_a v_a^2$ is the dynamic pressure of free stream air. In the above equation, the first term contributes the steady flow due to local flow inclination; the second unsteady term refers to aerodynamic damping.

FINITE ELEMENT FORMULATION

An eight-noded isoparametric element with 5 degrees of freedom at each node, namely u^0, v^0, w, θ_x, and θ_y, was used in the present analysis. The displacements are expressed in terms of their nodal values using element shape functions as follows:

$$\bar{u} = \sum_{i=1}^{8} N_i \bar{u}_i, \quad \bar{v} = \sum_{i=1}^{8} N_i \bar{v}_i, \quad w = \sum_{i=1}^{8} N_i w_i, \quad \theta_x = \sum_{i=1}^{8} N_i \theta_{xi}, \quad \theta_y = \sum_{i=1}^{8} N_i \theta_{yi}. \tag{9}$$

ELEMENT STIFFNESS MATRIX

The linear stain matrix $\{\varepsilon\}$ is obtained by substituting Eq. (9) into (5), and is expressed as

$$\{\varepsilon\} = [B]\{d_e\}, \tag{10}$$

where

$$\{d_e\} = \{\bar{u}_1, \bar{v}_1, \bar{w}_1, \theta_{x1}, \theta_{y1}, \ldots, \bar{u}_8, \bar{v}_8, w_8, \theta_{x8}, \theta_{y8}\}^T,$$

$$[B] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ 0 & N_{,y} & 0 & 0 & 0 \\ N_{i,y} & N_{,x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & -N_{i,y} & 0 \\ 0 & 0 & 0 & -N_{i,x} & N_{i,y} \\ 0 & 0 & N_{i,x} & 0 & N_i \\ 0 & 0 & N_{i,y} & -N_i & 0 \end{bmatrix}$$

is the strain-displacement matrix.

The element stiffness matrix is given by

$$[K_e] = \iint [B]^T[D][B]\, dx\, dy. \tag{11}$$

ELEMENT INITIAL STRESS STIFFNESS MATRIX

The nonlinear strains defined in Eq. (7) are represented in a matrix form

$$\{\varepsilon_{nl}\} = \{\varepsilon_{xnl}, \ \varepsilon_{ynl}, \ \gamma_{xynl}, \ \gamma_{xznl}, \ \gamma_{yznl}\}^T = [R]\{d\}/2, \tag{12}$$

where

$$\{d\} = \{\bar{u}_{,x}, \bar{u}_{,y}, \bar{v}_{,x}, \bar{v}_{,y}, w_{,x}, w_{,y}, \theta_{x,x}, \theta_{x,y}, \theta_{y,x}, \theta_{y,y}, \theta_x, \theta_y\}^T,$$

and $[R]$ is obvious from Eqs. (7) and (12).

Using Eq. (9), $\{d\}$ may be expressed as

$$\{d\} = [G]\{d_e\}, \tag{13}$$

where

$$[G] = \sum_{i=1}^{8} \begin{bmatrix} N_{i,x} & 0 & 0 & 0 & 0 \\ N_{i,y} & 0 & 0 & 0 & 0 \\ 0 & N_{i,x} & 0 & 0 & 0 \\ 0 & N_{i,y} & 0 & 0 & 0 \\ 0 & 0 & N_{i,x} & 0 & 0 \\ 0 & 0 & N_{i,y} & 0 & 0 \\ 0 & 0 & 0 & N_{i,x} & 0 \\ 0 & 0 & 0 & N_{i,y} & 0 \\ 0 & 0 & 0 & 0 & N_{i,x} \\ 0 & 0 & 0 & 0 & N_{i,y} \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}.$$

The initial stress stiffness matrix is given by

$$[K_{\sigma e}] = \iint [G]^T[S][G]\, dx\, dy, \tag{14}$$

where

$$[S] = \begin{bmatrix} S_{11} & & & & & & & & & & & \\ S_{21} & S_{22} & & & & & & & & & & \\ 0 & 0 & S_{33} & & & & & & & & & \\ 0 & 0 & S_{43} & S_{44} & & & & & & & & \\ 0 & 0 & 0 & 0 & S_{55} & & & & & & & \\ 0 & 0 & 0 & 0 & S_{65} & S_{66} & & & & & & \\ 0 & 0 & S_{73} & S_{74} & 0 & 0 & S_{77} & & \text{Symmetric} & & & \\ 0 & 0 & S_{83} & S_{84} & 0 & 0 & S_{87} & S_{88} & & & & \\ S_{91} & S_{92} & 0 & 0 & 0 & 0 & 0 & 0 & S_{99} & & & \\ S_{101} & S_{102} & 0 & 0 & 0 & 0 & 0 & 0 & S_{109} & S_{1010} & & \\ 0 & 0 & S_{113} & S_{114} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \\ S_{121} & S_{122} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

in which

$$S_{11} = S_{33} = S_{55} = N_x^i, \quad S_{22} = S_{44} = S_{66} = N_y^i,$$
$$S_{21} = S_{43} = S_{65} = N_{xy}^i, \quad S_{77} = S_{99} = N_x^i t^2/12,$$
$$S_{88} = S_{1010} = N_y^i t^2/12, \quad S_{87} = S_{109} = N_{xy}^i t^2/12,$$
$$-S_{73} = S_{91} = M_x^i, \quad -S_{84} = S_{102} = M_y^i,$$
$$-S_{74} = -S_{83} = S_{92} = S_{101} = M_{xy}^i,$$
$$-S_{113} = S_{121} = Q_x^i, \quad -S_{114} = S_{122} = Q_y^i.$$

ELEMENT MASS MATRIX

The element mass matrix is obtained from the integral

$$[M_e] = \iint [N]^T[P][N]\, dx\, dy, \qquad (15)$$

where

$$[N] = \sum_{i=1}^{8} \begin{bmatrix} N_i & 0 & 0 & 0 & 0 \\ 0 & N_i & 0 & 0 & 0 \\ 0 & 0 & N_i & 0 & 0 \\ 0 & 0 & 0 & N_i & 0 \\ 0 & 0 & 0 & 0 & N_i \end{bmatrix},$$

$$[P] = \begin{bmatrix} P & 0 & 0 & 0 & 0 \\ 0 & P & 0 & 0 & 0 \\ 0 & 0 & P & 0 & 0 \\ 0 & 0 & 0 & I & 0 \\ 0 & 0 & 0 & 0 & I \end{bmatrix},$$

in which

$$P = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \rho_m\, dz \quad \text{and} \quad I = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} z^2 \rho_m\, dz.$$

ELEMENT LOAD VECTOR

The element load vector due to external transverse static load f per unit area is given by

$$\{P_e\} = \iint N_i\{f\}\, dx\, dy. \qquad (16)$$

The element load vector due to hygrothermal forces and moments is given by

$$\{P_e^N\} = \iint [B]^T\{F^N\}\, dx\, dy. \qquad (17)$$

ELEMENT AERODYNAMIC MATRICES

The work done by the aerodynamic forces is given by

$$W = \int p(x, y, t)w \, dx \, dy, \tag{18}$$

where $p(x, y, t)$ is the aerodynamic pressure given in Eq. (8) and can be expressed as

$$p(x, y, t) = -\left(\lambda \frac{dw}{dx} + g \frac{dw}{dt}\right), \tag{19}$$

where

$$\lambda = \frac{2Q}{(M^2 - 1)^{1/2}}$$

is the dynamic pressure parameter, and

$$g = \frac{\lambda(M^2 - 2)}{v_a(M^2 - 1)^{3/2}}$$

is the aerodynamic damping parameter.

The work done by aerodynamic forces is given by

$$W = \left[\lambda \int_A \frac{dw}{dx} w \, dx \, dy + g \int_A \frac{dw}{dt} w \, dx \, dy\right]. \tag{20}$$

The deflection w is expressed in terms of nodal displacements as

$$w = [N]\{d_e\}. \tag{21}$$

Substituting Eq. (21) in Eq. (20) yields

$$W = [\lambda\{d_e\}^T\{A_1\}_e\{d_e\} + g\{A^2\}_e\{\dot{d}_e\}], \tag{22}$$

where

$$\{A_1\}_e = \iint [N]^T[N,_x] \, dx \, dy \tag{23}$$

is the aerodynamic stiffness matrix, and

$$\{A_2\}_e = \iint [N]^T[N] \, dx \, dy \tag{24}$$

is the aerodynamic damping matrix.

The governing dynamic equation for each element is then given by

$$[K]_e\{d_e\} + [K_\sigma]_e\{d_e\} + [M]_e\{\ddot{d}_e\} + g[A_2]_e\{\dot{d}_e\} + \lambda[A_1]_e\{d_e\} = 0 \tag{25}$$

where $\{d_e\}$ is a function of time variable.

Assume $\{d_e = \{\bar{d}_e\}e^{\omega t}$ and Eq. (25) reduces to

$$[[K]_e + [K_\sigma]_e + \omega^2[M]_e + g\omega[A_2]_e + \lambda[A_1]_e]\{\bar{d}_e\} = 0. \tag{26}$$

IMPOSITION OF SKEW BOUNDARY CONDITIONS

A skew panel can be conveniently modeled using isoparametric elements. However, care should be taken in fitting boundary conditions at the skew edges where the boundary conditions are described in skew coordinates, but the stiffness matrix is obtained in the x, y coordinate system. So transformation of the axis system is required to fit the boundary conditions along the skew edges. The required transformation matrix is given by

$$[T_s] = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \cos\psi & \sin\psi \\ 0 & 0 & 0 & -\sin\psi & \cos\psi \end{bmatrix}. \tag{27}$$

So the transformed properties of the element stiffness matrix in skew coordinates (x_1, y_1) are given by

$$[K_s]_e = [T_s]_e^T[K]_e[T_s]_e. \tag{28}$$

The transformation matrices $[T_s]_e$ for the left and right edge (Fig. 1) are given, respectively, as

$$[T_s]_e = \begin{bmatrix} [T_s] & & & & & & & 0 \\ & [I] & & & & & & \\ & & [I] & & & & & \\ & & & [I] & & & & \\ & & & & [I] & & & \\ & & & & & [I] & & \\ & & & & & & [T_s] & \\ 0 & & & & & & & [T_s] \end{bmatrix}$$

and

$$[T_s]_e = \begin{bmatrix} [I] & & & & & & & 0 \\ & [I] & & & & & & \\ & & [T_s] & & & & & \\ & & & [T_s] & & & & \\ & & & & [T_s] & & & \\ & & & & & [I] & & \\ 0 & & & & & & [I] & \\ & & & & & & & [I] \end{bmatrix}.$$

For the entire structure, using the standard assembly technique of FEM and applying the appropriate skew boundary conditions, one obtains

$$[[K] + [K_\sigma] + \omega^2[M] + g\omega[A_2] + \lambda[A_1]]\{\bar{d}\} = 0. \tag{29}$$

SOLUTION PROCESS

The stiffness matrix, the initial stress stiffness matrix, and mass matrix, the load vector, and the aerodynamic matrices are evaluated by expressing integrals in local natural coordinates ξ and η of the element and performing numerical integration using Gaussian quadrature. Then the element matrices are assembled to obtain respective global matrices.

The first part of the solution is to obtain the initial stress resultants induced by the external transverse static load and by moisture and temperature in static conditions. The initial displacements are found from the equilibrium condition given by

$$[K]\{d^i\} = \{P\} + \{P^N\}.$$

Then the initial stress resultants N_x^i, N_y^i, N_{xy}^i, M_x^i, M_y^i, and Q_x^i and Q_y^i are obtained from Eqs. (1)–(7) and Eqs. (9) and (10).

The second part of the solution is the determination of flutter boundaries. The problem defined in Eq. (29) is an eigenvalue problem and its solution gives eigenvalues for a specified value of dynamic pressure. For $\lambda = 0$, the problem degenerates into that of finding the free vibration frequencies of the panel. For $\lambda > 0$, the nonsymmetric aerodynamic matrix comes into picture and some of the eigenvalues eventually become complex for a certain range of dynamic pressure parameters λ. The lowest value of λ, for which a pair of complex conjugate eigenvalues appears, is denoted as critical dynamic pressure λ_{cr}. In the absence of aerodynamic damping, the flutter boundary simply corresponds to the lowest value of the dynamic pressure parameter where the first coalescence occurs. The normal mode method is used to reduce the system of aeroelastic equations. The first 15 frequencies are considered for the analysis. The governing equations are solved by using the standard IMSL complex eigenvalue solver on a Cyber 180/840 system. The search for critical dynamic pressure is carried out by increasing the value of λ until a pair of complex conjugate eigenvalues are found. Then λ is decreased until eigenvalues become all real again and the process is repeated until the gap between the values of λ bracketing λ_{cr} is sufficiently small.

RESULTS AND DISCUSSION

The analysis described in the previous section is applicable to the determination of stability boundaries of laminated rectangular and skew panels exposed to moisture and temperature through the volume of the plate. Two types of boundary conditions, simply supported and clamped, were studied. The air flow is assumed parallel to the x axis (Fig. 1). The nondimensional flutter Ω and Λ are defined as follows.

For isotropic panels the nondimensional frequency is expressed as $\Omega = \omega a^2(\rho_m t/D)^{0.5}$ and the nondimensional dynamic pressure is defined as $\Lambda = \lambda(a^3/D)$. For laminated composite panels it is assumed that $\Omega = \omega a^2/(\rho_m/E_2 t^2)^{0.5}$ and $\Lambda = \lambda(a^3/E_2 t^3)$. Lamina material properties at elevated temperature and moisture concentration used in the present study were taken from fig. 8.17 of Tsai and Hahn (1980) and are presented in Tables 1 and 2. Because the evaluation of shear correction factors from the exact theory of elasticity is difficult, in the present case a commonly used value of 5/6 was assumed. A convergence study indicated that the 6 × 6 mesh was sufficient to find reasonably convergent flutter results (Table 3). The present results are also found to be in reasonably good agreement with those of Xue and Mei (1993).

Table 1. Elastic Moduli of Graphite/Epoxy Lamina at Different Moisture Concentrations

Elastic Moduli (GPa)	Moisture Concentration ($C\%$)						
	0.0	0.25	0.50	0.75	1.00	1.25	1.50
E_1	130	130	130	130	130	130	130
E_2	9.5	9.25	9.0	8.75	8.5	8.5	8.5
G_{12}	6.0	6.0	6.0	6.0	6.0	6.0	6.0

$G_{13} = G_{12}$, $G_{23} = 0.5\, G_{12}$, $\nu_{12} = 0.3$, $\beta_i = 0$, and $\beta_2 = 0.44$. Adapted from fig. 8.17 of Tsai and Hahn (1980).

Table 3. Comparison of Flutter Boundary of Simply Supported Rectangular Panels with Different Temperatures

$\Delta T/\Delta T_{cr}$	Critical Dynamic Pressure (Λ_{cr})	
	Xue and Mei (1993)	Present Study
0.8	371.093	371.270 (6 × 6)
		369.290 (4 × 4)
1.2	309.117	304.520 (6 × 6)
		302.620 (4 × 4)

Effect of Moisture and Temperature

The moisture concentration and temperature distribution had a destabilizing effect on the stability boundary for all the laminates considered in the present investigation. Some panels, depending upon boundary conditions and lamination schemes, experienced static buckling. This type of static instability was not observed in the earlier investigation (Chowdary et al., 1994). The static buckled zone may be attributed to initial stress developed due to moisutre and temperature.

The intersection of flutter boundary with the Λ_{cr} axis (Fig. 2) is the value of Λ_{cr} associated with zero moisture content. In general, three regions were observed, depending upon the value of frequency parameter ω. In the region (DAC) labeled "panel flat, no flutter," ω is real. In the region (DAB) labeled "flutter," ω is complex, indicating that at least one root will lead to an oscillating divergent motion. In the region (CAB) "panel buckled, no flutter," the frequency parameter is purely imaginary, indicating static instability. The three regions are separated by two boundaries. The first is the buckling loop (CAB) that is the locus of points for which the frequency is zero. The second is the flutter boundary DAB, that is the locus of points for which two frequencies coalesce. The instability boundaries are found by specifying the moisture content and temperature and varying the value of flow velocity in small quantities until the first mode frequency changes.

The stability behavior of rectangular laminated panels with different boundary conditions and lamination schemes is shown in Figs. 2–9. It is observed that angle-ply $(\pm 45)_2$ clamped panels exposed to moisture and temperature did not undergone static buckling whereas simply supported panels experienced static buckling at approximately 0.5% moisutre content (C) and approximately 375 K temperature (T). Furthermore, the flutter boundary is observed to fall rapidly once the panel buckles. In both the cases, the clamped panel offered higher flutter resistance. The stability behavior of cross-ply $(0/90)_2$ panels exposed to moisture and temperature are illustrated in Figs. 4 and 5. In this case, simply supported panels exposed to moisture experi-

Table 2. Elastic Moduli of Graphical/Epoxy Lamina at Different Temperatures

Elastic Moduli (GPa)	Temperature (K)					
	300	325	350	375	400	425
E_1	130	130	130	130	130	130
E_2	9.5	8.5	8.0	7.5	7.0	6.75
G_{12}	6.0	6.0	5.5	5.0	4.75	4.5

$G_{13} = G_{12} = 0.5\, G_{12}$, $\nu_{12} = 0.3$, $\alpha_1 = -0.3 \times 10^{-6}/K$, and $\alpha_2 = 28.1 \times 10^{-6}/K$. Adapted from fig. 8.17 of Tsai and Hahn (1980).

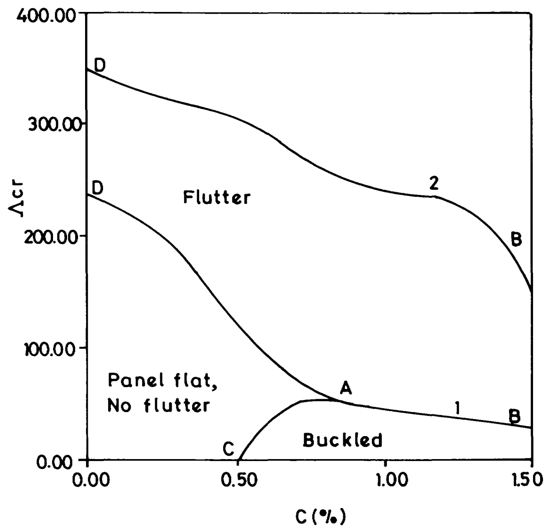

FIGURE 2 Effect of moisture on the stability boundary of an angle-ply $(\pm 45)_2$ laminated square panel: 1. simply supported and 2. clamped.

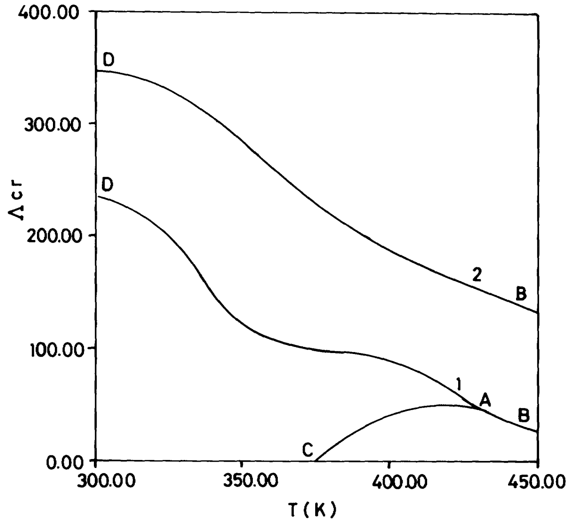

FIGURE 3 Effect of temperature on the stability boundary of an angle-ply $(\pm 45)_2$ laminated square panel: 1. simply supported and 2. clamped.

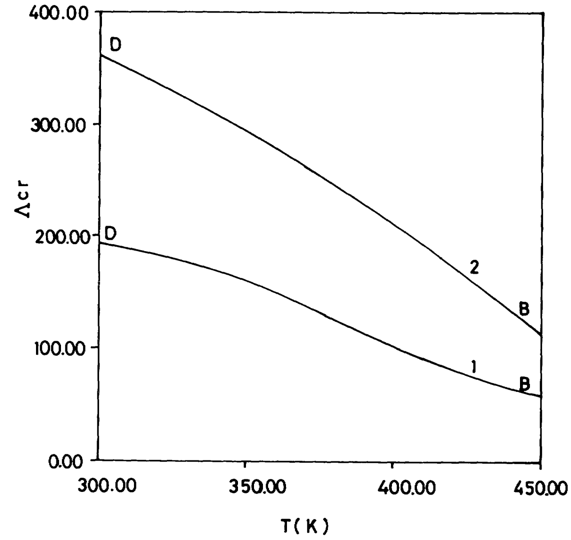

FIGURE 5 Effect of temperature on the stability boundary of a cross-ply $(0/90)_2$ laminated square panel: 1. simply supported and 2. clamped.

enced static buckling; but the same panels subjected to temperature up to 450 K did not undergone static buckling. Again, the buckling region is small and occurs at much higher moisture content than that of angle-ply laminated panels. In this case also, once buckling starts, the flutter boundary falls rapidly. The stability characteristics of symmetric laminates (0/45/0) under hygrothermal environment are illustrated in Figs. 6 and 7. The simply supported panels experienced static buckling at approximately 0.8% moisture content, whereas similar instability was not observed for clamped panels. The flutter behavior of 0/±45/0 laminated panels under hygrothermal environ-

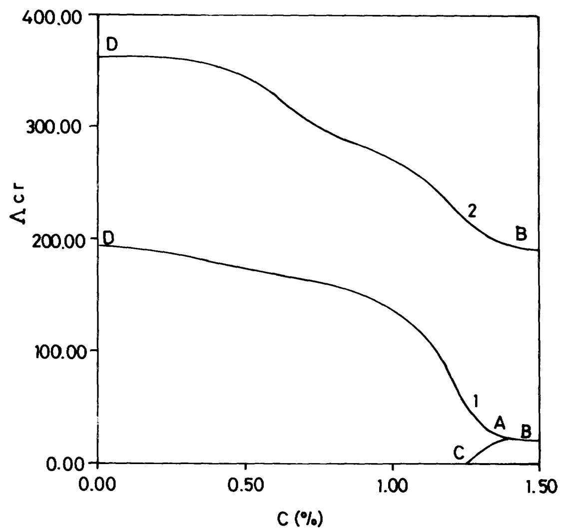

FIGURE 4 Effect of moisture on the stability boundary of a cross-ply $(0/90)_2$ laminated square panel: 1. simply supported and 2. clamped.

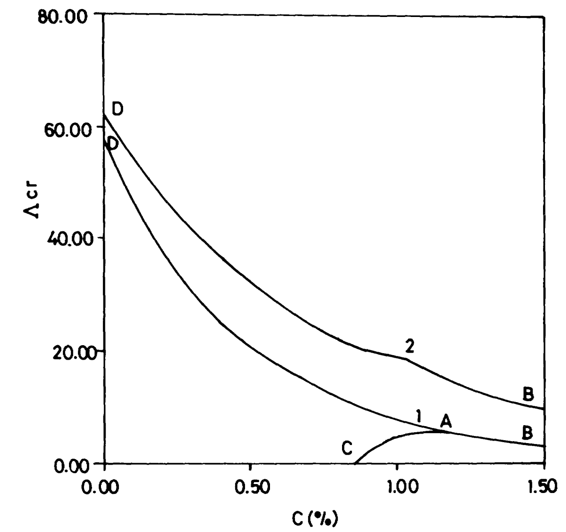

FIGURE 6 Effect of moisture on the stability boundary of a symmetrically (0/45/0) laminated square panel: 1. simply supported and 2. clamped.

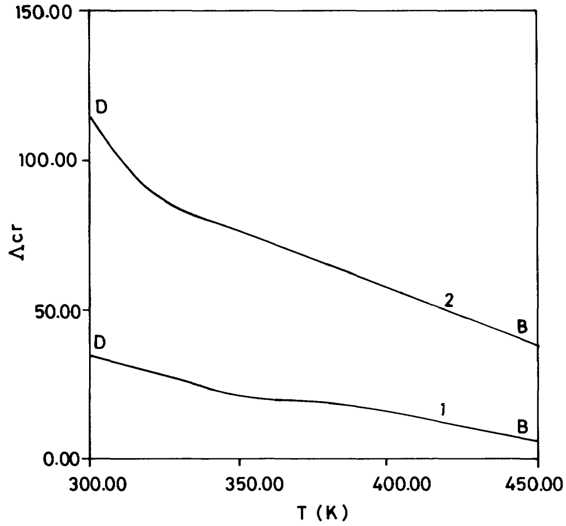

FIGURE 7 Effect of temperature on the stability boundary of a symmetrically (0/45/0) laminated square panel: 1. simply supported and 2. clamped.

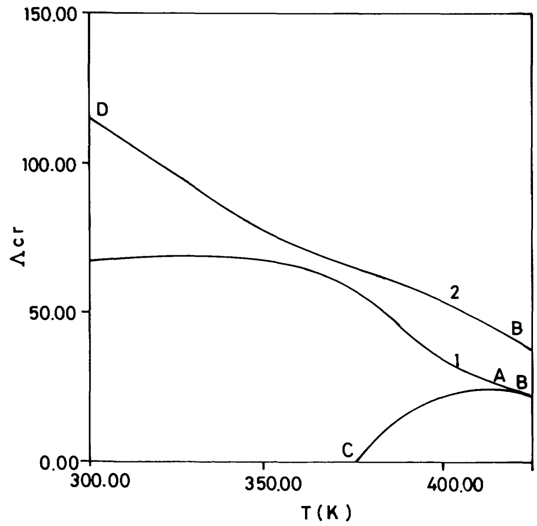

FIGURE 9 Effect of temperature on the stability boundary of an unsymmetric (0/+45/0) laminated square panel: 1. simply supported and 2. clamped.

ment is depicted in Figs. 8 and 9. For both the boundary conditions the flutter boundary is not observed after moisture content (C) exceeds approximately 1.0%, whereas simply supported panels subjected to temperature experienced buckling.

Further investigations were made to study the effect of skew angle on stability boundaries of angle-ply and cross-ply panels in a hygrothermal environment. The results are illustrated in Figs. 10–13. It is observed that none of the skew panels experienced static buckling. Furthermore, a cursory look at the figures reveals that skew angle had a stabilizing effect on the flutter boundary. Angle-ply panels subjected to temperature experienced a sudden fall in flutter boundary at approximately 360 K.

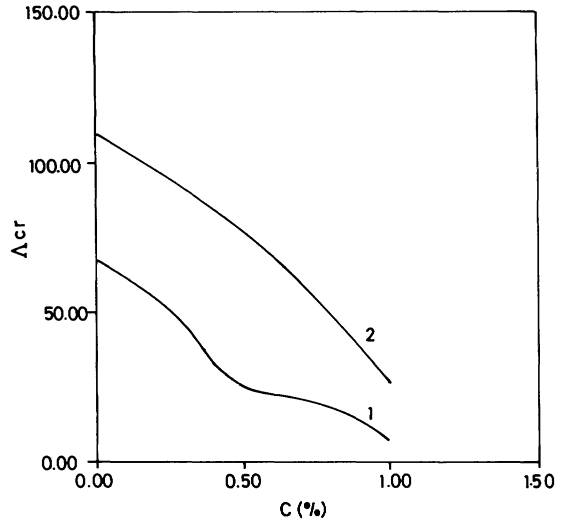

FIGURE 8 Effect of moisture on the stability boundary of an unsymmetric (0/±45/0) laminated square panel: 1. simply supported and 2. clamped.

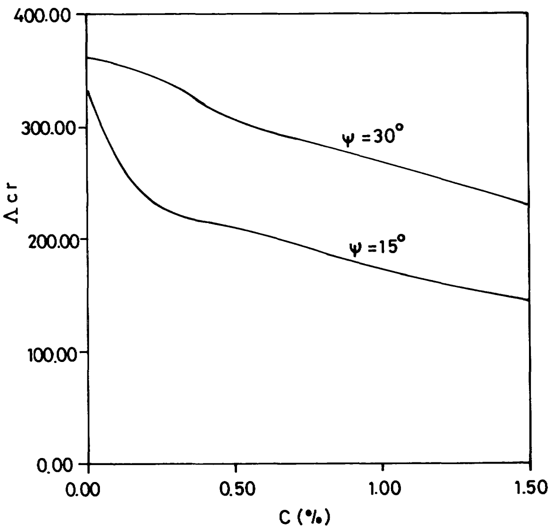

FIGURE 10 Effect of skew angle on the stability boundary of an angle-ply $(\pm 45)_2$ laminated square panel subjected to moisture (simply supported).

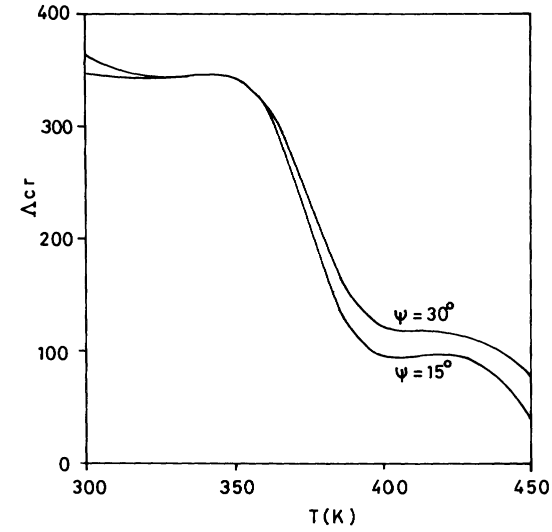

FIGURE 11 Effect of skew angle on the stability boundary of an angle-ply $(\pm 45)_2$ laminated square panel subjected to moisture (clamped).

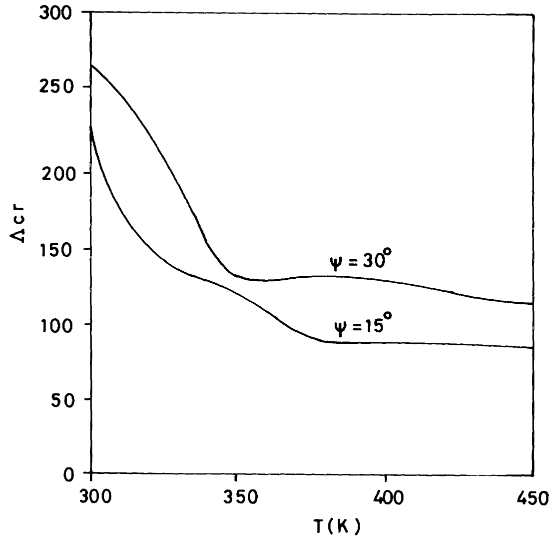

FIGURE 13 Effect of skew angle on the stability boundary of a cross-ply $(0/90)_2$ laminated square panel subjected to temperature (clamped).

CONCLUSION

The conventional finite element formulation was modified to include the hygrothermal effects on the flutter boundaries of laminated composite rectangular and skew panels. Depending upon boundary conditions and lamination schemes, the panels can have different regions: flat, buckled, or flutter. The range of these regions depend also on boundary conditions and lamination scheme. Moisture content and temperature have destabilizing effects on the flutter boundary, whereas skew angle a stabilized flutter boundary. Static buckling was not observed for panels skewed relative to the flow.

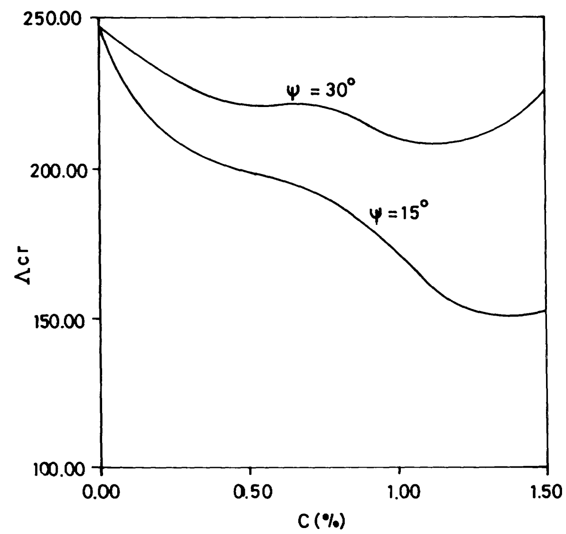

FIGURE 12 Effect of skew angle on the stability boundary of a cross-ply $(0/90)_2$ laminated square panel subjected to moisture (simply supported).

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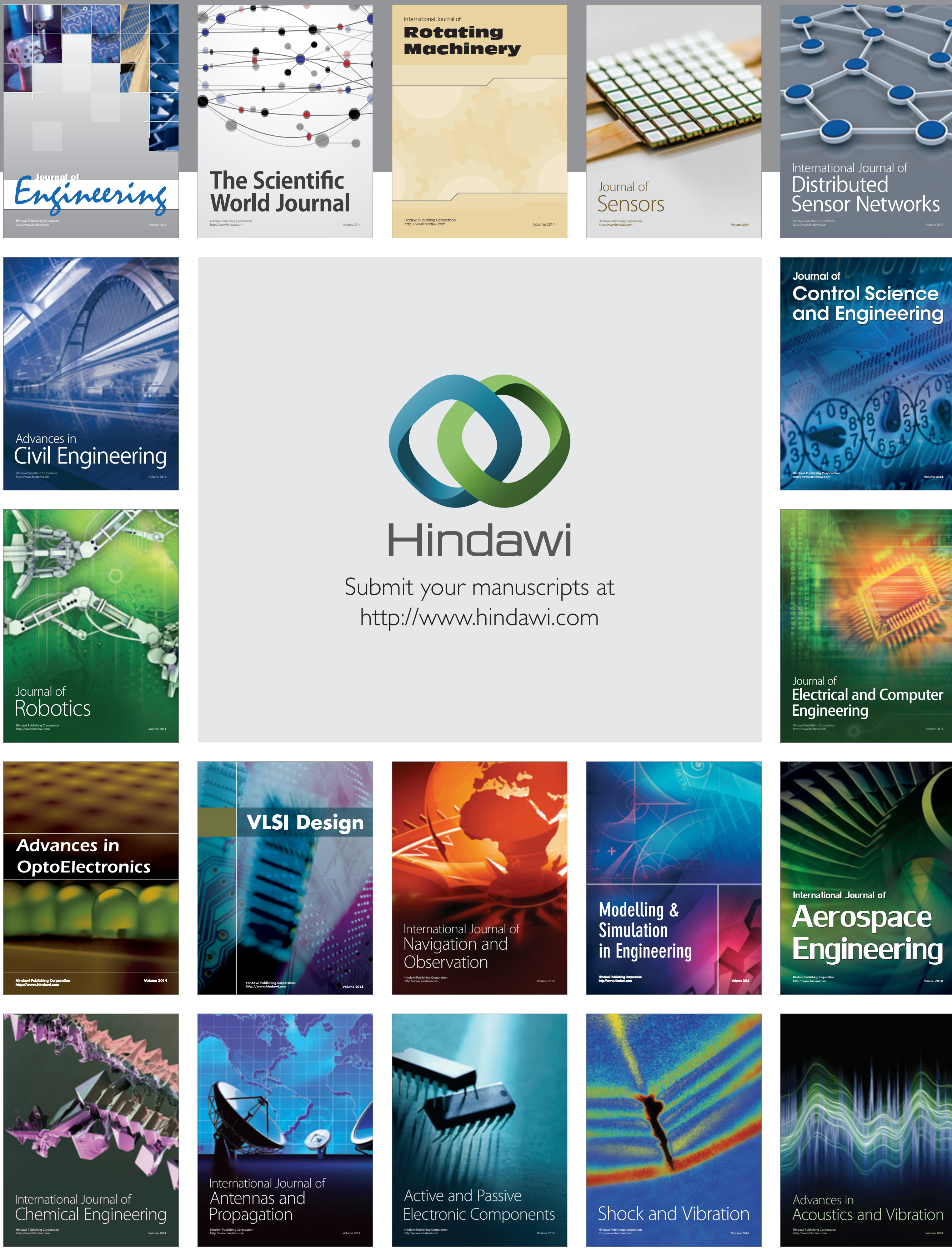
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