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Vibration Analysis of Viscoelastically Damped Sandwich Shells

The simplified governing equations and corresponding boundary conditions of vibration of viscoelastically damped unsymmetrical sandwich shells are given. The asymptotic solution to the equations is then discussed. If only the first terms of the asymptotic solution of all variables are taken as an approximate solution, the result is identical with that obtained from the modal strain energy method. By taking more terms of the asymptotic solution with successive calculations and use of the Padé approximants method, accuracy of natural frequencies and modal loss factors of sandwich shells can be improved. The lowest three or four natural frequencies and modal loss factors of simply supported cylindrical sandwich shells are calculated.

INTRODUCTION

It is well known that structural vibration can be reduced by utilizing layers of viscoelastic damping material. The constrained layer damping treatment is an effective approach and is described in Torvik (1980) and Nashif et al. (1985). Flexural vibrations of damped sandwich plates and structures have been investigated by a number of authors. The governing equations of flexural vibration of symmetrical and unsymmetrical sandwich plates were given in Mead (1972) and Rao and Nakra (1973). Flexural vibration of damped sandwich plates was also analyzed in Lu et al. (1979). In Johnson and Kienholz (1982) the modal strain energy (MSE) method was suggested for analysis of viscoelastically damped sandwich plates and structures. In He and Ma (1988) the simplified governing equations and corresponding boundary conditions of flexural vibration of damped unsymmetrical sandwich plates were given. The analytical exact solution and an asymptotic solution were obtained for simply supported rectangular plates. Ma and He (1992) gave a finite element analysis associated with an asymptotic solution method for the harmonic flexural vibration of damped unsymmetrical sandwich plates. Calculations were carried out for rectangular plates with either simply supported or clamped edges. The numerical results verify the reliability of the finite element analysis associated with the asymptotic solution method given there.

The constrained layer damping treatment can also be used to abate vibration of shells. In Vaswani et al. (1984) the governing differential equations of motion for flexural vibrations of a doubly curved sandwich panel, consisting of stiff face layers sandwiching soft viscoelastic core, were

Received July 14, 1995; Accepted May 20, 1996
Shock and Vibration, Vol. 3, No. 6, pp. 403–417 (1996)

CCC 1070-9622/96/060403-15

derived using variational principles. Alam and Asnani (1984a) presented the governing equations of motion of a general multilayered cylindrical shell. The solution for a radially simply supported shell was obtained. Numerical results were reported in Alam and Asnani (1984b). In El-Raheb and Wagner (1986) and Lu et al. (1991) analyses of the response of damped cylindrical shells carrying discontinuously constrained viscoelastic layers were presented. El-Raheb and Wagner (1993) extended their earlier techniques (1986) to analyze the damping of a toroidal segment of shell included in configurations enclosing an acoustic fluid. In Ramesh and Ganesan (1993) a finite element analysis of vibration and damping of conical shells with a constrained viscoelastic damping treatment was given.

In the present work a set of simplified governing equations and corresponding boundary conditions of vibration of viscoelastically damped unsymmetrical sandwich shells are given, and its asymptotic solution is then discussed. Harmonic vibrations of cylindrical sandwich shells with simply supported ends are calculated. The lowest three or four natural frequencies and modal loss factors of the shells are given.

GOVERNING EQUATIONS OF VIBRATION

To derive the governing equations of vibration of unsymmetrical sandwich shells, the following assumptions are made:

1. The thickness of the shell is small compared with the radii of curvature of the middle surface;
2. the face layers are elastic and isotropic and suffer no transverse shear deformation;
3. the core carries transverse shear, but no tangential stresses; it is linearly viscoelastic and has a complex shear modulus;
4. no slip occurs at the interfaces of the core and face layers and all points on a normal to the shell move with the same normal displacement;
5. although the metallic material of the structural layer and the constraining layer may be different, e.g., steel or aluminum, the values of their Poisson ratios may be approximately equal;
6. when the sandwich shell is in flexural vibration, the rotatory inertia effects of the shell are ignored and only the inertia effects due to the normal and tangential displacements are considered.

These assumptions are used to establish a set of simplified governing equations of unsymmetrical sandwich shells.

The unsymmetrical sandwich shell configuration is shown in Fig. 1. The thicknesses of the constraining layer (face 1), the viscoelastic layer (face 2), and the structural layer (face 3) are t_1, t_2, and t_3, respectively. The thickness of the shell is h, $h = t_1 + t_2 + t_3$. A system of orthogonal curvilinear coordinates is defined by the coordinates α_1^* and α_2^* corresponding to the lines of curvature on the geometrically middle surface of the sandwich shell and the coordinate z along the normal to the middle surface. The Lamé parameters and the normal radii of curvature in the directions of α_1^* and α_2^* of the middle surface are denoted by A_1^*, A_2^* and R_1^*, R_2^*, respectively. The tangential displacements of the points in face 1 and face 3 are given as follows:

$$
\begin{aligned}
u^{(1)}(\alpha_1^*, \alpha_2^*, z, t) &\approx U_m^{(1)} - (z - z_1)\left(\frac{\partial W}{A_1^*\partial\alpha_1^*} - \frac{U_m^{(1)}}{R_1^*}\right),\\
v^{(1)}(\alpha_1^*, \alpha_2^*, z, t) &\approx V_m^{(1)} - (z - z_1)\left(\frac{\partial W}{A_2^*\partial\alpha_2^*} - \frac{V_m^{(1)}}{R_2^*}\right),\\
u^{(3)}(\alpha_1^*, \alpha_2^*, z, t) &\approx U_m^{(3)} - (z - z_3)\left(\frac{\partial W}{A_1^*\partial\alpha_1^*} - \frac{U_m^{(3)}}{R_1^*}\right),\\
v^{(3)}(\alpha_1^*, \alpha_2^*, z, t) &\approx V_m^{(3)} - (z - z_3)\left(\frac{\partial W}{A_2^*\partial\alpha_2^*} - \frac{V_m^{(3)}}{R_2^*}\right).
\end{aligned}
\tag{1}
$$

Here $U_m^{(1)}$, $V_m^{(1)}$, z_1 and $U_m^{(3)}$, $V_m^{(3)}$, z_3 are the tangential displacements and the value of the coordinate z of the points at the middle surface of face 1 and face 3, respectively. $W(\alpha_1, \alpha_2, t)$ is the normal displacement of the shell. As in He and Ma (1988) one can introduce

$$
\begin{aligned}
&U_m(\alpha_1^*, \alpha_2^*, t)\\
&= [1/(\gamma_1 + \gamma_3)][\gamma_1 U_m^{(1)}(\alpha_1^*, \alpha_2^*, t) + \gamma_3 U_m^{(3)}(\alpha_1^*, \alpha_2^*, t)],\\
&V_m(\alpha_1^*, \alpha_2^*, t)\\
&= [1/(\gamma_1 + \gamma_3)][\gamma_1 V_m^{(1)}(\alpha_1^*, \alpha_2^*, t) + \gamma_3 V_m^{(3)}(\alpha_1^*, \alpha_2^*, t)],\\
&\psi_1(\alpha_1^*, \alpha_2^*, t) \qquad (2)\\
&= (1/c)[U_m^{(1)}(\alpha_1^*, \alpha_2^*, t) - U_m^{(3)}(\alpha_1^*, \alpha_2^*, t)],\\
&\psi_2(\alpha_1^*, \alpha_2^*, t)\\
&= (1/c)[V_m^{(1)}(\alpha_1^*, \alpha_2^*, t) - V_m^{(3)}(\alpha_1^*, \alpha_2^*, t)],
\end{aligned}
$$

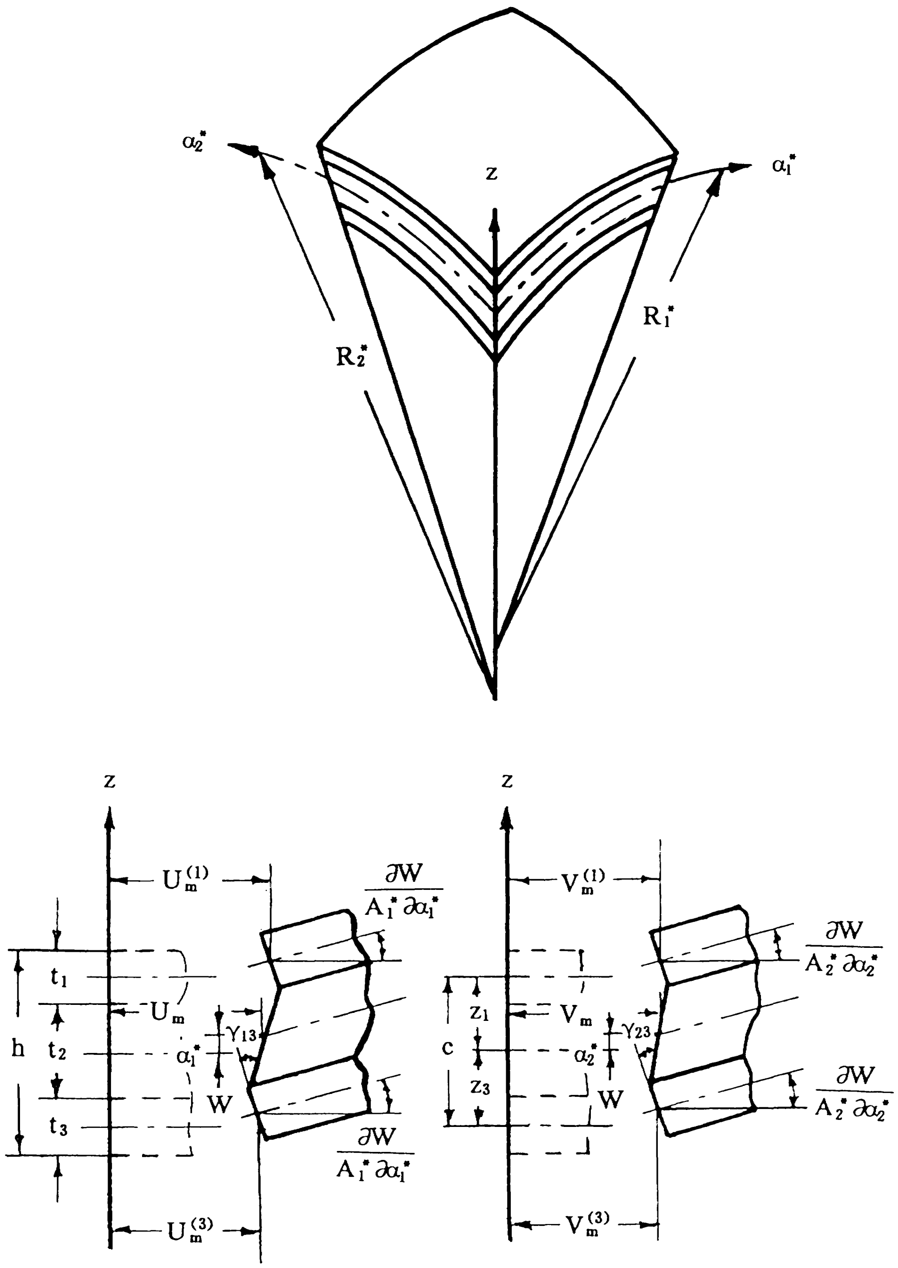

FIGURE 1 The configuration and the displacements of a damped sandwich shell.

where

$$\gamma_1 = E_1 t_1/(1-\nu^2),$$
$$\gamma_3 = E_3 t_3/(1-\nu^2), \qquad (3)$$
$$c = t_2 + \frac{1}{2}(t_1 + t_3),$$

in which E_1 and E_3 are the elastic moduli of the faces 1 and 3, ν is their common Poisson ratio, U_m and V_m may be regarded as the weighted mean tangential displacements of the unsymmetrical sandwich shell, and ψ_1 and ψ_2 are the rotatory angles of a line connecting the two corresponding points at the middle surfaces of faces 1 and 3 after deformation. According to the assumptions, there are uniform transverse shear strain components γ_{13} and γ_{23} in the viscoelastic layer. They can be expressed approximately as

$$\gamma_{13} \approx c(\psi_1 - \varphi_1)/t_2, \quad \gamma_{23} \approx c(\psi_2 - \varphi_2)/t_2, \qquad (4)$$

where

$$\varphi_1(\alpha_1^*, \alpha_2^*, t) = -\frac{\partial W}{A_1^*\partial\alpha_1^*} + \frac{U_m}{R_1^*},$$
$$\varphi_2(\alpha_1^*, \alpha_2^*, t) = -\frac{\partial W}{A_2^*\partial\alpha_2^*} + \frac{V_m}{R_2^*}. \tag{5}$$

The expression of the strain energy density per unit middle surface area of the shell can be obtained as

$$U_0 = \frac{1}{2}[N_1^*\varepsilon_1 + N_2^*\varepsilon_2 + S^*\gamma_{12} + M_1'^*\kappa_1'^* + M_2'^*\kappa_2'^* + 2H'^*\kappa_{12}'^* + M_1''^*\kappa_1''^* + M_2''^*\kappa_2''^* + 2H''^*\kappa_{12}''^* + Q_1'^*(\psi_1 - \varphi_1) + Q_2'^*(\psi_2 - \varphi_2)], \tag{6}$$

where

$$\varepsilon_1 = \frac{\partial U_m}{A_1^*\partial\alpha_1^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_1^*}{\partial\alpha_2^*}V_m + \frac{W}{R_1^*},$$

$$\varepsilon_2 = \frac{\partial V_m}{A_2^*\partial\alpha_2^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_2^*}{\partial\alpha_1^*}U_m + \frac{W}{R_2^*},$$

$$\gamma_{12} = \frac{A_2^*}{A_1^*}\frac{\partial}{\partial\alpha_1^*}\left(\frac{V_m}{A_2^*}\right) + \frac{A_1^*}{A_2^*}\frac{\partial}{\partial\alpha_2^*}\left(\frac{U_m}{A_1^*}\right),$$

$$\kappa_1'^* = \frac{\partial\psi_1}{A_1^*\partial\alpha_1^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_1^*}{\partial\alpha_2^*}\psi_2,$$

$$\kappa_2'^* = \frac{\partial\psi_2}{A_2^*\partial\alpha_2^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_2^*}{\partial\alpha_1^*}\psi_1, \tag{7}$$

$$\kappa_{12}'^* = \frac{1}{2}\left[\frac{A_2^*}{A_1^*}\frac{\partial}{\partial\alpha_1^*}\left(\frac{\psi_2}{A_2^*}\right) + \frac{A_1^*}{A_2^*}\frac{\partial}{\partial\alpha_2^*}\left(\frac{\psi_1}{A_1^*}\right)\right],$$

$$\kappa_1''^* = \frac{\partial\varphi_1}{A_1^*\partial\alpha_1^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_1^*}{\partial\alpha_2^*}\varphi_2,$$

$$\kappa_2''^* = \frac{\partial\varphi_2}{A_2^*\partial\alpha_2^*} + \frac{1}{A_1^*A_2^*}\frac{\partial A_2^*}{\partial\alpha_1^*}\varphi_1,$$

$$\kappa_{12}'^* = \frac{1}{2}\left[\frac{A_2^*}{A_1^*}\frac{\partial}{\partial\alpha_1^*}\left(\frac{\varphi_2}{A_2^*}\right) + \frac{A_1^*}{A_2^*}\frac{\partial}{\partial\alpha_2^*}\left(\frac{\varphi_1}{A_1^*}\right)\right].$$

Here $N_1^*, N_2^* \cdots Q_2'^*$ are all generalized internal forces. They can be expressed in terms of deformation as

$$N_1^* = \lambda(D_1 + D_3)(\varepsilon_1 + \nu\varepsilon_2)/a^2,$$
$$N_2^* = \lambda(D_1 + D_3)(\varepsilon_2 + \nu\varepsilon_1)/a^2,$$
$$S^* = \lambda(1 - \nu)(D_1 + D_3)\gamma_{12}/2a^2,$$
$$M_1'^* = Y(D_1 + D_3)(\kappa_1'^* + \nu\kappa_2'^*),$$
$$M_2'^* = Y(D_1 + D_3)(\kappa_2'^* + \nu\kappa_1'^*),$$
$$H'^* = (1 - \nu)\cdot Y(D_1 + D_3)\kappa_{12}'^*, \tag{8}$$
$$M_1''^* = (D_1 + D_3)(\kappa_1''^* + \nu\kappa_2''^*),$$
$$M_2''^* = (D_1 + D_3)(\kappa_2''^* + \nu\kappa_1''^*),$$
$$H''^* = (1 - \nu)(D_1 + D_3)\kappa_{12}''^*,$$
$$Q_1'^* = Yg(D_1 + D_3)(\psi_1 - \varphi_1)/a^2,$$
$$Q_2'^* = Yg(D_1 + D_3)(\psi_2 - \varphi_2)/a^2,$$

where a is a tangential dimension of the shell,

$$D_1 = \frac{E_1t_1^3}{12(1 - \nu^2)}, \quad D_3 = \frac{E_3t_3^3}{12(1 - \nu^2)},$$
$$\lambda = (\gamma_1 + \gamma_3)a^2/(D_1 + D_3), \tag{9}$$
$$Y = \gamma_1\gamma_3c^2/(\gamma_1 + \gamma_3)(D_1 + D_3),$$
$$g = \gamma_2(\gamma_1 + \gamma_3)a^2/\gamma_1\gamma_3t_2^2, \quad \gamma_2 = G_2t_2,$$

in which G_2 is the shear modulus of the core 2, which is taken as a real quantity temporarily. Y, λ, and g are called two "geometric parameters" and the "shear parameter," respectively.

For the sake of convenience later on, the following dimensionless variables are introduced:

$$A_1d\alpha_1 = A_1^*d\alpha_1^*/a, \quad A_2d\alpha_2 = A_2^*d\alpha_2^*/a,$$
$$u_m = U_m/a, \quad v_m = V_m/a, \quad w = W/a,$$
$$R_1 = R_1^*/a, \quad R_2 = R_2^*/a, \tag{10}$$
$$\kappa_1' = a\kappa_1'^*, \quad \kappa_2' = a\kappa_2'^*, \quad \kappa_{12}' = a\kappa_{12}'^*,$$
$$\kappa_1'' = a\kappa_1''^*, \quad \kappa_2'' = a\kappa_2''^*, \quad \kappa_{12}'' = a\kappa_{12}''^*.$$

Here α_1 and α_2 dimensionless coordinates. A_1, A_2 and R_1, R_2 denote the dimensionless Lamé parameters and the dimensionless normal radii of curvature, respectively. Then the strain energy U of the shell can be written as

$$U = \frac{1}{2}(D_1 + D_3) \iint [N_1\varepsilon_1 + N_2\varepsilon_3 + S\gamma_{12} + M_1'\kappa_1' + M_2'\kappa_2' + 2H'\kappa_{12}' + M_1''\kappa_1'' + M_2''\kappa_2'' + 2H''\kappa_{12}'' + Q_1'(\psi_1 - \varphi_1) + Q_2'(\psi_2 - \varphi_2)]A_1A_2\, d\alpha_1\, d\alpha_2. \tag{11}$$

The dimensionless generalized internal forces $N_1 \cdots Q_2'$ are expressed as

$$\begin{aligned} &N_1 = \lambda(\varepsilon_1 + \nu\varepsilon_2), \quad N_2 = \lambda(\varepsilon_2 + \nu\varepsilon_1), \\ &S = \lambda(1 - \nu)\gamma_{12}/2, \\ &M_1' = Y(\kappa_1' + \nu\kappa_2'), \quad M_2' = Y(\kappa_2' + \nu\kappa_1'), \\ &H' = Y(1 - \nu)\kappa_{12}', \\ &M_1'' = \kappa_1'' + \nu\kappa_2'', \quad M_2'' = \kappa_2'' + \nu\kappa_1'', \quad H'' = (1 - \nu)\kappa_{12}'', \\ &Q_1' = Yg(\psi_1 - \varphi_1), \quad Q_2' = Yg(\psi_2 - \varphi_2). \end{aligned} \tag{12}$$

The kinetic energy of the shell T is approximately

$$T = \frac{1}{2}\rho a^4 \iint \left[\left(\frac{\partial u_m}{\partial t}\right)^2 + \left(\frac{\partial v_m}{\partial t}\right)^2 + \left(\frac{\partial w}{\partial t}\right)^2\right] A_1A_2\, d\alpha_1\, d\alpha_2, \tag{13}$$

where ρ is the mass per unit area of the shell.

According to Hamilton's principle, one can obtain a set of simplified governing equations of motion of unsymmetrical sandwich shells (in dimensionless form)

$$\begin{aligned} &-\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(N_1A_2) + \frac{\partial}{\partial\alpha_2}(SA_1) + S\frac{\partial A_1}{\partial\alpha_2} - N_2\frac{\partial A_2}{\partial\alpha_1}\right] - \frac{1}{R_1}(Q_1' + Q_1'') + \frac{\rho a^4}{D_1 + D_3}\frac{\partial^2 u_m}{\partial t^2} = 0, \\ &-\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(SA_2) + \frac{\partial}{\partial\alpha_2}(N_2A_1) + S\frac{\partial A_2}{\partial\alpha_1} - N_1\frac{\partial A_1}{\partial\alpha_2}\right] - \frac{1}{R_2}(Q_2' + Q_2'') + \frac{\rho a^4}{D_1 + D_3}\frac{\partial^2 v_m}{\partial t^2} = 0, \\ &-\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(Q_1'A_2 + Q_1''A_2) + \frac{\partial}{\partial\alpha_2}(Q_2'A_1 + Q_2''A_1)\right] + \frac{N_1}{R_1} + \frac{N_2}{R_2} + \frac{\rho a^4}{D_1 + D_3}\frac{\partial^2 w}{\partial t^2} = 0, \\ &\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(M_1'A_2) + \frac{\partial}{\partial\alpha_2}(H'A_1) + H'\frac{\partial A_1}{\partial\alpha_2} - M_2'\frac{\partial A_2}{\partial\alpha_1}\right] - Q_1' = 0, \\ &\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(H'A_2) + \frac{\partial}{\partial\alpha_2}(M_2'A_1) + H'\frac{\partial A_2}{\partial\alpha_1} - M_1'\frac{\partial A_1}{\partial\alpha_2}\right] - Q_2' = 0, \end{aligned} \tag{14}$$

where

$$\begin{aligned} &Q_1'' = \frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(M_1''A_2) + \frac{\partial}{\partial\alpha_2}(H''A_1) + H''\frac{\partial A_1}{\partial\alpha_2} - M_2''\frac{\partial A_2}{\partial\alpha_1}\right], \\ &Q_2'' = \frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(H''A_2) + \frac{\partial}{\partial\alpha_2}(M_2''A_1) + H''\frac{\partial A_2}{\partial\alpha_1} - M_1''\frac{\partial A_1}{\partial\alpha_2}\right]. \end{aligned} \tag{15}$$

Equation (14) can be expressed in terms of $u_m(\alpha_1, \alpha_2, t)$, $v_m(\alpha_1, \alpha_2, t)$, $w(\alpha_1, \alpha_2, t)$, $\psi_1(\alpha_1, \alpha_2, t)$, and $\psi_2(\alpha_1, \alpha_2, t)$. It is a set of 12th-order partial differential equations with respect to α_1 and α_2.

When a shell is in simple harmonic vibration, the forms

$$\begin{aligned} u_m(\alpha_1, \alpha_2, t) &= u_m(\alpha_1, \alpha_2)e^{i\omega t}, \\ v_m(\alpha_1, \alpha_2, t) &= v_m(\alpha_1, \alpha_2)e^{i\omega t}, \\ w(\alpha_1, \alpha_2, t) &= w(\alpha_1, \alpha_2)e^{i\omega t}, \\ \psi_1(\alpha_1, \alpha_2, t) &= \psi_1(\alpha_1, \alpha_2)e^{i\omega t}, \\ \psi_2(\alpha_1, \alpha_2, t) &= \psi_2(\alpha_1, \alpha_2)e^{i\omega t}, \end{aligned} \tag{16}$$

are introduced. The circular frequency ω can be expressed in dimensionless form as

$$\Omega = \omega\sqrt{\rho a^4/(D_1 + D_3)}. \tag{17}$$

Equation (16) can now be substituted into Eq. (14). However, as the shell is assumed to vibrate harmonically, the shear modulus G_2 of the core must be changed into the complex modulus $G_2(1 + i\beta)$; here β is the loss factor of the viscoelastic material. The inertia terms in Eq. (14) $[\rho a^4/(D_1 + D_3)]\partial^2(u_m, v_m, w)/\partial t^2 = -\Omega^2(u_m, v_m, w)$

must also change correspondingly to $-\Omega^2(1 + i\eta^*)(u_m, v_m, w)$; here the quantity η^* is the modal loss factor of the shell. The physical significance of the complex frequency $\Omega^2(1 + i\eta^*)$ was discussed in Mead and Markus (1969). The amplitudes $u_m(\alpha_1, \alpha_2)$, $v_m(\alpha_1, \alpha_2)$, $w(\alpha_1, \alpha_2)$, $\psi_1(\alpha_1, \alpha_2)$, and $\psi_2(\alpha_1, \alpha_2)$ are all complex quantities. They must satisfy the following equations:

$$
\begin{aligned}
&-\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(N_1A_2) + \frac{\partial}{\partial\alpha_2}(SA_1) + S\frac{\partial A_1}{\partial\alpha_2} - N_2\frac{\partial A_2}{\partial\alpha_1}\right]\\
&-\frac{1}{R_1}Yg(1+i\beta)(\psi_1 - \varphi_1)\\
&-\frac{1}{R_1}Q_1'' - \Omega^2(1+i\eta^*)u_m = 0,\\
&-\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(SA_2) + \frac{\partial}{\partial\alpha_2}(N_2A_1) + S\frac{\partial A_2}{\partial\alpha_1} - N_1\frac{\partial A_1}{\partial\alpha_2}\right]\\
&-\frac{1}{R_2}Yg(1+i\beta)(\psi_2 - \varphi_2)\\
&-\frac{1}{R_2}Q_2'' - \Omega^2(1+i\eta^*)v_m = 0,\\
&-\frac{1}{A_1A_2}\frac{\partial}{\partial\alpha_1}[Yg(1+i\beta)A_2(\psi_1 - \varphi_1) + A_2Q_1'']\\
&-\frac{1}{A_1A_2}\frac{\partial}{\partial\alpha_2}[Yg(1+i\beta)A_1(\psi_2 - \varphi_2) + A_1Q_2'']\\
&+\frac{N_1}{R_1} + \frac{N_2}{R_2} - \Omega^2(1+i\eta^*)w = 0,\\
&\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(M_1'A_2) + \frac{\partial}{\partial\alpha_2}(H'A_1) + H'\frac{\partial A_1}{\partial\alpha_2} - M_2'\frac{\partial A_2}{\partial\alpha_1}\right]\\
&-Yg(1+i\beta)(\psi_1 - \varphi_1) = 0,\\
&\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(H'A_2) + \frac{\partial}{\partial\alpha_2}(M_2'A_1) + H'\frac{\partial A_2}{\partial\alpha_1} - M_1'\frac{\partial A_1}{\partial\alpha_2}\right]\\
&-Yg(1+i\beta)(\psi_2 - \varphi_2) = 0.
\end{aligned}
\tag{18}
$$

Here the amplitudes $N_1, N_2, \ldots, Q_2', Q_2''$ are expressed in terms of deformation calculated by the amplitudes u_m, v_m, w, ψ_1, and ψ_2 as Eq. (12). They are all complex quantities too. Typical homogeneous boundary conditions for edge $\alpha_1 = \overline{\alpha}_1$ (constant) are as follows:
simply supported edge (type I),

$$
\begin{aligned}
&N_1 = 0, \quad v_m = 0, \quad M_1' = 0,\\
&\psi_2 = 0, \quad w = 0, \quad M_1'' = 0;
\end{aligned}
\tag{19}
$$

simply supported edge (type II),

$$
\begin{aligned}
&N_1 = 0, \quad S = 0, \quad M_1' = 0,\\
&H' = 0, \quad w = 0, \quad M_1'' = 0;
\end{aligned}
\tag{20}
$$

clamped edge,

$$
\begin{aligned}
&u_m = 0, \quad v_m = 0, \quad \psi_1 = 0,\\
&\psi_2 = 0, \quad w = 0, \quad \varphi_1 = 0;
\end{aligned}
\tag{21}
$$

free edge,

$$
\begin{aligned}
&N_1 = 0, \quad S = 0, \quad M_1' = 0, \quad H' = 0,\\
&Q_1' + Q_1'' + \frac{\partial H''}{A_2\partial\alpha_2} = 0, \quad M_1'' = 0.
\end{aligned}
\tag{22}
$$

At a corner there is

$$
w = 0 \quad \text{or} \quad M_{ns}''(s+0) - M_{ns}''(s-0) = 0. \tag{23}
$$

Thus, to investigate transverse vibration of a viscoelastically damped unsymmetrical sandwich shell, one must first solve Eq. (18) for a given set of boundary conditions to find the natural frequencies Ω, the modal loss factors η^*, and the corresponding complex modes u_m, v_m, w, ψ_1, and ψ_2.

ASYMPTOTIC SOLUTION OF GOVERNING EQUATIONS

Solving Eq. (18) is not easy. The exact solution can be obtained only in some particular cases. To obtain an approximate and practical solution and to avoid calculation with complex values, an asymptotic solution with $\mu = i\beta$ as a complex parameter can be introduced. The same procedure was used in He and Ma (1988) to find the loss factors of sandwich plates. One first expands the solution in the power series

$$
\begin{aligned}
u_m &= u_{m0} + \mu u_{m1} + \mu^2 u_{m2} + \mu^3 u_{m3}\\
&\quad + \mu^4 u_{m4} + \mu^5 u_{m5} + \cdots,\\
v_m &= v_{m0} + \mu v_{m1} + \mu^2 v_{m2} + \mu^3 v_{m3}\\
&\quad + \mu^4 v_{m4} + \mu^5 v_{m5} + \cdots,\\
w &= w_0 + \mu w_1 + \mu^2 w_2 + \mu^3 w_3\\
&\quad + \mu^4 w_4 + \mu^5 w_5 + \cdots,
\end{aligned}
$$

$$\psi_1 = \psi_{1,0} + \mu\psi_{1,1} + \mu^2\psi_{1,2} + \mu^3\psi_{1,3} + \mu^4\psi_{1,4} + \mu^5\psi_{1,5} + \cdots,$$
$$\psi_2 = \psi_{2,0} + \mu\psi_{2,1} + \mu^2\psi_{2,2} + \mu^3\psi_{2,3} + \mu^4\psi_{2,4} + \mu^5\psi_{2,5} + \cdots,$$
$$\Omega^2 = \Omega_0^2 + \mu^2\Omega_2^2 + \mu^4\Omega_4^2 + \cdots,$$
$$i\eta^* = \mu\eta_1^* + \mu^3\eta_3^* + \mu^5\eta_5^* + \cdots. \quad (24)$$

The amplitudes $N_1, N_2, \ldots, Q_2', Q_2''$ are also expanded in the power series. Substituting Eq. (24) into Eq. (18) gives the successive equations that the asymptotic solution must satisfty

$$L_{1,0} \equiv L_1(u_{m0}, v_{m0}, w_0, \psi_{1,0}, \psi_{2,0}) \equiv -\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(N_{1,0}A_2) + \frac{\partial}{\partial\alpha_2}(S_0A_1) + S_0\frac{\partial A_1}{\partial\alpha_2} - N_{2,0}\frac{\partial A_2}{\partial\alpha_1}\right] - \frac{1}{R_1}Yg(\psi_{1,0} - \varphi_{1,0}) - \frac{1}{R_1}Q''_{1,0} - \Omega_0^2u_{m0} = 0,$$

$$L_{2,0} \equiv L_2(u_{m0}, v_{m0}, w_0, \psi_{1,0}, \psi_{2,0}) \equiv -\frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(S_0A_2) + \frac{\partial}{\partial\alpha_2}(N_{2,0}A_1) + S_0\frac{\partial A_2}{\partial\alpha_1} - N_{1,0}\frac{\partial A_1}{\partial\alpha_2}\right] - \frac{1}{R_2}Yg(\psi_{2,0} - \varphi_{2,0}) - \frac{1}{R_2}Q''_{2,0} - \Omega_0^2v_{m0} = 0,$$

$$L_{3,0} \equiv L_3(u_{m0}, v_{m0}, w_0, \psi_{1,0}, \psi_{2,0}) \equiv -\frac{1}{A_1A_2}\left\{\frac{\partial}{\partial\alpha_1}[YgA_2(\psi_{1,0} - \varphi_{1,0}) + A_2Q''_{1,0}] + \frac{\partial}{\partial\alpha_2}[YgA_1(\psi_{2,0} - \varphi_{2,0}) + A_1Q''_{2,0}]\right\} + \frac{N_{1,0}}{R_1} + \frac{N_{2,0}}{R_2} - \Omega_0^2w_0 = 0,$$

$$L_{4,0} \equiv L_4(u_{m0}, v_{m0}, w_0, \psi_{1,0}, \psi_{2,0}) \equiv \frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(M'_{1,0}A_2) + \frac{\partial}{\partial\alpha_2}(H'_0A_1) + H'_0\frac{\partial A_1}{\partial\alpha_2} - M'_{2,0}\frac{\partial A_2}{\partial\alpha_1}\right] - Yg(\psi_{1,0} - \varphi_{1,0}) = 0,$$

$$L_{5,0} \equiv L_5(u_{m0}, v_{m0}, w_0, \psi_{1,0}, \psi_{2,0}) \equiv \frac{1}{A_1A_2}\left[\frac{\partial}{\partial\alpha_1}(H'_0A_2) + \frac{\partial}{\partial\alpha_2}(M'_{2,0}A_1) + H'_0\frac{\partial A_2}{\partial\alpha_1} - M'_{1,0}\frac{\partial A_1}{\partial\alpha_2}\right] - Yg(\psi_{2,0} - \varphi_{2,0}) = 0, \quad (25)$$

$$L_{1,i+1} \equiv L_1(u_{m,i+1}, v_{m,i+1}, w_{i+1}, \psi_{1,i+1}, \psi_{2,i+1}) = \frac{1}{R_1}Yg(\psi_{1,i} - \varphi_{1,i}) + R_i(u_m),$$
$$L_{2,i+1} \equiv L_2(u_{m,i+1}, v_{m,i+1}, w_{i+1}, \psi_{1,i+1}, \psi_{2,i+1}) = \frac{1}{R_2}Yg(\psi_{2,i} - \varphi_{2,i}) + R_i(v_m),$$
$$L_{3,i+1} \equiv L_3(u_{m,i+1}, v_{m,i+1}, w_{i+1}, \psi_{1,i+1}, \psi_{2,i+1}) = \frac{Yg}{A_1A_2}\left\{\frac{\partial}{\partial\alpha_1}[A_2(\psi_{1,i} - \varphi_{1,i})] + \frac{\partial}{\partial\alpha_2}[A_1(\psi_{2,i} - \varphi_{2,i})]\right\} + R_i(w), \quad (26)$$
$$L_{4,i+1} \equiv L_4(u_{m,i+1}, v_{m,i+1}, w_{i+1}, \psi_{1,i+1}, \psi_{2,i+1}) = Yg(\psi_{1,i} - \varphi_{1,i}),$$
$$L_{5,i+1} \equiv L_5(u_{m,i+1}, v_{m,i+1}, w_{i+1}, \psi_{1,i+1}, \psi_{2,i+1}) = Yg(\psi_{2,i} - \varphi_{2,i}),$$
$$(i = 0, 1, 2, 3, 4, \ldots),$$

where

$$R_0(w) = \Omega_0^2\eta_1^*w_0,$$
$$R_1(w) = \Omega_0^2\eta_1^*w_1 + \Omega_2^2w_0,$$
$$R_2(w) = \Omega_0^2(\eta_1^*w_2 + \eta_3^*w_0) + \Omega_2^2(\eta_1^*w_0 + w_1),$$
$$R_3(w) = \Omega_0^2(\eta_1^*w_3 + \eta_3^*w_1) + \Omega_2^2(\eta_1^*w_1 + w_2) + \Omega_4^2w_0, \quad (27)$$
$$R_4(w) = \Omega_0^2(\eta_1^*w_4 + \eta_3^*w_2 + \eta_5^*w_0) + \Omega_2^2(\eta_1^*w_2 + \eta_3^*w_0 + w_3) + \Omega_4^2(\eta_1^*w_0 + w_1), \ldots$$

(the expressions of $R_i(w)$ $(i > 4)$ are omitted). Here L_1, L_2, L_3, L_4, and L_5 are linear partial differential operators defined in Eq. (25) where all generalized internal forces can be expressed in terms of the displacements and the rotatory angles in accordance with Eqs. (12) and (7). The expressions of $R_i(u_m)$ and $R_i(v_m)$ are similar to these of $R_i(w)$ in Eq. (27) with only w_j replaced by u_{mj} and $v_{mj}(j = 0, 1, 2, 3, 4)$. Substituting Eq. (24) into the boundary conditions (19)–(23), one finds that, except for the fifth of Eq. (22) for free edges that must be expanded in the power series of μ, the expressions for all other types of boundary conditions that the successive terms u_{mi}, v_{mi}, w_i, $\psi_{1,i}$, and $\psi_{2,i}$ $(i = 0, 1, 2, 3, 4, 5, \ldots)$ must satisfy are the same form as before. Therefore, from Eq. (25) and the boundary conditions with respect to u_{m0}, v_{m0}, w_0, $\psi_{1,0}$, and $\psi_{2,0}$, one can solve first a real eigenvalue problem and obtain all the eigenvalues Ω_0^2 and corresponding modes. The modes can be normalized as follows.

If all the boundary conditions of the shell are homogeneous, one can prove that orthogonality exists between two complex modes. This means that if there are two different complex eigenvalues $\Omega_I^2(1 + i\eta_I^*)$ and $\Omega_{\rm I}^2(1 + i\eta_{\rm I}^*)$ and their corresponding complex modes u_{mI}, v_{mI} w_I, ψ_{1I}, ψ_{2I} and $u_{m\rm I}$, $v_{m\rm I}$, $w_{\rm I}$, $\psi_{1\rm I}$, $\psi_{2\rm I}$, then

$$\iint (u_{mI}u_{m\rm I} + v_{mI}v_{m\rm I} + w_I w_{\rm I})A_1A_2\,d\alpha_1\,d\alpha_2 = 0. \quad (28)$$

Therefore, the normalization of the complex modes can be stated as

$$\iint (u_m^2 + v_m^2 + w^2)A_1A_2\,d\alpha_1\,d\alpha_2 = 1. \quad (29)$$

Substituting the first three equations of Eq. (24) into Eq. (29), one can obtain

$$\iint (u_{m0}^2 + v_{m0}^2 + w_0^2)A_1A_2\,d\alpha_1\,d\alpha_2 = 1, \quad (30)$$

$$\iint 2(u_{m0}u_{m1} + v_{m0}v_{m1} + w_0w_1)\;A_1A_2\,d\alpha_1\,d\alpha_2 = 0, \quad (31)$$

$$\iint [2(u_{m0}u_{m2} + v_{m0}v_{m2} + w_0w_2) + (u_{m1}^2 + v_{m1}^2 + w_1^2)]\;A_1A_2\,d\alpha_1\,d\alpha_2 = 0, \quad (32)$$

$$\iint [2(u_{m0}u_{m3} + v_{m0}v_{m3} + w_0w_3) + 2(u_{m1}u_{m2} + v_{m1}v_{m2} + w_1w_2)]\;A_1A_2\,d\alpha_1\,d\alpha_2 = 0, \quad (33)$$

$$\iint [2(u_{m0}u_{m4} + v_{m0}v_{m4} + w_0w_4) + 2(u_{m1}u_{m3} + v_{m1}v_{m3} + w_1w_3) + (u_{m2}^2 + v_{m2}^2 + w_2^2)]\;A_1A_2\,d\alpha_1\,d\alpha_2 = 0, \quad (34)$$

$$\iint [2(u_{m0}u_{m5} + v_{m0}v_{m5} + w_0w_5) + 2(u_{m1}u_{m4} + v_{m1}v_{m4} + w_1w_4) + 2(u_{m2}u_{m3} + v_{m2}v_{m3} + w_2w_3)]\;A_1A_2\,d\alpha_1\,d\alpha_2 = 0, \quad (35)$$

successively. In accordance with Eq. (30) one can normalize the real mode u_{m0}, v_{m0}, w_0, $\psi_{1,0}$, and $\psi_{2,0}$. From Eq. (25)

$$\iint (u_{m0}L_{1,0} + v_{m0}L_{2,0} + w_0L_{3,0} - \psi_{1,0}L_{4,0} - \psi_{2,0}L_{5,0})A_1A_2\,d\alpha_1\,d\alpha_2 = 0. \quad (36)$$

Through integrations by parts and other calculations one obtains

$$\begin{aligned}\Omega_0^2 = \iint \{&\lambda[(\varepsilon_{1,0} + \varepsilon_{2,0})^2 - 2(1-\nu)(\varepsilon_{1,0}\varepsilon_{2,0} - \tfrac{1}{4}\gamma_{12,0}^2)] \\ &+ Y[(\kappa'_{1,0} + \kappa'_{2,0})^2 - 2(1-\nu)(\kappa'_{1,0}\kappa'_{2,0} - \kappa'^2_{12,0})] \\ &+ [(\kappa''_{1,0} + \kappa''_{2,0})^2 - 2(1-\nu)(\kappa''_{1,0}\kappa''_{2,0} - \kappa''^2_{12,0})] \\ &+ Yg[(\psi_{1,0} - \varphi_{1,0})^2 + (\psi_{2,0} - \varphi_{2,0})^2]\} \\ &A_1A_2\,d\alpha_1\,d\alpha_2. \quad (37)\end{aligned}$$

Next one solves Eq. (26) for $i = 0$. First the value of $\eta_{\rm I}^*$ in the expressions of $R_0(u_m)$, $R_0(v_m)$, and $R_0(w)$ must be determined. By using a similar procedure as before when Eq. (37) was obtained, one can obtain

$$\eta_{\rm I}^* = \frac{1}{\Omega_0^2}\iint Yg[(\psi_{1,0} - \varphi_{1,0})^2 + (\psi_{2,0} - \varphi_{2,0})^2]A_1A_2\,d\alpha_1\,d\alpha_2. \quad (38)$$

$\eta_{\rm I}^*$ is the fraction of strain energy attributable to the viscoelastic core when the damped shell deforms in the mode u_{m0}, v_{m0}, w_0, $\psi_{1,0}$, and $\psi_{2,0}$. All the first terms of each expression in Eq. (24) can be regarded as an approximate solution, which are the same as the results obtained by means of the MSE method suggested in Johnson and Kienholz (1982).

To improve accuracy, one must calculate successive terms of the asymptotic solution. Having determined the value of $\eta_{\rm I}^*$, one can obtain the unique solution for u_{m1}, v_{m1}, w_1, $\psi_{1,1}$, and $\psi_{2,1}$ of all modes in accordance with Eq. (26) for $i = 0$ and corresponding boundary conditions. The general solution for u_{m1}, v_{m1}, w_1, $\psi_{1,1}$, and $\psi_{2,1}$ may be written in the following form:

$$\begin{aligned} u_{m1} &= u_{m1p} + k_1 u_{m0}, \quad v_{m1} = v_{m1p} + k_1 v_{m0}, \\ w_1 &= w_{1p} + k_1 w_0, \quad \psi_{1,1} = \psi_{1,1p} + k_1 \psi_{1,0}, \\ \psi_{2,1} &= \psi_{2,1p} + k_1 \psi_{2,0}. \end{aligned} \tag{39}$$

Here u_{m1p}, v_{m1p}, w_{1p}, $\psi_{1,1p}$, $\psi_{2,1p}$ are a set of particular solutions where k_1 is an undetermined constant. The value of k_1 can be given by means of the normalization condition, Eq. (31). Then the expressions for u_{m1}, v_{m1}, w_1, $\psi_{1,1}$, and $\psi_{2,1}$ can be determined completely.

Through calculations similar to previous ones, the solutions of the successive Eq. (26) for $i = 1, 2, 3, 4$ can be obtained. The expressions for Ω_2^2, η_3^*, Ω_4^2, and η_5^* are as follows:

$$\begin{aligned} \Omega_2^2 = \iint & Yg[(\psi_{1,0} - \varphi_{1,0})(\psi_{1,1} - \varphi_{1,1}) \\ & + (\psi_{2,0} - \varphi_{2,0})(\psi_{2,1} - \varphi_{2,1})] A_1 A_2 \, d\alpha_1 \, d\alpha_2, \end{aligned} \tag{40}$$

$$\begin{aligned} \eta_3^* = \frac{1}{\Omega_0^2} \iint & \{Yg[(\psi_{1,0} - \varphi_{1,0})(\psi_{1,2} - \varphi_{1,2}) \\ & + (\psi_{2,0} - \varphi_{2,0})(\psi_{2,2} - \varphi_{2,2})] \\ & - \eta_1^* \Omega_0^2 (u_{m0} u_{m2} + v_{m0} v_{m2} \\ & + w_0 w_2)\} A_1 A_2 \, d\alpha_1 \, d\alpha_2 - \eta_1^* \frac{\Omega_2^2}{\Omega_0^2}, \end{aligned} \tag{41}$$

$$\begin{aligned} \Omega_4^2 = \iint & \{Yg[(\psi_{1,0} - \varphi_{1,0})(\psi_{1,3} - \varphi_{1,3}) \\ & + (\psi_{2,0} - \varphi_{2,0})(\psi_{2,3} - \varphi_{2,3})] \\ & - \eta_1^* \Omega_0^2 (u_{m0} u_{m3} + v_{m0} v_{m3} + w_0 w_3) \\ & - \Omega_2^2 (u_{m0} u_{m2} + v_{m0} v_{m2} \\ & + w_0 w_2)\} A_1 A_2 \, d\alpha_1 \, d\alpha_2, \end{aligned} \tag{42}$$

$$\begin{aligned} \eta_5^* = \frac{1}{\Omega_0^2} \iint & \{Yg[(\psi_{1,0} - \varphi_{1,0})(\psi_{1,4} - \varphi_{1,4}) \\ & + (\psi_{2,0} - \varphi_{2,0})(\psi_{2,4} - \varphi_{2,4})] \\ & - \eta_1^* \Omega_0^2 (u_{m0} u_{m4} + v_{m0} v_{m4} + w_0 w_4) \\ & - (\eta_1^* \Omega_2^2 + \eta_3^* \Omega_0^2)(u_{m0} u_{m2} + v_{m0} v_{m2} \\ & + w_0 w_2)\} A_1 A_2 \, d\alpha_1 \, d\alpha_2 \\ & - \frac{1}{\Omega_0^2} (\eta_1^* \Omega_4^2 + \eta_3^* \Omega_2^2). \end{aligned} \tag{43}$$

Using the normalization conditions (32)–(35), one can obtain the unique solutions for $u_{m,i+1}$, $v_{m,i+1}$, w_{i+1}, $\psi_{1,i+1}$, and $\psi_{2,i+1} (i = 1, 2, 3, 4)$ successively in accordance with Eq. (26) and corresponding boundary conditions. If the calculation is not to be continued, one is left with the expressions for the asymptotic solution given explicitly in Eq (24), without the residual terms indicated by $(\cdot\ \cdot\ \cdot)$ on their right sides. The last two solutions of Eq. (24) may be rewritten as

$$\Omega^2 = \Omega_0^2 \left[1 + \beta^2 \left(\frac{\Omega_4^2}{\Omega_2^2} - \frac{\Omega_2^2}{\Omega_0^2} \right) \right] \Big/ \left(1 + \beta^2 \frac{\Omega_4^2}{\Omega_2^2} \right), \tag{44}$$

$$\eta^* = \eta_1^* \beta \left[1 + \beta^2 \left(\frac{\eta_5^*}{\eta_3^*} - \frac{\eta_3^*}{\eta_1^*} \right) \right] \Big/ \left(1 + \beta^2 \frac{\eta_5^*}{\eta_3^*} \right). \tag{45}$$

Equation (44) and (45) can be obtained by using the Padé approximants method (see appendix B in Ma and He, 1992). By calculating with Eqs. (44) and (45), one can often obtain better results.

EXAMPLE

Consider a circular viscoelastically damped sandwich cylindrical shell of radius a and length 1 with simple support (type I) ends. Introduce the dimensionless coordinates $\alpha_1 = x/a = \xi$ and $\alpha_2 = \theta$ and let the ends of the shell be $\xi = 0$ and $\xi = \xi_1 = 1/a$. The following solution form that satisfies the boundary conditions for the ends is assumed:

$$\begin{aligned} u_m &= A \cos(\pi\xi/\xi_1) \sin m\theta, \\ v_m &= B \sin(\pi\xi/\xi_1) \cos m\theta, \\ w &= C \sin(\pi\xi/\xi_1) \sin m\theta, \\ \psi_1 &= D \cos(\pi\xi/\xi_1) \sin m\theta, \\ \psi_2 &= E \sin(\pi\xi/\xi_1) \cos m\theta. \end{aligned} \tag{46}$$

Here we only attempt the lowest several natural frequencies and take the number of axial half waves to equal one. However, the circumferential wave number m is to be selected to associate with the lowest several natural frequencies. Substituting Eq. (46) into Eq. (18) leads an algebraic complex eigenvalue equation. The exact solution in the form as Eq. (46) can be obtained directly from the algebraic equation. We are only interested in the transverse modes of vibration. The motions are mostly radial and the amplitudes C are predominant. The asymptotic solution is also calcu-

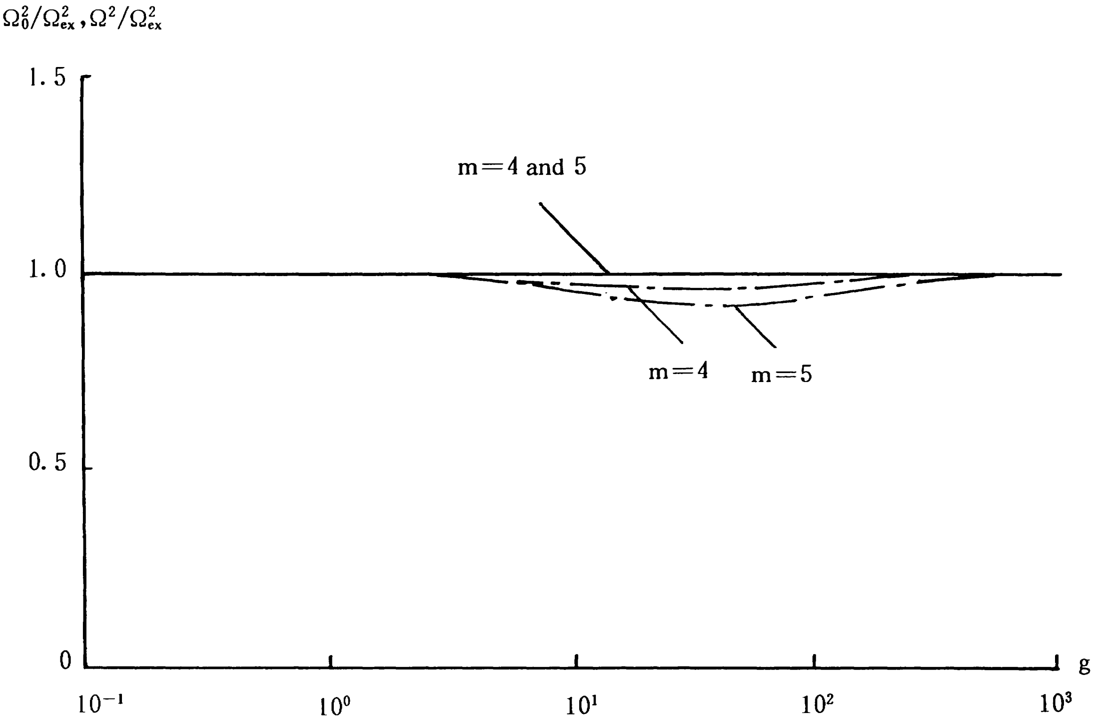

FIGURE 2 Ratio of the asymptotic solutions of natural frequencies of simply supported cylindrical shells ($Y = 3.5$, $\lambda = 12 \times 10^4$) to the exact ones vs. the variable g ($m = 4$ and 5). (- · -) Ω_0^2/Ω_{ex}^2; (—)Ω^2/Ω_{ex}^2.

lated for comparison. It can be obtained by expanding the amplitudes in Eq. (46) in the power series and using the procedure as given in the preceding section.

Numerical calculation was done for a shell with $a = 100$ cm and $1 = 200$ cm. Both the structural layer and the constraining layer are made of aluminum. Their common Poisson ratio ν equals 0.3. The thicknesses of the layers are $t_1 = t_3 = 1$ cm and $t_2 = 0.08$ cm. The loss factor of the core β equals 1. The two geometric parameters are $Y = 3.5$ and $\lambda = 12 \times 10^4$. The value of the shear parameter g varies from 0.1 to 1000. The circumferential wave number m associated with the lowest natural frequency depends on the value of the variable g. When $g = 0.1$, 1, and 10, $m = 5$. When $g = 100$ and 1000, then $m = 4$. In Fig. 2 curves showing the variation Ω_0^2/Ω_{ex}^2 and Ω^2/Ω_{ex}^2 with the variable g are given with $m = 4$ and 5. The values of Ω^2 are calculated according to Eq. (44). Ω_{ex}^2 is the exact solution obtained from the algebraic complex eigenvalue equation. Figure 3 shows η^*/β as a function of the variable g, where the values of η^* were calculated according to $\eta_1^*\beta$ or Eq. (45). The latter in this example is nearly identical with the exact value η_{ex}^*, which was also obtained from the complex eigenvalue equation. Table 1 gives the values of Ω_0^2, Ω_2^2, Ω_4^2, and Ω^2 calculated according to Eq. (44) for corresponding values of g of 1, 10, 100, and 1000 and m of 3, 4, 5, and 6. The exact values Ω_{ex}^2 are also given for comparison. The values of η_1^*, η_3^*, η_5^*, and η^* calculated according to Eq. (45) and the exact values η_{ex}^* for the same values of g and m are given in Table 2.

For comparison numerical calculation was also done for a simply supported (type I) thinner cylindrical shell with $a = 50$ cm and $1 = 200$ cm. The thicknesses of the layers are $t_1 = t_2 = t_3 = 0.25$ cm. However, the materials of the face layers and the core are the same. The two geometric parameters of the shell are quite different from those of the above-mentioned shell. They are $Y = 12$ and $\lambda = 48 \times 10^4$. The value of g varies as well. The values of Ω_0^2, Ω_2^2, Ω_4^2, and Ω^2 calculated according to Eq. (44) and the exact values Ω_{ex}^2 for corresponding values of g to be 0.1, 1, 10, 100, and 1000 and m to be selected to associate with the lowest three natural frequencies are given in Table 3. The values of η_1^*, η_3^*, η_5^*, and η^* calculated according to Eq. (45) and the exact values η_{ex}^* for the same values of g and m are

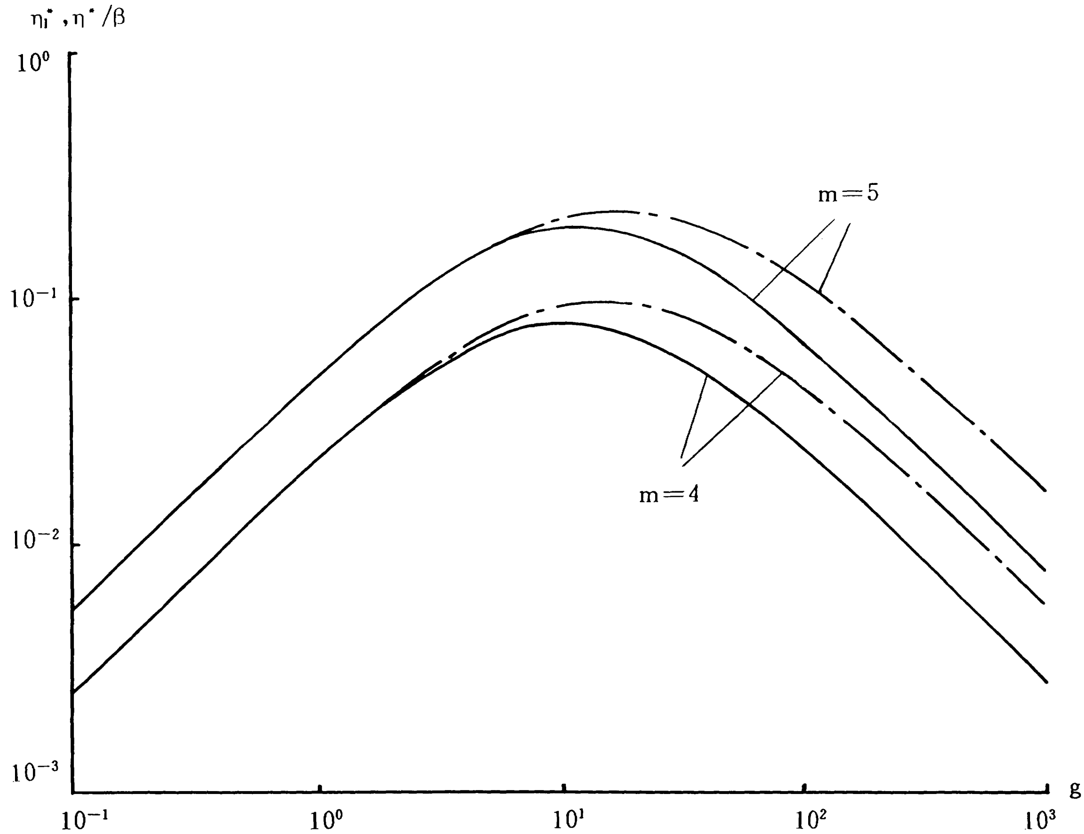

FIGURE 3 Ratio of η^* to β vs. the variable g (m = 4 and 5) for simply supported cylindrical shells (Y = 3.5, λ = 12 × 10^4). (- · -) η_I^*; (—) η^*/β.

Table 1. Dimensionless Frequencies of Simply Supported Cylindrical Shells

g	m	Ω_0^2	Ω_2^2	Ω_4^2	Ω^2	Ω_{ex}^2
1	3	4628.37	−2.026	−0.0134	4630.39	4630.37
	4	2158.26	−2.497	−0.0066	2160.75	2160.74
	5	1596.75	−2.794	−0.0035	1599.54	1599.55
	6	1918.90	−2.983	−0.0019	1921.88	1921.87
10	3	4759.90	−39.534	−8.593	4792.37	4792.37
	4	2457.06	−79.764	−9.851	2528.06	2528.05
	5	2140.39	−122.507	−8.732	2254.75	2254.75
	6	2781.56	−161.059	−6.859	2936.04	2936.03
100	3	4906.89	−28.213	−22.709	4922.52	4922.52
	4	2947.94	−110.623	−78.827	3012.54	3012.54
	5	3354.33	−311.017	−191.434	3546.85	3546.86
	6	5240.13	−690.522	−360.179	5693.94	5693.93
1000	3	4938.08	−3.776	−3.691	4939.99	4939.99
	4	3085.13	−17.409	−16.783	3094.00	3093.99
	5	3796.95	−59.380	−56.248	3827.44	3827.46
	6	6387.46	−163.680	−151.780	6472.39	6472.37

β = 1.0, Y = 3.5, λ = 12 × 10^4.

Table 2. Modal Loss Factors of Simply Supported Cylindrical Shells

g	m	η_1^*	η_3^*	η_5^*	η^*	η_{ex}^*
1	3	0.005441	0.000038	0.000000	0.005404	0.005404
	4	0.022499	0.000086	0.000000	0.022414	0.022414
	5	0.049794	0.000149	0.000000	0.049646	0.049645
	6	0.061344	0 000135	0.000000	0.061210	0.061210
10	3	0.017822	0.004020	0.000908	0.014542	0.014542
	4	0.092391	0.014407	0.002248	0.079929	0.079929
	5	0.214412	0.027552	0.003541	0.189998	0.189998
	6	0.280598	0.028196	0.002833	0.254976	0.254977
100	3	0.006409	0.005195	0.004212	0.003539	0.003539
	4	0.044454	0.033345	0.025012	0.025401	0.025401
	5	0.118185	0.083702	0.059280	0.069186	0.069185
	6	0.182459	0.119215	0.077893	0.110355	0.110355
1000	3	0.000773	0.000757	0.000740	0.000391	0.000391
	4	0.005747	0.005573	0.005404	0.002918	0.002918
	5	0.016068	0.015472	0.014898	0.008186	0.008186
	6	0.026611	0.025358	0.024164	0.013626	0.013626

$\beta = 1.0$, $Y = 3.5$, $\lambda = 12 \times 10^4$.

given in Table 4. In Figs. 4 and 5 the variations Ω_0^2/Ω_{ex}^2 and Ω^2/Ω_{ex}^2 and η^*/β with the variable g are respectively given with $m = 4$.

From Figs. 2 to 5 and Tables 1 to 4 it is evident that for the both types of cylindrical shells with quite different values of Y and λ the errors in the values of Ω_0^2 and η_1^* are somewhat appreciable in a certain range of the value of g in comparison with the exact values of Ω_{ex}^2 and η_{ex}^*, while the values of Ω^2 and η^* obtained from Eqs. (44) and (45) are nearly equal to the exact ones.

From the results calculated it is shown that the maximum damping occurs at somewhere in the range of the value of g for a definite circumferential wave number m. It is quite analogous to the vibration analysis of viscoelastically damped sandwich plates of He and Ma (1988) and sandwich beams of Plunkett and Lee (1970). Therefore, careful attention must be paid to geometric configuration of a sandwich shell to achieve maximum effectiveness for vibration control.

CONCLUSIONS

In this article a set of simplified governing equations and corresponding boundary conditions of viscoelastically damped unsymmetrical sandwich

Table 3. Dimensionless Frequencies of Thinner Cylindrical Shells

g	m	Ω_0^2	Ω_2^2	Ω_4^2	Ω^2	Ω_{ex}^2
0.1	4	809.812	−0.0976	-0.3507×10^{-5}	809.909	809.918
	5	851.937	−0.1051	-0.1591×10^{-5}	852.042	852.074
	6	1393.931	−0.1095	-0.0802×10^{-5}	1394.041	1393.957
1	3	1745.583	−6.314	−0.0571	1751.910	1751.898
	4	949.117	−8.340	−0.0269	957.429	957.433
	5	1087.008	−9.483	−0.0134	1096.478	1096.446
10	3	2075.220	−101.045	−26.271	2155.415	2155.396
	4	1823.935	−241.720	−34.129	2035.756	2035.739
	5	2784.250	−395.705	−31.200	3151.035	3151.031
100	2	6220.304	−4.892	−4.470	6222.860	6222.831
	3	2394.386	−57.892	−48.181	2425.982	2425.966
	4	3145.337	−287.345	−211.299	3310.921	3310.884
1000	2	6225.100	−0.552	−0.547	6225.378	6225.313
	3	2456.394	−7.409	−7.269	2460.134	2460.146
	4	3491.275	−43.371	−41.965	3513.317	3513.300

$\beta = 1.0$, $Y = 12$, $\lambda = 48 \times 10^4$.

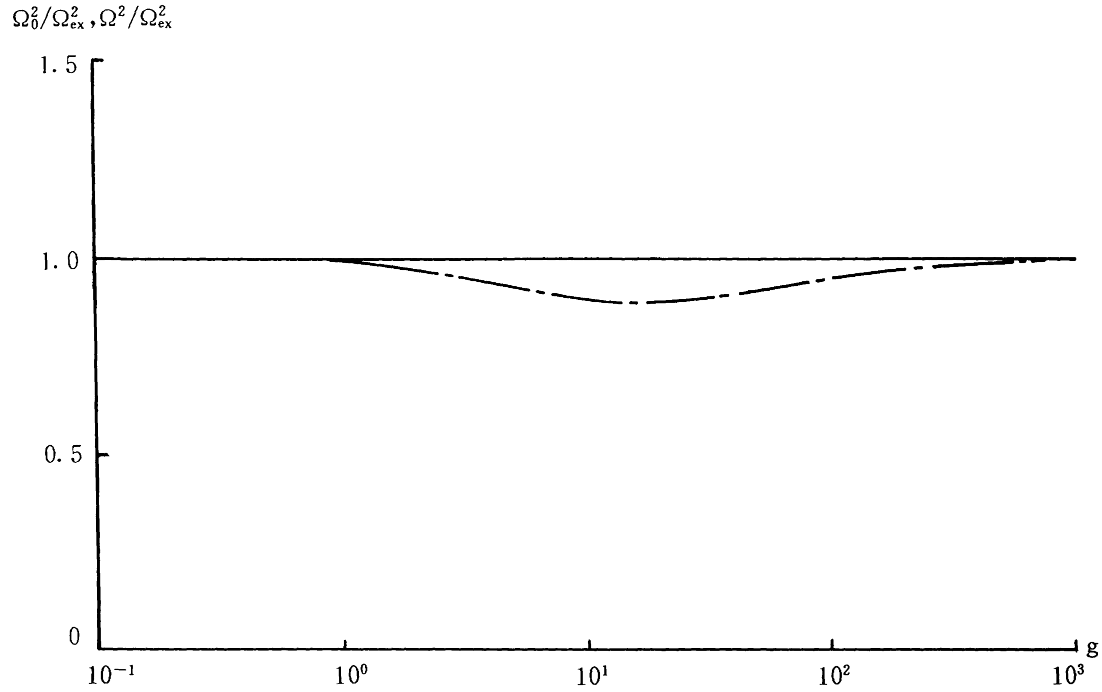

FIGURE 4 Ratio of the asymptotic solutions of natural frequencies of thinner cylindrical shells ($Y = 12$, $\lambda = 48 \times 10^4$) to the exact ones vs. the variable g ($m = 4$). (- · -) Ω_0^2/Ω_{ex}^2; (—) Ω^2/Ω_{ex}^2.

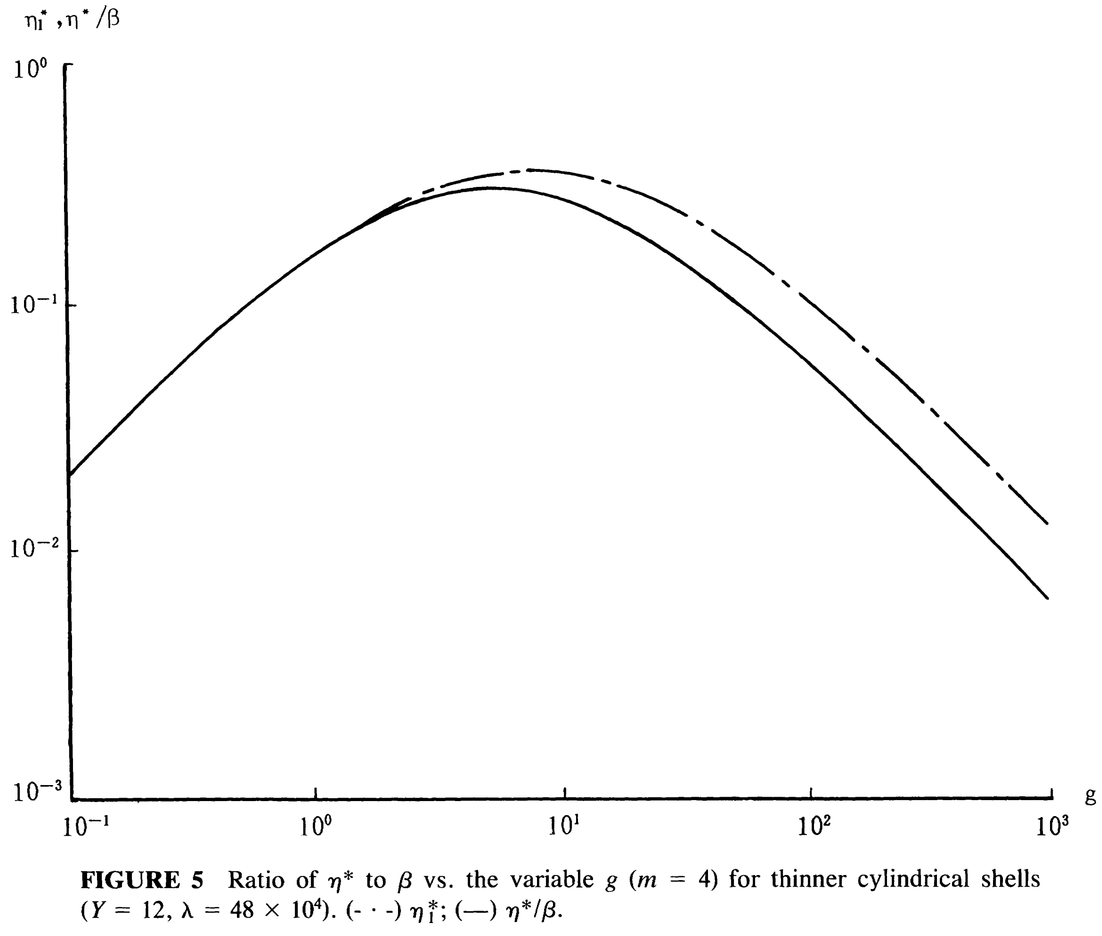

FIGURE 5 Ratio of η^* to β vs. the variable g ($m = 4$) for thinner cylindrical shells ($Y = 12$, $\lambda = 48 \times 10^4$). (- · -) η_I^*; (—) η^*/β.

Table 4. Modal Loss Factors of Thinner Cylindrical Shells

g	m	η_1^*	η_3^*	η_5^*	η^*	η_{ex}^*
0.1	4	0.020143	0.000003	0.000000	0.020140	0.020140
	5	0.031732	0.000004	0.000000	0.031727	0.031726
	6	0.028833	0.000002	0.000000	0.028830	0.028832
1	3	0.038781	0.000487	0.000006	0.038300	0.038301
	4	0.154747	0.001859	0.000022	0.152910	0.152910
	5	0.232158	0.002353	0.000024	0.229828	0.229835
10	3	0.095501	0.029477	0.009100	0.072978	0.072978
	4	0.352706	0.096539	0.026426	0.276913	0.276915
	5	0.506151	0.111843	0.024714	0.414549	0.414550
100	2	0.000823	0.000752	0.000688	0.000430	0.000430
	3	0.026503	0.022698	0.019440	0.014276	0.014276
	4	0.106535	0.088072	0.072810	0.058321	0.058321
1000	2	0.000089	0.000088	0.000088	0.000045	0.000044
	3	0.003045	0.002997	0.002949	0.001535	0.001532
	4	0.012629	0.012376	0.012129	0.006378	0.006376

$\beta = 1.0$, $Y = 12$, $\lambda = 48 \times 10^4$.

shells in vibration are given. To avoid calculation with complex values, an asymptotic solution of the simplified governing equations was introduced, with the loss factor of the viscoelastic material of the core as a parameter. As examples, calculations were carried out for two types of circular cylindrical shells with simply supported ends. The lowest three or four natural frequencies and modal loss factors of the shells are given. If in the asymptotic solution only the first terms of all quantities are adopted, then the result is identical with that as given in accordance with the MSE method. However, the results of the examples indicate that the errors in the values of the natural frequencies and modal loss factors are somewhat appreciable in a certain range of the value of the shear parameter g. By taking more terms of the asymptotic solution, with successive calculations and use of the Padé approximants method, accuracy can be improved. For the sample problems, the values of Ω^2 and η^* calculated according to Eqs. (44) and (45) give the accurate prediction. Although the asymptotic solution in analytical form can only be obtained in simpler cases, in other cases one can use approximate methods, e.g., finite element methods, to obtain numerical solutions. With the character of the free transverse vibrations of a shell thus calculated, it is then possible to analyze further the response of the shell to various types of dynamic normal loads in order to provide a reliable basis for design.

The support by the National Science Foundation of China is gratefully acknowledged.

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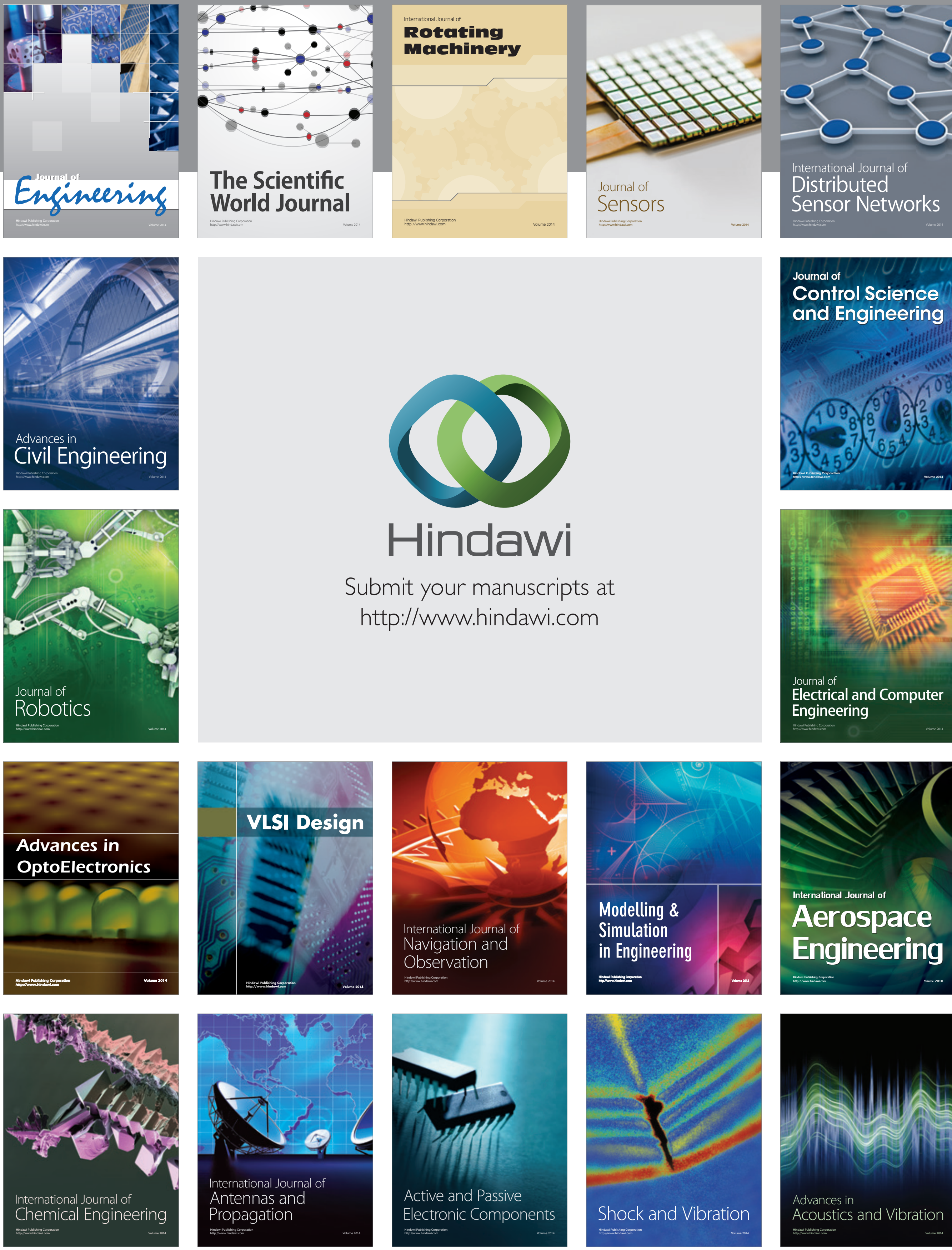
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