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Vibration Isolation Review: II. Shock Excitation

This is the second part of a two part review of shock and vibration isolation. It covers three distinct categories of shock excitation—pulselike shock, velocity shock, and complex shock—and discusses the means that are available in each case to measure the effectiveness of shock mitigation by the imposition of flexible connections between the isolated system and its base © *1996 John Wiley & Sons, Inc.*

INTRODUCTION

In a previous article vibration isolation against sinusoidal and random excitation was reviewed (Nelson, 1994). This article continues that review by extending the consideration of vibration isolation to systems excited by mechanical shock.

A mechanical shock generally appears as an acceleration applied to the base of a machine, structure, or piece of equipment. In addition, the acceleration is applied suddenly, reaches a high level, and often persists for only a short time. As such, the system response is far from the steady-state or stationary conditions considered in part I and, in particular, the concept of transmissibility introduced there must be redefined. This redefinition is less codified than for the steady-state/stationary case and a variety of measures of shock mitigation are in use.

ISOLATION AGAINST PULSELIKE SHOCK

The canonical pulselike shock is the rectangular pulse. As shown in Fig. 1, a rectangular pulse of acceleration applied to the base of a lumped parameter oscillator is dynamically equivalent to a rectangular pulse of force applied to the same oscillator with a fixed base.

Subsequent discussion will be in terms of this fixed-base, force-shock model. The analytical solution for the problem of Fig. 1(b) can be found in many places. There is also a simple phase plane solution (Jacobsen and Ayre, 1958) that can be extended to pulses of arbitrary shape. For most purposes, it is best to represent these solutions in the frequency domain, in particular by means of the shock spectrum (SS). An SS is the locus of the global response maximums (i.e., the maximum of all the local response maximums) of a single degree of freedom (SDOF) oscillator plotted in a nondimensional frequency space. As shown in Fig. 2, one can separately plot an SS for the period of time during which the shock force is acting, the initial shock spectrum (ISS), and an SS for the period of time after the force has been removed, the residual shock spectrum (RSS). The maximax SS is understood to be the upper bound of the combined ISS and RSS. The SS of a wave form should not be confused with its Fourier spectrum: the Fourier spectrum is an input quantity and the SS is an output quantity. However, if the quantity of interest is acceleration, then it can be shown that the undamped

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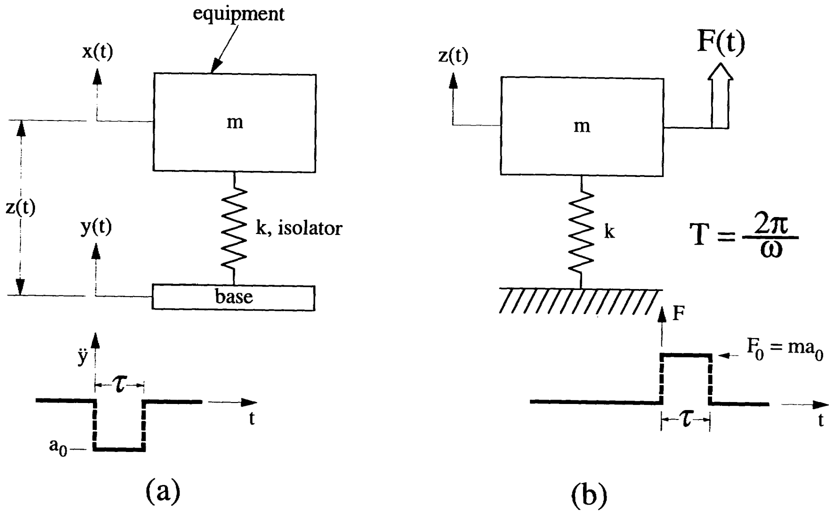

FIGURE 1 Dynamic equivalence of (a) base motion shock and (b) force shock.

RSS $= \omega|F(\omega)|$ where $F(\omega)$ is the Fourier spectrum.)

The shock transmissibility (T_S) can be defined as

$$T_S = \frac{\text{maximax}[kz]}{F_0}, \tag{1}$$

or, more generally, as

$$T_S = \frac{D(\omega)}{Z_0}, \tag{2}$$

where $D(\omega)$ is the maximax shock spectrum in terms of relative displacement as a function of the frequency, ω, and Z_0 is the maximum static deflection that would result if the force pulse were applied slowly.

The variation of T_S with τ/T for a rectangular pulse is shown in Fig. 3 where τ is the pulse duration in time and T is the natural period of the SDOF oscillator. T_S for other pulse shapes are shown in Fig. 4.

A more detailed study of Fig. 2 shows that $T_S \leq 1$ (shock isolation) if $\tau/T \leq 1/6$ or

$$\omega \leq \frac{\pi}{3\tau}. \tag{3}$$

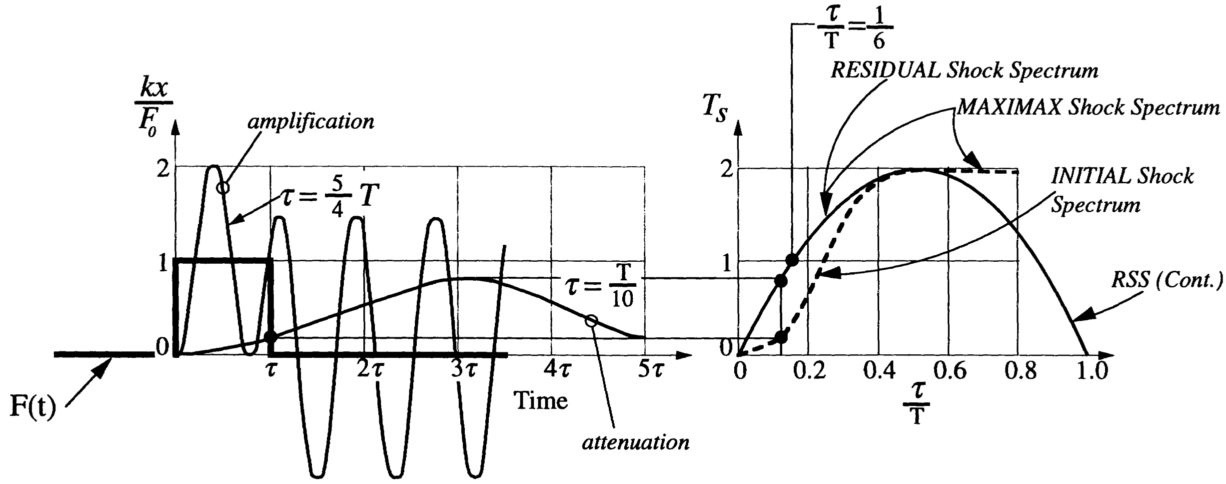

FIGURE 2 Points on the initial shock spectrum (ISS) and the residual shock spectrum (RSS) for $\tau/T = 1/10$. Adapted from Norris et al. (1959).

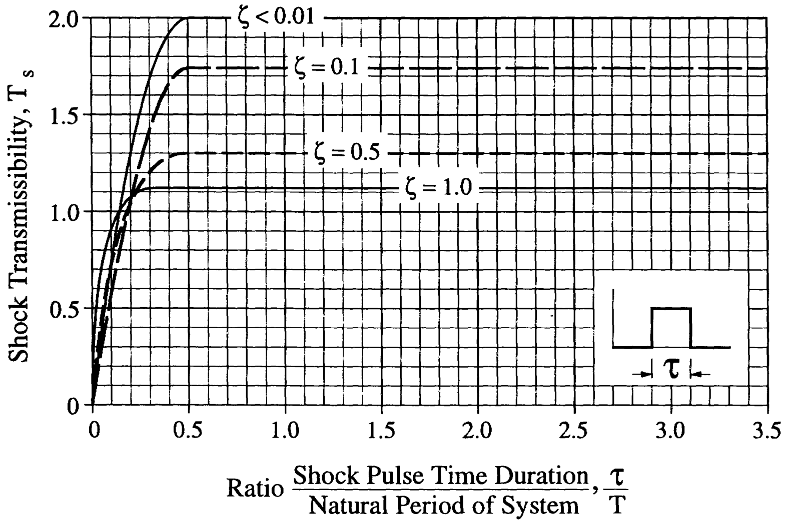

FIGURE 3 The shock spectrum (SS) for a rectangular shock pulse; ζ = viscous damping ratio. Used by permission from Barry Controls (1984).

The slightly conservative form $\omega \leq 1/\tau$ is easier to remember.

Now consider an arbitrary (arb.) force pulse, $F(t)$, that is circumscribed by the rectangular (rect.) force pulse F_0, τ (see Fig. 5). Let T_S (arb.) be the shock transmissibility associated with $F(t)$ and T_S (rect.) be the shock transmissibility for the circumscribing rectangular pulse. For an oscillator initially at rest, it can be shown (Frolov and Furman, 1990) that if T_S (rect.) ≤ 1, then T_S (arb.) ≤ 1. In other words, if the circumscribing rectangular pulse leads to isolation, the pulse $F(t)$ will also be isolated. This result becomes obvious if τ is much less than T. For this limiting case, the oscillator response is governed solely by the impulse associated with each force pulse, i.e., by the area under their respective force vs. time functional forms; clearly the area under $F(t)$ is less than F_0, τ. On this basis, the rectangular pulse is widely used for the preliminary design of shock isolation systems even though it is unlikely to be encountered in practice.

It can also be shown that $\omega^2 D(\omega) \approx A(\omega)$, where $A(\omega)$ is the SS in terms of absolute acceleration and where the equality is close for small damping and exact for zero damping. The term $V(\omega) = \omega D(\omega)$ is equal to neither the absolute nor the relative velocity SS and hence is usually referred to as the pseudovelocity SS. These three SS, $D(\omega)$, $V(\omega)$, and $A(\omega)$, can be compactly displayed on a four-way logarithmic chart just like their sinusoidal counterparts (see Schiff, 1990).

ISOLATION AGAINST VELOCITY SHOCK

The classical problem of velocity shock is a package falling freely through a distance h and impacting a rigid surface (see Fig. 6). The classic article on this problem is that by Mindlin (1945). Following Mindlin, the expression for conservation of energy of a cushioned package in contact with a rigid surface is

$$\frac{1}{2} m\dot{x}^2 + \int_0^x P(\xi)\, d\xi = mg(h + x), \qquad (4)$$

where $P(\xi)$ is the force in the cushion. The maximum cushion displacement, x_m, will occur when $\dot{x} = 0$. If, in addition to $\dot{x} = 0$, $h \gg x_m$, Eq. (4) simplifies to

$$\int_0^{x_m} P(\xi)\, d\xi = mgh, \qquad (5)$$

which gives x_m if $P(\xi)$ is known. The maximum acceleration, $\ddot{x}_m$, will also occur at $\dot{x} = 0$, hence

$$\ddot{x}_m = \frac{P_m}{m}, \qquad (6)$$

where P_m is $P(\xi)$ evaluated at $\xi = x_m$. If G_m is defined as the maximum acceleration in gs, then

$$G_m = \frac{P_m}{mg}. \qquad (7)$$

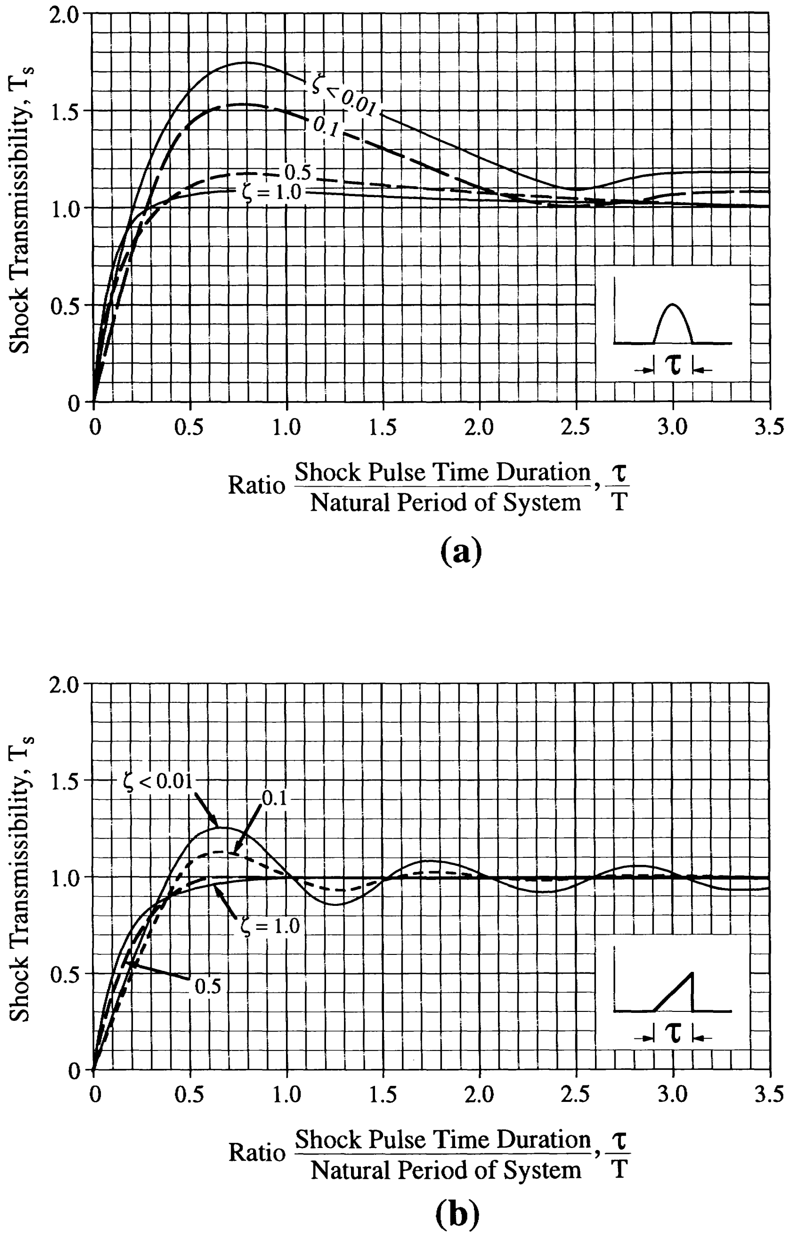

FIGURE 4 The shock spectrum (SS) for (a) a half-sine shock pulse and (b) a terminal sawtooth shockpulse; ζ = viscous damping ratio. Note that for these pulses, $T_s \rightarrow 1$ as $\tau/T \rightarrow \infty$, i.e., for quasistatic loading. Used by permission from Barry Controls (1984).

The procedure for determining G_m is then

1. find x_m from Eq. (5),
2. find P_m from $P_m = P(x_m)$, and
3. find G_m from Eq. (7).

For the case of a linear cushion, i.e., $P = kx$, this procedure yields

$$G_m = \sqrt{\frac{2h\omega_n^2}{g}}, \tag{8}$$

where $\omega_n = [k/m]^{1/2}$ is the natural circular frequency of the cushion-mass system. Rather than linear cushions, most packaged equipment is supported by a cushion that is made of a strongly nonlinear material such as plastic foam or latex-

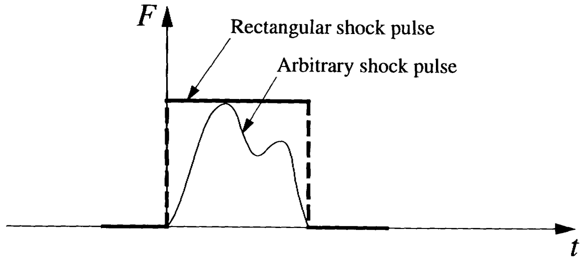

FIGURE 5 A rectangular shock pulse (rect.) that circumscribes an arbitrary shock pulse (arb.).

bound fibers. While the above procedure is the same, its implementation is different. The nonlinear equivalent of Eq. (8) was formulated by Janssen (1952) as

$$G_m = J\frac{h}{t}, \tag{9}$$

where h is the drop height, t is the thickness of the cushion, and J is the so-called cushion factor, the ratio of the peak stress developed in the cushion to the energy stored per unit volume of the cushion.

Mustin (1968) provided a proper derivation of Eq. (9) as well as a comprehensive review of velocity shock for items mounted on nonlinear cushions.

As cushion strain increases, G_m decreases because of the increased ability of the cushion to absorb energy; however, at increased levels of strain, many nonlinear materials, such as plastic foam, begin to stiffen and this increases G_m. Hence, for such material J will have a minimum, denoted by J_0. This inference is supported by test results (see Fig. 7, which is taken from Henny and Leslie, 1962). Mustin shows how to determine J_0 from a knowledge of the stress–strain behavior of the cushion material.

If geometric and material constraints allow, the package engineer should design the cushion system to operate in the vicinity of J_0 and to provide a G_m that does not exceed the fragility of the packaged item. In this sense, the figure of merit for velocity shock mitigation becomes G_m rather than transmissibility.

The fragility, or damage sensitivity, of a piece of equipment or structure is the locus of motion parameters that first induce failure, malfunction, or, in the case of consumer products, loss of acceptable appearance. This locus is plotted in a space of G_m vs. Δv where Δv is the velocity change of the package falling from rest to impact. Figure 8 is a schematic fragility curve. A discussion of such curves is given by Kornhauser (1964), and a testing protocol for measuring shock fragility is given in ASTM D3332 (1993). (Note that Kornhauser plots Δv vs. G_m.) An acceptable value of G_m is one that falls safely into the no-damage region of the equipment's fragility curve.

Lacking the certainty of an experimentally determined fragility curve, it is reasonable to substitute the values contained in the qualification test specification for the packaged mechanical component or electrical equipment.

ISOLATION AGAINST COMPLEX SHOCK

Pulselike shocks and velocity shocks are not oscillatory. However, the shock inputs associated with several types of shock events are highly os-

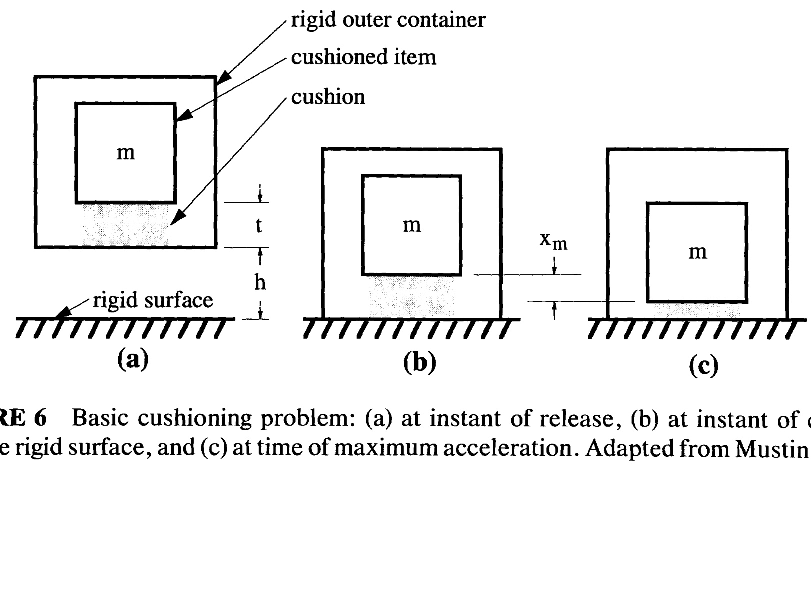

FIGURE 6 Basic cushioning problem: (a) at instant of release, (b) at instant of contact with the rigid surface, and (c) at time of maximum acceleration. Adapted from Mustin (1968).

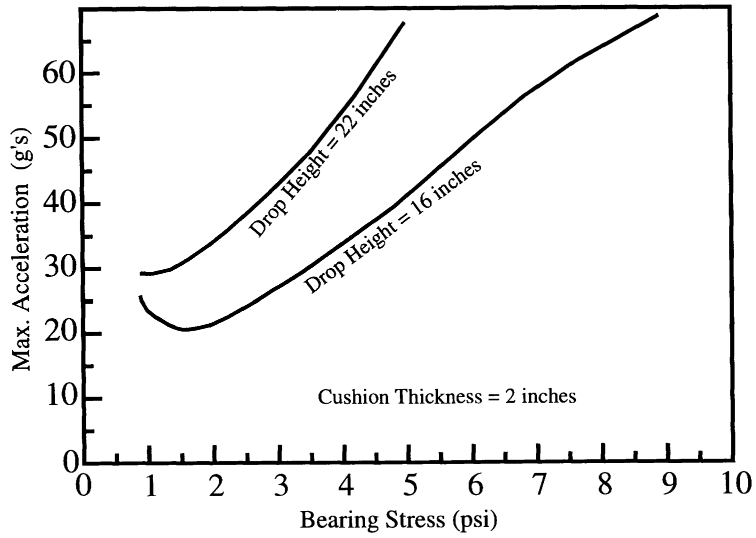

FIGURE 7 Maximum acceleration versus bearing stress curves for a polyurethane foam.

cillatory. Such events are referred to as complex shocks. A specialized procedure was developed to assess the response of structures and equipment subjected to complex shocks. This specialized procedure, called the response spectrum method (RSM), was first suggested by Biot in 1933.

In the 1950s the RSM was adapted by the Naval Research Laboratory to predict equipment "hardness" against the shocks (i.e., explosions) encountered by surface vessels and submarines during combat (see Belsheim and O'Hara, 1960). A review of the subsequent evolution of the RSM in the Navy was provided by Remmers (1982). In the 1970s it was widely used in the nuclear power industry to ensure protection of power stations from earthquakes (see Gupta, 1990). In the 1980s the method was extended to cover the situation of large structures, e.g., buildings, that were isolated from earthquake motions by rubber blocks (see Skinner et al., 1993; Kelly, 1993). The use of elastomeric shock isolators for entire buildings seems to have started in England (Waller, 1969). Now there are seismically isolated buildings in more than 17 countries and over 100 such structures have been built or are under construction; for a complete review see Buckle and Mayes (1990). For a discussion of resiliently mounted

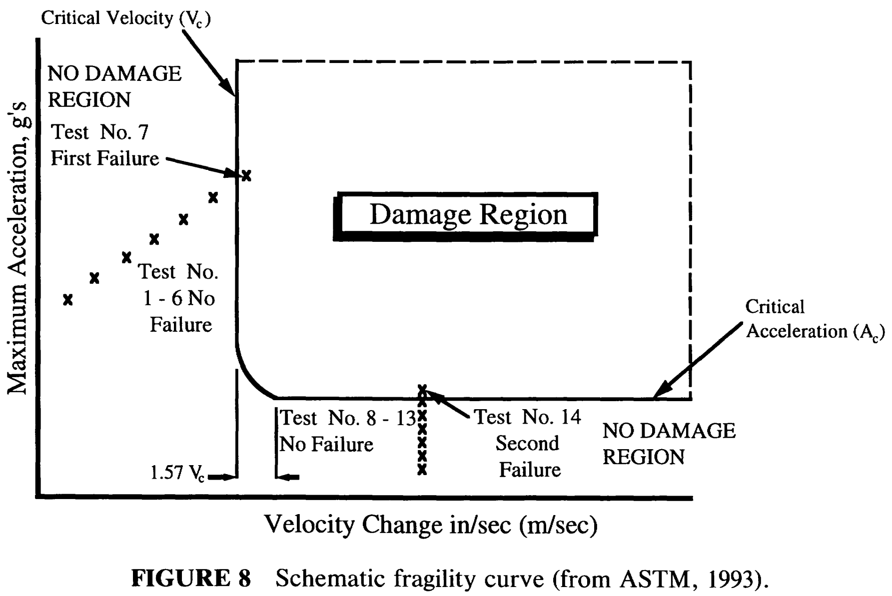

FIGURE 8 Schematic fragility curve (from ASTM, 1993).

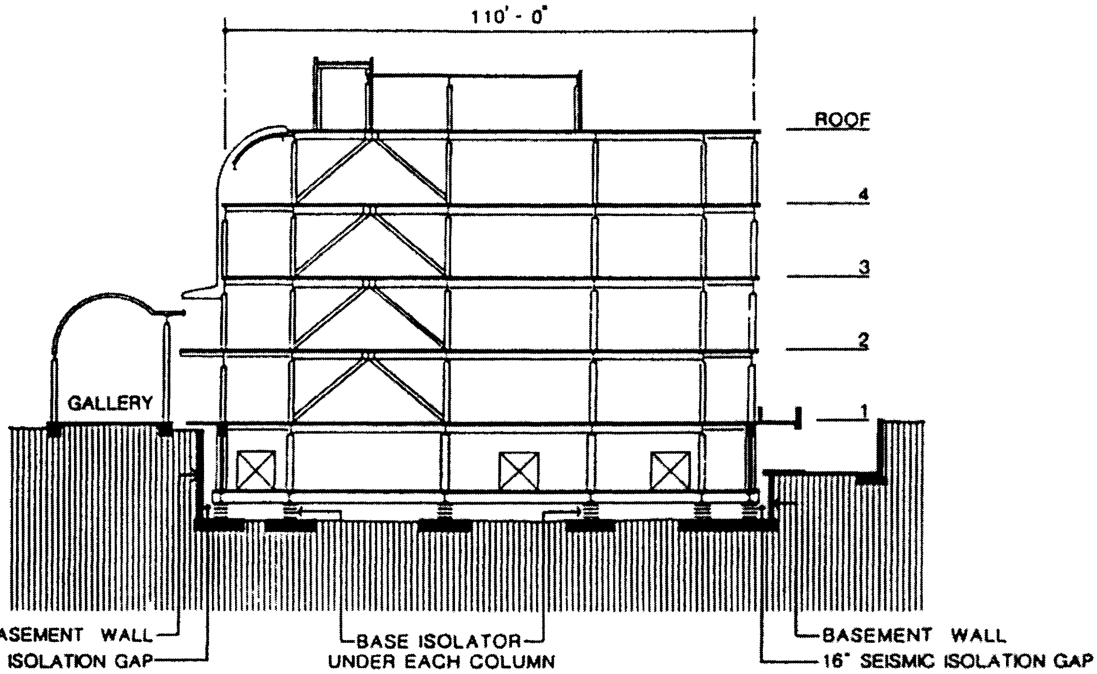

FIGURE 9 Cross section of the Foothill Communities Law and Justice Center showing isolators in the subbasement. Used by permission from Kelly (1993).

mechanical equipment subjected to seismic shock, see Lama (1994). The first base-isolated building in the United States is the Foothill Community Law and Justice Center in Rancho Cucamonga, California, built in 1985 (see Fig. 9).

To illustrate the RSM for a base-isolated structure, consider the problem of Fig. 10, which is adapted from Kelly (1993). The equations of motion for the undamped, base-isolated structure in Fig. 10 are

$$\begin{bmatrix} M & m_b \\ m_b & m_b \end{bmatrix} \begin{Bmatrix} \ddot{z}_s \\ \ddot{z}_b \end{Bmatrix} + \begin{bmatrix} k_I & O \\ O & k_b \end{bmatrix} \begin{Bmatrix} z_s \\ z_b \end{Bmatrix} = - \begin{bmatrix} M & m_b \\ m_b & m_b \end{bmatrix} \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \ddot{y}, \tag{10}$$

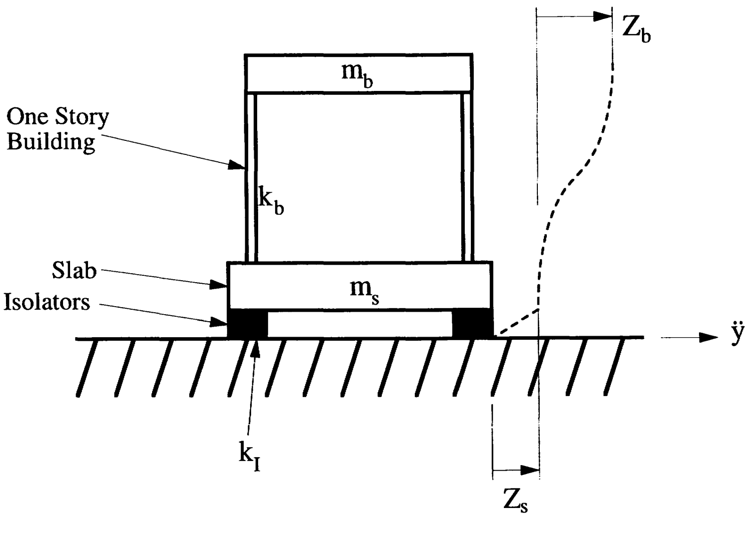

FIGURE 10 Model of a single-story shear building on isolators; $\ddot{y}$ is the ground acceleration due to earthquake.

where $M = m_s + m_b$, the total mass; $\ddot{y}$ is the seismic excitation time history of the ground; and z the relative displacement with respect to the ground.

Equation (10) can be rewritten in the form

$$[M]\{\ddot{z}\} + [K]\{z\} = -[M]\{r\}\ddot{y}. \quad (11)$$

Denote the eigenvectors (normal modes) associated with Eq. (11) by $\{\varphi_1\}$, $\{\varphi_2\}$ and the eigenvalues (natural frequencies) by ω_1, ω_2. Using the coordinate transformation

$$\{z(t)\} = [\{\varphi_1\}\{\varphi_2\}]\{q(t)\}, \quad (12)$$

where

$$\{q(t)\} = \{q_1(t), q_2(t)\}^T \quad (13)$$

are the modal coordinates, Eq. (11) is converted into two, uncoupled SDOF equations, i.e.,

$$\ddot{q}_i + \omega_i^2 q_i = -L_i\ddot{y} \quad i = 1, 2 \quad (14)$$

where

$$L_i = \frac{\{\varphi_i\}^T[M]\{r\}}{\{\varphi_i\}^T[M]\{\varphi_i\}} \quad i = 1, 2 \quad (15)$$

are the modal participation factors. Further study of Eq. (14) reveals that $i = 1$ corresponds closely to a mode wherein the building acts as a rigid body on flexible isolators while the deformation of the building structure is confined principally to the $i = 2$ mode. In consequence, the usual design situation would be $\omega_2 \gg \omega_1$. Using Eq. (15) one can show that if $\omega_2 \gg \omega_1$, then $L_2 \ll L_1$. This result is the essence of the seismic isolation scheme shown in Fig. 10: low shear flexibility in the seismic isolators insures low participation by the modes that contain significant structural deformation.

Guided by previous discussions, it can be shown that

$$\text{maximax}\{z_i\} = \{\varphi_i\}L_i D(\omega_i), \quad (16)$$

where $D(\omega_i)$ is the relative displacement shock spectrum associated with $\ddot{y}$ evaluated at ω_i. If R_{ij} is the maximax response of the jth DOF when the structure is vibrating in its ith mode, Eq. (16) can be reduced to

$$R_{ij} = \varphi_{ij}L_i D(\omega_i), \quad (17)$$

where φ_{ij} and L_i come from the lumped parameter model of the structure and $D(\omega_i)$ is deduced from a study of measured complex excitations, be they earthquakes or explosions.

There remains one uncertainty with the use of Eq. (17). The maximax operator used in Eq. (16) removes all phase information among the modes. So it is uncertain how best to sum R_{ij} with respect to the modal index i. The obvious choice, a sum of the absolute values of R_{ij}, is usually much too conservative for cost-effective design. The most common choice is the square root of the sum of squares, i.e.,

$$R_j = \left[\sum_i R_{ij}^2\right]^{1/2}. \quad (18)$$

This choice effectively assumes that the various modes are uncorrelated, and it therefore works best when the modes are widely spaced. A discussion of how to proceed when the modes are closely spaced can be found in Chopra (1995).

There seems no general agreement about a figure of merit for seismic excitation. One can borrow a term from acoustics and define an insertion loss

$$\mathrm{IL}_j = 20 \log \frac{R_{j,h}}{R_{j,s}}, \quad (19)$$

where IL_j is the insertion loss for the jth DOF, $R_{j,s}$ is the combined modal response at the jth DOF for a soft mounted (i.e., isolated) structure, and $R_{j,h}$ is the combined modal response at the jth DOF for a hard-mounted structure (i.e., a structure with the isolators replaced by rigid connections). A 20-dB insertion loss would then correspond to an order of magnitude reduction in response due to the presence of isolators.

The design of large (1-m diameter) seismic shock isolators to carry large axial loads (e.g., 1000 tons) while permitting a low natural frequency in shear (e.g., 1 Hz) is a specialized but well-developed technology. A design of growing acceptance is the use of a highly filled natural rubber prism with embedded horizontal steel plates for enhanced vertical stiffness. For example, the design in Fig. 9 uses 98 such isolators. Design details and test results can be found in Skinner et al. (1993) and Kelly (1993).

CONCLUSION

Three of the major fields of mechanical shock were surveyed: pulselike shocks, velocity shocks, and complex shocks. Shock mitigation figures of merit appropriate to each field were defined and discussed. There was no discussion of shock testing; however, a sampling of this field can be found in Kao (1975), Harris (1988), and Hudson (1991).

REFERENCES

ASTM, *D3332,* 1993, *Standard Test Methods for Mechanical-Shock Fragility of Products, Using Shock Machines,* Vol. 15.09, ASTM, Philadelphia, PA.

Barry Controls, 1984, *Application Selection Guide,* Barry Wright Corp, Watertown, MA.

Belsheim, R. O., and O'Hara, G. J., 1960, *Shock Design of Shipboard Equipment: Part I—Dynamic Design-Analysis Method,* Naval Research Lab Report 5545, Washington, DC.

Biot, M. A., 1933, "Theory of Elastic Systems under Transient Loading with an Application to Earthquake Proof Buildings," *Proceedings of the National Academy of Sciences,* Vol. 19, pp. 262–268.

Buckle, I. G., and Mayes, R. L., 1990, "Seismic Isolation: History, Application and Performance—A World View" *Earthquake Spectra,* Vol. 6, pp. 161–201.

Chopra, A. K., 1995, *Dynamics of Structures,* Prentice–Hall, Englewood Cliffs, NJ.

Frolov, K. V., and Furman, F. A., 1990, *Applied Theory of Vibration Isolation Systems,* Hemisphere Pub. Co., Washington, DC.

Gupta, A. K., 1990, *Response Spectrum Method,* Blackwell Scientific Dist. C, London, UK.

Harris, C. M. (Ed.), 1988, *Shock & Vibration Handbook,* 3rd ed., *Shock Testing Machines,* McGraw–Hill, New York, Chap. 26.

Henny, C., and Leslie, F., 1962, *An Approach to the Solution of Shock and Vibration Isolation Problems as Applied to Package Cushioning Materials,* Shock and Vibration Bullletin 30, Naval Research Laboratory, Washington, DC.

Hudson, D., 1991, "Shock Test Guide for the Evaluation Engineer," *Shock and Vibration Digest,* Vol. 23, No. 2, pp. 15–23.

Jacobsen, L. S., and Ayre, R. S., 1958, *Engineering Vibrations,* McGraw–Hill, New York.

Janssen, R. R., 1952, *A Method for the Proper Selection of a Package Cushion Material and its Dimensions,* Report NA-51-1004, North American Aviation, Inc., Los Angeles, CA.

Kao, G. C., 1975, "Testing Techniques for Simulating Earthquake Motion," *Journal of Environmental Sciences,* March/April, pp. 22–48.

Kelly, J. M., 1993, *Earthquake-Resistant Design with Rubber,* Springer–Verlag, New York.

Kornhauser, M., 1964, *Structural Effects of Impact,* Spartan Books, Inc., Baltimore, MD.

Lama, P. J., 1994, "Effects of Seismic Inputs on Resiliently Mounted Mechanical Equipment," *Sound and Vibration,* July.

Mindlin, R., 1945, "Dynamics of Package Cushioning," *Bell System Technical Journal,* Vol. 24, pp. 353–461.

Mustin, G. S., 1968, *Theory and Practice of Cushion Design,* Shock and Vibration Information Center, SVM-2, Washington, DC.

Nelson, F. C., 1994, "Vibration Isolation: A Review, I. Sinusoidal and Random Excitations," *Shock and Vibration,* Vol. 1, pp. 485–493.

Norris, C. H., Hansen, R. J., Holley, M. J., Biggs, J. M., Namyet, S., and Minani, J. K., 1959, *Structural Design for Dynamic Loads,* McGraw–Hill, New York.

Remmers, G., 1982, Maurice Biot 50th Anniversary Lecture—*The Evolution of Spectral Techniques in Navy Shock Design,* Shock and Vibration Bulletin 50, Naval Research Laboratory, Washington, DC.

Schiff, P., 1990, *Dynamic Analysis and Failure Modes of Simple Structures,* Wiley, New York.

Skinner, R. I., Robinson, W. H., and McVerry, G. H., 1993, *An Introduction to Seismic Isolation,* Wiley, New York.

Waller, R. A., 1969, *Buildings on Springs,* Pergamon Press, Oxford, UK.