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Research Article

A Novel Discrete Grey Model and Its Application

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This paper aims to further increase the prediction accuracy of the grey model based on the existing discrete grey model, DGM(1,1). Herein, we begin by studying the connection between forecasts and the first entry of the original series. The results comprehensively show that the forecasts are independent of the first entry in the original series. On this basis, an effective method of inserting an arbitrary number in front of the first item of the original series to extract messages is applied to produce a novel grey model, which is abbreviated as FDGM(1,1) for simplicity. Incidentally, the proposed model can even forecast future data using only three historical data. To demonstrate the effectiveness of the proposed model, two classical examples of the tensile strength and life of the product are employed in this paper. The numerical results indicate that FDGM(1,1) has a better prediction performance than most commonly used grey models.

1. Introduction

Professor Deng [1] pioneered the grey system theory in 1982; it was regarded as an appreciative approach for dealing with poor information and small samples. The grey system model, which is a crucial fraction of grey system theory, has been widely used in numerous fields, particularly in energy field. For example, to forecast quarterly hydropower production of China, Wang et al. [2] proposed a grey forecasting method based on a data grouping approach. In 2016, Zeng and Li [3] studied a self-adapting intelligent grey model to accurately forecast the natural gas demand in China. Ding et al. [4] investigated a rolling grey model based on the optimization of the initial condition for forecasting China's electricity consumption. Wang et al. [5] used an optimized NGBM(1,1) model to forecast the qualified discharge rate of industrial wastewater in China. In particular, the traditional grey model was considered as a standard model for most applications, which is often abbreviated as GM(1,1) where the first "1" represents the first-order and the second "1" represents the univariate. Based on this, most researchers focused on the number of variables and proposed the multivariate grey model, for example, Ma et al. [6–8], Zeng et al. [9, 10] and others [11]. Additionally, the research results related to the multivariate grey model appeared continuously.

In most applications thus far, the primary characteristics of the grey system theory are known to be the accumulative generating operation (AGO) [12]. In other words, the AGO is essential in the whole grey system, which helps transform the nonnegative smooth discrete function into a series obeying the pure exponential law or an approximate one [13]. It reduces the randomness of the original data and strengthens the statistical rule hidden in the original data. In most of the existing literature, it is easy to see these models apply the integer-order accumulative generating operation, the first-order accumulative generating operation (1-AGO) in particular, which results in a less flexible-model time series in real applications. First, Wu et al. [14] pointed out the novel grey system model with fractional order accumulation. Further, the numerical results demonstrated in their work significantly contributed to an improved prediction performance in the grey system model and theory. Moreover, their study was applied in various fields and further improved. More recently, Wu et al. [15] investigated a novel conformable fractional nonhomogeneous grey model which is based on the new definitions of the conformable fractional accumulation and difference for forecasting carbon dioxide emissions of BRICS countries. Similarly, the results of research conducted using the fractional grey system model are emerging continually.

Meanwhile, the discrete grey system model is always the focus. Xie and Liu [16] initially proposed the discrete grey model DGM(1,1), which is based on the traditional grey model GM(1,1). Further, he analyzed the relationship between DGM(1,1) and GM(1,1). Subsequently, Ma and Liu [17] applied their thought to GM(1,1), consequently proposing a discrete GM(1,1) model. Additionally, Tien [18] reported the appealed research results, which state that modeling data and forecasts are independent of the first entry of the original series. Thus, the first entry by the GM(1,1) and GM(1,n) models separately is inefficient, implying the existence of a fairly good approach to increase prediction performance by inserting an arbitrary number before the first entry to extract the messages.

On these theoretical bases, we propose a novel discrete grey system model, FDGM(1,1). The main contributions of this paper are summarized as follows:

(1) The relation between forecasts and the first entry of the original series is studied.

(2) Proof of the forecasts from DGM(1,1) being independent of the first entry of the original series is presented.

(3) The novel discrete grey system model FDGM(1,1) is discussed.

(4) The two empirical examples are used to confirm the accuracy of the proposed model.

The remainder of this paper is organized as follows. Section 2 describes the modeling procedure of the existing discrete grey system model. Section 3 gives a detailed analysis of the connection between forecasts and the first entry of the original series and presents a novel grey modeling method. The modeling evaluation criteria and detailed computational steps are detailed in Section 4. Section 5 confirms the effectiveness and applicability of the proposed model compared with other commonly used grey system models, and the main conclusions and future research potential are listed in the last section.

2. The Description of DGM(1,1)

Suppose

$$X^{(0)} = \left\{x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right\}, \quad (1)$$

is a nonnegative series, and the first-order accumulative generating operator (1-AGO) series is

$$X^{(1)} = \left\{x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right\}, \quad (2)$$

where $x^{(1)}(k) = \sum_{i=1}^{k} x^{(0)}(i)$, $k = 1, 2, \ldots, n$. The equation

$$x^{(1)}(k+1) = \beta_1 x^{(1)}(k) + \beta_2, \quad (3)$$

is called the basic DGM(1,1). Additionally, the system parameters β_1 and β_2 could be estimated by using the least square method, which are

$$(\beta_1, \beta_2)^{\mathrm{T}} = \left(B^{\mathrm{T}} B\right)^{-1} B^{\mathrm{T}} Y, \quad (4)$$

where

$$B = \begin{bmatrix} x^{(1)}(1) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{bmatrix},$$
$$Y = \left[x^{(1)}(2), x^{(1)}(3), \ldots, x^{(1)}(n)\right]^{\mathrm{T}}. \quad (5)$$

Then, the recursive function of (3) could be written as follows, given that the initial condition is $x^{(1)}(1) = x^{(0)}(1)$:

$$\widehat{x}^{(1)}(k+1) = \beta_1^k \left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right) + \frac{\beta_2}{1-\beta_1}. \quad (6)$$

Using the first-order inverse accumulative generating operation (IAGO), the restored values of $x^{(0)}(k)$ are obtained as follows:

$$\widehat{x}^{(0)}(k) = \left(\beta_1^k - \beta_1^{k-1}\right)\left(x^{(0)}(1) - \frac{\beta_2}{1-\beta_1}\right). \quad (7)$$

3. Methodology

3.1. Study on the Connection between Forecasts and the First Entry of Original Series. To investigate the connection between forecasts and the first entry of the original series, according to [12], we added an arbitrary constant c to the first entry, i.e., and we further obtained $x^{(0)}(1) + c$. Correspondingly, matrices B and Y became

$$D = \begin{bmatrix} x^{(1)}(1)+c & 1 \\ x^{(1)}(2)+c & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1)+c & 1 \end{bmatrix},$$
$$Y_R = \left[x^{(1)}(2)+c, x^{(1)}(3)+c, \ldots, x^{(1)}(n)+c\right]^{\mathrm{T}}. \quad (8)$$

The assumption that the forecasts are independent of the first entry of the original series will hold if forecasts obtained using $x^{(0)}(1) + c$ equal to those obtained using $x^{(0)}(1)$. We incidentally introduced the product theory of the determinant as a lemma because we need it to complete this computational process.

Lemma 1 (see [19]). *If* $\begin{bmatrix} A & 0 \\ -I & C \end{bmatrix}$ *and* $\begin{bmatrix} I & A \\ 0 & I \end{bmatrix}$ *are both partitioned matrices, where A and C are two matrices of the orders* $p \times q$ *and* $q \times p$*, respectively, then, the following equations hold true:*

$$\begin{bmatrix} I & A \\ 0 & I \end{bmatrix}\begin{bmatrix} A & 0 \\ -I & C \end{bmatrix} = \begin{bmatrix} 0 & AC \\ -I & C \end{bmatrix},$$
$$\begin{vmatrix} A & 0 \\ -I & C \end{vmatrix} = \begin{vmatrix} 0 & AC \\ -I & C \end{vmatrix} = |AC|. \quad (9)$$

Therefore, the adjoint matrix of $D^{\mathrm{T}} D$ was written as $(D^{\mathrm{T}} D)^*$. The multiplier $(D^{\mathrm{T}} D)^{-1}$ could be written as $(D^{\mathrm{T}} D)^{-1} = (D^{\mathrm{T}} D)^* / |D^{\mathrm{T}} D|$. Further, (4) could be rewritten as

$$(\gamma_1, \gamma_2)^{\mathrm{T}} = \frac{(D^{\mathrm{T}}D)^*}{|D^{\mathrm{T}}D|} D^{\mathrm{T}} Y. \quad (10)$$

According to Lemma 1, $|D^{\mathrm{T}}D|$ can be changed to

$$\left|D^{\mathrm{T}}D\right| = \begin{vmatrix} x^{(1)}(1)+c & x^{(1)}(2)+c & \cdots & x^{(1)}(n-1)+c & 0 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & x^{(1)}(1)+c & 1 \\ 0 & -1 & 0 & 0 & x^{(1)}(2)+c & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & x^{(1)}(n-1)+c & 1 \end{vmatrix}. \quad (11)$$

(11) was easily yielded

$$\left|D^{\mathrm{T}}D\right| = \begin{vmatrix} x^{(1)}(1) & x^{(1)}(2) & \cdots & x^{(1)}(n-1) & 0 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & x^{(1)}(1) & 1 \\ 0 & -1 & 0 & 0 & x^{(1)}(2) & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & x^{(1)}(n-1) & 1 \end{vmatrix} = \left|B^{\mathrm{T}}B\right|. \quad (12)$$

Naturally, the system parameters γ_1 and γ_2 could be rewritten, respectively, as

$$\gamma_1 = \frac{\Gamma_1}{|D^{\mathrm{T}}D|}, \qquad \gamma_2 = \frac{\Gamma_2}{|D^{\mathrm{T}}D|}, \quad (13)$$

where Γ_1 and Γ_2 are corresponding determinants obtained by replacing the first and second row of $|D^{\mathrm{T}}D|$ by Y_R. Then,

$$\gamma_1 = \frac{1}{|D^{\mathrm{T}}D|} \begin{vmatrix} x^{(1)}(2)+c & x^{(1)}(3)+c & \cdots & x^{(1)}(n)+c & 0 & 0 \\ 1 & 1 & \cdots & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 & x^{(1)}(1)+c & 1 \\ 0 & -1 & 0 & 0 & x^{(1)}(2)+c & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & x^{(1)}(n-1)+c & 1 \end{vmatrix},$$

$$\gamma_2 = \frac{1}{|D^{\mathrm{T}}D|} \begin{vmatrix} x^{(1)}(1)+c & x^{(1)}(2)+c & \cdots & x^{(1)}(n-1)+c & 0 & 0 \\ x^{(1)}(2)+c & x^{(1)}(3)+c & \cdots & x^{(1)}(n)+c & 0 & 0 \\ -1 & 0 & 0 & 0 & x^{(1)}(1)+c & 1 \\ 0 & -1 & 0 & 0 & x^{(1)}(2)+c & 1 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & -1 & x^{(1)}(n-1)+c & 1 \end{vmatrix}. \quad (14)$$

By solving these equations, we obtained

$$\gamma_1 = \beta_1, \quad \gamma_2 = \beta_2 + c - c\beta_1. \tag{15}$$

From (6), we know that

$$\left(\gamma_1^k - \gamma_1^{k-1}\right)\left(x^{(0)}(1) - \frac{\gamma_2}{1-\gamma_1}\right) = \left(\beta_1^k - \beta_1^{k-1}\right)\left(x^{(0)}(1) + c - \frac{\beta_2 + c - c\beta_1}{1-\beta_1}\right) = \hat{x}^{(0)}(k). \tag{16}$$

Forecasts obtained using $x^{(0)}(1)+c$ equal those obtained using the first entry; this implies that forecasts are independent of the first entry of original series.

3.2. The Presentation of FDGM(1,1). Based on the above proof, we could use the method proposed by Tien, to rebuild the novel discrete grey model, FDGM(1,1), by inserting an arbitrary number (generally, 0 is considered for simplicity) before the first entry of the original series to extract messages ultimately enhancing the prediction ability of the grey model. We consider that this model shares a similar modeling procedure as DGM(1,1) with some modification. Thus, we only list the modified fraction of the model. First, the modeling data become

$$X^{(0)} = \left\{x^{(0)}(0), x^{(0)}(1), \ldots, x^{(0)}(n)\right\}. \tag{17}$$

Remember that the traditional grey model requires at least four data to model; however, we insert a constant in front of the first entry of the original series herein. This implies that the novel grey model only requires three data to build the prediction model.

Correspondingly, the matrices B and Y should become

$$B = \begin{bmatrix} x^{(1)}(0) & 1 \\ x^{(1)}(2) & 1 \\ \vdots & \vdots \\ x^{(1)}(n-1) & 1 \end{bmatrix}, \quad Y = \left[x^{(1)}(1), x^{(1)}(3), \ldots, x^{(1)}(n)\right]^{\mathrm{T}}, \tag{18}$$

respectively. Consequently, the recursive function also changed to

$$\hat{x}^{(1)}(k+1) = \beta_1^k\left(x^{(0)}(0) - \frac{\beta_2}{1-\beta_1}\right) + \frac{\beta_2}{1-\beta_1}. \tag{19}$$

4. Model Evaluation Indices and Computational Steps

To assess the prediction accuracies of prediction models, two statistical indices are used in this paper, including the mean absolute percentage error (MAPE) and root mean squared error (RMSE), which are defined as

$$\mathrm{MAPE} = \frac{1}{n-1}\sum_{i=2}^{n}\frac{|e(i)|}{x^{(0)}(i)} \times 100\%, \quad \mathrm{RMSE} = \sqrt{\frac{1}{n-1}\sum_{i=2}^{n}(e(i))^2}, \tag{20}$$

respectively, where $e(i) = \hat{x}^{(0)}(i) - x^{(0)}(i)$ represents the simulative residual at time i. Further, we also give the MAPE criteria for measuring forecasting levels of grey models, as listed in Table 1.

The detailed computational steps can be conducted as follows:

Step 1: obtain the original series $X^{(0)}$ and its 1-AGO series $X^{(1)}$.

Step 2: compute the system parameters using the least square method from matrices B and Y mentioned in Subsection 3.2.

Step 3: compute simulative values of $X^{(1)}$, $\hat{X}^{(1)}$, using (19).

Step 4: calculate restored values of $X^{(1)}$, $\hat{X}^{(1)}$, using IAGO.

5. Validation of FDGM(1,1)

This section provides two classical examples for verifying the effectiveness and applicability of FDGM(1,1). In addition, comparative models such as GM(1,1), DGM(1,1), and FGM(1,1) are built in this section.

5.1. Case 1. We consider an example presented by [18]. In this case, it should be noticed that when temperature increases, the hardness and strength of materials have changeable trend with monotonically increasing characteristics. The experimental data on tensile strength from 400°C to 1100°C are shown in Table 2. Empirically, the first six data are used to build prediction models and the remaining data are used to evaluate the prediction accuracies of these models.

The simulated and predicted values are listed in Table 3.

Ignoring the first-fitted value, in Table 3, the GM(1,1) has the worst prediction performance because the largest MAPE was 0.46% and the largest RMSE was 43.28 in the training stage. Additionally, it has the worst prediction accuracy by examining the statistical indices in the verification stage. The

Table 1: MAPE criteria for modeling examination.

MAPE	< 10	10 ~ 20	20 ~ 50	>50
Forecasting ability	Excellent	Good	Reasonable	Weak

Table 2: Raw data of tensile strength from 400°C to 1100°C.

Temperature	Strength (MPa)	Temperature	Strength (MPa)
400	1931	800	1207
500	1724	900	1069
600	1517	1000	952
700	1345	1100	848

Table 3: Fitted and predicted values obtained by different grey system models.

Raw data	GM(1,1)	FGM(1,1)	DGM(1,1)	FDGM(1,1)
1931				
1724	1715.06	1714.10	1716.98	1715.76
1517	1522.70	1522.32	1524.19	1523.58
1345	1351.92	1352.01	1353.04	1352.93
1207	1200.29	1200.74	1201.12	1201.39
1069	1065.67	1066.40	1066.25	1066.83
MAPE	0.46	0.38	0.45	**0.37**
RMSE	43.28	37.06	41.57	**35.28**
952	946.15	947.09	946.52	947.34
848	840.03	841.13	840.24	841.23
MAPE	0.77	0.67	0.75	**0.65**
RMSE	48.81	35.61	45.10	**33.77**

MAPE (0.37% and 0.65%) and RMSE values (35.28 and 33.77) of the FDGM(1,1) are smaller than those of the others in the training or the verification stages. This implies that the FDGM(1,1) performs the best among the commonly used grey models. The relative error generated from these grey models is plotted in Figure 1, revealing the same finding.

5.2. Case 2. We consider an example presented by [20] to further demonstrate the effectiveness of FDGM(1,1), where the raw data describe the failure time during the product. As mentioned in Case 1, we divide raw data into two groups: the first nine data are used to build models, and the others are used to evaluate the prediction accuracies of these models. In this case, the results are given in Table 4.

In Table 4, it is easy to know that the MAPE values of these models are 4.05%, 5.27%, 4.06%, and 3.97%, respectively. The RMSE values of them are 0.92, 0.96, 0.93, and 0.97, respectively. By the MAPE criteria given in Table 1, these four grey models all work quite well in this case. Nonetheless, we can find that FDGM(1,1) still has the best prediction performance with the lowest MAPE values either in the training period or in testing period, meaning FDGM(1,1) could be regarded as a fairly appropriate model to predict lifetime of product in this case.

Table 4 shows the MAPE values of these models at 4.05%, 5.27%, 4.06%, and 3.97%, respectively; the corresponding RMSE values are 0.92, 0.96, 0.93, and 0.97, respectively. The MAPE criteria given in Table 1 imply that the four grey models work quite well in this case. Nonetheless, FDGM(1,1) has the best prediction performance with the lowest MAPE values in the training or testing periods, which implies that FDGM(1,1) could be regarded as optimal for predicting the lifetime of the product in this case.

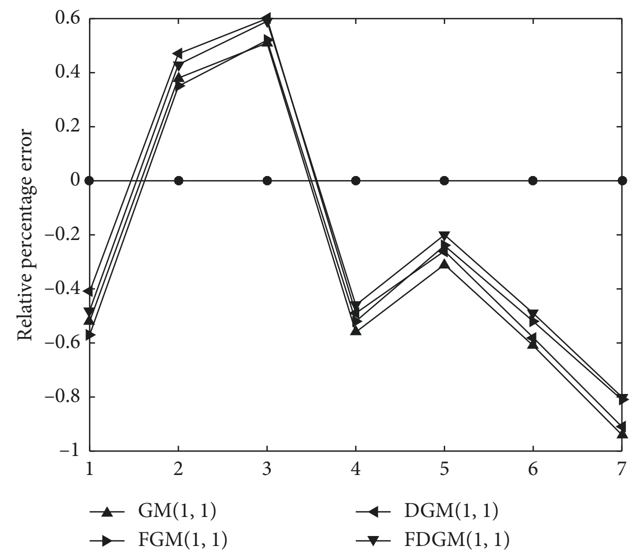

Figure 1: Relative percentage errors generated from different grey models.

Table 4: Fitted and predicted values obtained by different grey system models.

Raw data	GM(1,1)	FGM(1,1)	DGM(1,1)	FDGM(1,1)
7				
9.4	10.52	10.18	10.56	10.23
12.5	12.21	11.90	12.25	11.95
14.0	14.17	13.90	14.22	13.96
15.9	16.44	16.25	16.50	16.32
19.3	19.09	18.99	19.15	19.07
24.1	22.15	22.19	22.23	22.29
25.8	25.71	25.93	25.80	26.04
28.7	29.84	30.30	29.94	30.43
MAPE	4.05	5.27	4.06	**3.97**
RMSE	**0.92**	0.96	0.93	0.97
39.6	34.63	35.41	34.75	35.57
42.2	40.19	41.38	40.33	41.56
MAPE	8.67	6.26	8.33	**5.85**
RMSE	3.79	3.02	3.67	**2.88**

5.3. Further Discussion. As can be observed from the aforementioned studies, inserting an arbitrary number before the first entry to extract messages could enhance the forecasting ability of the grey system model to some extent. However, in some situations, forecasts do not depend on the first entry of the original series. Owing to the high prediction accuracy of the proposed model, we could use it to further predict future data in the next two or more periods. The proposed model could be employed in other fields, such as industry, energy, and economics. This could help engineering planning decision-makers obtain more valuable information and design better strategies to face changes in advance.

6. Conclusion and Future Research

This paper presents a novel grey model by inserting an arbitrary number before the first entry of an original series to extract messages to increase the prediction accuracy of the existing DGM(1,1). However, forecasts do not depend on the first entry of the original series. Therefore, we start by studying the connection between forecasts and the first entry. Nevertheless, forecasts are independent of the first entry; therefore, we propose the novel grey model, which is written as FDGM(1,1) for simplicity. Then, two classical examples were used to evaluate the prediction performance of the proposed model. The numerical results imply that the proposed model has better prediction performance than other commonly used models.

Thus far, we have discussed the advantage of FDGM(1,1); however, some issues must be solved in future research. For instance, the discrete-time response function is an approximate solution to DGM(1,1), which means there is some scope to increase prediction accuracy. However, the rolling grey model shows an appreciative forecasting ability in some cases, which is the another respective we should consider in the future work.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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