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Research Article

Complex Fuzzy Power Aggregation Operators

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A complex fuzzy set is an extension of the fuzzy set, of which membership grades take complex values in the complex unit disk. We present two complex fuzzy power aggregation operators including complex fuzzy weighted power (CFWP) and complex fuzzy ordered weighted power (CFOWP) operators. We then study two geometric properties which include rotational invariance and reflectional invariance for these complex fuzzy aggregation operators. We also apply the new proposed aggregation operators to decision making and illustrate an example to show the validity of the new approach.

1. Introduction

Complex fuzzy set (CFS) is introduced by Ramot et al. [1] as a generalization of the traditional fuzzy set [2]. It is characterized by a complex-valued membership grade including amplitude and phase terms. Therefore, the CFS can describe two features of data. Hence, it is more general than the traditional fuzzy set. For example, if we ask the way and we obtain the data with distance and direction simultaneously, then we may can use the complex-valued membership grades, of which the amplitude and phase parts, respectively, characterize the distance and direction of the destination. To deal with complex fuzzy information, Ramot et al. [1, 3] introduced complex fuzzy operations and complex fuzzy relations. Hu et al. [4–6] introduced the approximate parallelity and orthogonality relations for CFSs. Zhang et al. [7] introduced the δ-equalities between CFSs. Bi et al. [8] introduced two classes of entropy measures for CFSs, Hu et al. [9, 10] and Alkouri and Salleh [11] introduced several distance measures for CFSs.

Recently, some complex fuzzy aggregation operators have been given to aggregate complex fuzzy information. Ramot et al. [3] used vector aggregation to aggregate complex fuzzy information. Ma et al. [12] proposed a product-sum aggregation operator and used it to multiple periodic factor prediction. Bi et al. [13, 14] provided complex fuzzy geometric aggregation operators and complex fuzzy arithmetic aggregation operators. Power mean is one of the famous classical means in classical averaging functions. Motivated by the success of the power mean, many types of power aggregation operators are proposed for different fuzzy environments, including the generalized power aggregation operators [15, 16], intuitionistic fuzzy power aggregation operator [17–19], the interval-valued intuitionistic fuzzy power aggregation operator [20, 21], Pythagorean fuzzy power aggregation operator [22], neutrosophic fuzzy power aggregation operator [23–25], and hesitant fuzzy power aggregation operator [26].

However, the arguments of these power aggregation operators are exact real values. To the best of our knowledge, no research has been conducted on power aggregation operators in the complex fuzzy environment. Therefore, it is necessary to extend the power aggregation operators to the complex fuzzy environment. As mentioned by Ramot et al. [1], the phase term of CFSs is the key feature which essentially distinguish CFSs from other extensions of traditional fuzzy sets. Rotational invariance and reflectional invariance [27]are two of the concepts that mostly depend on the phase term. Complex fuzzy logics, both with and without rotational invariance, are examined by Dick [27]. Interestingly, all complex fuzzy aggregation operators in [3, 13, 14] are rotationally invariant and reflectionally invariant. Therefore, it is necessary to consider complex fuzzy aggregation operators without rotational invariance (or without reflectional invariance).

The aim of this paper is to develop the power aggregation operators for situations with complex fuzzy information. We first present the complex fuzzy weighted power (CFWP) operator which is a generalization of the CFWA operator and develop the complex fuzzy ordered weighted power (CFOWP) operator, which is a generalization of the complex fuzzy ordered weighted arithmetic (CFOWA) operator. These aggregation operators can be used in the complex fuzzy environment. We also present an application of the complex fuzzy power aggregation operators in a decision-making problem concerning the evaluation of a target location.

This paper is organized as follows. In Section 2, we briefly review some basic concepts of complex fuzzy sets. In Section 3, we present the CFWP operator on CFSs. In Section 4, we present the CFOWP operator. In Section 5, we present an application example in decision making. Conclusions are made in Section 6.

2. Preliminaries

2.1. Complex Fuzzy Sets and Complex Fuzzy Operations. First we recall some basic concepts of complex fuzzy sets and complex fuzzy operations [1, 3].

Let D be the set of complex numbers on the complex unit disk, i.e.,

$$D = \{x \in \mathbb{C} \,\|x| \leq 1\}. \quad (1)$$

Suppose U is a fixed universe. A complex fuzzy set A on U is a mapping A: $U \longrightarrow D$. For $x \in U$, the mapping value $A(x)$ can be denoted as

$$r_A(x) \cdot e^{j\nu_A(x)}, \quad (2)$$

where $j = \sqrt{-1}$, the amplitude term $r_A(x) \in [0,1]$, and the phase term $\nu_A(x) \in \mathbb{R}$.

For convenience, this paper only considers the complex numbers on D, which are also called complex fuzzy values (CFVs). Let $a = r_a \cdot e^{j\nu_a}$ be a CFV, in which the amplitude term is $r_a \in [0,1]$ and the phase term is $\nu_a \in \mathbb{R}$. The modulus of a is r_a, also denoted by $|a|$.

Three unary operators of CFVs which include power, rotation, and reflection are defined as follows.

(i) Power of a CFV $a \in D$:

$$a^t = r_a^t \cdot e^{jt\nu_a}, \quad t \neq 0. \quad (3)$$

(ii) Rotation of a CFV $a \in D$ with λ angle:

$$\mathrm{Rot}_\lambda(a) = r_a \cdot e^{j(\nu_a+\lambda)}. \quad (4)$$

(iii) Reflection of a CFV $a \in D$:

$$\mathrm{Ref}(a) = r_a \cdot e^{j-\nu_a}. \quad (5)$$

Since complex fuzzy operators are defined on the complex plane, some authors introduced several geometric properties for complex fuzzy operators [4, 5, 13, 27]. Among these geometric properties, rotational invariance and reflectional invariance are the two commonly used, which are defined as follows [13, 27].

(i) A complex fuzzy operator ν: $D^n \longrightarrow D$ is rotationally invariant if and only if, for any θ,

$$\nu(\mathrm{Rot}_\theta(a_1), \ldots, \mathrm{Rot}_\theta(a_n)) = \mathrm{Rot}_\theta(\nu(a_1, \ldots, a_n)). \quad (6)$$

(ii) A complex fuzzy operator ν: $D^n \longrightarrow D$ is reflectionally invariant if and only if

$$\nu(\mathrm{Ref}(a_1), \ldots, \mathrm{Ref}(a_n)) = \mathrm{Ref}(\nu(a_1, \ldots, a_n)). \quad (7)$$

Rotational invariance [27] and reflectional invariance [13] indicate that an operator is invariant under a rotation and a reflection, respectively, as shown in Figure 1.

Theorem 1. *The power operator is reflectionally invariant.*

Proof. Let $a = r_a \cdot e^{j\nu_a} \in D$ and $t > 0$. Since

$$\begin{aligned}\mathrm{Ref}(a^t) &= \mathrm{Ref}(r_a^t \cdot e^{jt\nu_a}) = r_a^t \cdot e^{-jt\nu_a},\\ (\mathrm{Ref}(a))^t &= (r_a \cdot e^{-j\nu_a})^t = r_a^t \cdot e^{-jt\nu_a}.\end{aligned} \quad (8)$$

Thus, the power operator is reflectionally invariant. □

Theorem 2. *Suppose $t \neq 1$, the power operator is not rotationally invariant.*

Proof. Let $a = r_a \cdot e^{j\nu_a} \in D$ and $t > 0$. Then,

$$\begin{aligned}\mathrm{Rot}_\theta(a^t) &= \mathrm{Rot}_\theta(r_a^t \cdot e^{jt\nu_a}) = r_a^t \cdot e^{j(t\nu_a+\theta)},\\ (\mathrm{Rot}_\theta(a))^t &= \left(r_a \cdot e^{j(\nu_a+\theta)}\right)^t = r_a^t \cdot e^{jt(\nu_a+\theta)}.\end{aligned} \quad (9)$$

Since $(t\nu_a + \theta) \neq t(\nu_a + \theta)$. So, the power operator is not rotationally invariant. □

2.2. Complex Fuzzy Aggregation Operations. Ramot et al. [3] defined the complex fuzzy aggregation operation by a mapping

$$f\colon D^n \longrightarrow D. \quad (10)$$

It is termed as a vector aggregation operator. Then, they gave the complex fuzzy weighted arithmetic (CFWA) operator, i.e.,

$$\mathrm{CFWA}(a_1, a_2, \ldots, a_n) = \sum_{i=1}^{n} w_i a_i, \quad (11)$$

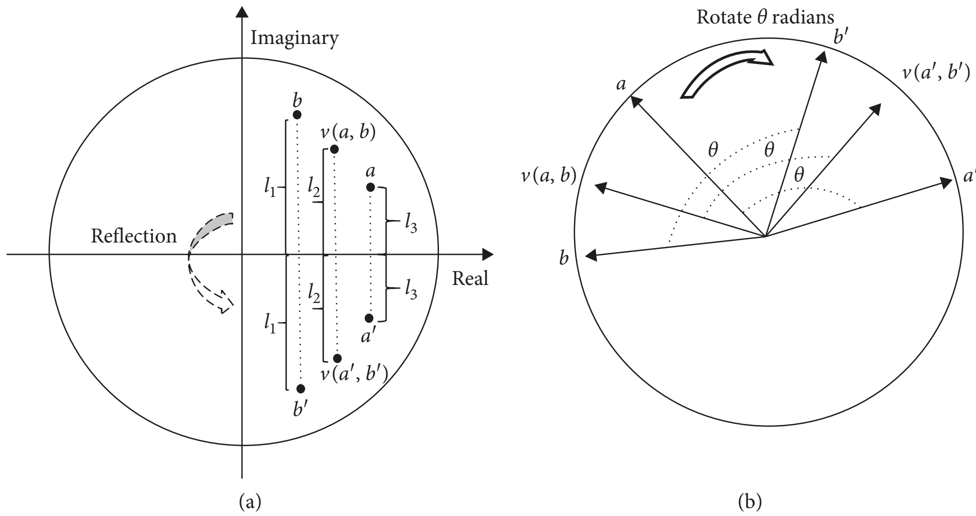

Figure 1: (a) Reflectional invariance and (b) rotational invariance [13].

for any $a_1, a_2, \ldots, a_n \in D$, where $w_i \in D$ for all i and $\sum_{i=1}^{n} |w_i| = 1$. Note that the complex weights are used to make the definition as general as possible [3]. In this paper, we only consider the real-valued weights.

Bi et al. [14] studied CFWA with real-valued weights in details. They [13] also gave the complex fuzzy weighted geometric (CFWG) operator, i.e.,

$$\mathrm{CFWA}(a_1, a_2, \ldots, a_n) = \prod_{i=1}^{n} w_i^{a_i}, \tag{12}$$

for any $a_1, a_2, \ldots, a_n \in D$, where $w_i \in [0,1]$ for all i and $\sum_{i=1}^{n} w_i = 1$.

Both CFWA and CFWG operators have the following results.

Theorem 3 (see [13, 14]). *Both CFWA and CFWG operators are rotationally invariant and reflectionally invariant.*

We see that aggregation operators in [3, 13, 14] are rotationally invariant and reflectionally invariant. Dick [27] firstly introduced rotational invariance for complex fuzzy operations. Interestingly, Dick also examined complex fuzzy logics without rotational invariance based on the framework of vector logic.

3. Complex Fuzzy Weighted Power Aggregation Operators

In this section, we introduce the weighted power aggregation operators in the complex fuzzy environment and discuss their fundamental properties.

Definition 1. Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, a complex fuzzy weighted power (CFWP) operator is defined as follows:

$$\mathrm{CFWG}(a_1, a_2, \ldots, a_n) = \left(\sum_{i=1}^{n} w_i a_i^t \right)^{1/t}, \tag{13}$$

where $w_i \in [0,1]$ for all i, $\sum_{i=1}^{n} w_i = 1$, and t is a parameter such that $t \in (0, +\infty)$.

When $w_i = 1/n$ $(i = 1, 2, \ldots, n)$, then the CFWP operator is denoted by the complex fuzzy power average (CFPA) operator, i.e.,

$$\mathrm{CFPA}(a_1, a_2, \ldots, a_n) = \left(\sum_{i=1}^{n} \frac{1}{n} a_i^t \right)^{1/t}. \tag{14}$$

When $a_i \in [0,1]\,(i = 1, 2, \ldots, n)$, the CFWP operator can reduce to a traditional fuzzy weighted power operator of real numbers on unit interval [0,1].

When $a_i \in [0,1]$ and $w_i = 1/n$ $(i = 1, 2, \ldots, n)$, the CFWP operator is the power mean of real numbers on unit interval [0,1].

When $t = 1$, the CFWP operator can reduce to the complex fuzzy weighted arithmetic (CFWA) operator [3]. When $t = 2$, then the CFWP operator is denoted by the complex fuzzy weighted quadric averaging (CFWQA) operator, i.e.,

$$\mathrm{CFWQA}(a_1, a_2, \ldots, a_n) = \left(\sum_{i=1}^{n} w_i a_i^2 \right)^{1/2}. \tag{15}$$

When $t = 2$ and $w_i = 1/n$ $(i = 1, 2, \ldots, n)$, then the CFWP operator is denoted by the complex fuzzy quadric averaging (CFQA) operator, i.e.,

$$\mathrm{CFQA}(a_1, a_2, \ldots, a_n) = \left(\sum_{i=1}^{n} \frac{1}{n} a_i^2 \right)^{1/2}. \tag{16}$$

Theorem 4. *Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, then the aggregated value $CFWP(a_1, a_2, \ldots, a_n)$ is also a complex fuzzy value.*

Proof. Since $|a_i| \le 1\ \ (i = 1, 2, \ldots, n)$, then for $t > 0$, $|a_i^t| \le 1$:

$$\begin{aligned}\sum\nolimits_{i=1}^{n} w_i a_i^t &= \left|w_1 a_1^t + w_2 a_2^t + \cdots + w_n a_n^t\right| \\ &\le w_1\left|a_1^t\right| + w_2\left|a_2^t\right| + \cdots + w_n\left|a_n^t\right| \\ &\le w_1 + w_2 + \cdots + w_n = 1.\end{aligned} \tag{17}$$

Then, for $1/t > 0$, we have $|(\sum_{i=1}^{n} w_i a_i^t)^{1/t}| \le 1$. Thus, the aggregated value $\mathrm{CFWP}(a_1, a_2, \ldots, a_n)$ is also a complex fuzzy value.

The CFWP operator is closed on D for $t > 0$, but it is not closed on D for $t < 0$. See the following example.

Example 1. Assume that the parameter is $t = -1$, two CFVs are $a_1 = 1$ and $a_2 = 1 \cdot e^{j2\pi/3}$ and weights are $w_1 = w_2 = 0.5$. Then,

$$\begin{aligned}\mathrm{CFWP}(a_1, a_2) &= \frac{2}{(1/a_1) + (1/a_2)} \\ &= \frac{2}{(1/1) + (1/1 \cdot e^{j2\pi/3})} \\ &= \frac{2}{1 \cdot e^{j5\pi/3}} \\ &= 2 \cdot e^{j\pi/3}.\end{aligned} \tag{18}$$

Thus, we have $\mathrm{CFWP}(a_1, a_2) \notin D$.

Moreover, the CFWP operator satisfies the following properties. □

Theorem 5. *Let $a_i\,(i = 1, 2, \ldots, n)$ and $b_i\,(i = 1, 2, \ldots, n)$ be two collections of CFVs, the weights be $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$, and $\sum_{i=1}^{n} w_i = 1$. Then,*

(1) Idempotency: if $a_i = t$ for all $(i = 1, 2, \ldots, n)$, then

$$\mathrm{CFWP}(a_1, a_2, \ldots, a_n) = t. \tag{19}$$

(2) Amplitude boundedness:

$$\left|\mathrm{CFWP}(a_1, a_2, \ldots, a_n)\right| \le \max_i \left|a_i\right|. \tag{20}$$

Proof. (1) Trivial.

(2) Let $r = \max_i |a_i|$. Since $w_i \in [0, 1]\ \ (i = 1, 2, \ldots, n)$, then

$$\begin{aligned}\sum\nolimits_{i=1}^{n} w_i a_i^t &= \left|w_1 a_1^t + w_2 a_2^t + \cdots + w_n a_n^t\right| \\ &\le \left|w_1 a_1^t\right| + \left|w_2 a_2^t\right| + \cdots + \left|w_n a_n^t\right| \\ &= w_1\left|a_1^t\right| + w_2\left|a_2^t\right| + \cdots + w_n\left|a_n^t\right| \\ &\le w_1 r^t + w_2 r^t + \cdots + w_n r^t \\ &= r^t.\end{aligned} \tag{21}$$

Then, $|(\sum_{i=1}^{n} w_i a_i^t)^{1/t}| \le (r^t)^{1/t} = r$.

However, the CFWP operator does not satisfy the property of amplitude monotonicity. See the following example.

Example 2. Assume that the parameter is $t = 1$, CFVs are $a_1 = 0.8$, $a_2 = 0.8 \cdot e^{j2\pi/3}$, and $b_1 = b_2 = 0.6$ and weigths are $w_1 = w_2 = 0.5$. Then, we have

$$\begin{aligned}\mathrm{CFWP}(a_1, a_2) &= 0.5 \cdot 0.8 + 0.5 \cdot 0.8 \cdot e^{j2\pi/3} \\ &= 0.4 \cdot e^{j\pi/3},\end{aligned} \tag{22}$$

and $\mathrm{CFWP}(b_1, b_2) = 0.6$. So, $|a_1| > |b_1|$ and $|a_2| > |b_2|$, but $|\mathrm{CFWP}(a_1, a_2)| < |\mathrm{CFWP}(b_1, b_2)|$. □

Theorem 6. *The CFWP operator is reflectionally invariant.*

Proof. From Ref. [13], the complex fuzzy weighted arithmetic operator is reflectionally invariant, then

$$\mathrm{Ref}\left(\sum_{i=1}^{n} w_i a_i^t\right) = \sum_{i=1}^{n} w_i \mathrm{Ref}\left(a_i^t\right). \tag{23}$$

Since the power operator is reflectionally invariant, we have

$$\mathrm{Ref}\left(\left(\sum_{i=1}^{n} w_i a_i^t\right)^{1/t}\right) = \left(\mathrm{Ref}\left(\sum_{i=1}^{n} w_i a_i^t\right)\right)^{1/t}, \tag{24}$$

and for all $i = 1, 2, \ldots, n$,

$$\mathrm{Ref}\left(a_i^t\right) = \left(\mathrm{Ref}\left(a_i\right)\right)^t. \tag{25}$$

For any collection of CFVs $a_i\,(i = 1, 2, \ldots, n)$, from the above three equations, we have

$$\begin{aligned}&\mathrm{Ref}\left(\mathrm{CFWP}(a_1, a_2, \ldots, a_n)\right) \\ &= \mathrm{Ref}\left(\left({}^{n}\sum_{i=1} w_i a_i^t\right)^{1/t}\right) \\ &= \left(\mathrm{Ref}\left(\sum_{i=1}^{n} w_i a_i^t\right)\right)^{1/t} \quad \text{(from Equation 12)} \\ &= \left(\sum_{i=1}^{n} w_i \mathrm{Ref}\left(a_i^t\right)\right)^{1/t} \quad \text{(from Equation 11)} \\ &= \left(\sum_{i=1}^{n} w_i \mathrm{Ref}\left(a_i\right)^t\right)^{1/t} \quad \text{(from Equation 13)} \\ &= \mathrm{CFWP}\left(\mathrm{Ref}(a_1), \mathrm{Ref}(a_2), \ldots, \mathrm{Ref}(a_n)\right).\end{aligned} \tag{26}$$

Then, the CFWP operator is reflectionally invariant.

From [13], the CFWA operator (the case of $t = 1$ of the CFWP operator) is reflectionally invariant. However, the CFWP operator is not rotationally invariant for the parameter $t \ne 1$. See the following example.

Example 3. Assume that the parameter is $t = 2$, CFVs are $a_1 = 1$ and $a_2 = 1 \cdot e^{j2\pi/3}$ and weigths are $w_1 = w_2 = 0.5$. Then, we have

$$\begin{aligned}
&\mathrm{CFWP}\left(\mathrm{Rot}_{\pi/3}(a_1), \mathrm{Rot}_{\pi/3}(a_2)\right) \\
&\quad = \sqrt{w_1\left(\mathrm{Rot}_{\pi/3}(a_1)\right)^2 + w_2\left(\mathrm{Rot}_{\pi/3}(a_2)\right)^2} \\
&\quad = \sqrt{0.5 \cdot e^{j2\pi/3} + 0.5 \cdot e^{j6\pi/3}} \\
&\quad = \sqrt{0.5 \cdot e^{j\pi/3}} \\
&\quad = \sqrt{0.5} \cdot e^{j\pi/6}, \\
&\mathrm{Rot}_{\pi/3}\left(\mathrm{CFWP}(a_1, a_2)\right) \\
&\quad = \mathrm{Rot}_{\pi/3}\left(\sqrt{w_1 a_1^2 + w_2 a_2^2}\right) \\
&\quad = \mathrm{Rot}_{\pi/3}\left(\sqrt{0.5 + 0.5 \cdot e^{j4\pi/3}}\right) \\
&\quad = \mathrm{Rot}_{\pi/3}\left(\sqrt{0.5 \cdot e^{j5\pi/3}}\right) \\
&\quad = \mathrm{Rot}_{\pi/3}\left(\sqrt{0.5} \cdot e^{j5\pi/6}\right) \\
&\quad = \sqrt{0.5} \cdot e^{j7\pi/6}.
\end{aligned} \tag{27}$$

Then, $\sqrt{0.5} \cdot e^{j\pi/6} \neq \sqrt{0.5} \cdot e^{j7\pi/6}$. □

4. Complex Fuzzy Ordered Weighted Power Aggregation Operators

Based on the partial ordering of complex fuzzy values [1] and the ordered weighted averaging (OWA) operator [28], we define a complex fuzzy ordered weighted power (CFOWP) operator as follows.

Definition 2. Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, then a CFOWP operator is defined as

$$\mathrm{CFOWG}(a_1, a_2, \ldots, a_n) = \left(\sum_{i=1}^{n} w_i a_{\sigma(i)}^t\right)^{1/t}, \tag{28}$$

where $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$, $(\sigma(1), \sigma(2), \ldots, \sigma(n))$ is a permutation of $(1, 2, \ldots, n)$ such that $|a_{\sigma(i-1)}| > |a_{\sigma(i)}|$ (or $|a_{\sigma(i-1)}| = |a_{\sigma(i)}|$, $\nu_{a_{\sigma(i-1)}} = \nu_{a_{\sigma(i)}}$) for all i.

Especially, when $w_i = 1/n\,(i = 1, 2, \ldots, n)$, then the CFOWP operator is reduced to the CFWP operator.

Similar to the CFWP operator, the CFOWP operator has the following properties.

Theorem 7. *Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, then the aggregated value $\mathrm{CFOWP}(a_1, a_2, \ldots, a_n)$ is also a CFV.*

Theorem 8. *Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, CFOWP weights be $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$ and $\sum_{i=1}^{n} w_i = 1$. Then, we have the following properties.*

(1) Idempotency: if $a_i = t$ for all $i = 1, 2, \ldots, n$, then

$$\mathrm{CFOWP}(a_1, a_2, \ldots, a_n) = t. \tag{29}$$

(2) Amplitude boundedness:

$$\left|\mathrm{CFOWP}(a_1, a_2, \ldots, a_n)\right| \leq \max_i |a_i|. \tag{30}$$

Theorem 9. *The CFOWP operator is reflectionally invariant.*

Theorem 10. *Let $a_i\,(i = 1, 2, \ldots, n)$ be a collection of CFVs, CFOWP weights be $w_i \in [0, 1]$ $(i = 1, 2, \ldots, n)$, and $\sum_{i=1}^{n} w_i = 1$. Then, we have the following properties.*

(1) If $w = (1, 0, \ldots, 0)$. Then,

$$\left|\mathrm{CFOWP}(a_1, a_2, \ldots, a_n)\right| = \max_i |a_i|. \tag{31}$$

(2) If $w = (0, 0, \ldots, 1)$. Then,

$$\left|\mathrm{CFOWP}(a_1, a_2, \ldots, a_n)\right| = \min_i |a_i|. \tag{32}$$

(3) If $w_i = 1, w_k = 0, k \neq i$. Then,

$$\left|\mathrm{CFOWP}(a_1, a_2, \ldots, a_n)\right| = \left|a_{\sigma(i)}\right|, \tag{33}$$

where $a_{\sigma(i)}$ is ith largest of $a_i\,(i = 1, 2, \ldots, n)$ based on modulus of complex numbers.

Arithmetic mean, geometric mean, and power mean are three commonly used averaging functions. Now these functions are extended to the complex domain, such as complex fuzzy weighted arithmetic (CFWA) operator in [3, 13], complex fuzzy weighted geometric (CFWG) operator in [13], and complex fuzzy weighted power (CFWP) operator in this paper. Now we give a brief summary of these three operators. The results can be summarized as in Table 1, in which √ and × represent the corresponding property holds and does not hold, respectively.

Note that for the parameter $t \neq 1$, the CFWP operator does not have the property of rotational invariance, when the parameter $t = 1$, it is the CFWA operator and have the property of rotational invariance.

5. Approach to Decision Making with the CFWP Operator

In this section, we present an approach based on the CFWP operator to decision making with complex fuzzy information.

Let $E = \{e_1, e_2, \ldots, e_m\}$ be a set of experts and $X = \{x_1, x_2, \ldots, x_n\}$ be a set of alternatives. Then, the decision maker provides a decision matrix $A = (a_{ik})_{n\times m}$, where a_{ik} is a complex fuzzy value given by the expert e_k for alternative x_i. The process can be summarized as follows:

Step 1: transform the matrix A into the normalized complex fuzzy matrix $C = (c_{ik})_{m\times n}$, by $c_{ik} = a_{ik}/d$, where $d = \max_{ik}|a_{ik}|$.

Step 2: aggregate all the CFVs $a_{ik}\,(k = 1, 2, \ldots, m)$ and get the overall CFV b_i corresponding to the alternative x_i by the CFWP:

Table 1: A comparison of CFWA, CFWG, and CFWP operators.

	Idempotency	Amplitude boundedness	Amplitude monotonicity	Rotational invariance	Reflectional invariance
CFWA	√	√	×	√	√
CFWG	√	√	√	√	√
CFWP	√	√	×	×	√

√ and ×, respectively, represent the corresponding property holds and does not hold.

Table 2: Decision matrix.

	e_1	e_2	e_3	e_4	e_5
x_1	$0.69 \cdot e^{j0.11\pi}$	$0.69 \cdot e^{j0.12\pi}$	$0.71 \cdot e^{j0.09\pi}$	$0.70 \cdot e^{j0.13\pi}$	$0.69 \cdot e^{j0.12\pi}$
x_2	$0.72 \cdot e^{j1.11\pi}$	$0.70 \cdot e^{j1.12\pi}$	$0.72 \cdot e^{j1.09\pi}$	$0.73 \cdot e^{j1.13\pi}$	$0.68 \cdot e^{j1.12\pi}$
x_3	$0.69 \cdot e^{j0.61\pi}$	$0.68 \cdot e^{j0.62\pi}$	$0.69 \cdot e^{j0.64\pi}$	$0.74 \cdot e^{j0.61\pi}$	$0.72 \cdot e^{j0.62\pi}$
x_4	$0.68 \cdot e^{j1.41\pi}$	$0.67 \cdot e^{j1.42\pi}$	$0.74 \cdot e^{j1.39\pi}$	$0.72 \cdot e^{j1.41\pi}$	$0.68 \cdot e^{j1.42\pi}$

$$b_i = \mathrm{CFWP}\left(a_{i1}, a_{i2}, \ldots, a_{in}\right). \tag{34}$$

Step 3: rank all CFVs $b_i\,(i = 1, 2, \ldots, n)$ based on the modulus of complex numbers.

Next, we give an example to illustrate the above approach.

Example 4. In real life, we ask strangers for directions, such as where is the nearest supermarket? Suppose that there are four alternatives $x_i\,(i = 1, 2, 3, 4)$ and five strangers $e_i\,(i = 1, 2, 3, 4, 5)$ with the same weight. The decision maker obtains the decision matrix $C = (a_{ik})_{4\times 5}$ (see Table 2), where $c_{ik} = r_{ik} \cdot e^{\nu_{ik}}$ is complex number, in which r_{ik} and ν_{ik}, respectively, represent distance and direction.

Step 1: the complex fuzzy values c_{ik} do not need normalization.

Step 2: aggregate the complex fuzzy values b_i of the alternatives x_i by the CFPA operator: $b_1 = 0.69475 \cdot e^{j0.3576}$, $b_2 = 0.70889 \cdot e^{j0.35784}$, $b_3 = 0.70354 \cdot e^{-j1.1947}$, and $b_4 = 0.69765 \cdot e^{j1.2857}$.

Step 3: rank the complex fuzzy values $b_i\,(i = 1, 2, 3, 4)$: $|b_1| < |b_4| < |b_3| < |b_2|$.

As we can see, depending on the aggregation used, x_1 is the nearest supermarket.

6. Conclusion

In this paper, we discussed two complex fuzzy power aggregation operators, the CFWP and the CFOWP operators. Obviously, they are not closed on D for $t < 0$ (see Example 1). We discussed the CFWP and the CFOWP operators for the parameter $t > 0$. The definition of the CFWP operator is more comprehensive than the CFWA operator [3] because the latter is based on a fixed parameter $t = 1$. Note that the CFWA operator is rotationally invariant, but the CFWP operator is not rotationally invariant when the parameter $t \neq 1$.

We have applied the CFWP operator to decision making with complex fuzzy information. In future research, we expect to develop more extensions of the complex fuzzy aggregation operators and their application to other decision making problems.

Data Availability

The data used to support the findings of this study are included in the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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