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Research Article

Hybrid Solutions of (3 + 1)-Dimensional Jimbo-Miwa Equation

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The rational solutions, semirational solutions, and their interactions to the $(3+1)$-dimensional Jimbo-Miwa equation are obtained by the Hirota bilinear method and long wave limit. The hybrid solutions contain rogue wave, lump solution, and the breather solution, in which the breathers which are manifested as growing and decaying periodic line waves show different dynamics in different planes. Rogue waves are localized in time and are obtained theoretically as a long wave limit of breathers with indefinitely larger periods; they arise from a constant background at $t \ll 0$ and then disappear in the constant background when time goes on. More importantly, the interactions between some hybrid solutions are demonstrated in detail by the three-dimensional figures, such as hybrid solution between the stripe soliton and breather and hybrid solution between stripe soliton and lump solution.

1. Introduction

In soliton theory, the study of integrability to nonlinear evolution is always a hot topic of interest, which can be regarded as a key step of their exact solvability. Many areas of integrable systems are researched, such as Painlevé analysis [1], Hamiltonian structure [2–5], Lax pair [6–8], Bäcklund transformation (BT) [9–12], infinite conservation laws [13–16], and bilinear integrability [17–20]. Based on the bilinear methods, we have got many kinds of solutions, such as lump solutions [21–24] and Pfaffian solution [25]. Recently, rogue wave solutions of lots of nonlinear evolutional equations have been gained, for example, the Boussinesq equation [26] and KP equation [27, 28]. Rogue waves, which were originally coined for vivid description of transient gigantic ocean waves of extreme amplitudes that seem to appear out of nowhere and disappear without a trace, have taken the responsibility for numerous marine disasters. In recent year, the rogue wave phenomenon has appeared in a class of social and scientific contexts, ranging from geophysics and hydrodynamics [29] to oceanography, Bose-Einstein condensation, plasma physics [30], nonlinear optics [31, 32], and financial markets [33, 34]. Mathematically, rogue waves are a kind of rational solutions which are localized in both space and time [35]. The first-order or fundamental rogue waves of the nonlinear Schrödinger equation were first obtained by Peregrine in 1983 [36], and higher-order rogue waves of the NLS equation are presented recently. In addition to the NLS equations, a mass of complex systems possesses rogue wave solutions, such as the Hirota equation [37], the Sasa-Satsuma equation [38], multicomponent Yajima-Oikawa systems [39], and AB system [40]. More interestingly, the research on rogue waves varies from rational solutions to semirational solutions in the relevant studies [41, 42]. Whatever, the semirational solutions exhibit a range of more interesting and more complicated dynamic behavior, such as bright-dark rogue wave pair, rogue waves interacting with solitons [41], or breathers [42].

In this article, we focus on the $(3+1)$-dimensional Jimbo-Miwa equation

$$u_{xxxy} + 3u_y u_{xx} + 3u_x u_{xy} + 2u_{yt} - 3u_{xz} = 0, \tag{1}$$

which is a second member in the entire Kadomtsev-Petviashvili (KP) hierarchy [43]. Although (1) is nonintegrable, many types of solutions have been given. In [44], Zhang and Chen have gained one kind of rogue waves by the interaction between positive quadratic function and hyperbolic cosine function. However, in order to boost the possible applications of the $(3+1)$-dimensional Jimbo-Miwa equation in ocean studies and other fields, it is still necessary to find analytical form of the rogue waves for this equation.

The structure of the paper is as follows: In Section 2, we present the evolution breather to (3 + 1)-dimensional Jimbo-Miwa equation by the parameter perturbation method, and their typical dynamics are analysed and illustrated. In Section 3, lump solution and line rogue wave have been gained by long wave limit which show different dynamics behaviors in each plane by choosing different parameters. In Section 4, we discuss the interaction between soliton and other localized waves, which includes the soliton and breather, the soliton and lump solution, and soliton and line rogue wave.

2. The Evolution Breather to (3 + 1)-Dimensional Jimbo-Miwa Equation

Through the variable transformation

$$u = 2(\ln f)_x, \tag{2}$$

(1) will be changed into its bilinear form as

$$\left(\left(D_x^3 + 2D_t\right)D_y - 3D_xD_z\right) f \cdot f = 0, \tag{3}$$

where f is a real function with respect to x, y, z, and t and the operator D is the classic Hirota bilinear operator defined as

$$\begin{aligned} P\left(D_x, D_y, D_t\right) F(x, y, t, \ldots) G(x, y, t, \ldots) &= P\left(\partial_x - \partial'_x, \partial_y - \partial'_y, \partial_t - \partial'_t, \ldots\right) F(x, y, t, \ldots) \\ &\quad \cdot G\left(x', y', t', \ldots\right)\Big|_{x'=x, y'=y' t'=t}, \end{aligned} \tag{4}$$

where P is a polynomial of $D_x, D_y, D_t, \ldots$.

By the parameter perturbation method, the two-soliton solution to the Jimbo-Miwa equation can be written as

$$u = 2(\ln f)_x, \tag{5}$$

with

$$f = 1 + e^{\eta_1} + e^{\eta_2} + A_{12} e^{\eta_1 + \eta_2}, \tag{6}$$

where

$$\begin{aligned} A_{12} &= \frac{(q_1 - q_2)(m_1q_2 - m_2q_1) - q_1q_2(p_1q_1 - p_2q_2)(p_1 - p_2)}{(q_1 - q_2)(m_1q_2 - m_2q_1) - q_1q_2(p_1q_1 + p_2q_2)(p_1 + p_2)}, \\ \eta_i &= p_i\left(x + q_iy + m_iz + k_it + \eta_i^0\right), \quad (i = 1, 2) \\ k_i &= \frac{3m_i - p_i^2 q_i}{2q_i} \quad (i = 1, 2). \end{aligned} \tag{7}$$

As reported in the earlier work, the two-soliton solution will be reduced to the breather under appropriate constraints to the parameters, such as Davey-Stewartson (DS) equation and (3 + 1)-dimensional Kadomtsev-Petviashvili (KP) equation and the similar breather also existed in the Jimbo-Miwa equation by choosing

$$\begin{aligned} p_1 &= p_2^* = ia_1, \\ q_1 &= q_2^* = a + ib, \\ m_1 &= m_2 = b_1, \\ \eta_1^0 &= \eta_2^{0*}, \end{aligned} \tag{8}$$

in (5), where $*$ indicates the conjugate operator. Without loss of generality, taking

$$\begin{aligned} p_1 &= i, \\ p_2 &= -i, \\ q_1 &= 1 + i, \\ q_2 &= 1 - i, \\ m_1 &= 2, \\ m_2 &= 2, \\ \eta_1^0 &= \eta_2^0 = 0, \end{aligned} \tag{9}$$

the corresponding function f can be written as

$$\begin{aligned} f = 1 &+ 2\left(\cosh\left(\frac{3t}{2} - y\right) + \sinh\left(\frac{3t}{2} - y\right)\right) \\ &\cdot \left(\cos(x + y + z + 2t)\right) \\ &+ 2\left(\cosh(3t - 2y) + \sinh(3t - 2y)\right). \end{aligned} \tag{10}$$

From the framework of function f, we know that the spatial variable y is different from the spatial variable x and z in essence. When $x = 0$ or $z = 0$, the solution will be changed into the breather and when $y = 0$, the solution will be transferred into the period line wave under a given time t. Furthermore, the period line waves will go back to the constant uniformly when $t \to \pm\infty$. So we can call these period line waves as line breather. The corresponding dynamics properties to solution u are depicted in Figures 1 and 2.

Furthermore, we can get the two breathers by some constraints to the four-soliton solutions. The four-soliton solutions are

$$\begin{aligned} f_4 = 1 &+ \sum_{i=1}^{4} e^{p_i(x+q_iy+m_iz+k_it+\eta_i^0)} + \sum_{i=2}^{4} A_{1i} e^{p_1(x+q_1y+m_1z+k_1t+\eta_1^0)+p_i(x+q_iy+m_iz+k_it+\eta_i^0)} \\ &+ \sum_{i=3}^{4} A_{2i} e^{p_2(x+q_2y+m_2z+k_2t+\eta_2^0)+p_i(x+q_iy+m_iz+k_it+\eta_i^0)} + A_{34} e^{p_3(x+q_3y+m_3z+k_3t+\eta_3^0)+p_4(x+q_4y+m_4z+k_4t+\eta_4^0)} \end{aligned}$$

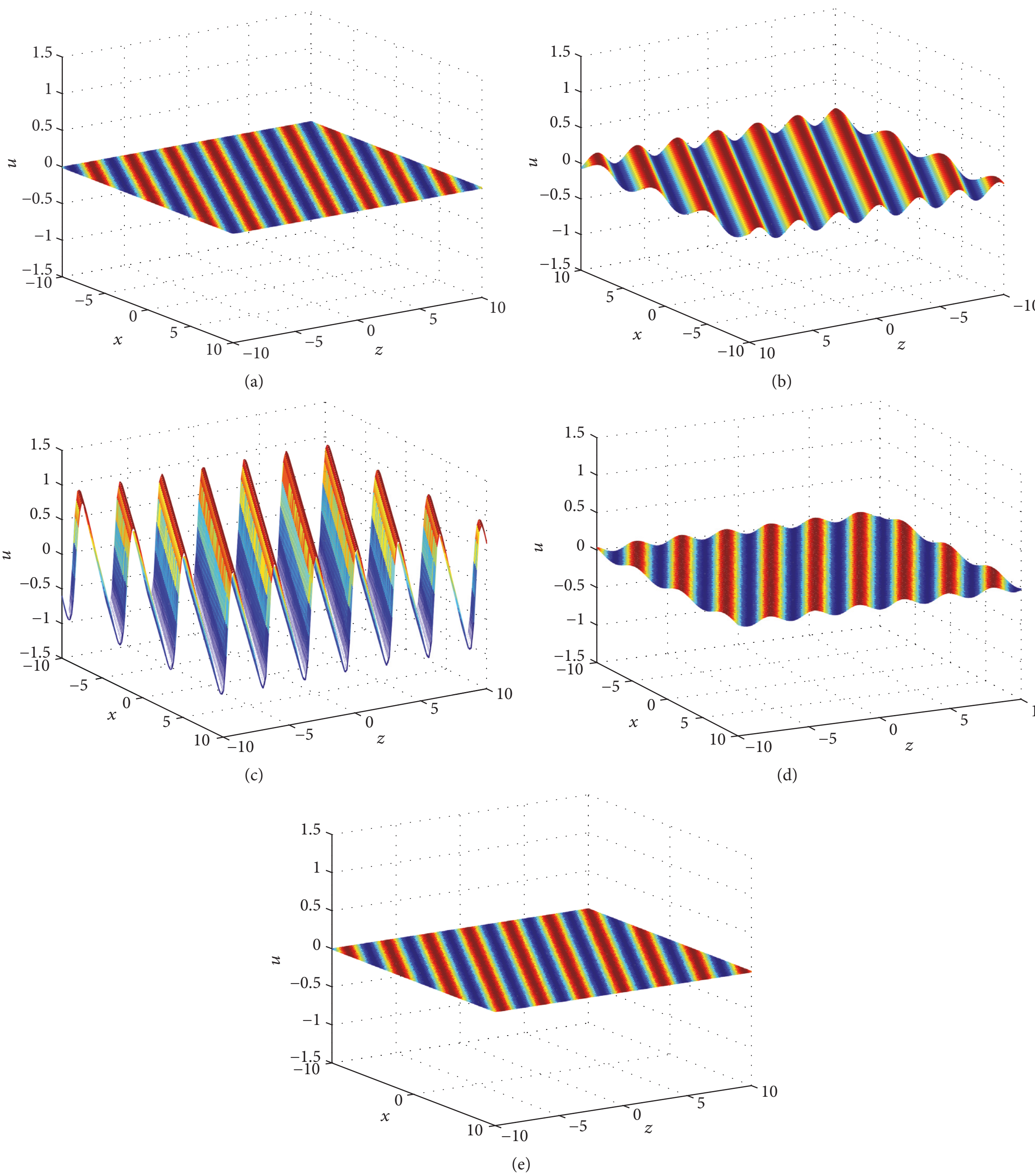


Figure 1: The evolution dynamics line period wave to (1): (a) $t = -10$, (b) $t = -2$, (c) $t = 0$, and (d) $t = 10$.

$$
\begin{aligned}
&+ A_{12}A_{13}A_{23}e^{p_1(x+q_1y+m_1z+k_1t+\eta_1^0)+p_2(x+q_2y+m_2z+k_2t+\eta_2^0)+p_3(x+q_3y+m_3z+k_3t+\eta_3^0)} \\
&+ A_{12}A_{14}A_{24}e^{p_1(x+q_1y+m_1z+k_1t+\eta_1^0)+p_2(x+q_2y+m_2z+k_2t+\eta_2^0)+p_4(x+q_4y+m_4z+k_4t+\eta_4^0)} \\
&+ A_{13}A_{14}A_{34}e^{p_1(x+q_1y+m_1z+k_1t+\eta_1^0)+p_3(x+q_3y+m_3z+k_3t+\eta_3^0)+p_4(x+q_4y+m_4z+k_4t+\eta_4^0)} \\
&+ A_{23}A_{24}A_{34}e^{p_2(x+q_2y+m_2z+k_2t+\eta_2^0)+p_3(x+q_3y+m_3z+k_3t+\eta_3^0)+p_4(x+q_4y+m_4z+k_4t+\eta_4^0)} + A_{12}A_{13}A_{14}A_{23}A_{24}A_{34}e^{A},
\end{aligned}
\tag{11}
$$

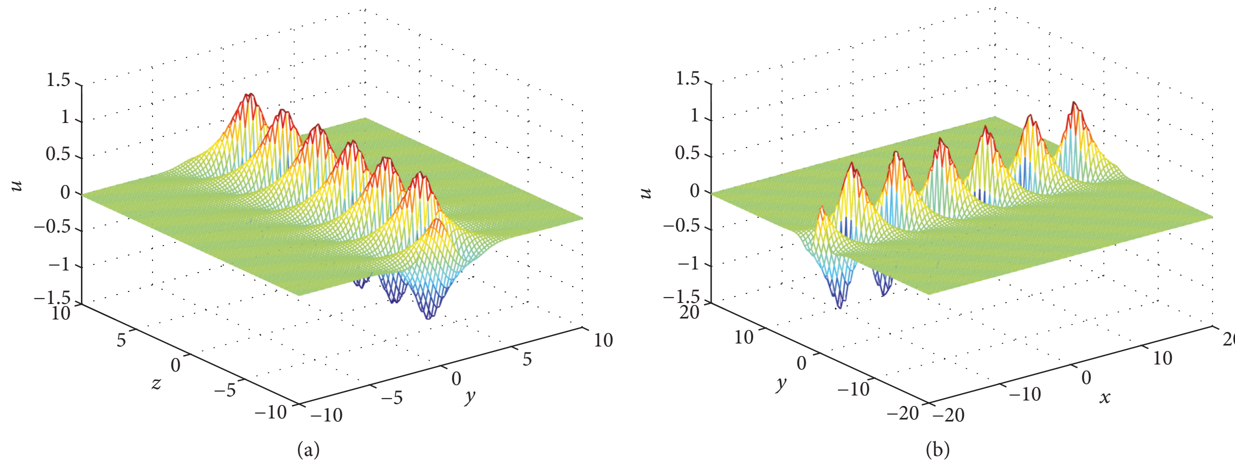


FIGURE 2: The breather to (1) in (y, z)-plane and (x, y)-plane.

where

$$A = \sum_{i=1}^{4} p_i \left(x + q_i y + m_i z + k_i t + \eta_i^0 \right). \tag{12}$$

Similar to the skills of one breather, we can deal with the four-soliton solution by the following parameters choices:

$$\begin{aligned} p_1 &= a_2 I, \\ p_2 &= -a_2 I, \\ q_1 &= b_2 + c_2 I, \\ q_2 &= b_2 - c_2 I, \\ m_1 &= m_2 = d_2, \\ p_3 &= a_3 I, \\ p_4 &= -a_3 I, \\ q_3 &= b_3 + c_3 I, \\ q_4 &= b_3 - c_3 I, \\ m_3 &= m_4 = d_3, \\ \eta_1^0 &= \eta_2^0 = \eta_3^0 = \eta_4^0 = 0. \end{aligned} \tag{13}$$

Without loss of generality, these parameters can be chosen as

$$\begin{aligned} p_1 &= I, \\ p_2 &= -I, \\ q_1 &= 1 + I, \\ q_2 &= 1 - I, \\ m_1 &= m_2 = 1, \\ p_3 &= 2I, \\ p_4 &= -2I, \\ q_3 &= 1 + I, \\ q_4 &= 1 - I, \\ m_3 &= m_4 = 2; \end{aligned} \tag{14}$$

then the function f can be changed to

$$\begin{aligned} f = {} & 1 + 2e^{3t/4-y} \cos\left(x + y + z + \frac{5t}{4} \right) \\ & + 2e^{3t-2y} \cos(2x + 2y + 4z + 7t) \\ & + \frac{2}{9} e^{15t/4-3y} \cos\left(3x + 3y + 5z + \frac{33t}{4} \right) \\ & + 2e^{15t/4-3y} \sin\left(x + y + 3z + \frac{23t}{2} \right) \\ & + 4e^{15t/4-3y} \cos\left(x + y + 3z + \frac{23t}{4} \right) \\ & + \frac{4}{3} e^{9t/2-4y} \cos(2x + 2y + 4z + 7t) \\ & + \frac{2}{3} e^{9t/2-4y} \cos(2x + 2y + 4z + 7t) \\ & + \frac{20}{9} e^{27t/4-5y} \cos\left(x + y + z + \frac{5t}{4} \right) \\ & + \frac{10}{9} e^{27t/4-5y} \sin\left(x + y + z + \frac{5t}{4} \right) + 3e^{3t/2-2y} \\ & + \frac{25}{27} e^{15t/2-6y} + 5e^{6t-4y}. \end{aligned} \tag{15}$$

The corresponding solutions u in the (x, y) are shown in Figure 3. It can be seen that the two period line waves arise from the constant background, and when time goes on, they

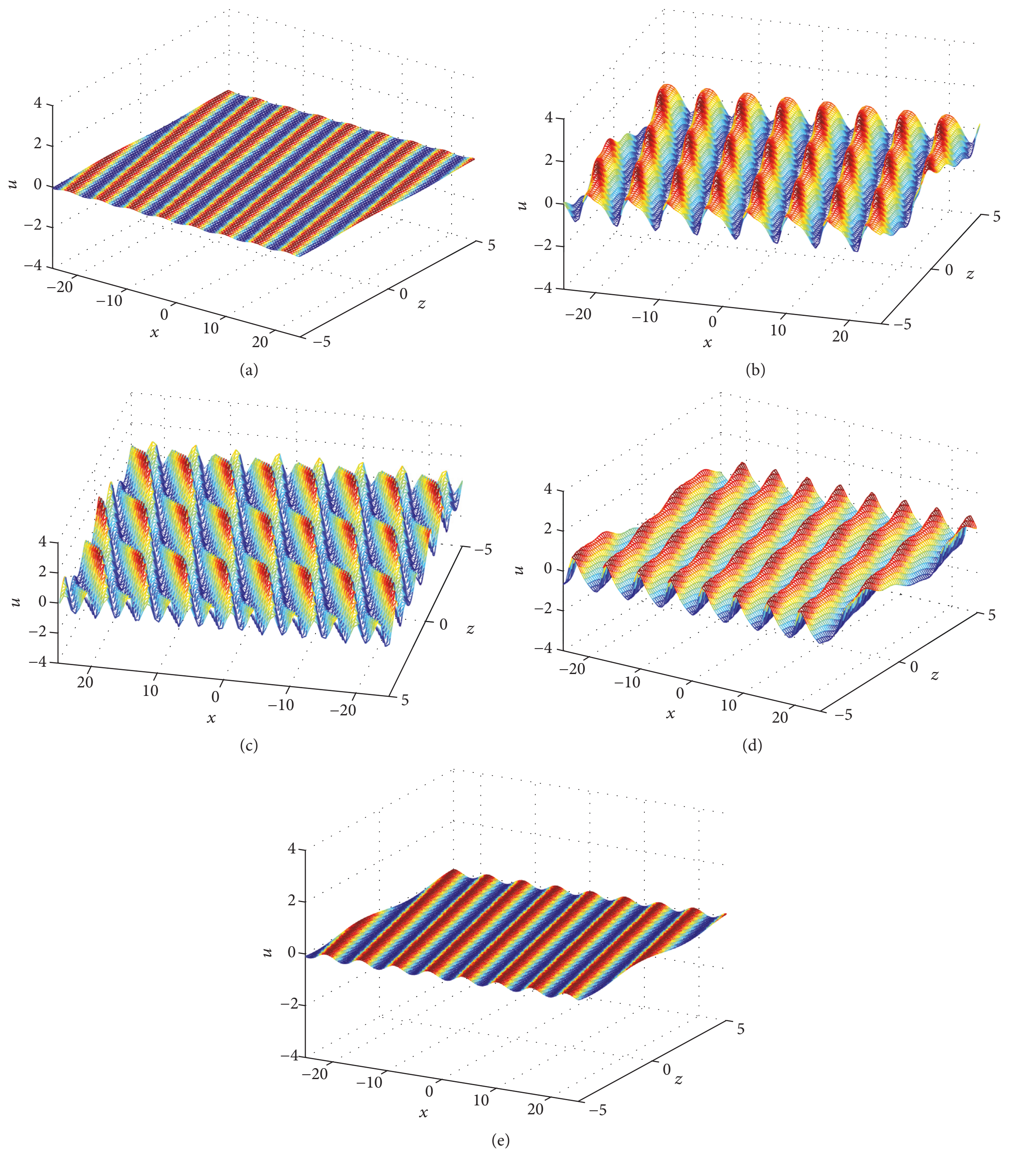


Figure 3: The time evolution of two period line solitons of the 3D Jimbo-Miwa equation in the (x, z)-plane for parameters $q_1 = 1+I$, $q_2 = 1-I$, $m_1 = 2$, and $m_2 = 2$, and the time is (a) $t = -5$, (b) $t = -1$, (c) $t = 0$, (d) $t = 1$, and (e) $t = 5$.

both return back to the constant background. In addition, with similar parameters choices, these period solitons will have different behaviors in other planes, which can be seen in Figure 4.

3. Lump Solution and Line Rogue Wave

Based on the theory of long wave limit to the function f, we can get the rogue wave to (1). Taking $p_1 = m_1\epsilon$, $p_2 = m_2\epsilon$, and

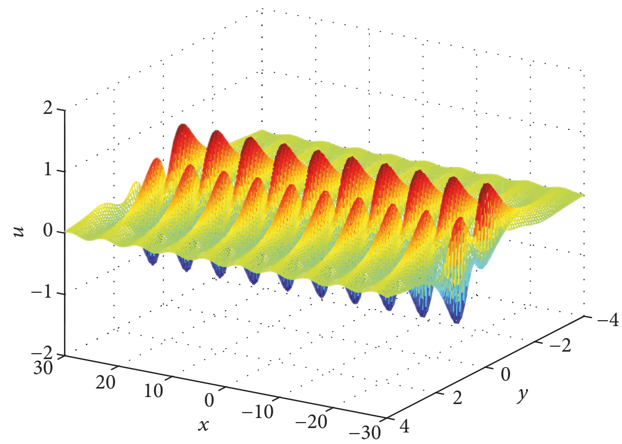


Figure 4: Two breathers to 3D Jimbo-Miwa equation by choosing $p_1 = I$, $p_2 = -I$, $q_1 = 1 + I$, $q_2 = 1 - I$, $m_1 = m_2 = 1$, $p_3 = 2I$, $p_4 = -2I$, $q_3 = 2 + I$, $q_4 = 2 - I$, and $m_3 = m_4 = 4$.

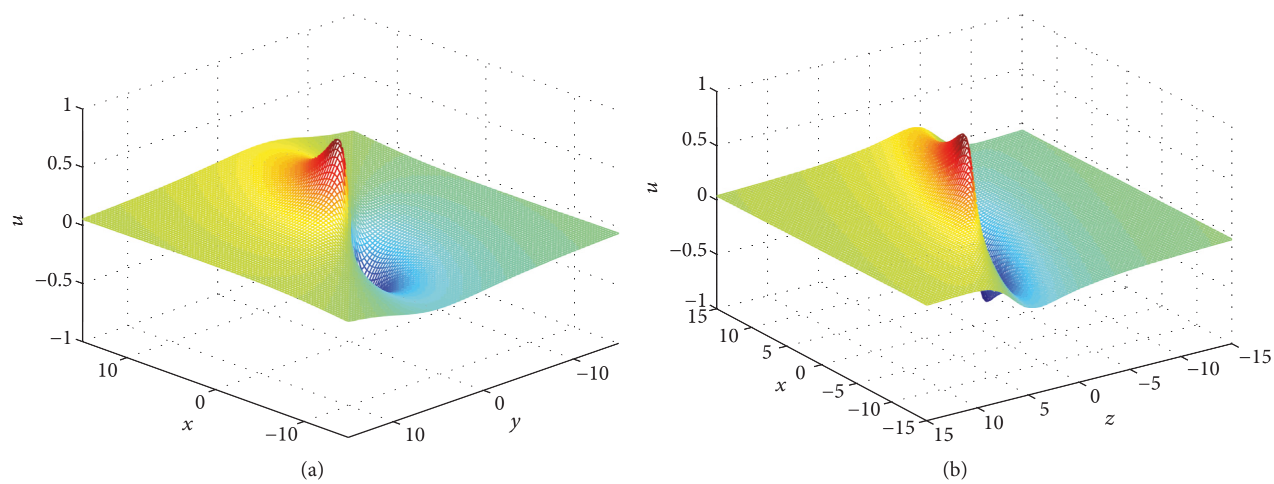


Figure 5: Lump solution of the 3D Jimbo-Miwa equation by choosing $q_1 = 1+I$, $q_2 = 1-I$, $m_1 = 2+I$, and $m_2 = 2-I$: (a) is the (x, y)-plane and (b) is the (x, z)-plane.

$\eta_1^0 = \eta_2^{0*} = I\pi$, in (6) and letting $\epsilon \to 0$ can get the solutions of (6) as

$$f = (\theta_1\theta_2 + \theta_0)\, a_1 a_2 \epsilon^2 + O(\epsilon^3), \tag{16}$$

where

$$\begin{aligned} \theta_i &= x + q_i y + m_i z + \frac{3m_i t}{2q_i}, \quad (i = 1, 2) \\ \theta_0 &= \frac{2q_1 q_2 (q_1 + q_2)}{(q_1 - q_2)(m_1 q_2 - m_2 q_1)}; \end{aligned} \tag{17}$$

then the corresponding solution u to (1) can be expressed as

$$u = 2\frac{\theta_1 + \theta_2}{\theta_1\theta_2 + \theta_0}. \tag{18}$$

Under some constraints to these parameters, such as $q_1 = q_2^*$, $m_1 = m_2^*$, and $\theta_0 > 0$, (18) is nonsingular. There will be different dynamics behaviors in each plane by choosing different parameters. For simplicity, we only concentrate on the (x, y)-plane as an example. Set $q_1 = a+bI$ and $m_1 = c+dI$, and a, b, c, d are all real constants. Then we can get the lump solution and rogue wave, respectively.

3.1. Lump Solution. It is obvious that (18) will be a constant along a trajectory defined by

$$\begin{aligned} x + ay + cz + \frac{3(ac+bd)^2}{2(a^2+b^2)} &= 0, \\ by + dz + \frac{3(ad-bc)}{2(a^2+b^2)} &= 0. \end{aligned} \tag{19}$$

Furthermore, at any given t and z, the solution u will tend to 0 when (x, y) tends to infinity. So this kind of solution keeps moving on the constant background. Its dynamics properties are demonstrated in Figure 5.

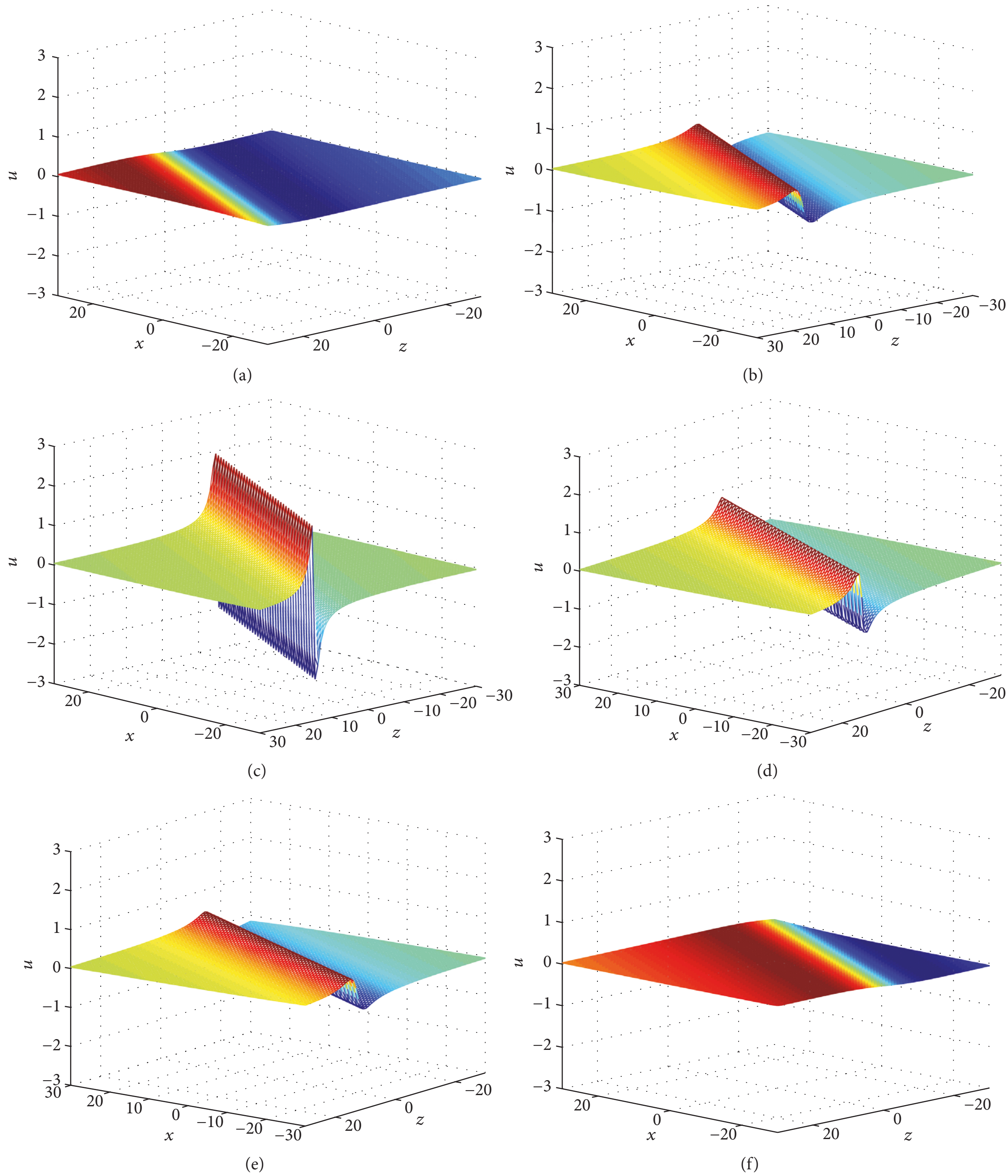


FIGURE 6: The time evolution of kink line rogue wave of the 3D Jimbo-Miwa equation in the (x, z)-plane for parameters $q_1 = 1 + I$, $q_2 = 1 - I$, $m_1 = 2$, and $m_2 = 2$, and the time is (a) $t = -20$, (b) $t = -3$, (c) $t = 0$, (d) $t = 1.5$, (e) $t = 3$, and (f) $t = 20$.

3.2. Rogue Wave. Due to the constraint of θ_0, there only exists one kind of rogue wave in (x, z)-plane by choosing some special parameters. When $d = 0$, that is, m_1 and m_2 are real constants, then the solution changes into the rogue wave in the (x, z)-plane. If $t \to \pm\infty$, it approaches 0, and as time goes on, it will have a height peak wave and then disappear, which is different from the general soliton, whose dynamics behavior is depicted in Figure 6.

Moreover, we can present the interaction between lump solution and line rogue wave by considering the four-soliton

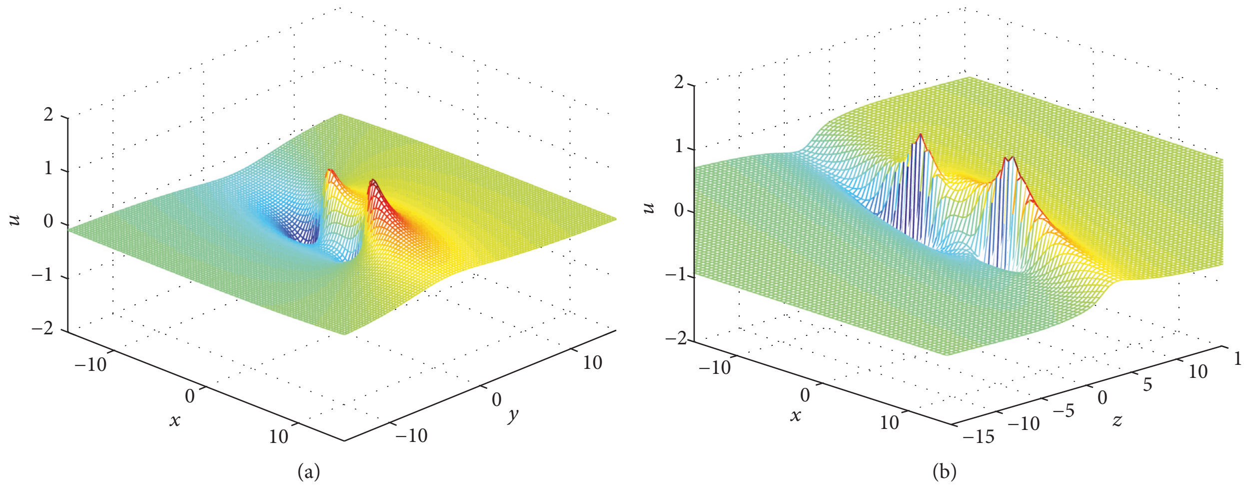


FIGURE 7: Two lump solution to 3D Jimbo-Miwa equation by choosing $q_1 = 1 + I$, $q_2 = 1 - I$, $m_1 = 2 + I$, $m_2 = 2 - I$, $q_3 = 1 + 2I$, $q_4 = 1 - 2I$, $m_3 = 3 + I$, and $m_4 = 3 - I$: (a) is the (x, y)-plane under $t = 0$ and $z = 0$ and (b) is the (x, z)-plane under $t = 0$ and $y = 0$.

solutions. Still, we use the long wave limit to the function f, putting

$$\begin{aligned} p_1 &= a_1\epsilon, \\ p_2 &= a_2\epsilon, \\ p_3 &= a_3\epsilon, \\ p_4 &= a_4\epsilon, \\ \eta_1 &= \eta_3 = \eta_2^* = \eta_4^* = I\pi, \end{aligned} \tag{20}$$

and setting the limit $\epsilon = 0$; then the corresponding solution f can be written as

$$\begin{aligned} f = {} & (\eta_1\eta_2\eta_3\eta_4 + a_{12}\eta_3\eta_4 + a_{13}\eta_2\eta_4 + a_{14}\eta_2\eta_3 \\ & + a_{23}\eta_1\eta_4 + a_{24}\eta_1\eta_3 + a_{34}\eta_1\eta_2)\, a_1a_2a_3a_4\epsilon^4 \\ & + (a_{12}a_{34} + a_{13}a_{24} + a_{14}a_{23})\, a_1a_2a_3a_4\epsilon^4 + O(\epsilon^5), \end{aligned} \tag{21}$$

where

$$\begin{aligned} \eta_i &= x + q_i y + m_i z + \frac{3m_i t}{2q_1}, \quad (i = 1, 2, 3, 4) \\ a_{ij} &= 2\frac{q_iq_j(q_i + q_j)}{(q_i - q_j)(m_iq_j - m_jq_i)} \quad (i = 1, 2, 3, 4). \end{aligned} \tag{22}$$

Similarly, taking $q_2 = q_1^*$, $q_3 = q_4^*$, $m_2 = m_1^*$, and $m_3 = m_4^*$, we can obtain the lump solution and rogue wave of (1). Suppose $q_1 = a + bI$, $q_2 = a - bI$, $m_1 = c + dI$, $m_2 = c - dI$, $q_3 = a_1 + b_1 I$, $q_3 = a_1 - b_1 I$, $m_3 = c_1 + d_1 I$, and $m_4 = c_1 - d_1 I$, and a, b, c, d, a_1, b_1, c_1, d_1 are all real constants. Then we can get three kinds of cases about the rogue wave and lump solution.

(i) Two Lump Solutions. When q_1, q_3, m_1, m_3 are all imaginary numbers, two lump solutions will appear and are demonstrated in Figure 7.

(ii) One Lump Solution and One Rogue Wave. Similar to the one rogue wave, if the pair of m_1, m_2 are imaginary numbers and m_3, m_4 are real numbers or the opposite, the exciting interaction between lump solution and line rogue wave will appear, which is shown in Figure 8.

(iii) Two Rogue Waves. If m_1, m_2, m_3, m_4 are all real numbers, that is, their imaginary parts are 0, the interaction between two kink line rogue waves will appear and is depicted in Figure 9.

4. The Interaction between Soliton and Other Localized Waves

In this section, we will discuss the interaction between soliton and other localized waves, which includes three cases: the first is the soliton and breather, the second is the soliton and lump solution, and the third is soliton and line rogue wave.

Firstly, we study the interaction between soliton and breather. As we all know, with some constraints to the parameters for the two-soliton solution, we can get the breather. As to the three-soliton solutions, we will obtain the soliton and breather by using the same constraints, such as the three-soliton solutions

$$f = 1 + \sum_{i=1}^{3} e^{p_i(x+q_iy+m_iz+k_it+\eta_i^0)} + \sum_{i=2}^{3} A_{1i}e^{p_1(x+q_1y+m_1z+k_1t+\eta_1^0)+p_i(x+q_iy+m_iz+k_it+\eta_i^0)}$$

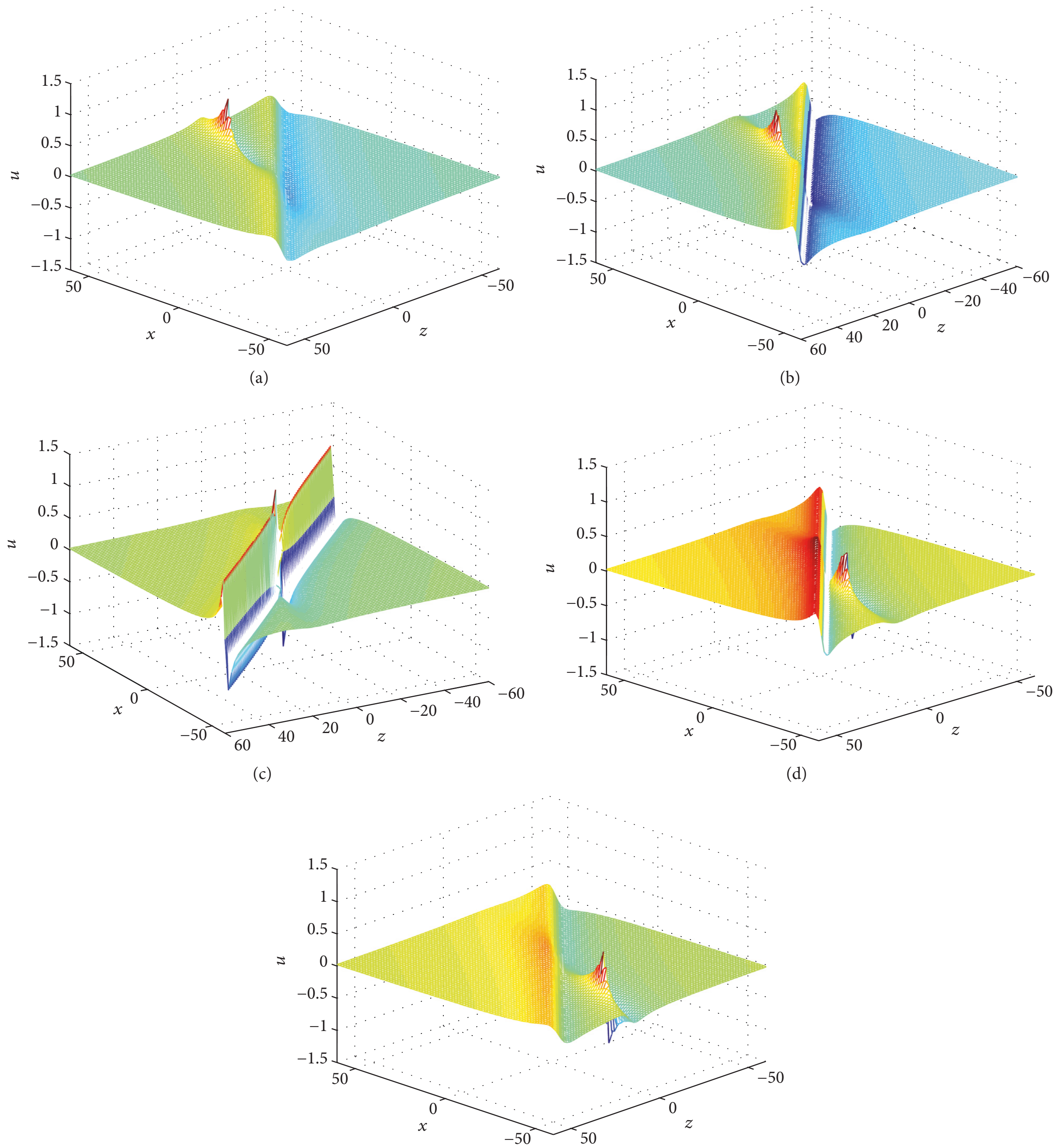


Figure 8: The time evolution interaction of kink line rogue wave and lump solution of the 3D Jimbo-Miwa equation in the (x, z)-plane for parameters $q_1 = 1 + I, q_2 = 1 - I, m_1 = 2 + I, m_2 = 2 - I, q_3 = 1 + 2I, q_4 = 1 - 2I, m_3 = 1, m_4 = 1$, and the time is (a) $t = -10$, (b) $t = -5$, (c) $t = 0$, (d) $t = 5$, and (e) $t = 10$.

$$+ A_{23} e^{p_2(x+q_2 y+m_2 z+k_2 t+\eta_2^0)+p_3(x+q_3 y+m_3 z+k_3 t+\eta_3^0)}$$

$$+ A_{12}A_{13}A_{23} e^{p_1(x+q_1 y+m_1 z+k_1 t+\eta_1^0)+p_2(x+q_2 y+m_2 z+k_2 t+\eta_2^0)+p_3(x+q_3 y+m_3 z+k_3 t+\eta_3^0)}. \quad (23)$$

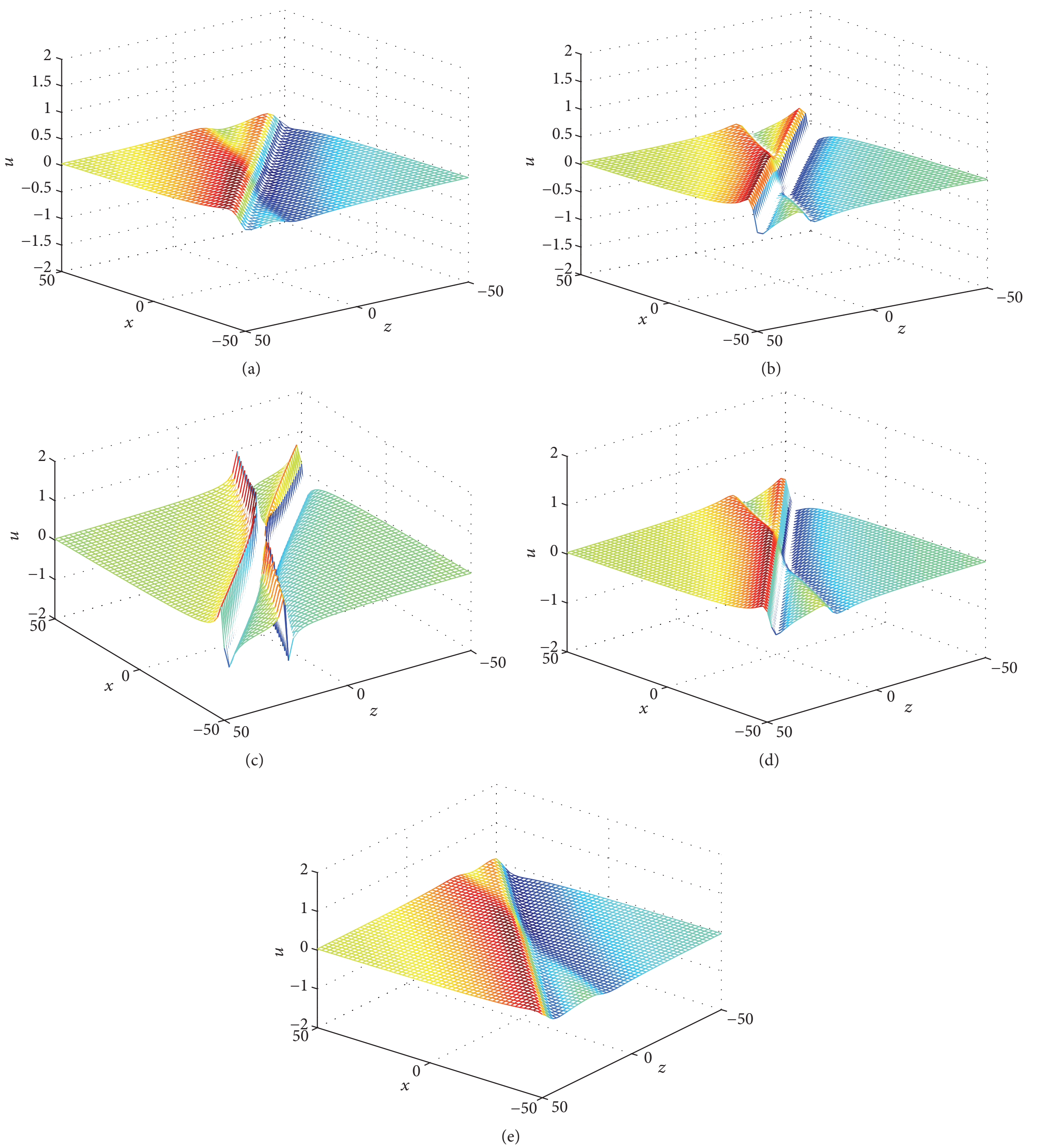


FIGURE 9: The time evolution interaction of two kink line rogue waves of the 3D Jimbo-Miwa equation in the (x, z)-plane for parameters $q_1 = 1 + I, q_2 = 1 - I, m_1 = 2, m_2 = 2, q_3 = 1 + 2I, q_4 = 1 - 2I, m_3 = 1, m_4 = 1$, and the time is (a) $t = -10$, (b) $t = -5$, (c) $t = 0$, (d) $t = 5$, and (e) $t = 10$.

Taking the skills described in Section 2, we still use these constraints for these parameters. For simplicity, these parameters are chosen as

$$p_1 = I,$$

$$p_2 = -I,$$

$$q_1 = 1 + I,$$

$$q_2 = 1 - I,$$

$$m_1 = 2,$$

$$m_2 = 2,$$

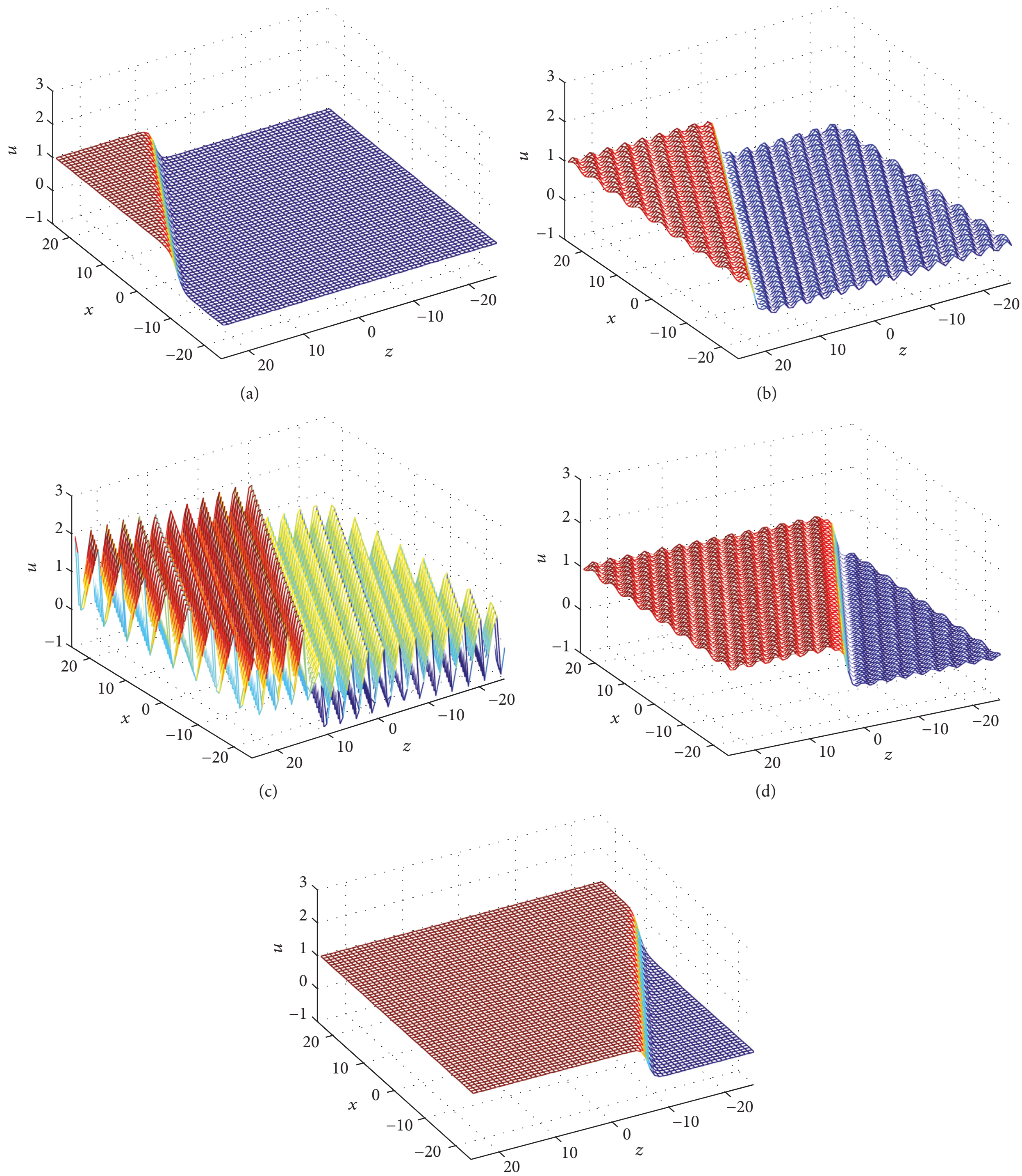


FIGURE 10: The evolution period wave and one stripe soliton: the corresponding time is (a) $t = -4$, (b) $t = -2$, (c) $t = 0$, (d) $t = 2$, and (e) $t = 4$.

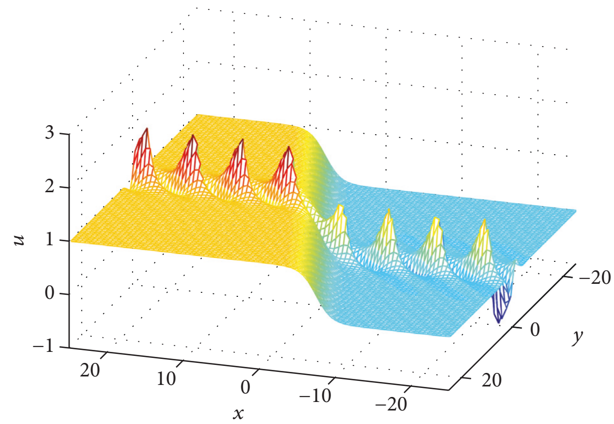
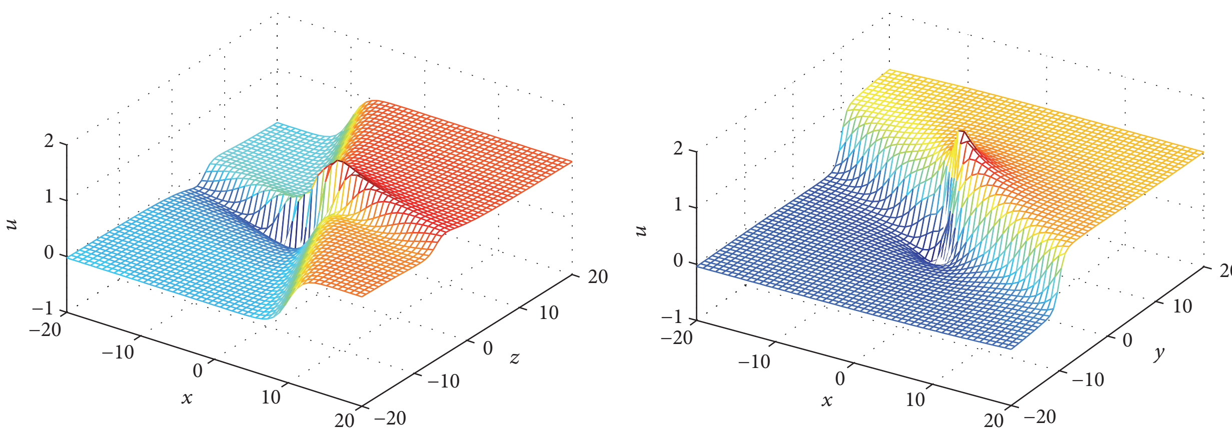


Figure 11: Lump solution of the 3D Jimbo-Miwa equation by choosing $q_1 = 1 + I, q_2 = 1 - I, m_1 = 2 + I, m_2 = 2 - I$, in the (x, y)-plane.



(a) (b)

Figure 12: Lump solution of the 3D Jimbo-Miwa equation by choosing $q_1 = 1 + I, q_2 = 1 - I, m_1 = 3 + I, m_2 = 3 - I, p_3 = 1, q_3 = 2, m_3 = 0.5$: (a) is the (x, z)-plane and (b) is the (x, y)-plane.

$$\begin{aligned} p_3 &= 1, \\ q_3 &= \frac{3}{10}, \\ m_3 &= 2, \\ \eta_1^0 &= \eta_2^0 = \eta_3^0 = 0; \end{aligned} \tag{24}$$

then the corresponding function f will be written as

$$\begin{aligned} f = {} & 1 + 2e^{3t/2-y}\cos(x + y + 2z + 2t) \\ & + \frac{14062}{10361}e^{x-(7/10)y+11t+2z}\cos(x + y + 2z + 2t) \\ & + \frac{7482}{10361}e^{x-(7/10)y+11t+2z}\sin(x + y + 2z + 2t) \\ & + 2e^{3t-2y} + \frac{12244}{10361}e^{x-(17/10)y+2z+(25/2)t}. \end{aligned} \tag{25}$$

It is obvious that the expression of (1) contains one stripe soliton and period wave, but, in different planes, the period wave will have different dynamics behaviors: one is the period line wave and the other is the normal breather. So there will be two different interactions between soliton and period wave, which are depicted in Figures 10 and 11.

What is more, based on the long wave limit, the three-soliton solutions can also be transformed into the rogue wave and one stripe soliton, lump solution, and stripe solution. Putting

$$\begin{aligned} p_1 &= a_1\epsilon, \\ p_2 &= a_2\epsilon, \\ \eta_1^0 &= \eta_2^{0*} = I\pi, \\ \eta_3^0 &= 0, \\ q_2 &= q_1^*, \\ m_2 &= m_1^*, \end{aligned} \tag{26}$$

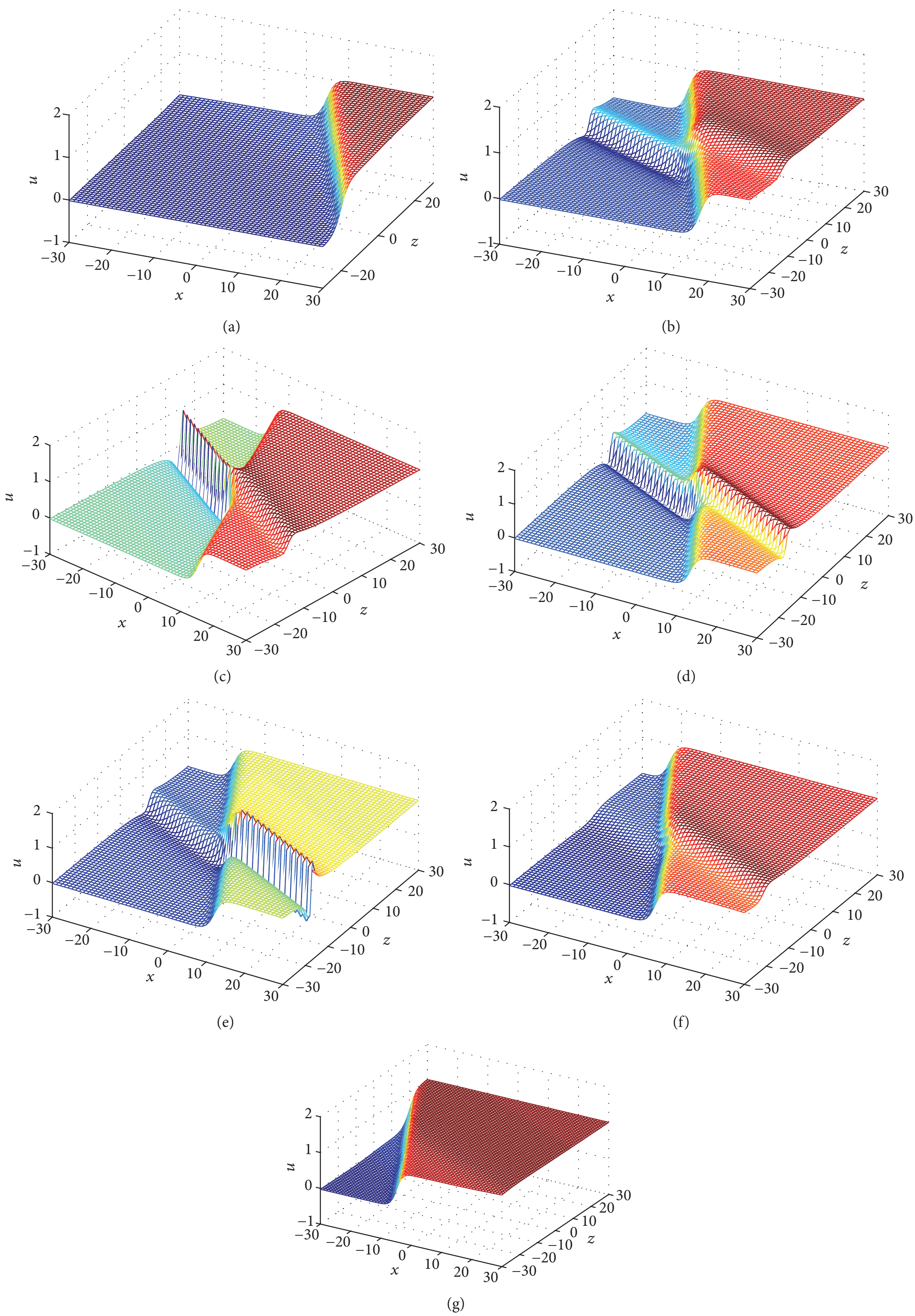


Figure 13: The evolution rogue wave and stripe soliton with the parameters choices $q_1 = 1 + I, q_2 = 1 - I, m_1 = 2, m_2 = 2, p_3 = 1, q_3 = 0.5, m_3 = 0.5$, and the time is (a) $t = -20$, (b) $t = -3$, (c) $t = 0$, (d) $t = 1.5$, (e) $t = 3$, (f) $t = 8$, and (g) $t = 20$.

and taking the limit $\epsilon \to 0$, we can get the mix soliton solutions.

(i) Lump Solution and Stripe Soliton. When the imaginary part of m_1 is nonzero, this kind of solution will be the combination of stripe soliton and lump solution; it is shown in Figure 12.

(ii) Rogue Wave and Stripe. When the imaginary part of m_1 is zero, this kind of solution will be changed into the rogue wave and soliton. When $t \to \pm\infty$, there is only one stripe soliton. As time goes on, the rogue wave appears gradually and disappears soon afterwards, which affects the characteristics of rogue wave; it is shown in Figure 13.

5. Conclusion

In summary, line breathers in the (3 + 1)-dimensional Jimbo-Miwa equation (1) have been derived by the bilinear transformation and been demonstrated by 3D figures. By a long wave limit of breathers, localized analytical solutions in rational form for the (3 + 1)-dimensional Jimbo-Miwa equation are proposed and the explicit forms of rational and semirational solutions have been presented. More importantly, we gain line rogue waves and lumps by modifying the internal parameters from the rational solution. We have shown that the line rogue waves possess a growing and decaying line profile that arises from a constant background and disappears in the initial constant background again. Furthermore, the obtained semirational solutions composed of solitons, breathers, lump solutions, and rogue waves exhibit a range of interesting and complicated dynamic behaviors. Our results are more comprehensive as more various rogue waves and the complicated interaction between breathers, lump solutions, rogue waves, and solitons are demonstrated in this paper. The combination of rogue waves, solitons, and breathers obtained in this paper is a new kind of solutions, which reveals the potential rich dynamic behavior in rogue wave solutions and helps to promote our understanding of rogue wave phenomena. Hence, we will study the higher-order line rogue waves and more new kinds of hybrid solutions for the (3 + 1)-dimensional Jimbo-Miwa equation in the future.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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