Hindawi
Mathematical Problems in Engineering
Volume 2017, Article ID 2159281, 14 pages
https://doi.org/10.1155/2017/2159281

*Research Article*

# A Multiproduct Single-Period Inventory Management Problem under Variable Possibility Distributions

**Zhaozhuang Guo,[1,2] Shengnan Tian,[1] and Yankui Liu[1]**

[1] *Risk Management & Financial Engineering Laboratory, College of Management, Hebei University, Baoding, Hebei 071002, China*
[2] *Fundamental Science Department, North China Institute of Aerospace Engineering, Langfang, Hebei 065000, China*

Correspondence should be addressed to Zhaozhuang Guo; zhaozhuang2004@163.com

Received 2 August 2017; Revised 21 September 2017; Accepted 4 December 2017; Published 20 December 2017

Academic Editor: Jean-Pierre Kenne



In multiproduct single-period inventory management problem (MSIMP), the optimal order quantity often depends on the distributions of uncertain parameters. However, the distribution information about uncertain parameters is usually partially available. To model this situation, a MSIMP is studied by credibilistic optimization method, where the uncertain demand and carbon emission are characterized by variable possibility distributions. First, the uncertain demand and carbon emission are characterized by generalized parametric interval-valued (PIV) fuzzy variables, and the analytical expressions about the mean values and second-order moments of selection variables are established. Taking second-order moment as a risk measure, a new credibilistic multiproduct single-period inventory management model is developed under mean-moment optimization criterion. Furthermore, the proposed model is converted to its equivalent deterministic model. Taking advantage of the structural characteristics of the deterministic model, a domain decomposition method is designed to find the optimal order quantities. Finally, a numerical example is provided to illustrate the efficiency of the proposed mean-moment credibilistic optimization method. The computational results demonstrate that a small perturbation of the possibility distribution can make the nominal optimal solution infeasible. In this case, the decision makers should employ the proposed credibilistic optimization method to find the optimal order quantities.

## 1. Introduction

The MSIMP is a classical inventory management problem. In order to maximize (minimize) the total expected profit (cost), the decision makers have to make the optimal order quantities at the beginning of the period. At the end of the selling period, either stock-out or excess inventory will occur. The two possibilities should be considered during the decision-making process. The popularity of the MSIMP is due to its applicability in retailing and manufacturing industries. Hadley and Whitin [1] first considered a MSIMP with storage capacity or budget constraints and proposed a dynamic programming solution procedure to find the optimal order quantities. Since then, many researchers have developed stochastic MSIMP. For instance, Nahmias and Schmidt [2] discussed the MSIMP under the linear and deterministic constraints on budget or space. H.-S. Lau and A. H. L. Lau [3] extended the MSIMP to handle multiconstraint and presented a Lagrangian-based numerical solution procedure for the MSIMP. When the conditions of closed-form expressions did not hold, Erlebacher [4] proposed an effective heuristic solution. Moon and Silver [5] dealt with the MSIMP subject to not only a budget constraint on the total value of the replenishment quantities but also fixed costs for nonzero replenishment. Furthermore, Abdel-Malek et al. [6] considered a MSIMP under a budget constraint with probabilistic demand and random yield. Zhang [7] considered the MSIMP with both supplier quantity discounts and a budget constraint and formulated it as a mixed integer nonlinear programming model. In order to deal with the possible shortage of limited capacity, Zhang and Du [8] discussed zero lead time outsourcing strategy and nonzero lead time outsourcing strategy. They also developed the structural properties and solution procedures for their profit-maximization models. Abdel-Malek and Montanari [9] proposed a methodology for studying the dual of the solution space of the MSIMP with two constraints and introduced an approach to obtain the optimal order quantities of each product. In addition, Huang et al. [10] studied a competitive MSIMP with shortage penalty cost and partial product substitution. In view of risk preference, Özler et al. [11] proposed the MSIMP under a Value at Risk constraint. Van Ryzin and Mahajan [12] reviewed the contributions to multiproduct inventory problem with demand substitution. Under mean-variance and utility function approaches, Van Mieghem [13] studied multiproduct single-period networks' problems in probabilistic framework.

When the exact probability distribution of demand is unavailable, probabilistic robust optimization method [14] is a tool to deal with the corresponding uncertainty in inventory management problem. Based on the assumption that demand was described by discrete or interval scenarios, Vairaktarakis [15] discussed several minimax regret formulations for the MSIMP with a budget constraint. When the distribution of demand had known support, mean, and variance, Kamburowski [16] presented the theoretical foundations for analyzing the inventory management problem. They derived the closed-form formulas for the worst-case and best-case order quantities. Shu et al. [17] considered the distribution-free single-period inventory management problem by borrowing an economic theory from transportation disciplines. Moon et al. [18] found the differences between normal distribution approaches and distribution-free approaches in four scenarios with mean and variance. Under interval demand uncertainty, Solyali et al. [19] proposed a new robust formulation which could solve the intractability issue for large problem instances. As for recent development in stochastic inventory management problems, the interested reader may further refer to [20–24].

Most of the extensions of inventory management problem have been made in the probabilistic framework, where uncertain parameters are characterized by random variables. However, in some cases, there are not enough data to determine the exact probability distribution of random variable because of economic reason or technical difficulty. In such a case, the variable is approximately specified based on the experiences and subjective judgments of the experts in related fields, so fuzzy inventory management problem is also an active research area. Fuzzy set theory was applied in the early inventory management literature [25, 26]. In the area of fuzzy MSIMP, Mandal and Roy [27] considered a multiproduct displayed inventory model under shelf-space constraint in fuzzy environment, where the demand rate of a product was considered as a function of the displayed inventory level. Under fuzzy demand environment, Ji and Shao [28] studied the MSIMP and formulated three kinds of models. Dutta [29] formulated a fuzzy MSIMP model whose objective was to maximize the total profit by considering fuzzy demands. In fuzzy-stochastic environment, Saha et al. [30] developed multiproduct multiobjective supply chain models with budget and risk constraints, where the manufacturing costs of the items were fuzzy variables and the demands for the products were random variables. Based on credibility measure, Guo [31] proposed two single-period inventory models, where the uncertain demands were characterized by discrete and continuous possibility distributions, respectively. Tian and Guo [32] formulated a credibilistic optimization model for a single-product single-period inventory problem with two suppliers.

The work mentioned above studied inventory management problem under the assumption that the exact possibility distribution of fuzzy variable was available, which motivates us to study the MSIMP from a new perspective. The motivation of this paper is based on the following considerations. First, shorter product life cycles and growing innovation rates make the market demand extremely variable. In this case, the distribution information about market demand is only partially available. It is reasonable to assume that the exact possibility distribution is embodied in a zonal area for a practical MSIMP, so the interval-valued fuzzy variable is introduced to characterize uncertain market demand. Second, the optimal order quantities for different products are heavily influenced by the carbon emission constraint. In some practical inventory management problems, it is difficult to determine the exact carbon emission during logistic activities. Under credibilistic carbon emission constraint, a parametric credibilistic optimization model is developed for MSIMP. To the best of our knowledge, this issue has not been addressed in the literature.

This paper studies MSIMP by parametric credibilistic optimization method, where uncertain market demand and uncertain carbon emission are characterized by generalized PIV possibility distributions. Decision makers can make informed decisions based on a tradeoff model between the mean total profit and the second-order moment of total profit under budget constraint and uncertain carbon emission constraint. The strength of the proposed method is that the distributions of market demand and carbon emission can be tailored to the partial information at hand. That is, when the distribution information about uncertain parameters is partially available, the proposed method is more convenient for modeling uncertain demand and carbon emission in a practical MSIMP. The proposed credibilistic optimization method differs from the existing MSIMP literature in the following several aspects. (i) A novel method is introduced to model the perturbation distributions of uncertain demand and carbon emission, which is different from the existing literature. (ii) For PIV fuzzy variable, its lambda selection variable is introduced as its representative; the possibility distribution of lambda selection can traverse the entire support of the PIV fuzzy variable as the lambda parameter varies its values. (iii) On the basis of L-S multiple integral, two new optimization indexes, mean and second-order moment, about the total profit are defined to build a parametric credibilistic optimization model under credibilistic constraint of carbon emission. (iv) A domain decomposition method is designed to divide the original credibilistic optimization model into several equivalent parametric programming submodels, which can be solved by conventional optimization software.

The remainder of this paper is organized as follows. After introducing some basic concepts in fuzzy possibility theory, Section 2 discusses the properties about generalized PIV fuzzy variable and its selection variable. In Section 3, a new parametric credibilistic optimization model is first developed for MSIMP, where uncertain demand and uncertain carbon emission are characterized by variable possibility distributions. Then the equivalent deterministic model of the proposed parametric credibilistic optimization model is discussed in this section. A new domain decomposition method is also designed in this section to find the optimal order quantities. In Section 4, some numerical experiments are conducted to demonstrate the validity of the proposed credibilistic optimization method. Section 5 gives the conclusion of the paper.

## 2. Generalized PIV Fuzzy Variables

First, in this section, some basic concepts in fuzzy possibility theory are recalled [33–36].

Let $\Gamma$ be the universe of discourse, $\mathscr{P}(\Gamma)$ the power set of $\Gamma$, and $\widetilde{\text{Pos}}: \mathscr{P}(\Gamma) \mapsto \mathscr{R}([0,1])$ a fuzzy possibility measure. The triplet $(\Gamma, \mathscr{P}(\Gamma), \widetilde{\text{Pos}})$ is called a fuzzy possibility space.

Let $\xi$ be a type 2 fuzzy variable defined on the space $(\Gamma, \mathscr{P}(\Gamma), \widetilde{\text{Pos}})$. If, for any $r \in \Re$, the secondary possibility distribution function $\tilde{\mu}_\xi(r) = \widetilde{\text{Pos}}\{\xi = r\}$ is a subinterval $[\mu_{\xi^L}(r;\theta_l), \mu_{\xi^U}(r;\theta_r)]$ of $[0,1]$, then $\xi$ is called a PIV fuzzy variable, where $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that $\xi$ takes the value $r$.

A type 2 fuzzy variable $\xi$ is called a generalized PIV normal fuzzy variable [36], if its secondary possibility distribution is the subinterval

$$\left[(1-\theta_l)e^{-(r-\mu)^2/2\sigma^2}, e^{-(r-\mu)^2/2\sigma^2} + \left(1 - e^{-(r-\mu)^2/2\sigma^2}\right)\theta_r\right] \quad (1)$$

of $[0,1]$ for $r \in \Re$, where $\mu \in \Re$, $\sigma > 0$ and $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that $\xi$ takes on the value $r$. When $\theta_l = \theta_r = 0$, the corresponding fuzzy variable is denoted by $\xi^n$, whose possibility distribution is called the *nominal possibility distribution* of $\xi$. In the following, $\xi \sim n(\mu, \sigma^2; \theta_l, \theta_r)$ means that $\xi$ is a generalized PIV normal fuzzy variable.

A type 2 fuzzy variable $\eta$ is called a generalized PIV triangular fuzzy variable [36], if its secondary possibility distribution is the subinterval $[(r-r_1)/(r_2-r_1) - \theta_l((r-r_1)/(r_2-r_1)), (r-r_1)/(r_2-r_1) + \theta_r((r_2-r)/(r_2-r_1))]$ of $[0,1]$, for $r \in [r_1, r_2]$, and the subinterval $[(r_3-r)/(r_3-r_2) - \theta_l((r_3-r)/(r_3-r_2)), (r_3-r)/(r_3-r_2) + \theta_r((r-r_2)/(r_3-r_2))]$ of $[0,1]$ for $r \in [r_2, r_3]$, where $r_1 < r_2 < r_3$ are real numbers and $\theta_l, \theta_r \in [0,1]$ are two parameters characterizing the degree of uncertainty that $\eta$ takes on the value $r$. When $\theta_l = \theta_r = 0$, the corresponding fuzzy variable is denoted by $\eta^n$, whose possibility distribution is called the nominal possibility distribution of $\eta$. In the following, $\eta \sim \text{Tri}(r_1, r_2, r_3; \theta_l, \theta_r)$ means that $\eta$ is a generalized PIV triangular fuzzy variable.

For a PIV fuzzy variable, its lambda selection is defined in [34]. Assume that $\xi$ is a PIV fuzzy variable with the secondary possibility distribution $\tilde{\mu}_\xi(r) = [\mu_{\xi^L}(r;\theta_l), \mu_{\xi^U}(r;\theta_r)]$. For any $\lambda \in [0,1]$, a fuzzy variable $\xi^\lambda$ is called a lambda selection of $\xi$ if $\xi^\lambda$ has the following generalized parametric possibility distribution:

$$\mu_{\xi^\lambda}(r;\theta_l,\theta_r) = (1-\lambda)\mu_{\xi^L}(r;\theta_l) + \lambda\mu_{\xi^U}(r;\theta_r). \quad (2)$$

Obviously, the possibility distribution of lambda selection variable depends on the parameter $\lambda$. That is, the possibility distribution of lambda selection variable can traverse the entire support of PIV fuzzy variable as the lambda parameter varies its value in the interval $[0,1]$.

Based on L-S integral [37], the mean value of a fuzzy variable $\xi$ is defined as

$$E(\xi) = \int_{(-\infty,+\infty)} r \,\mathrm{d}\text{Cr}\{\xi \le r\}, \quad (3)$$

where the credibility $\text{Cr}\{\xi \le r\}$ is computed by

$$\text{Cr}\{\xi \le r\} = \frac{1}{2}\left\{\sup_{x\in\Re}\mu_\xi(x;\theta_l,\theta_r) + \sup_{x\le r}\mu_\xi(x;\theta_l,\theta_r) - \sup_{x>r}\mu_\xi(x;\theta_l,\theta_r)\right\}. \quad (4)$$

In addition, the second-order moment of a fuzzy variable $\xi$ is defined as

$$M(\xi) = \int_{(-\infty,+\infty)} [r - E(\xi)]^2 \,\mathrm{d}\text{Cr}\{\xi \le r\}, \quad (5)$$

where $E(\xi)$ is the mean value of $\xi$ defined by (3).

For lambda selection variable, its mean value and second-order moment are important optimization indices in the MSIMP. The following theorems establish their analytical expressions, which will be used in the rest of the paper. For the sake of presentation, the proofs of the following theorems are provided in the appendix.

**Theorem 1.** *Let $\xi^\lambda$ be a lambda selection of the generalized PIV normal fuzzy variable $n(\mu,\sigma^2;\theta_l,\theta_r)$. Then the mean value of the lambda selection $\xi^\lambda$ is*

$$E(\xi^\lambda) = \omega\mu, \quad (6)$$

*where $\omega = 1 - (1-\lambda)\theta_l - \lambda\theta_r$.*

**Theorem 2.** *Let $\eta^\lambda$ be a lambda selection of the generalized PIV triangular fuzzy variable $(r_1, r_2, r_3; \theta_l, \theta_r)$. Then the mean value of the lambda selection $\eta^\lambda$ is*

$$E(\eta^\lambda) = [1 - (1-\lambda)\theta_l]\frac{r_1 + 2r_2 + r_3}{4} + \frac{1}{4}\lambda\theta_r(r_1 - 2r_2 + r_3). \quad (7)$$

**Theorem 3.** *Let $\xi^\lambda$ be a lambda selection of the generalized PIV normal fuzzy variable $n(\mu,\sigma^2;\theta_l,\theta_r)$. Then the second-order moment of the lambda selection $\xi^\lambda$ is*

$$M(\xi^\lambda) = \omega\left[2\sigma^2 + (\omega-1)^2\mu^2\right]. \quad (8)$$

**Theorem 4.** *Let $\eta^\lambda$ be a lambda selection of the generalized PIV triangular fuzzy variable $(r_1, r_2, r_3; \theta_l, \theta_r)$. Then the second-order moment of the lambda selection $\eta^\lambda$ is*

$$\begin{aligned} M\left(\eta^\lambda\right) &= \frac{1}{2}\lambda\theta_r\left[\left(r_1 - m\right)^2 + \left(r_3 - m\right)^2\right] \\ &+ \frac{1-(1-\lambda)\theta_l - \lambda\theta_r}{6\left(r_2 - r_1\right)}\left[\left(r_2 - m\right)^3 - \left(r_1 - m\right)^3\right] \\ &+ \frac{1-(1-\lambda)\theta_l - \lambda\theta_r}{6\left(r_3 - r_2\right)}\left[\left(r_3 - m\right)^3 - \left(r_2 - m\right)^3\right], \end{aligned} \quad (9)$$

*where $m = E(\eta^\lambda)$.*

In the next section, the distribution information about uncertain demand and uncertain carbon emission is partially available and characterized by generalized PIV normal fuzzy variable and triangular fuzzy variable, respectively.

## 3. Credibilistic Optimization Model for MSIMP

In order to model MSIMP, some necessary notations are provided in the following subsection.

*3.1. Notations*

*Fixed Parameters*

$n$: number of products

$i$: product index, $i = 1, 2, \ldots, n$

$c_i$: procurement cost for unit product $i$

$g_i$: goodwill cost for unit unmet demand of product $i$

$p_i$: retailer's sales price for unit product $i$

$s_i$: salvage value for unit residual product $i$

$\theta_{li}$: downward perturbation degree of nominal possibility distribution for product $i$

$\theta_{ri}$: upward perturbation degree of nominal possibility distribution for product $i$

$\lambda_i$: lambda selection parameter of demand distribution for product $i$

$\mu_{\lambda_i}$: mean value of the lambda selection variable for product $i$

$D_i$: largest market demand for product $i$

$B$: total investment amount

$K$: total carbon emission allowance from government

$\beta$: predetermined confidence level

$N^+$: the set of nonnegative integers

*Decision Variables*

$Q_i$: retailer's order quantity for product $i$

*Uncertain Parameters*

$\xi_i$: uncertain market demand with variable possibility distribution

$\eta_i$: uncertain carbon emission due to logistic activities for product $i$

$\pi$: uncertain profit for retailer

*3.2. Credibilistic Optimization Model and Its Equivalent Deterministic Form.* In this subsection, a MSIMP is studied, where the uncertain demand and uncertain carbon emission are characterized by generalized PIV fuzzy variables. At the beginning of selling season, the retailer is interested in determining the order quantity $Q_i$ for product $i$ to satisfy customer demand for each product. For product $i$, the distribution information of uncertain demand is only partially known based on the experts' experiences or subjective judgments. Assume that the uncertain demand for product $i$ ($i = 1, 2, \ldots, n$) is characterized by generalized PIV normal fuzzy variable $\xi_i = n(\mu_i, \sigma_i^2; \theta_{li}, \theta_{ri})$, $i = 1, 2, \ldots, n$, and the largest market demand for product $i$ is no more than $D_i$. At the end of the period, if $Q_i \geq \xi_i$, then $Q_i - \xi_i$ units are salvaged for a per-unit revenue $s_i$, and if $Q_i < \xi_i$, then $\xi_i - Q_i$ units represent lost sales cost for a per-unit cost $g_i$.

The profit for the retailer stemming from the sales of product $i$ is represented as

$$\begin{aligned} \pi\left(Q_i, \xi_i\right) = \left(p_i - c_i\right)\xi_i - \left(c_i - s_i\right)\left(Q_i - \xi_i\right)^+ \\ - \left(p_i - c_i + g_i\right)\left(\xi_i - Q_i\right)^+ \end{aligned} \quad (10)$$

for $i = 1, 2, \ldots, n$, respectively.

The profit function for product $i$ cannot be directly maximized because it is a fuzzy variable. In order to transform the fuzzy objective into a crisp one, the mean profit of $\pi(Q_i, \xi_i)$ is computed by

$$E\left[\pi\left(Q_i, \xi_i\right)\right] = \int_{[0,D_i]} \pi\left(Q_i, r\right) \mathrm{dCr}\left\{\xi_i \leq r\right\}. \quad (11)$$

Since $\xi_i$ has an interval-valued possibility distribution, robust optimization method (see [38–41]) can be used to model the MSIMP.

In this paper, the lambda selection variable $\xi_i^\lambda$ is employed to represent the generalized PIV fuzzy variable $\xi_i$. In this case, the mean value of profit $\pi(Q_i, \xi_i^\lambda)$ is computed by

$$\begin{aligned} E\left[\pi\left(Q_i, \xi_i^\lambda\right)\right] &= \int_{[0,D_i]} \pi\left(Q_i, r\right) \mathrm{dCr}\left\{\xi_i^\lambda \leq r\right\} \\ &= \left(p_i + g_i - c_i\right) h_i Q_i \\ &- \left(p_i + g_i - s_i\right) \int_0^{Q_i} \mathrm{Cr}\left\{\xi_i^\lambda \leq r\right\} \mathrm{d}r \\ &- g_i \mu_{\lambda_i}, \end{aligned} \quad (12)$$

where

$$h_i = \mathrm{Cr}\left\{\xi_i^\lambda \le D_i\right\},$$
$$\mu_{\lambda_i} = \int_{[0,D_i]} r\,\mathrm{dCr}\left\{\xi^\lambda \le r\right\}$$
$$= D_i \mathrm{Cr}\left\{\xi_i^\lambda \le D_i\right\} - \int_0^{D_i} \mathrm{Cr}\left\{\xi_i^\lambda \le r\right\} \mathrm{d}r. \quad (13)$$

Furthermore, the second-order moment of profit $\pi(Q_i, \xi_i^\lambda)$ is computed by

$$M\left[\pi\left(Q_i, \xi_i^\lambda\right)\right] = \int_{[0,D_i]} \{\pi\left(Q_i, r\right) - E\left[\pi\left(Q_i, r\right)\right]\}^2 \mathrm{dCr}\left\{\xi_i^\lambda \le r\right\}$$
$$= h\left[\left(c_i - p_i - g_i\right)^2 Q_i^2 + 2g_i D_i\left(c_i - p_i - g_i\right) Q_i + g_i^2 {D_i}^2\right] + 2\left\{\left[c_i\left(p_i - s_i\right) + s_i^2\right] Q_i - p_i s_i\right\}$$
$$\cdot \int_0^{Q_i} \mathrm{Cr}\left\{\xi_i^\lambda \le r\right\} \mathrm{d}r + 2g_i Q_i\left(p_i + g_i - c_i\right)$$
$$\cdot \int_{Q_i}^{D_i} \mathrm{Cr}\left\{\xi_i^\lambda \le r\right\} \mathrm{d}r - 2\left[\left(p_i - s_i\right)^2 + {g_i}^2\right]$$
$$\cdot \int_0^{Q_i} r\mathrm{Cr}\left\{\xi_i^\lambda \le r\right\} \mathrm{d}r - 2g_i^2 \int_{Q_i}^{D_i} r\mathrm{Cr}\left\{\xi_i^\lambda \le r\right\} \mathrm{d}r$$
$$+ m_i^2\left(h_i - 2\right), \quad (14)$$

where $m_i = E[\pi(Q_i, \xi_i^\lambda)]$.

As a result, the total profit of the retailer in MSIMP is

$$\Pi\left(Q, \xi^\lambda\right) = \sum_{i=1}^{n} \pi\left(Q_i, \xi_i^\lambda\right). \quad (15)$$

Based on L-S multiple integral, the mean total profit of the retailer is computed by

$$E\left[\Pi\left(Q, \xi^\lambda\right)\right] = \int \cdots \int_{\mathscr{R}^n} \Pi\left(Q, \xi^\lambda\right) \mathrm{d}\left(\mathrm{Cr}\left\{\xi_1^\lambda \le r\right\} \times \cdots \times \mathrm{Cr}\left\{\xi_n^\lambda \le r\right\}\right), \quad (16)$$

while the second-order moment of the total profit is computed by

$$M\left[\Pi\left(Q, \xi^\lambda\right)\right] = \int \cdots \int_{\mathscr{R}^n} \left\{\Pi\left(Q, \xi^\lambda\right) - E\left[\Pi\left(Q, \xi^\lambda\right)\right]\right\}^2 \mathrm{d}\left(\mathrm{Cr}\left\{\xi_1^\lambda \le r\right\} \times \cdots \times \mathrm{Cr}\left\{\xi_n^\lambda \le r\right\}\right). \quad (17)$$

In order to find the optimal order quantity $Q_i$, the retailer should take into account the allocation of emission allowance $K$, which will be received before the selling season. It is well-known that transportation mode has a significant influence on carbon emission per ton-mile. For product $i$, it is usually difficult to determine the exact carbon emission during logistic activities. Based on the retailer' experience, assume that the carbon emission for product $i$ $(i = 1, 2, \ldots, n)$ is characterized by generalized PIV triangular fuzzy variable $\eta_i \sim \mathrm{Tri}(r_{1i}, r_{2i}, r_{3i}; \theta_{li}, \theta_{ri})$.

According to [36], $\sum_{i=1}^{n} Q_i \eta_i$ is also a generalized PIV triangular fuzzy variable; its lambda selection variable is denoted as $(\sum_{i=1}^{n} Q_i \eta_i)^\lambda$.

Under mean-moment optimization criterion, a new parametric credibilistic optimization model for the MSIMP is formally built as

$$\max_{Q} \quad E\left[\Pi\left(Q, \xi^\lambda\right)\right] - \gamma\sqrt{M\left[\Pi\left(Q, \xi^\lambda\right)\right]} \quad (E\text{-}M\ 1) \quad (18)$$
$$\text{s.t.} \quad \mathrm{Cr}\left\{\left(\sum_{i=1}^{n} Q_i \eta_i\right)^\lambda \le K\right\} \ge \beta \quad (19)$$
$$\sum_{i=1}^{n} c_i Q_i \le B \quad (20)$$
$$0 \le Q_i \le D_i, \quad i = 1, 2, \ldots, n \quad (21)$$
$$Q_i \in N^+, \quad i = 1, 2, \ldots, n. \quad (22)$$

Objective function (18) in model ($E$-$M$ 1) is to maximize the tradeoff between the mean total profit and the standard second-order moment of the total profit, where $\gamma$ is some nonnegative constant that reflects the decision maker's degree of risk aversion. Constraint (19) means that the carbon emission due to logistic activities is less than the total carbon emission $K$ with a predetermined confidence level $\beta$. Constraint (20) represents the fact that the investment amount on total production cost has an upper limit on the maximum investment. Constraints (21) and (22) ensure that decision variables $Q_i$ $(i = 1, 2, \ldots, n)$ are nonnegative integers in a reasonable range.

In order to solve model ($E$-$M$ 1), its equivalent deterministic model is discussed in the following theorem. For the sake of presentation, the proof of the following theorem is also provided in the appendix.

**Theorem 5.** *Let $\eta_i$ be mutually independent fuzzy variables. Then model (E-M 1) is equivalent to the following deterministic programming model:*

$$\max_{Q} \quad \Pi^e(Q) - \gamma\sqrt{\Pi^m(Q)} \quad (E\text{-}M\ 2) \tag{23}$$

$$\text{s.t.} \quad \left(\sum_{i=1}^{n} Q_i\eta_i\right)^{\lambda}_{\inf}(\beta) \le K \tag{24}$$

$$\sum_{i=1}^{n} c_i Q_i \le B \tag{25}$$

$$0 \le Q_i \le D_i, \quad i = 1, 2, \ldots, n \tag{26}$$

$$Q_i \in N^+, \quad i = 1, 2, \ldots, n, \tag{27}$$

*where*

$$\Pi^e(Q) = \sum_{i=1}^{n}\left(m_i \prod_{k\neq i} h_k\right),$$

$$h_i = \text{Cr}\left\{\xi_i^{\lambda} \le D_i\right\}, \quad m_i = E\left[\pi\left(Q_i, \xi_i^{\lambda}\right)\right],$$

$$\Pi^m(Q) = \sum_{i=1}^{n}\left[\prod_{k\neq i} h_k \int_{[0,D_i]} \{\pi(Q_i, r)\}^2 \, d\text{Cr}\left\{\xi_i^{\lambda} \le r\right\} + \left(\prod_{k\neq i} h_k m_i\right)^2 \left(\prod_{k=1}^{n} h_k - 2\right)\right] + \prod_{k\neq i\neq j} h_k \left[\sum_{j\neq i} m_i m_j \left(1 - \prod_{k=1}^{n} h_k\right)^2\right]. \tag{28}$$

In Theorem 5, model ($E$-$M$ 2) is a parametric programming model with respect to parameter $\lambda$. The value of parameter lambda determines the location and shape of the possibility distribution of selection variable. According to the definition of lambda selection variable, parameter $\lambda$ may change its value from 0 to 1. It is highlighted that the possibility distribution of lambda selection variable can traverse the entire support of PIV fuzzy variables as the lambda parameter changes its value in the interval $[0, 1]$. For any given $\lambda \in [0, 1]$, the corresponding integer programming model ($E$-$M$ 2) can be solved by conventional optimization software.

*3.3. Domain Decomposition Method.* Note that the analytical expressions of $\Pi^e(Q)$ and $\sqrt{\Pi^m(Q)}$ include the integral $\int_0^{Q_i} \text{Cr}\{\xi_i^{\lambda} \le r\}dr$. According to the definition of $\text{Cr}\{\xi \le r\}$, the integral $\int_0^{Q_i} \text{Cr}\{\xi_i^{\lambda} \le r\}dr$ is a piecewise function with respect to $Q_i$. Since decision makers do not know in advance which subregion the global optimal solution locates in, to solve $2^n$ submodels by optimization software to obtain $2^n$ local optimal solutions is required. By comparing the objective values of the obtained local optimal solutions, the global optimal solutions, $Q_i^*$, $i = 1, 2, \ldots, n$, can be found. Given the values of distribution parameters $\theta_{li}$, $\theta_{ri}$, and $\lambda_i$, the process of domain decomposition method is summarized as follows.

*Step 1.* Solve parametric programming submodels of model (E-M 2) by software Matlab 7.1. Let $0 \le Q_i^1 \le \mu_i$, $\mu_i \le Q_i^2 \le M_i$, $i = 1, 2, \ldots, n$. Given a set of values $Q_{it}^k$, $k = 1$ or $2$, $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, 2^n$, denote the corresponding local optimal solutions as $Q_{it}^*$, $i = 1, 2, \ldots, n$.

*Step 2.* Compare the local objective values $v_t = E[\pi(Q, \xi^{\lambda})]$ at local optimal solution $Q_{it}^*$ and find the global maximum profit by the following formula:

$$v_l = \max_{1\le t\le 2^n} v_t, \tag{29}$$

where $E[\pi(Q, \xi^{\lambda})]$ is the mean profit of $\pi(Q, \xi^{\lambda})$.

*Step 3.* Return $Q_{il}^*$ as the global optimal solution to model ($E$-$M$ 2) with the global optimal value $E[\pi(Q_{il}^*, \xi^{\lambda})]$.

In the next section, the effectiveness of the proposed domain decomposition method is demonstrated by a practical multiproduct single-period inventory management problem.

## 4. Numerical Experiments

*4.1. Problem Statement.* In order to illustrate the proposed credibilistic optimization model (E-M 2), a two-product single-period inventory problem is provided with generalized PIV normal demand variables. The retailer's optimal strategy will be obtained by the proposed credibilistic optimization method. Before a hot summer, the retailer needs to order two kinds of products: air-conditioning (Product 1) and evaporative air cooler (Product 2). The retailer is interested in determining the order quantity of air-conditioning $Q_1$ and the order quantity of evaporative air cooler $Q_2$ to satisfy customer demand. For product $i$ ($i = 1, 2$), the distribution information of uncertain demand is partially available based on the experts' experiences. Suppose that the uncertain demand $\xi_i$ for product $i$ follows generalized PIV normal possibility distribution $n(\mu_i, \sigma_i^2; \theta_{li}, \theta_{ri})$, $i = 1, 2$. Based on the practical background of inventory problem, the largest market demand for product $i$ is no more than $D_i$. At the end of the period, if $Q_i \ge \xi_i$, then $Q_i - \xi_i$ units are salvaged for a per-unit revenue $s_i$, and if $Q_i < \xi_i$, then $\xi_i - Q_i$ units represent lost sales cost for a per-unit cost $g_i$. In view of the carbon emission constraint, the retailer receives the allocation of emission allowance $K = 251000$ grams before the summer. For product $i$ ($i = 1, 2$), the distribution information about the unit carbon emission during logistic activities is partially available based on the experts' experiences. Assume that the unit carbon emissions for two products follow generalized PIV triangular possibility distributions $\text{Tri}(85, 100, 110; 0.25, 0.15)$ and $\text{Tri}(40, 50, 65; 0.25, 0.15)$, respectively. Due to logistic activities, the sum of emissions is less than the predetermined total emission $K$ with confidence level $\beta = 0.9$. Additionally,

Table 1: Parameters for a two-product single-period inventory problem.

| Product $i$ | $c_i$ (\$) | $p_i$ (\$) | $s_i$ (\$) | $g_i$ (\$) | $\xi_i$ | $D_i$ |
|---|---|---|---|---|---|---|
| Product 1 | 220 | 300 | 205 | 90 | $n(800, 55^2; 0.3, 0.25)$ | 3000 |
| Product 2 | 105 | 160 | 90 | 55 | $n(2400, 75^2; 0.15, 0.2)$ | 6000 |

Table 2: The local optimal solutions with different domains of $Q_1$ and $Q_2$.

| The range of $Q_1$ | The range of $Q_2$ | $Q_1^*$ | $Q_2^*$ | $E^*$ |
|---|---|---|---|---|
| $0 \le Q_1 \le 800$ | $0 \le Q_2 \le 2400$ | 800 | 2378 | 115885.79 |
| $0 \le Q_1 \le 800$ | $2400 \le Q_2 \le 6000$ | 800 | 2400 | 116558.62 |
| $800 \le Q_1 \le 3000$ | $0 \le Q_2 \le 2400$ | 814 | 2400 | 117240.53 |
| $800 \le Q_1 \le 3000$ | $2400 \le Q_2 \le 6000$ | 813 | 2410 | 117491.83 |

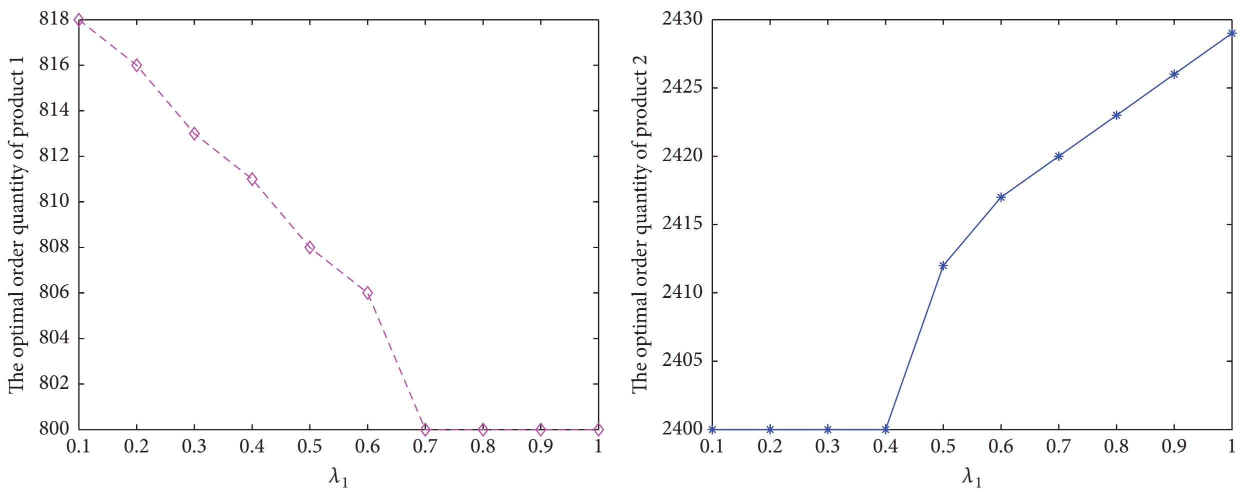


Figure 1: The optimal order quantities of product 1 and product 2 with $\lambda_2 = 0.6$.

the available maximum investment for the retailer is $B$ = \$432000. The other pertinent data for the products are given in Table 1.

*4.2. Computational Results.* In numerical experiments, it is assumed that $\lambda_1 = 0.6$, $\lambda_2 = 0.8$, and $\gamma = 0.3$. According to the proposed domain decomposition method, the feasible region of the above inventory management problem can be decomposed into four disjoint subregions of decision variables $Q_i$, $i = 1, 2$. Matlab 7.1 optimization software is employed to solve the corresponding parametric programming submodels. The numerical experiments are conducted on a personal computer (Lenovo with Intel Pentium(R) Dual-Core E5700 3.00 GHz CPU and RAM 4.00 GB) by using the Microsoft Windows 10 operating system. The computational results are reported in Table 2. By comparing the obtained local optimal solutions, the global optimal solution $(Q_1^*, Q_2^*) = (813, 2410)$ is found with the maximum mean total profit 117491.83.

*4.3. Sensitivity Analysis for Parameter Lambda.* By the meanings of parameters $\lambda_1$ and $\lambda_2$, the two parameters determine the location and shape about the possibility distribution of selection variable in the support of uncertain demand and uncertain carbon emission. A decision maker may prescribe the values of parameters $\lambda_1$ and $\lambda_2$ based on his experience or knowledge. If the decision maker cannot identify the values of parameters $\lambda_1$ and $\lambda_2$, he may generate randomly their values from some prescribed subintervals of $[0, 1]$. In our experiments, to identify the influence of perturbation distribution on solution results, the optimal solutions are first computed by adjusting the selection parameter $\lambda_1$ in the optimization problem with fixed $\lambda_2 = 0.6$. When $\lambda_1$ increases its value from 0.1 to 1 with step 0.1, the computational results about the optimal order quantities of product 1 and product 2 are plotted in Figure 1, and the corresponding mean total profits are plotted in Figure 2. From Figures 1 and 2, it is found that the optimal order quantities and mean total profit vary while the selection parameter $\lambda_1$ varies. Specifically, the optimal order quantity of product 1 is monotone decreasing with respect to parameter $\lambda_1$, while the optimal order quantity of product 2 is monotone increasing with respect to parameter $\lambda_1$. As a result, the mean total profit is monotone increasing with respect to parameter $\lambda_1$.

In the following, the optimal solutions are computed by adjusting the selection parameter $\lambda_2$ in the optimization problem with fixed $\lambda_1 = 0.6$. When $\lambda_2$ increases its value from 0.1 to 1 with step 0.1, the computational results of the

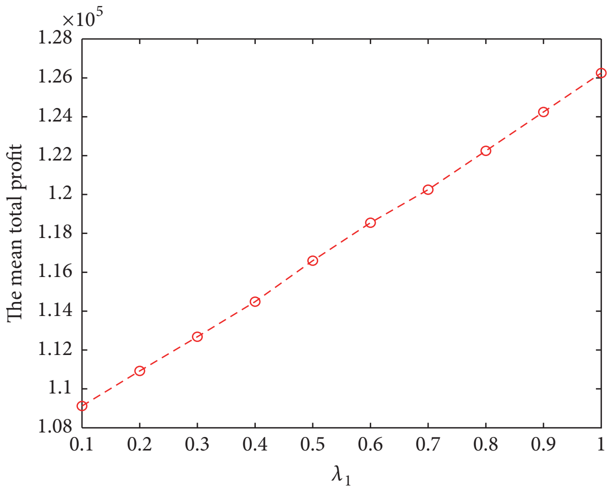


Figure 2: The mean total profit.

optimal order quantities of product 1 and product 2 are plotted in Figure 3, and the corresponding mean total profits are plotted in Figure 4. From Figures 3 and 4, it is concluded that the optimal order quantity of product 1 is monotone increasing with respect to parameter $\lambda_2$, while the optimal order quantity of product 2 is monotone decreasing with respect to parameter $\lambda_2$. As a result, the mean total profit is monotone decreasing with respect to parameter $\lambda_2$.

The above computational results demonstrate that the optimal order quantities of product 1 and product 2 depend heavily on the location parameters $\lambda_1$ and $\lambda_2$. That is, the optimal order quantities of our multiproduct single-period inventory problem depend heavily on the possibility distribution of uncertain demand.

### 4.4. Comparison Study

*4.4.1. Comparing with Stochastic Optimization Method.* In this subsection, the credibilistic optimization method is compared with stochastic optimization method, where the stochastic demands of product 1 and product 2 follow normal probability distributions $\mathcal{N}(800, 55^2)$ and $\mathcal{N}(2400, 75^2)$, respectively. According to the stochastic optimization method for MSIMP, the optimal order quantities for product 1 and product 2 are 815 and 2407 with the maximum mean profit 189530. The solution result is totally different from our credibilistic optimal solutions reported in Figures 1 and 3. Compared with our credibilistic optimal solutions, the optimal solutions 815 and 2407 to stochastic model are not feasible solutions to the deterministic programming model in Theorem 5. That is, the stochastic optimal solution does not satisfy carbon emission constraint (24) and the investment amount constraint (25).

*4.4.2. Comparing with Fuzzy Optimization Method under Fixed Possibility Distribution.* In this subsection, the credibilistic optimization method is compared with fuzzy optimization method, where the uncertain demands of product 1 and product 2 follow fixed possibility distributions. For the sake of comparison, the fixed possibility distributions are taken as the nominal possibility distributions of uncertain demands corresponding to $\theta_{li} = \theta_{ri} = 0$, $i = 1, 2$. By solving the fuzzy optimization model, the obtained nominal optimal order quantities are 800 and 2400 with the nominal maximum mean total profit 183748.85. Obviously, the nominal maximum mean total profit is larger than the optimal mean total profits obtained in Figures 2 and 4. The computational results imply that a small perturbation of the nominal possibility distribution may heavily affect the quality of optimal solution.

To further analyze the influence of the perturbation parameters, some additional experiments are conducted with different values of perturbation parameters $\theta_{li}$ and $\theta_{ri}$. The computational results are reported in Tables 3–6, in which the robust value is defined as the reduction from the nominal optimal profit to the optimal profits with different values of perturbation parameters. The computational results imply that the robust value is increasing with respect to perturbation parameters $\theta_l$ or $\theta_r$; that is, the larger the perturbation parameter, the larger the uncertainty degree embedded in the generalized PIV possibility distribution of uncertain demand. The decision makers can adjust the values of perturbation parameters according to their obtained distribution information. As a consequence, the considered MSIMP depends heavily on the location parameter $\lambda$ and perturbation parameter $\theta$. For practical inventory management problems, if decision makers cannot identify the values of parameters $\lambda$ and $\theta$, they may generate randomly their values from some prescribed subintervals of $[0, 1]$. The computational results demonstrate the advantages of variable possibility distributions over fixed possibility distributions.

The comparison studies described in Sections 4.4.1 and 4.4.2 lead to the following observations.

Firstly, stochastic optimization method for MSIMP is based on the assumption that the market demands are of stochastic nature, and the probability distributions of uncertain parameters are available. When the probability distributions of uncertain market demands cannot be determined, the stochastic optimization method cannot be used to determine the optimal order quantities.

Secondly, in fuzzy MSIMP, it is usually assumed that the nominal possibility distributions of uncertain parameters can be determined exactly and a small perturbation of nominal possibility distribution will not affect significantly the solution quality. The comparison study shows that the robust value is increasing with respect to perturbation parameters. The decision makers should adjust the values of perturbation parameters according to their obtained distribution information.

Finally, it should be highlighted that under given perturbation parameters the optimal order quantities depend heavily on the values of location parameter lambda. The proposed parametric credibilistic optimization method is capable of detecting cases when perturbation distributions can heavily affect the quality of the nominal solution. In these cases, the decision makers should employ the proposed credibilistic optimization method to find the optimal order

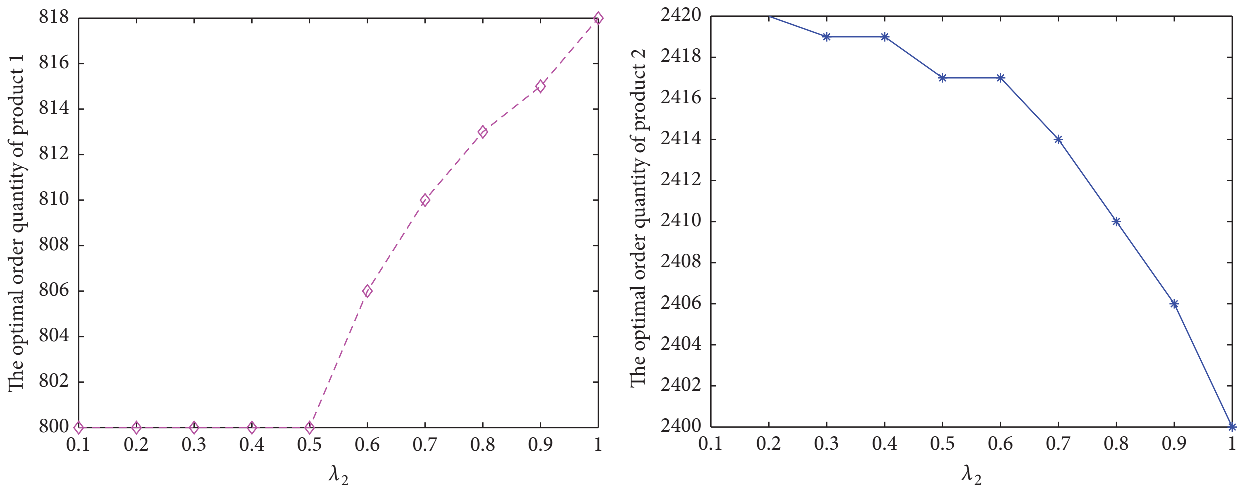


FIGURE 3: The optimal order quantities of product 1 and product 2 with $\lambda_1 = 0.6$.

TABLE 3: The influence of perturbation parameter $\theta_{l1}$ with $\theta_{r1} = 0.25$, $\theta_{l2} = 0.15$, and $\theta_{r2} = 0.2$.

| $\theta_{l1}$ | $Q_1^*$ | $Q_2^*$ | The mean total profit | The robust value |
|---|---|---|---|---|
| 0.05 | 810 | 2415 | 134003.40 | 49745.45 |
| 0.10 | 809 | 2416 | 130918.55 | 52830.30 |
| 0.15 | 809 | 2417 | 127886.71 | 55862.14 |
| 0.20 | 808 | 2417 | 124771.49 | 58977.36 |
| 0.30 | 806 | 2417 | 118548.70 | 65200.15 |

TABLE 4: The influence of perturbation parameter $\theta_{r1}$ with $\theta_{l1} = 0.3$, $\theta_{l2} = 0.15$, and $\theta_{r2} = 0.2$.

| $\theta_{r1}$ | $Q_1^*$ | $Q_2^*$ | The mean total profit | The robust value |
|---|---|---|---|---|
| 0.05 | 818 | 2400 | 131554.32 | 52194.53 |
| 0.15 | 816 | 2406 | 125167.66 | 58581.19 |
| 0.18 | 813 | 2409 | 123169.52 | 60579.33 |
| 0.25 | 806 | 2417 | 118548.70 | 65200.15 |
| 0.35 | 800 | 2421 | 111953.16 | 71795.69 |

TABLE 5: The influence of perturbation parameter $\theta_{l2}$ with $\theta_{l1} = 0.3$, $\theta_{r1} = 0.25$, and $\theta_{r2} = 0.2$.

| $\theta_{l2}$ | $Q_1^*$ | $Q_2^*$ | The mean total profit | The robust value |
|---|---|---|---|---|
| 0.10 | 807 | 2419 | 121531.99 | 62216.86 |
| 0.15 | 806 | 2417 | 118548.70 | 65200.15 |
| 0.20 | 804 | 2414 | 115493.36 | 68255.49 |
| 0.25 | 800 | 2411 | 112352.60 | 71396.25 |
| 0.30 | 800 | 2406 | 109349.89 | 74398.96 |

TABLE 6: The influence of perturbation parameter $\theta_{r2}$ with $\theta_{l1} = 0.3$, $\theta_{r1} = 0.25$, and $\theta_{l2} = 0.15$.

| $\theta_{r2}$ | $Q_1^*$ | $Q_2^*$ | The mean total profit | The robust value |
|---|---|---|---|---|
| 0.10 | 800 | 2422 | 126656.59 | 57092.26 |
| 0.15 | 800 | 2420 | 122469.12 | 61279.73 |
| 0.20 | 806 | 2417 | 118548.70 | 65200.15 |
| 0.25 | 812 | 2412 | 114563.42 | 69185.43 |
| 0.30 | 816 | 2400 | 110287.01 | 73461.84 |

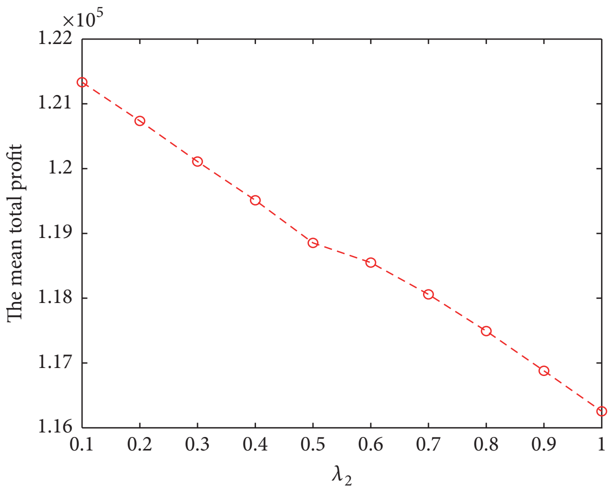


Figure 4: The mean total profit.

quantities, which may immunize against the effect of perturbation distribution.

## 5. Conclusions

In this paper, the MSIMP has been studied from a new perspective. The major new results include the following several aspects.

(i) When the distribution information about uncertain demand and uncertain carbon emission was partially available, these uncertain parameters were characterized by generalized PIV fuzzy variables. For their selection variables, the analytical expressions of the mean and second-order moment have been established.

(ii) Two new indexes, mean and second-order moment, about the total profit were defined based on L-S multiple integral, and their analytical expressions have been established. Furthermore, a new parametric credibilistic optimization model was developed for MSIMP.

(iii) The equivalent deterministic model of the proposed credibilistic MSIMP has been established. According to the structural characteristics of the equivalent deterministic model, a domain decomposition method was designed to find the optimal order quantities.

(iv) In numerical experiments, the proposed optimization method was compared with stochastic optimization method and fuzzy optimization method under fixed possibility distribution. The computational results demonstrated that a small perturbation of the demand distribution could make the nominal optimal solution infeasible and thus practically meaningless. In this case, the decision makers should employ the proposed credibilistic optimization method to find the optimal order quantities, which may immunize against the effect of perturbation distribution.

The developed parametric credibilistic optimization model for MSIMP addressed the effect of perturbation possibility distributions. In the model process, the generalized PIV fuzzy variables were represented by their lambda selections. For a practical MSIMP, based on the uncertain distribution sets of generalized PIV fuzzy variables, distributionally robust optimization method will be studied in our future research. Extension to considering decision makers' risk tolerance fuzziness [42] for the MSIMP is another interesting research direction. In addition, hybrid uncertainty and their solution method [43] can be introduced to tackle the MSIMP.

## Appendix

## Proofs of Main Theorems

*Proof of Theorem 1.* Since $\xi \sim n(\mu,\sigma^2;\theta_l,\theta_r)$, the generalized possibility distribution of $\xi^\lambda$ is

$$\mu_{\xi^\lambda}(r;\theta_l,\theta_r) = e^{-(r-\mu)^2/2\sigma^2}\left[1-(1-\lambda)\theta_l-\lambda\theta_r\right] + \lambda\theta_r. \tag{A.1}$$

According to the definition of credibility measure [44], the credibility $\mathrm{Cr}\{\xi^\lambda \le r\}$ is computed by

$$\mathrm{Cr}\left\{\xi^\lambda \le r\right\} = \frac{1}{2}\left\{\sup_{x\in\Re}\mu_{\xi^\lambda}(x;\theta_l,\theta_r) + \sup_{x\le r}\mu_{\xi^\lambda}(x;\theta_l,\theta_r) - \sup_{x>r}\mu_{\xi^\lambda}(x;\theta_l,\theta_r)\right\}, \tag{A.2}$$

which can generate a measure using the method discussed in [45]. By calculation, one has

$$\mathrm{Cr}\left\{\xi^\lambda \le r\right\} = \begin{cases} \dfrac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2} + \dfrac{\lambda\theta_r}{2}, & r\in(-\infty,\mu) \\ 1-(1-\lambda)\theta_l - \dfrac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2} - \dfrac{\lambda\theta_r}{2}, & r\in[\mu,+\infty). \end{cases} \tag{A.3}$$

According to (3) and (A.3), one has

$$E\left(\xi^\lambda\right) = \int_{(-\infty,+\infty)} r\,\mathrm{d}\mathrm{Cr}\left\{\xi^\lambda\le r\right\}$$

$$= \int_{(-\infty,\mu)} r\,\mathrm{d}\left\{\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2} + \frac{\lambda\theta_r}{2}\right\} + \int_{[\mu,+\infty)} r\,\mathrm{d}\left\{1-(1-\lambda)\theta_l\right.$$

$$-\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2}-\frac{\lambda\theta_r}{2}\Big\}=[1-(1-\lambda)\theta_l-\lambda\theta_r]\mu=\omega\mu. \tag{A.4}$$

The proof of theorem is complete. □

*Proof of Theorem 2.* Since $\eta \sim \mathrm{Tri}(r_1, r_2, r_3; \theta_l, \theta_r)$, the generalized possibility distribution of the lambda selection variable $\eta^\lambda$ is

$$\mu_{\eta^\lambda}(r,\theta)=\begin{cases}\lambda\theta_r+\dfrac{(r-r_1)[1-(1-\lambda)\theta_l-\lambda\theta_r]}{r_2-r_1}, & r\in[r_1,r_2]\\ \lambda\theta_r+\dfrac{(r_3-r)[1-(1-\lambda)\theta_l-\lambda\theta_r]}{r_3-r_2}, & r\in[r_2,r_3].\end{cases} \tag{A.5}$$

According to the definition of credibility measure, one has

$$\mathrm{Cr}\{\eta^\lambda\le r\}=\begin{cases}0, & r\in(-\infty,r_1)\\ \dfrac{1}{2}\lambda\theta_r+\dfrac{(r-r_1)[1-(1-\lambda)\theta_l-\lambda\theta_r]}{2(r_2-r_1)}, & r\in[r_1,r_2)\\ 1-(1-\lambda)\theta_l-\dfrac{1}{2}\left\{\lambda\theta_r+\dfrac{(r_3-r)[1-(1-\lambda)\theta_l-\lambda\theta_r]}{r_3-r_2}\right\}, & r\in[r_2,r_3)\\ 1-(1-\lambda)\theta_l & r\in[r_3,+\infty).\end{cases} \tag{A.6}$$

By calculation, one has

$$\begin{aligned}E(\eta^\lambda)&=\int_{[r_1,r_3]}r\,\mathrm{dCr}\{\eta^\lambda\le r\}\\&=\int_{[r_1,r_1]}r\,\mathrm{dCr}\{\eta^\lambda\le r\}+\int_{(r_1,r_2]}r\,\mathrm{dCr}\{\eta^\lambda\le r\}+\int_{(r_2,r_3)}r\,\mathrm{dCr}\{\eta^\lambda\le r\}+\int_{[r_3,r_3]}r\,\mathrm{dCr}\{\eta^\lambda\le r\}\\&=[1-(1-\lambda)\theta_l]\frac{r_1+2r_2+r_3}{4}+\frac{1}{4}\lambda\theta_r(r_1-2r_2+r_3).\end{aligned} \tag{A.7}$$

The proof of theorem is complete. □

*Proof of Theorem 3.* According to (5) and (6), the second-order moment of $\xi^\lambda$ is computed as follows:

$$\begin{aligned}M(\xi^\lambda)&=\int_{(-\infty,+\infty)}[r-\omega\mu]^2\,\mathrm{dCr}\{\xi^\lambda\le r\}\\&=\int_{(-\infty,\mu)}[r-\omega\mu]^2\,\mathrm{d}\left\{\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2}+\frac{\lambda\theta_r}{2}\right\}\\&\quad+\int_{[\mu,+\infty)}[r-\omega\mu]^2\,\mathrm{d}\left\{1-(1-\lambda)\theta_l-\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{2}e^{-(r-\mu)^2/2\sigma^2}-\frac{\lambda\theta_r}{2}\right\}\\&=\frac{\omega}{2}\left\{\int_{(-\infty,\mu)}[r-\omega\mu]^2\,\mathrm{d}\left\{e^{-(r-\mu)^2/2\sigma^2}\right\}-\int_{[\mu,+\infty)}[r-\omega\mu]^2\,\mathrm{d}\left\{e^{-(r-\mu)^2/2\sigma^2}\right\}\right\}=\omega\left[2\sigma^2+(\omega-1)^2\mu^2\right].\end{aligned} \tag{A.8}$$

The proof of theorem is complete. □

*Proof of Theorem 4.* In the following, the mean value of $\eta^\lambda$ is denoted as $m$; that is, $m = E(\eta^\lambda)$. According to (5) and (7), the second-order moment of the lambda selection variable $\eta^\lambda$ is computed by

$$\begin{aligned}M(\eta^\lambda)&=\int_{[r_1,r_4]}\left[r-E(\eta^\lambda)\right]^2\mathrm{dCr}\{\eta^\lambda\le r\}\\&=\int_{(r_1,r_2)}[r-m]^2\,\mathrm{d}\left\{\frac{1}{2}\lambda\theta_r+\frac{(r-r_1)[1-(1-\lambda)\theta_l-\lambda\theta_r]}{2(r_2-r_1)}\right\}\end{aligned}$$

$$
\begin{aligned}
&+\int_{(r_2,r_3)}[r-m]^2\,\mathrm{d}\left\{1-(1-\lambda)\theta_l-\frac{1}{2}\left[\lambda\theta_r+\frac{(r_3-r)\left[1-(1-\lambda)\theta_l-\lambda\theta_r\right]}{r_3-r_2}\right]\right\}+\frac{1}{2}\lambda\theta_r(r_1-m)^2\\
&+\frac{1}{2}\lambda\theta_r(r_3-m)^2\\
&=\frac{1}{2}\lambda\theta_r\left[(r_1-m)^2+(r_3-m)^2\right]+\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{6(r_2-r_1)}\left[(r_2-m)^3-(r_1-m)^3\right]\\
&+\frac{1-(1-\lambda)\theta_l-\lambda\theta_r}{6(r_3-r_2)}\left[(r_3-m)^3-(r_2-m)^3\right].
\end{aligned}
\tag{A.9}
$$

The proof of theorem is complete. □

*Proof of Theorem 5.* First, objective function (18) is equivalent to maximizing $\Pi^e(Q)-\gamma\sqrt{\Pi^m(Q)}$. By calculation the multiple L-S integrals, one has

$$
\begin{aligned}
\Pi^e(Q)&=E\left[\Pi\left(Q,\xi^\lambda\right)\right]\\
&=\int\cdots\int_{\mathscr{R}^n}\sum_{i=1}^{n}\pi(Q_i,r)\,\mathrm{d}\left(\mathrm{Cr}\{\xi_1^\lambda\le r\}\times\cdots\times\mathrm{Cr}\{\xi_n^\lambda\le r\}\right)\\
&=\sum_{i=1}^{n}\int\cdots\int_{\mathscr{R}^n}\pi(Q_i,r)\,\mathrm{d}\left(\mathrm{Cr}\{\xi_1^\lambda\le r\}\times\cdots\times\mathrm{Cr}\{\xi_n^\lambda\le r\}\right)\\
&=\sum_{i=1}^{n}\left(m_i\prod_{k\ne i}h_k\right).
\end{aligned}
\tag{A.10}
$$

Similarly, the second-order moment of the total profit is computed by

$$
\begin{aligned}
\Pi^m(Q)&=M\left[\Pi\left(Q,\xi^\lambda\right)\right]=\int\cdots\int_{\mathscr{R}^n}\left\{\sum_{i=1}^{n}\left[\pi(Q_i,r)-m_i\prod_{k\ne i}h_k\right]\right\}^2\mathrm{d}\left(\mathrm{Cr}\{\xi_1^\lambda\le r\}\times\cdots\times\mathrm{Cr}\{\xi_n^\lambda\le r\}\right)\\
&=\sum_{i=1}^{n}\int\cdots\int_{\mathscr{R}^n}\left[\pi(Q_i,r)-m_i\prod_{k\ne i}h_k\right]^2\mathrm{d}\left(\mathrm{Cr}\{\xi_1^\lambda\le r\}\times\cdots\times\mathrm{Cr}\{\xi_n^\lambda\le r\}\right)\\
&+\sum_{i\ne j}\int\cdots\int_{\mathscr{R}^n}\left(\pi(Q_i,r)-m_i\prod_{k\ne i}h_k\right)\left(\pi(Q_j,r)-m_j\prod_{k\ne j}h_k\right)\mathrm{d}\left(\mathrm{Cr}\{\xi_1^\lambda\le r\}\times\cdots\times\mathrm{Cr}\{\xi_n^\lambda\le r\}\right)\\
&=\sum_{i=1}^{n}\left[\prod_{k\ne i}h_k\int_{[0,D_i]}\{\pi(Q_i,r)\}^2\,\mathrm{dCr}\{\xi_i^\lambda\le r\}+\left(m_i\prod_{k\ne i}h_k\right)^2\left(\prod_{k=1}^{n}h_k-2\right)\right]+\prod_{k\ne i\ne j}h_k\left[\sum_{i\ne j}m_im_j\left(1-\prod_{k=1}^{n}h_k\right)^2\right].
\end{aligned}
\tag{A.11}
$$

Next, the equivalence between constraint (19) and constraint (24) is discussed. Since $\eta_i$ $(1\le i\le n)$ are mutually independent generalized PIV fuzzy variables, according to [36], one has

$$
\sum_{i=1}^{n}Q_i\eta_i=\mathrm{Tri}\left(\sum_{i=1}^{n}Q_ir_{1i},\sum_{i=1}^{n}Q_ir_{2i},\sum_{i=1}^{n}Q_ir_{3i};\theta_l,\theta_r\right),\tag{A.12}
$$

where $\theta_l=\max_{1\le i\le n}\theta_{li}$ and $\theta_r=\max_{1\le i\le n}\theta_{ri}$.

Since the credibility $\mathrm{Cr}\{\xi\le r\}$ is a monotone increasing function, $\mathrm{Cr}\{(\sum_{i=1}^{n}Q_i\eta_i)^\lambda\le K\}\ge\beta$ is equivalent to

$$
K\ge\inf\left\{x\mid\mathrm{Cr}\left\{\left(\sum_{i=1}^{n}Q_i\eta_i\right)^\lambda\le x\right\}\ge\beta\right\};\tag{A.13}
$$

that is, $K\ge(\sum_{i=1}^{n}Q_i\eta_i)^\lambda_{\inf}(\beta)$. The proof of theorem is complete. □

## Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

## Acknowledgments

This work is supported by the National Natural Science Foundation of China (no. 61773150).

## References

[1] G. Hadley and T. M. Whitin, *Analysis of Inventory Systems*, Prentice-Hall, Englewood Cliffs, NJ, USA, 1963.

[2] S. Nahmias and C. P. Schmidt, "An efficient heuristic for the multi-item newsboy problem with a single constraint," *Naval Research Logistics Quarterly*, vol. 31, no. 3, pp. 463–474, 1984.

[3] H.-S. Lau and A. H. L. Lau, "The multi-product multi-constraint newsboy problem: Applications, formulation and solution," *Journal of Operations Management*, vol. 13, no. 2, pp. 153–162, 1995.

[4] S. J. Erlebacher, "Optimal and heuristic solutions for the multi-item newsvendor problem with a single capacity constraint," *Production Engineering Research and Development*, vol. 9, no. 3, pp. 303–318, 2000.

[5] I. Moon and E. A. Silver, "The multi-item newsvendor problem with a budget constraint and fixed ordering costs," *Journal of the Operational Research Society*, vol. 51, no. 5, pp. 602–608, 2000.

[6] L. Abdel-Malek, R. Montanari, and D. Meneghetti, "The capacitated newsboy problem with random yield: The Gardener Problem," *International Journal of Production Economics*, vol. 115, no. 1, pp. 113–127, 2008.

[7] G. Zhang, "The multi-product newsboy problem with supplier quantity discounts and a budget constraint," *European Journal of Operational Research*, vol. 206, no. 2, pp. 350–360, 2010.

[8] B. Zhang and S. Du, "Multi-product newsboy problem with limited capacity and outsourcing," *European Journal of Operational Research*, vol. 202, no. 1, pp. 107–113, 2010.

[9] L. L. Abdel-Malek and R. Montanari, "On the multi-product newsboy problem with two constraints," *Computers & Operations Research*, vol. 32, no. 8, pp. 2095–2116, 2005.

[10] D. Huang, H. Zhou, and Q.-H. Zhao, "A competitive multiple-product newsboy problem with partial product substitution," *Omega* , vol. 39, no. 3, pp. 302–312, 2011.

[11] A. Özler, B. Tan, and F. Karaesmen, "Multi-product newsvendor problem with value-at-risk considerations," *International Journal of Production Economics*, vol. 117, no. 2, pp. 244–255, 2009.

[12] G. Van Ryzin and S. Mahajan, "On the relationship between inventory costs and variety benefits in retail assortments," *Management Science*, vol. 45, no. 11, pp. 1496–1509, 1999.

[13] J. A. Van Mieghem, "Risk mitigation in newsvendor networks: Resource diversification, flexibility, sharing, and hedging," *Management Science*, vol. 53, no. 8, pp. 1269–1288, 2007.

[14] S. Wang and W. Pedrycz, "Data-driven adaptive probabilistic robust optimization using information granulation," *IEEE Transactions on Cybernetics*, 2017.

[15] G. L. Vairaktarakis, "Robust multi-item newsboy models with a budget constraint," *International Journal of Production Economics*, vol. 66, no. 3, pp. 213–226, 2000.

[16] J. Kamburowski, "The distribution-free newsboy problem under the worst-case and best-case scenarios," *European Journal of Operational Research*, vol. 237, no. 1, pp. 106–112, 2014.

[17] M.-H. Shu, C.-W. Yeh, and Y.-C. Fu, "Impacts of transportation cost on distribution-free newsboy problems," *Mathematical Problems in Engineering*, vol. 2014, Article ID 307935, 2014.

[18] I. Moon, D. K. Yoo, and S. Saha, "The distribution-free newsboy problem with multiple discounts and upgrades," *Mathematical Problems in Engineering*, vol. 2016, Article ID 2017253, 2016.

[19] O. Solyali, J. F. Cordeau, and G. Laporte, "The impact of modeling on robust inventory management under demand uncertainty," *Management Science*, vol. 62, no. 4, pp. 1188–1201, 2016.

[20] R. Qiu and J. Shang, "Robust optimisation for risk-averse multi-period inventory decision with partial demand distribution information," *International Journal of Production Research*, vol. 52, no. 24, pp. 7472–7495, 2014.

[21] S. Fay and J. Xie, "Timing of product allocation: Using probabilistic selling to enhance inventory management," *Management Science*, vol. 61, no. 2, pp. 474–484, 2015.

[22] W. Luo and K. Shang, "Joint inventory and cash management for multidivisional supply chains," *Operations Research*, vol. 63, no. 5, pp. 1098–1116, 2015.

[23] X. Chen, P. Hu, S. Shum, and Y. Zhang, "Dynamic stochastic inventory management with reference price effects," *Operations Research*, vol. 64, no. 6, pp. 1529–1536, 2016.

[24] H. Mamani, S. Nassiri, and M. R. Wagner, "Closed-Form Solutions for Robust Inventory Management," *Management Science*, vol. 63, no. 5, pp. 1625–1643, 2017.

[25] J. Kacprzyk and P. Stanieski, "Long-term inventory policy-making through fuzzy decision-making models," *Fuzzy Sets and Systems*, vol. 8, no. 2, pp. 117–132, 1982.

[26] D. Petrović, R. Petrović, and M. Vujošević, "Fuzzy models for the newsboy problem," *International Journal of Production Economics*, vol. 45, no. 1–3, pp. 435–441, 1996.

[27] N. K. Mandal and T. K. Roy, "A displayed inventory model with L-R fuzzy number," *Fuzzy Optimization and Decision Making. A Journal of Modeling and Computation Under Uncertainty*, vol. 5, no. 3, pp. 227–243, 2006.

[28] X. Y. Ji and Z. Shao, "Fuzzy multi-product constraint newsboy problem," *Applied Mathematics and Computation*, vol. 180, no. 1, pp. 7–15, 2006.

[29] P. Dutta, "A multi-product newsboy problem with fuzzy customer demand and a storage space constraint," *International Journal of Operational Research*, vol. 8, no. 2, pp. 230–246, 2010.

[30] A. Saha, S. Kar, and M. Maiti, "Multi-item fuzzy-stochastic supply chain models for long-term contracts with a profit sharing scheme," *Applied Mathematical Modelling*, vol. 39, no. 10-11, pp. 2815–2828, 2015.

[31] Z.-Z. Guo, "Optimal inventory policy for single-period inventory management problem under equivalent value criterion," *Journal of Uncertain Systems*, vol. 10, no. 4, pp. 302–311, 2016.

[32] S.-N. Tian and Z.-Z. Guo, "A credibilistic optimization approach to single-product single-period inventory problem with two suppliers," *Journal of Uncertain Systems*, vol. 10, no. 3, pp. 223–240, 2016.

[33] Z.-Q. Liu and Y.-K. Liu, "Type-2 fuzzy variables and their arithmetic," *Soft Computing*, vol. 14, no. 7, pp. 729–747, 2010.

[34] Y. Liu and Y.-K. Liu, "The lambda selections of parametric interval-valued fuzzy variables and their numerical characteristics," *Fuzzy Optimization and Decision Making. A Journal of Modeling and Computation Under Uncertainty*, vol. 15, no. 3, pp. 255–279, 2016.

[35] Y. Chen and S. Shen, "The cross selections of parametric Interval-Valued fuzzy variables," *Journal of Uncertain Systems*, vol. 9, no. 2, pp. 156–160, 2015.

[36] Y.-K. Liu and Z.-Z. Guo, "Arithmetic about linear combinations of GPIV fuzzy variables," *Journal of Uncertain Systems*, vol. 11, no. 2, pp. 154–160, 2017.

[37] M. Carter and B. van Brunt, *The Lebesgue-Stieltjes Integral*, Spinger-Verlag, New York, NY, USA, 2000.

[38] Z. Guo, Y. Liu, and Y. Liu, "Coordinating a three level supply chain under generalized parametric interval-valued distribution of uncertain demand," *Journal of Ambient Intelligence and Humanized Computing*, vol. 8, no. 5, pp. 677–694, 2017.

[39] S. Wang, T. S. Ng, and M. Wong, "Expansion planning for waste-to-energy systems using waste forecast prediction sets," *Naval Research Logistics*, vol. 63, no. 1, pp. 47–70, 2016.

[40] X. Bai and Y. K. Liu, "Robust optimization of supply chain network design in fuzzy decision system," *Journal of Intelligent Manufacturing*, vol. 27, no. 6, pp. 1131–1149, 2016.

[41] Y. Liu and Y.-K. Liu, "Distributionally robust fuzzy project portfolio optimization problem with interactive returns," *Applied Soft Computing*, vol. 56, pp. 655–668, 2017.

[42] S. Wang, B. Wang, and J. Watada, "Adaptive Budget-Portfolio Investment Optimization Under Risk Tolerance Ambiguity," *IEEE Transactions on Fuzzy Systems*, vol. 25, no. 2, pp. 363–376, 2017.

[43] S. Wang and W. Pedrycz, "Robust granular optimization: a structured approach for optimization under integrated uncertainty," *IEEE Transactions on Fuzzy Systems*, vol. 23, no. 5, pp. 1372–1386, 2015.

[44] B. Liu and Y. Liu, "Expected value of fuzzy variable and fuzzy expected value models," *IEEE Transactions on Fuzzy Systems*, vol. 10, no. 4, pp. 445–450, 2002.

[45] Y.-K. Liu and Y. Liu, "Measure generated by joint credibility distribution function," *Journal of Uncertain Systems*, vol. 8, no. 9, pp. 239-240, 2014.