

## Research Article

# Reliability Evaluation of Bridges Based on Nonprobabilistic Response Surface Limit Method

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Due to many uncertainties in nonprobabilistic reliability assessment of bridges, the limit state function is generally unknown. The traditional nonprobabilistic response surface method is a lengthy and oscillating iteration process and leads to difficulty solving the nonprobabilistic reliability index. This article proposes a nonprobabilistic response surface limit method based on the interval model. The intention of this method is to solve the upper and lower limits of the nonprobabilistic reliability index and to narrow the range of the nonprobabilistic reliability index. If the range of the reliability index reduces to an acceptable accuracy, the solution will be considered convergent, and the nonprobabilistic reliability index will be obtained. The case study indicates that using the proposed method can avoid oscillating iteration process, make iteration process stable and convergent, reduce iteration steps significantly, and improve computational efficiency and precision significantly compared with the traditional nonprobabilistic response surface method. Finally, the nonprobabilistic reliability evaluation process of bridge will be built through evaluating the reliability of one PC continuous rigid frame bridge with three spans using the proposed method, which appears to be more simple and reliable when lack of samples and parameters in the bridge nonprobabilistic reliability evaluation is present.

## 1. Introduction

There are many unavoidable uncertainties in practical structure engineering. Traditionally, the probability model is utilized in the structural reliability analysis [1]. Probabilistic reliability analysis strongly depends on the probability distribution function, which relies on a large number of statistical data [2]. However, for some important and complicated structures, many uncertain parameters have little or no statistical data, which causes difficulties in accurate description of parameter distribution. In addition, probabilistic reliability is very sensitive to variations of model parameters. Small errors in statistical data can lead to considerable errors in the structure [2, 3].

Because of inadequate data, the probabilistic reliability is not useful for solving these practical problems. However, nonprobabilistic reliability can effectively deal with reliability problems when only few statistical data can be obtained. A nonprobabilistic convex model was first proposed in the 1990s by Ben-Haim [4, 5]. Ben-Haim and Elishakoff [6]

proposed a nonprobabilistic safety factor to measure the nonprobabilistic reliability index using an interval theory. The nonprobabilistic reliability theories presented by Ben-Haim [4, 5] and Ben-Haim and Elishakoff [6] were not involved in probability at all and could overcome the inextricable difficulties faced by the traditional probability model. Therefore, nonprobabilistic reliability is an appropriate method when the available data of uncertainties are limited or absent. This was illustrated by Guo et al. [7, 8], who contrasted the probabilistic and nonprobabilistic reliability methods through modeling concepts, model construction, and formulations for computation. Nonprobabilistic reliability of structures has become a new, exciting research direction, and the corresponding research approach has also aroused wide attention from theory and engineering circles.

In summary, nonprobabilistic reliability analysis was generally based on the interval model [9, 10] or the convex model [11–13]. The nonprobabilistic reliability index based on the interval model is actually the minimum norm of the coordinate vector in the standardized space, and solving the

reliability index is an optimum problem with the equality constraint. For the linear performance functions, the analytical expression of the nonprobabilistic reliability index can be easily obtained. However, the performance functions are generally nonlinear in practical engineering.

For simple nonlinear performance functions, Guo et al. [7, 8] suggested definition approach, transfer approach, and optimization approach to solve the nonprobabilistic reliability index. For complexly and strongly nonlinear performance functions, researchers frequently used an optimized iterative algorithm. For the hyperellipsoidal model, the nonprobabilistic reliability index is in accordance with the probabilistic reliability index in definition, so the design point method, successfully applied to the probabilistic reliability analysis, can be used for the nonprobabilistic reliability analysis [14]. For the reliability index defined by the Euclidean norm, the Most Probable failure Point (MPP) can be obtained along the normal direction of the limit state surface. For the nonprobabilistic reliability index based on the interval model defined by the infinite norm, the MPP might not be along the normal direction of the limit state surface.

In order to simplify the search process, other researchers suggested the one-dimensional optimization method [15, 16] and the space search algorithm [17]. These two methods are correct only for the linear performance function, because only part of the probable failure points is searched for the nonlinear performance function. If the performance functions are the normalized quadratic expression, the Sequence Quadratic Programming (SQP) can be used to solve the nonprobabilistic reliability index based on the interval model [18]. Recently, the Gradient Projection Method (GPM) was proposed to solve the nonprobabilistic reliability index [19]. GPM is the general method that is most suitable for the nonprobabilistic convex model, although the convergence process during the iteration needs special treatment. The interval model was used in the structural reliability optimization design [20, 21], although this issue is not discussed in this article in detail.

When the limit state function is not easily obtained, the above methods are not applicable. Jiang et al. [22] presented the nonprobabilistic response surface method based on the interval model, which contributed to solving the nonprobabilistic reliability index for the implicit performance function. Chen et al. [23] used the response surface method to build the explicit performance function, accepted the interval model and the ellipsoidal model to compute the nonprobabilistic reliability index, and compared the results from these two methods.

Until present, nonprobabilistic reliability analysis based on the convex model and the interval model has achieved some positive results and been applied in engineering practice. A great obstacle is solving the nonprobabilistic reliability index. The traditional nonprobabilistic response surface method is prone to lengthy and oscillating iteration processes, which leads to difficulty solving the nonprobabilistic reliability index. For these problems, this article will propose the nonprobabilistic response surface limit method based on the interval model and build the nonprobabilistic reliability evaluation process for bridges through evaluation of the

reliability of one PC continuous rigid frame bridge with three spans.

## 2. Nonprobabilistic Reliability Index Based on the Interval Model

Suppose  $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$  is the set of the basic interval variables for the structures, where  $x_i \in X_i^I$  ( $i = 1, 2, \dots, n$ ). The performance function is given as

$$M = g(\mathbf{x}) = g(x_1, \dots, x_n). \quad (1)$$

When  $g(\mathbf{x})$  is the continuous function of  $x_i$  ( $i = 1, 2, \dots, n$ ),  $M$  also is the interval variable. From the perspective of the nonprobabilistic reliability theory, the hypersurface  $g(\mathbf{x}) = 0$  is the failure surface of the structures. When  $g(\mathbf{x}) < 0$ , the structure is in the failure state; when  $g(\mathbf{x}) > 0$  the structure is safe.

If  $M^c$  and  $M^r$  represent the mean and dispersion of  $M$ , respectively, the nonprobabilistic reliability index is estimated as

$$\eta = \frac{M^c}{M^r} \in \begin{cases} [1, +\infty) & \text{Reliable and safe} \\ (-1, 1) & \text{Unreliable} \\ (-\infty, -1] & \text{Failure.} \end{cases} \quad (2)$$

Based on (2), when  $\eta > 1$ ,  $g(\mathbf{x}) > 0$  for any  $x_i \in X_i^I$  ( $i = 1, 2, \dots, n$ ), which means the structure is in the safe state; when  $\eta < -1$ ,  $g(\mathbf{x}) < 0$  for any  $x_i \in X_i^I$  ( $i = 1, 2, \dots, n$ ), which means the structure is failed; when  $-1 \leq \eta \leq 1$ ,  $g(\mathbf{x}) \geq 0$  and  $g(\mathbf{x}) \leq 0$  maybe occurs for any  $x_i \in X_i^I$  ( $i = 1, 2, \dots, n$ ), which means the structure may be safe or at risk. The greater  $\eta$ , the safer the structure.

Standardize the interval variable  $x_i$  through the following transformation formula:

$$x_i = x_i^c + x_i^r \delta_i \quad i = 1, 2, \dots, n, \quad (3)$$

where  $x_i^c = (x_i^u + x_i^l)/2$  and  $x_i^r = (x_i^u - x_i^l)/2$ , which represent the mean and dispersion of  $x_i$ , and  $x_i^u$  and  $x_i^l$  are the upper and lower values of  $x_i$ , respectively. Therefore, the interval variable sets  $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$  can be transformed into the standardized interval variable sets  $\delta = \{\delta_1, \delta_2, \dots, \delta_n\}$ . The region of variation of the standardized interval variable  $\delta_i$  is  $[-1, 1]$ , and the extended range of  $\delta_i$  is  $[-\infty, +\infty]$  [15]. Substitute  $\delta = \{\delta_1, \delta_2, \dots, \delta_n\}$  into the failure surface  $g(\mathbf{x}) = 0$ ; then the continuous performance function can be transformed into the standardized performance function:

$$M = g(x_1, x_2, \dots, x_n) = G(\delta_1, \delta_2, \dots, \delta_n) = 0. \quad (4)$$

In practice engineering, the performance function is generally expressed as

$$M = g(\mathbf{x}) = r - s, \quad (5)$$

where  $r$  is the resistance and  $s$  is the load.

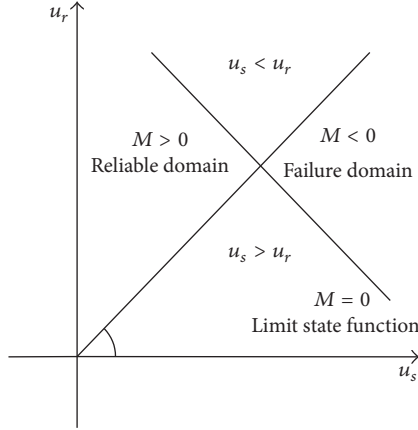


FIGURE 1: Reliability index of linear function.

(1) For the Linear Performance Function

$$M = \sum_{i=1}^n a_i r_i - \sum_{j=m+1}^n b_j s_j = 0. \quad (6)$$

Standardize  $r$  and  $s$ :  $r = R^c + R^r u_r$ ,  $s = S^c + S^r u_s$ , and then the linear performance function will be transformed into

$$M = \sum_{i=1}^m a_i R_i^r \delta_{r_i} - \sum_{j=m+1}^n b_j S_j^r \delta_{s_j} + \left( \sum_{i=1}^m a_i R_i^c - \sum_{j=m+1}^n b_j S_j^c \right) = 0 \quad (7)$$

and the nonprobabilistic reliability index is estimated as [24]

$$\eta = \begin{cases} \frac{\sum_{i=1}^m a_i R_i^c - \sum_{i=m+1}^n b_j S_j^c}{\sum_{i=1}^m |a_i| R_i^r - \sum_{i=m+1}^n |b_j| S_j^r}, & \sum_{i=1}^m a_i R_i^c - \sum_{i=m+1}^n b_j S_j^c > 0 \\ 0, & \sum_{i=1}^m a_i R_i^c - \sum_{i=m+1}^n b_j S_j^c \leq 0 \end{cases} \quad (8)$$

which can be also expressed using the geometric illustration in the two-dimensional surface (Figure 1).

(2) For the Nonlinear Performance Function. The nonprobabilistic reliability index based on the interval model is defined as [16, 24]

$$\eta = \min(\|\mathbf{u}\|_{\infty}). \quad (9)$$

Here the interval variables  $\mathbf{u}$  satisfy  $G(u_1, u_2, \dots, u_n) = 0$ , where  $\|\mathbf{u}\|_{\infty}$  is the infinite norm of the vector  $\mathbf{u}$ ; namely,

$$\|\mathbf{u}\|_{\infty} = \max(|u_1|, |u_2|, \dots, |u_n|). \quad (10)$$

In practice, the nonprobabilistic reliability index is the minimum distance from the origin of coordinates to the failure surface measured by the infinite norm in the regularized standard space. The above nonprobabilistic reliability index for the nonlinear performance function can be expressed using the geometric illustration in the two-dimensional surface (Figure 2).

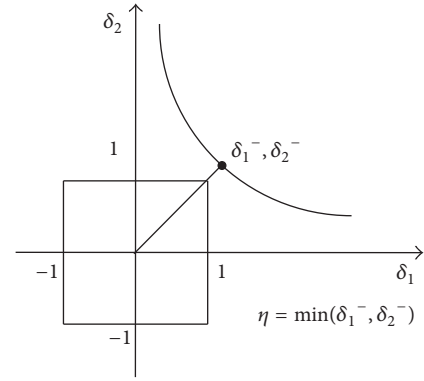


FIGURE 2: Reliability index of nonlinear function.

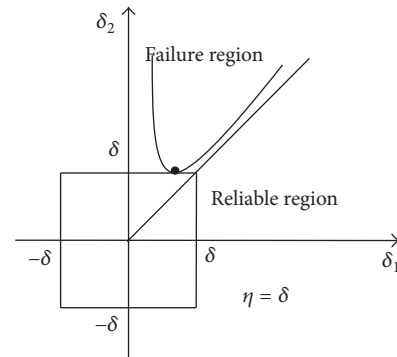


FIGURE 3: Most Probable failure Point above 45° line.

### 3. Nonprobabilistic Response Surface Limit Method

**3.1. Limit Method Basic Principle.** Fan and Chen [25] pointed out that all MPPs only exist among the extreme points and the root points of the limit state function. Obviously, the tangent line must exist at the extreme points of the failure surface. Taking the two-dimensional case as an example, the square is used to approximate the performance function curve when the length of side of the square, with the origin of coordinates as its center, is increasing from 0. With increased length of the side of the square, one side of the square will be tangent with the performance function curve, or one vertex of the square will intersect with the performance function curve. The maximum of the absolute values of coordinates for the earliest intersection is the desired nonprobabilistic reliability index.

For two-dimensional case, the MPPs may be in the region above the 45° line, in the region below the 45° line, or on the 45° line (the boundary of these two regions). The limit method will also be discussed for these three cases, respectively (Figures 3, 4, and 5).

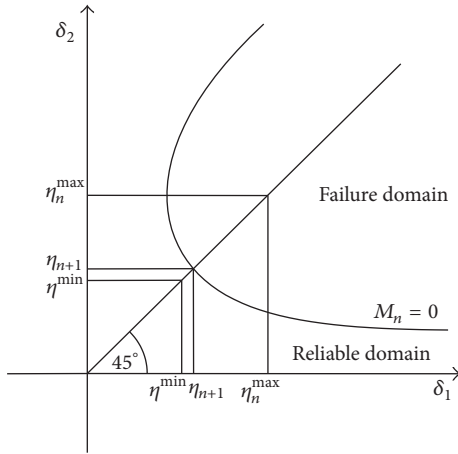
**3.2. Response Surface Method.** The response surface method was initially presented by Box and Wilson [26] and was mainly used to fit the limit state surface when the performance function was not known in practical engineering. Myers and Montgomery [27] perfected the proposed basic



TABLE I: Comparison of calculation results.

	Iterative steps	Nonprobabilistic reliability index	Relative error
Analytical method	—	1.7191	—
Traditional nonprobabilistic response surface method	5	1.7213	$1.27 \times 10^{-3}$
Presented nonprobabilistic response surface limit method	2	1.7191	0

Note. Relative error is obtained by comparing the nonprobabilistic reliability index calculated using other methods with that calculated using the analytical method.


 FIGURE 7:  $(n + 1)$ th iteration.

## 4. Results and Discussion

4.1. *Example 1.* Consider the following limit state function that includes three interval variables  $x_1$ ,  $x_2$ , and  $x_3$ :

$$G = 11x_1^2 + 7x_2^2 - 4x_3^2 + 3, \quad (14)$$

where  $x_1 \in [9, 13]$ ,  $x_2 \in [4, 6]$ , and  $x_3 \in [6.5, 11.5]$ .

Standardize the limit state function as shown by (14), so the performance function can be expressed as the following form again:

$$G = 11(11 + 2 \times \delta_1)^2 + 7(5 + 1 \times \delta_2)^2 - 4(9 + 2.5 \times \delta_3)^2 + 3. \quad (15)$$

First, the global optimal solution method is used, and the nonprobabilistic reliability index can be computed; that is,  $\eta = 1.7191$ . Next, the nonprobabilistic reliability index will be computed using the nonprobabilistic response surface limit method again. Letting  $f = 2$ , after selecting the initial point, the fitted performance function can be given as follows:

$$G = 3 + 2.65 \times 10^{-11}x_1 + 5.75 \times 10^{-11}x_2 + 1.57 \times 10^{-11}x_3 + 11x_1^2 + 7x_2^2 - 4x_3^2. \quad (16)$$

After the regularization of the above fitted performance function, the nonprobabilistic reliability indices are solved:  $\eta = (-9.6727, -4.7119, 2.2084, 20.6377, 1.7191, 26.512)$ . The maximum of the nonprobabilistic reliability indices, which

are smaller than  $f = 2$ , is 1.7191; that is,  $\eta = 1.7191$ . Then letting  $f = 1.7191$ , after the iterative computations,  $\eta = 1.7191$  can be computed again. This result satisfies the condition of convergence, so the nonprobabilistic reliability index is finally obtained:  $\eta = 1.7191$ .

The nonprobabilistic reliability index computed using the traditional nonprobabilistic response surface method and that computed using the presented nonprobabilistic response surface limit method are summarized in Table 1.

4.2. *Example 2.* Consider the following limit state function which includes two interval variables  $x_1$  and  $x_2$ :

$$G = x_1^3 + x_2^3 - 4, \quad (17)$$

where  $x_1 \in [2, 4]$  and  $x_2 \in [2.5, 4.5]$ .

Standardize the limit state function as shown by (17), so the performance function can be expressed as the following form again:

$$G = (3 + 1 \times \delta_1)^3 + (3.5 + 1 \times \delta_2)^3 - 4. \quad (18)$$

First, the Gradient Projection Method [19] is used, and the nonprobabilistic reliability index can be computed; that is,  $\eta = 2.0397$ . Next, the nonprobabilistic reliability index will be computed using the presented nonprobabilistic response surface limit method again. After selecting the initial point, the computation process indicates that there is no solution with  $f = 2$ . Then let  $f = 3$  again, and after four iterations the fitted performance function can be given as follows:

$$G = -0.3196 - 2.57x_1 - 6.103 \times 10^{-11}x_2 + 2.783 \times 10^{-11}x_1^2. \quad (19)$$

With the above function, the results  $\eta^{\min} = 1.962$  and  $\eta^{\max} = 2.178$  can be obtained. Meanwhile, the nonprobabilistic reliability indices can be also solved:  $\eta = (-2.5383, -2.7875, -3.339, -2.0397)$ . Due to the condition of  $\eta^{\min} < \eta < \eta^{\max}$ , the nonprobabilistic reliability index is  $\eta = 2.0397$ . Then letting  $f = 2.0397$ , after the iterative computations,  $\eta = 2.0397$  can be computed again. This result satisfies the condition of convergence, so the nonprobabilistic reliability index is finally determined:  $\eta = 2.0397$ .

The nonprobabilistic reliability index computed using the traditional nonprobabilistic response surface method and that computed using the nonprobabilistic response surface limit method are summarized in Table 2.



TABLE 2: Comparison of calculation results.

	Iterative steps	Nonprobabilistic reliability index	Relative error
Analytical method	—	2.0397	—
Gradient Projection Method	—	2.0397	0
Traditional nonprobabilistic response surface method	10	2.0396	$4.9 \times 10^{-5}$
Presented nonprobabilistic response surface limit method	4	2.0397	0

Note. Relative error is obtained by comparing the nonprobabilistic reliability index calculated using other methods with that calculated using the analytical method.

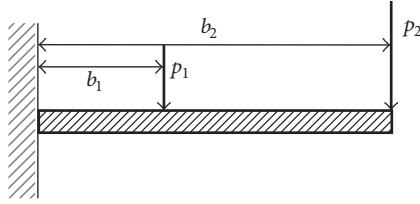


FIGURE 8: Schematic diagram of cantilever beam.

Table 2 indicates that, in solving the high nonlinear problems, the usage of the proposed nonprobabilistic response surface limit method will greatly reduce the iterative steps compared to the traditional nonprobabilistic response surface method. The computational efficiency is significantly improved when there is no difference in the solving accuracy.

**4.3. Example 3.** As shown in Figure 8, the concentrated loads  $p_1$  and  $p_2$  are applied on the cantilever beam, and the distances of these two concentrated loads from the fixed end are  $b_1$  and  $b_2$ , respectively. Suppose the cantilever beam is failed when  $|m_{\max}| \geq m_{\text{cr}}$ , where  $m_{\text{cr}}$  is the critical ultimate bending moment and  $m_{\max}$  is the maximum of the bending moment of beam. The basic interval variables are  $p_1 \in [4.4, 5.6]$  kN,  $p_2 \in [1.7, 2.3]$  kN,  $b_1 \in [1.8, 2.2]$  m,  $b_2 \in [4.5, 5.5]$  m, and  $m_{\text{cr}} \in [32, 40]$  kN·m. These parameters will be used in the estimation of the nonprobabilistic reliability for the structure.

Let  $f = 2$ ; the performance function is established using the quadratic form response surface:

$$G = 20 + x_1 - 2x_2 - 5x_3 - 5x_4 - 2x_5 + 6.046 \times 10^{-14}x_1^2 + 2.591 \times 10^{-12}x_2^2 + 9.266 \times 10^{-12}x_3^2 + 2.911 \times 10^{-11}x_4^2 + 3.187 \times 10^{-12}x_5^2 \quad (20)$$

After the regularization of the performance function, the nonprobabilistic reliability indices are solved:  $\eta = (2.807, 12.308, 3.721, 22.857, 4.324, -9.412, -3.404, -12.308, -3.721, -2.388, 2.540, 4.848, 1.8391, -6.957, 2.388)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 2$ , is 1.8391; that is,  $\eta = 1.8391$ . Then letting  $f = 1.8391$ , after the iterative computations,  $\eta = 1.8391$  can be computed again. This result satisfies the condition of convergence, so the nonprobabilistic reliability index is finally obtained:  $\eta = 1.8391$ .

Let  $f = 2$ ; the performance function is established using the primary response surface:

$$G = 1.1672 + 0.8836x_1 + 1.7194x_2 - 1.8054x_3 - 1.0768x_4 + 0.7384x_5 - 0.0191x_1x_2 - 0.0973x_1x_3 + 0.2446x_1x_4 - 0.0166x_1x_5 + 1.0812x_2x_3 - 3.3464x_2x_4 + 0.2995x_2x_5 + 2.0012x_3x_4 - 1.8198x_3x_5 - 1.837 \times 10^{-14}x_4x_5. \quad (21)$$

After the regularization of the performance function, the nonprobabilistic reliability indices are solved:  $\eta = (-5.423, 24.616, -2.414, -2.319, 5.003, 6.51, -9.103, 2.421, -33.967, 1.7446, -7.755, 20.966, -220.919, -8.065, 2.546, 4.132, 5.657, -11.984, 2.274, -51.899)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 2$ , is 1.7446; that is,  $\eta = 1.7446$ . Then letting  $f = 1.7446$ , after the iterative computations,  $\eta = 1.7446$  can be computed. These results satisfy the condition of convergence, so the nonprobabilistic reliability index is finally obtained:  $\eta = 1.7446$ .

The nonprobabilistic reliability index computed using the one-dimensional optimization algorithm [15, 16] is  $\eta = 1.7446$ . Finally, the nonprobabilistic reliability indices, computed using the one-dimensional optimization algorithm, the quadratic form response surface method, and the primary response surface method, are summarized in Table 3.

Table 3 indicates that the forms of the response surface functions have great influence on the final nonprobabilistic reliability index. The response surface function built greatly approximates the real limit state function, and the nonprobabilistic reliability index obtained greatly approximates the real value.

**4.4. Example 4.** A PC (prestressed reinforced concrete) continuous rigid frame bridge with three spans is investigated. Three spans are 75 m + 130 m + 75 m, respectively, and the substructure is twin thin-wall pier.

**4.4.1. Nonprobabilistic Reliability Evaluation Process.** The proposed nonprobabilistic response surface limit method will be used to evaluate the bridge reliability. The measured data is obtained for building the Finite Element Model (FEM). The nonprobabilistic reliability index is solved based on

TABLE 3: Comparison of calculation results.

	Iterative steps	Nonprobabilistic reliability index	Relative error
Analytical method	—	1.7446	—
One-dimensional optimization algorithm	—	1.7446	0
Quadratic form response surface method	2	1.8391	$5.4 \times 10^{-2}$
Primary response surface method	2	1.7446	0

Note. Relative error is obtained by comparing the nonprobabilistic reliability index calculated using other methods with that calculated using the analytical method.

TABLE 4: Initial value (before reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	4294.3	175427.3	179721.6
40	140	12996.5	165715.9	178712.4
50	140	-1072.5	181548.0	180475.5
45	130	-1784.9	169259.3	167474.3
45	150	10282.6	181596.3	191878.9

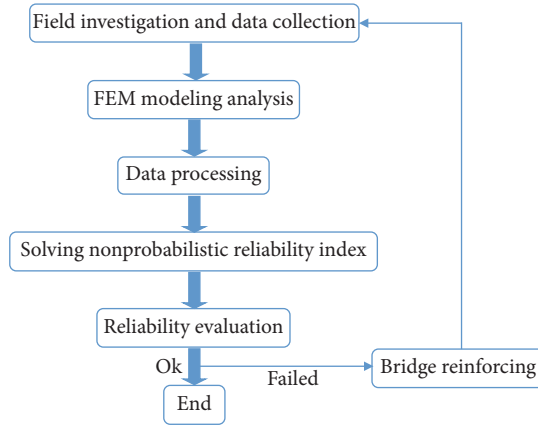


FIGURE 9: Reliability evaluation process.

the processed data using the Matlab software. The specific evaluation process is shown in Figure 9.

4.4.2. *Finite Element Model.* The measured relative parameters are as follows: height of main beam  $h \in [295, 305]$  cm; bridge deck width  $B \in [1645, 1655]$  cm; reinforced concrete weight  $\gamma \in [23, 25]$  kN/m<sup>3</sup>; concrete compressive strength  $C \in [40, 50]$  MPa; area of the single prestressed tendon  $A \in [135, 145]$  mm<sup>2</sup>.

The sensitivity analysis indicates that concrete compressive strength and area of the single prestressed tendon have great influence on the bending moment. Therefore, the reliability analysis will be conducted by selecting concrete compressive strength  $C$  and area of the single prestressed tendon  $A$  as the variables. The Finite Element Model is established using the Midas software, as shown in Figure 10. The maximum resistance and effect values of the control section will be also computed using the Midas software. The nonprobabilistic reliability analysis of the structure will be

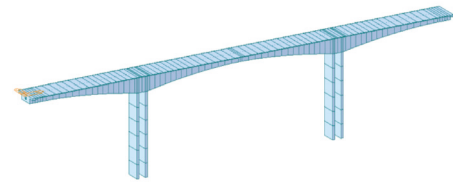


FIGURE 10: Finite Element Model.

performed by selecting the midspan section as the control section.

4.4.3. *Reliability Evaluation before Reinforcement.* Let  $f = 2$ ; the initial parameters of the model, the resistance ( $R$ ), and the effect ( $S$ ) computed using the Midas software are summarized in Table 4.

Now the proposed nonprobabilistic response surface limit method is utilized to solve the nonprobabilistic reliability index based on the parameters in Table 4. The fitted performance function can be given as follows:

$$G = 1.09 \times 10^5 - 7.41 \times 10^3 x_1 + 730.7 x_2 + 66.7 x_1^2 - 0.45 x_2^2. \quad (22)$$

After the regularization of the performance function, the nonprobabilistic reliability indices are solved:  $\eta = (4.218, 132.733, 15.425, 0.686)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 2$ , is 0.686; that is,  $\eta = 0.686$ . Then letting  $f = 0.686$ , after the first iterative computations, the iterative results are summarized in Table 5.

Next the proposed nonprobabilistic response surface limit method is again utilized to solve the nonprobabilistic reliability index based on the parameters in Table 5. The fitted performance function can be given as follows:

TABLE 5: First iterative computation results (before reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	4294.3	175427.3	179721.6
43.29	140	7042.5	171665.5	178708.1
46.72	140	1679.4	178796.7	180476.1
45	136.57	2214.5	173313.5	175528.0
45	143.43	6363.3	177541.4	183904.7

TABLE 6: Second iterative computation results (before reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	4294.3	175427.3	179721.6
43.43	140	6838.5	171869.4	178707.9
46.57	140	1829.7	178646.4	180476.1
45	136.86	2403.5	173501.7	175905.2
45	143.14	6199.6	177366.4	183566.0

TABLE 7: Third iterative computation results (before reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	4294.3	175427.3	179721.6
43.45	140	6809.7	171898.2	178707.9
46.55	140	1849.9	178626.2	180476.1
45	136.91	2437.0	173518.8	175955.8
45	143.1	6166.3	177349.3	183515.7

$$G = 3.22 \times 10^4 - 3.84 \times 10^3 x_1 + 733.8x_2 + 25.3x_1^2 - 0.46x_2^2. \quad (23)$$

After the regularization of the performance function as shown by (23), the nonprobabilistic reliability indices are obtained:  $\eta = (12.36, 131.24, 46.64, 0.628)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 0.686$ , is 0.628; that is,  $\eta = 0.628$ . Then letting  $f = 0.628$ , after the second iterative computations, the iterative results are summarized in Table 6.

Next the presented nonprobabilistic response surface limit method is again utilized to solve the nonprobabilistic reliability index based on the parameters in Table 6. The fitted performance function can be given as follows:

$$G = 3.85 \times 10^4 - 3.05 \times 10^3 x_1 + 398.6x_2 + 16.1x_1^2 + 0.74x_2^2. \quad (24)$$

After the regularization of the performance function as shown by (24), the nonprobabilistic reliability indices are computed:  $\eta = (19.768, -82.202, 58.167, 0.619)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 0.628$ , is 0.619; that is,  $\eta = 0.619$ . Then let  $f = 0.619$ , after the third iterative computations, the iterative results are summarized in Table 7.

Next the proposed nonprobabilistic response surface limit method is again utilized to solve the nonprobabilistic

reliability index based on the parameters in Table 7. The fitted performance function can be given as follows:

$$G = 3.07 \times 10^4 - 2.93 \times 10^3 x_1 + 476x_2 + 14.8x_1^2 - 0.45x_2^2. \quad (25)$$

After the regularization of the performance function as shown by (25), the nonprobabilistic reliability indices are computed:  $\eta = (21.680, -133.413, 67.108, 0.618)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 0.619$ , is 0.618; that is,  $\eta = 0.618$ . Here  $(0.619 - 0.618)/0.618 = 0.0016 < 0.01$ , which satisfies the condition of convergence. In summary, the nonprobabilistic reliability index is finally determined, which is  $\eta = 0.618$ .

**4.4.4. Reliability Evaluation after Reinforcement.** According to  $\eta = 0.618 < 1$ , the bridge is unreliable. Then 16 bundles of the external prestressed tendon are adopted to reinforce the unreliable bridge, and the nonprobabilistic reliability index for the reinforced bridge is solved again.

Let  $f = 2$ ; the initial parameters of the model, the resistance ( $R$ ), and the effect ( $S$ ) computed using the Midas software are summarized in Table 8.

Next the proposed nonprobabilistic response surface limit method is again utilized to solve the nonprobabilistic reliability index based on the parameters in Table 8. The fitted performance function can be given as follows:

$$G = 1.15 \times 10^5 - 7.62 \times 10^3 x_1 + 783.1x_2 + 68.7x_1^2 - 0.49x_2^2. \quad (26)$$



TABLE 8: Initial value (after reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	10879.6	179122.2	190001.8
40	140	19787.6	169093.5	188881.1
50	140	5404.2	185434.4	190838.6
45	130	4377.0	172708.7	177085.7
45	150	17283.8	185534.1	202817.9

TABLE 9: First iterative computation results (after reinforcement).

Compressive strength (MPa)	Area of the prestressed tendon (mm)	$M = R - S$ (kN·m)	$R$ (kN·m)	$S$ (kN·m)
45	140	10879.6	179122.2	190001.8
40.52	140	18633.3	170246.9	188880.2
49.48	140	5796.5	185042.3	190838.8
45	131.05	5053.5	173378.8	178432.3
45	148.96	16625.6	184865.4	201490.9

After the regularization of the performance function as shown by (26), the nonprobabilistic reliability indices are computed:  $\eta = (4.19, 131.191, 14.579, 1.791)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 2$ , is 1.791; that is,  $\eta = 1.791$ . Then letting  $f = 1.791$ , after the first iterative computations, the iterative results are summarized in Table 9.

Next the proposed nonprobabilistic response surface limit method is again utilized to solve the nonprobabilistic reliability index based on the parameters in Table 9. The fitted performance function can be given as follows:

$$G = 1.09 \times 10^5 - 7.42 \times 10^3 x_1 + 797.3x_2 + 66.5x_1^2 - 0.54x_2^2. \quad (27)$$

After the regularization of the performance function as shown by (27), the nonprobabilistic reliability indices are computed:  $\eta = (4.307, 119.671, 15.147, 1.785)$ . The maximum of the nonprobabilistic reliability indices, which are smaller than  $f = 1.791$ , is 1.785; that is,  $\eta = 1.785$ . Here  $(1.791 - 1.785)/1.785 = 0.0034 < 0.01$  satisfies the condition of convergence. In summary, the nonprobabilistic reliability index is finally determined; that is,  $\eta = 1.785$ , which indicates that the bridge is reliable.

## 5. Conclusions

This article is the first to successfully propose a nonprobabilistic response surface limit method to perform nonprobabilistic reliability analysis for the structures based on the interval model. Using this method, nonprobabilistic reliability analysis can be performed with the conditions of unknown performance function and data shortage. Compared with the traditional nonprobabilistic response surface method, there are two other main advantages of the proposed method: (1) it effectively avoids the tedious and oscillating iteration process; (2) it greatly enhances computational efficiency and precision and obtains the nonprobabilistic reliability index rapidly. The complete nonprobabilistic reliability evaluation process for

the bridge was illustrated and built through evaluating the reliability of one PC continuous rigid frame bridge with three spans. The results indicated that the reliability level of the selected PC continuous rigid frame bridge with three spans was very low, so that reinforcement measures were needed to improve the performance of this bridge. The further reliability estimation for the reinforced bridge demonstrated that the reliability level of the reinforced bridge was greatly improved, and the reinforcements were effective. In summary, this study illustrated the effectiveness and feasibility of the proposed nonprobabilistic response surface limit method in the bridge nonprobabilistic reliability estimation through theoretical analysis and example verification.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

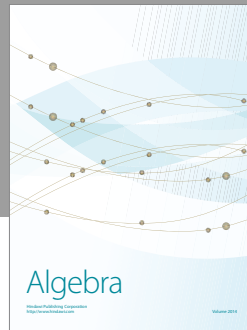
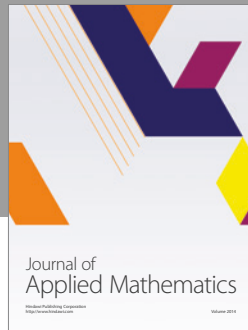
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