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Research Article

A Fuzzy Approach to Robust Control of Stochastic Nonaffine Nonlinear Systems

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This paper investigates the stabilization problem for a class of discrete-time stochastic non-affine nonlinear systems based on T-S fuzzy models. Based on the function approximation capability of a class of stochastic T-S fuzzy models, it is shown that the stabilization problem of a stochastic non-affine nonlinear system can be solved as a robust stabilization problem of the stochastic T-S fuzzy system with the approximation errors as the uncertainty term. By using a class of piecewise dynamic feedback fuzzy controllers and piecewise quadratic Lyapunov functions, robust semiglobal stabilization condition of the stochastic non-affine nonlinear systems is formulated in terms of linear matrix inequalities. A simulation example illustrating the effectiveness of the proposed approach is provided in the end.

1. Introduction

In recent years, Takagi-Sugeno (T-S) type dynamic fuzzy model [1] based control methodologies have attracted great attention from control community. T-S fuzzy models describe a nonlinear system by the “blending” of a set of local linear dynamic models. This relatively simple structure facilitates the systematic stability analysis and controller design of T-S fuzzy control systems in view of the powerful linear systems control theory [2–13]. By using a common quadratic Lyapunov function and LMI techniques, control design of T-S fuzzy systems can be formulated in a convex optimization problem, which can be effectively solved by various tools. However, common Lyapunov functions (CLFs) tend to be conservative and even might not exist for many highly complex nonlinear systems [3]. In order to reduce the conservatism of approaches based on CLFs, some results based on piecewise Lyapunov functions (PLFs) have been proposed [12, 13]. For the most recent advances on relevant topics, readers please refer to the book and the survey paper [2, 3] and the references therein for details.

Control design of nonlinear systems based on T-S fuzzy models can be typically summarized into two steps: (i) for a given nonlinear system, find its approximate T-S fuzzy model; and (ii) design a controller for the obtained T-S fuzzy model. It has been shown that T-S fuzzy models are universal function approximators in the sense that they are able to approximate any smooth nonlinear functions to any degree of accuracy in any convex compact region [14–16], which provides a theoretical foundation for utilizing the T-S fuzzy modeling method as an alternative approach to describing complex nonlinear systems approximately. However, it has been proved in [16] that the commonly used T-S fuzzy models where the control variables are not included in the premise variables are only able to approximate affine nonlinear systems to any degree of accuracy on any compact set. This implies that only the control design of affine nonlinear systems can be solved based on the commonly used T-S fuzzy models. To deal with more general nonlinear systems, that is, non-affine nonlinear systems, recently the coauthors proposed a class of generalized T-S fuzzy models which are universal function approximators of non-affine nonlinear systems [17, 18].

On another fruitful research frontier, stochastic control systems have been extensively studied because stochastic modeling plays a very important role in many branches of science and engineering [19–22]. Although many valuable results on stability analysis and controller synthesis of stochastic linear systems have been reported, most of the existing results on stochastic nonlinear control systems do not provide any systematic way of control design due to the difficulty in searching for suitable Lyapunov functions, especially for highly complex stochastic nonlinear systems. Motivated by the deterministic T-S fuzzy model based control techniques, the T-S fuzzy models have been extended to the stochastic case, where the local models are stochastic linear dynamic models instead of deterministic ones [23–26]. Especially, to deal with stochastic non-affine nonlinear systems (SNNS), the so-called generalized stochastic T-S fuzzy models were proposed in [26] by the co-authors.

In [26], the stabilization problem of continuous-time SNNS was studied based on the generalized stochastic T-S fuzzy models. However, it is noted that the approach proposed in [26] is based on common Lyapunov functions, which is very conservative. In this paper, we investigate the stabilization problem of discrete-time SNNS based on discrete-time generalized stochastic T-S fuzzy models. By using a piecewise Lyapunov function and a class of piecewise dynamic feedback fuzzy controllers, it is shown that the robust semiglobal stabilization condition of discrete-time SNNS can be formulated in terms of a set of linear matrix inequalities (LMIs) that are numerically efficient with commercially available software.

The rest of this paper is structured as follows. Section 2 is devoted to model description and problem formulation. In Section 3, robust controller design result for discrete-time stochastic non-affine nonlinear systems is presented. Simulation results are provided in Section 4 to demonstrate the effectiveness of the proposed approach. Conclusions are given in Section 5.

Notations. The notations used in this paper are fairly standard. The notation $\star$ is used to indicate the terms that can be induced by symmetry. "T" represents vector or matrix transpose. $\mathbf{I}_n$ and $\mathbf{0}_{m\times n}$ are used to denote the $n \times n$ identity matrix and the $m \times n$ zero matrix, respectively. The subscripts n and $m \times n$ are omitted when the size is not relevant or can be determined from the context. $P > 0$ means that matrix P is real, symmetric, and positive definite. For a matrix A, $\lambda\{A\}$ is the eigenvalue of A. Let $\mathbb{E}\{\cdot\}$ be the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$, and let $(\Omega, \mathcal{F}, \mathcal{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$.

2. Model Description and Problem Formulation

2.1. T-S Fuzzy Model Description of SNNS

In this paper, we consider the following discrete-time stochastic non-affine nonlinear system:

$$x(t+1) = f(x(t), u(t)) + g(x(t), u(t))W(t), \tag{2.1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in X \subset \Re^n$, $u(t) = [u_1(t), \ldots, u_m(t)]^T \in \mathcal{U} \subset \Re^m$, $X \times \mathcal{U}$ is a compact set on $\Re^n \times \Re^m$ containing the origin, and $W(t) = [W_1(t), W_2(t), \ldots, W_q(t)]^T$ is a q-dimensional Wiener process defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with

$$\mathbb{E}\{W_i(t)\} = 0, \qquad \mathbb{E}\left\{W_i^2(t)\right\} = 1. \tag{2.2}$$

Moreover, the noise processes $W_1(t), W_2(t), \ldots, W_q(t)$, the system state, and the control input are independent. It is assumed in this paper that the mappings $f \in \mathcal{C}^1$ and $g \in \mathcal{C}^1$ both vanish at zero, that is, $f(0,0) = 0$ and $g(0,0) = 0$. It is also assumed that f and g satisfy the usual linear growth and local Lipschitz conditions for existence and uniqueness of solutions to (2.1).

Our objective is to develop an approach to controlling the SNNS in (2.1) via T-S fuzzy modeling. In order to approximate the SNNS in (2.1), the following discrete-time generalized stochastic T-S fuzzy model is employed.

Plant rule $\mathcal{R}^l$

IF $x_1(t)$ is $\mathcal{U}_1^l$ AND... AND $x_n(t)$ is $\mathcal{U}_n^l$; $u_1(t)$ is $\mathcal{V}_1^l$ AND... AND $u_m(t)$ is $\mathcal{V}_m^l$; **THEN**

$$x(t+1) = A_l x(t) + B_l u(t) + \sum_{k=1}^{q} (C_{lk} x(t) + D_{lk} u(t)) W_k(t), \quad l \in \mathfrak{L} := \{1, 2, \ldots r\}, \tag{2.3}$$

where $\mathcal{R}^l$ denotes the lth rule, r the total number of rules, $\mathcal{U}_i^l$ and $\mathcal{V}_j^l$ the fuzzy sets, $x(t) \in \Re^n$ the state vector, $u(t) \in \Re^m$ the input vector, and $[A_l, B_l, C_{lk}, D_{lk}]$ the matrices of the lth local model.

Under the center-average defuzzifier, product inference, and singleton fuzzifier, the T-S fuzzy system in (2.3) can be expressed globally as

$$x(t+1) = \widehat{f}(x(t), u(t)) + \widehat{g}(x(t), u(t))W(t) \tag{2.4}$$

with

$$\hat{f}(x(t),u(t)) = \sum_{l=1}^{r}\mu_l(x,u)[A_l x(t) + B_l u(t)],$$
$$\hat{g}(x(t),u(t)) = \sum_{l=1}^{r}\mu_l(x,u)\times\left[C_{l1}x(t) + D_{l1}u(t),\ldots,C_{lq}x(t) + D_{lq}u(t)\right], \tag{2.5}$$
$$\mu_l(x,u) = \frac{\prod_{i=1}^{n}\mathcal{U}_i^l(x_i)\prod_{j=1}^{m}\mathcal{V}_j^l(u_j)}{\sum_{l=1}^{r}\prod_{i=1}^{n}\mathcal{U}_i^l(x_i)\prod_{j=1}^{m}\mathcal{V}_j^l(u_j)},$$

where $\mu_l(x,u)$ are the so-called normalized fuzzy membership functions satisfying $\sum_{l=1}^{r}\mu_l(x,u) = 1$ and $\mu_l(x,u) \geq 0$.

In the co-authors' recent work [26], the continuous-time counterpart of the stochastic T-S fuzzy models in (2.4) has been proved to be the universal function approximator to continuous-time SNNS. It has been also shown in [26] that the function approximation capability also holds for the discrete-time case, which is summarized in the following lemma.

Lemma 2.1 (see [26]). *For any given SNNS described by* (2.1) *and any two positive constants* ϵ_1 *and* ϵ_2*, there exist a set of fuzzy basis functions* $\mu_l(x,u)$ *and constant matrices* A_l*,* B_l*,* C_{lk}*, and* D_{lk}*,* $l \in \{1,\ldots,r\}$*,* $k \in \{1,\ldots,q\}$ *such that*

$$\hat{f}(x,u) = \sum_{l=1}^{r}\mu_l(x,u)[A_l x + B_l u] = f(x,u) + \epsilon_f(x,u),$$
$$\hat{g}(x,u) = \sum_{l=1}^{r}\mu_l(x,u)\left[C_{l1}x + D_{l1}u,\ldots,C_{lq}x + D_{lq}u\right] = g(x,u) + \epsilon_g(x,u), \tag{2.6}$$

where

$$\epsilon_f(x,u) = \Delta E_f(x,u)\overline{x},$$
$$\epsilon_g(x,u) = \left[\epsilon_{g1}(x,u),\ldots,\epsilon_{gq}(x,u)\right] = \left[\Delta E_{g1}(x,u)\overline{x},\ldots,\Delta E_{gq}(x,u)\overline{x}\right], \tag{2.7}$$

with

$$\left\|\Delta E_f(x,u)\right\| < \epsilon_1, \quad \left\|\Delta E_{gk}(x,u)\right\| < \epsilon_2, \quad k = \{1,\ldots,q\}. \tag{2.8}$$

From (2.6) in Lemma 2.1, an SNNS described by (2.1) can be exactly expressed in a compact set by a generalized stochastic T-S fuzzy model in (2.4) with the approximation errors as some norm-bounded uncertainties as follows:

$$x(t+1) = \sum_{l=1}^{r} \mu_l(x,u) \Bigg\{ A_l x(t) + B_l u(t) + \epsilon_f(x(t),u(t)) + \sum_{k=1}^{q} \left(C_{lk} x(t) + D_{lk} u(t) + \epsilon_{gk}(x(t),u(t)) \right) W_k(t) \Bigg\}, \tag{2.9}$$

where

$$\epsilon_f(x(t),u(t)) = \Delta E_f(x,u) \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \qquad \epsilon_{gk}(x(t),u(t)) = \Delta E_{gk}(x,u) \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}. \tag{2.10}$$

Therefore, one can easily conclude that the stabilization of an SNNS given in (2.1) can be actually solved as a robust stabilization problem of its corresponding stochastic generalized T-S fuzzy model with the approximation errors as the uncertainty terms.

2.2. System Formulation and Dynamic Fuzzy Controllers

It is noted that fuzzy system (2.4) induces a polyhedral partition of the premise space, which is dependent on both the system state x and control input u. As a result, the global fuzzy system can be viewed as a number of subsystems in a number of individual regions.

In this paper, the premise space is divided into a set of crisp regions and fuzzy regions. Denote the partitioned regions as $\{S_i\}_{i \in \Im}$ with $\Im$ as the set of region indices and define $\mathcal{I}(i)$ the indices of fired rules in each region S_i, then the crisp regions and fuzzy regions can be defined respectively by

$$\begin{aligned} S_i &:= \left\{ (x,u) \mid \mu_m(x,u) = 1,\ m \in \mathcal{I}(i) \right\}, \\ S_j &:= \left\{ (x,u) \mid 0 \leq \mu_m(x,u) < 1,\ m \in \mathcal{I}(j) \right\}, \end{aligned} \tag{2.11}$$

where $i, j \in \Im$.

Based on such a partition method, the fuzzy model (2.4), or the original SNNS (2.1), in each region can be rewritten by a blending of $m \in \mathcal{I}(i)$ subsystems,

$$\begin{aligned} x(t+1) = &\sum_{m \in \mathcal{I}(i)} \mu_m(x(t),u(t)) \\ &\times \Bigg\{ A_m x(t) + B_m u(t) + \epsilon_f(x(t),u(t)) \\ &+ \sum_{k=1}^{q} \left(C_{mk} x(t) + D_{mk} u(t) + \epsilon_{gk}(x(t),u(t)) \right) W_k(t) \Bigg\}, \quad (x,u) \in S_i,\ i \in \Im. \end{aligned} \tag{2.12}$$

In order to stabilize the nonlinear system in (2.12), we employ the following piecewise dynamic state feedback fuzzy controller:

$$u(t+1) = \sum_{m \in \mathcal{I}(i)} \mu_m(x,u)\{F_{mi}x(t) + G_{mi}u(t)\}, \quad (x,u) \in S_i,\ i \in \Im. \tag{2.13}$$

Remark 2.2. As it has been argued in [17], because the premise variables of the generalized stochastic T-S fuzzy system in (2.12) contain the system control input, the commonly used parallel distributed compensation (PDC) scheme cannot be directly applied. Instead, the dynamic fuzzy controller in (2.13) is proposed. It is noted that by using the dynamic state feedback controller in (2.13), the closed-loop control system can be expressed in the summation of one index which is different from the traditional static state feedback case where indices are used. This will lead to much less number of LMIs in controller design which will be shown subsequently.

Remark 2.3. When the local gains of the piecewise dynamic fuzzy controller (PDFC) in (2.13) are equal, that is, $[F_{mi}, G_{mi}] = [F_i, G_i]$, for $m \in \mathcal{I}(i)$, the fuzzy controller in (2.13) reduces to the so-called piecewise dynamic crisp controller (PDCC). It will be shown in Section 4 that the fuzzy controller in (2.13) achieves better performance than the piecewise dynamic state feedback controller.

Then the closed-loop control system consisting of (2.12) and (2.13) is given by

$$\begin{aligned} \overline{x}(t+1) &= \sum_{m \in \mathcal{I}(i)} \mu_m(\overline{x}(t)) \\ &\quad \times \Bigg\{ \left(\mathcal{A}_{mi} + R\Delta E_f(\overline{x}(t))\right)\overline{x}(t) \\ &\quad + \sum_{k=1}^{q} \left(\mathcal{C}_{mk} + R\Delta E_{gk}(\overline{x}(t))\right)\overline{x}(t)W_k(t) \Bigg\}, \quad \overline{x}(t) \in S_i, \end{aligned} \tag{2.14}$$

where $\mathcal{A}_{mi} = \overline{A}_m + \overline{B}\,\overline{K}_{mi}$, ΔE_f and ΔE_{gk} are defined in (2.10), and

$$\begin{gathered} \overline{x}(t) = \begin{bmatrix} x(t) \\ u(t) \end{bmatrix}, \qquad \overline{A}_m = \begin{bmatrix} A_m & B_m \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m} \end{bmatrix}, \\ \overline{B} = \begin{bmatrix} \mathbf{0}_{n\times m} \\ \mathbf{I}_m \end{bmatrix}, \qquad R = \begin{bmatrix} \mathbf{I}_n \\ \mathbf{0}_{m\times n} \end{bmatrix}, \\ \overline{K}_{mi} = \begin{bmatrix} F_{mi} & G_{mi} \end{bmatrix}, \qquad \mathcal{C}_{mk} = \begin{bmatrix} C_{mk} & D_{mk} \\ \mathbf{0}_{m\times n} & \mathbf{0}_{m\times m} \end{bmatrix}. \end{gathered} \tag{2.15}$$

In addition, we define a new set that represents all possible state transitions among regions of the closed-loop T-S fuzzy system in (2.15) as follows:

$$\Omega := \left\{ (i,j) \mid \overline{x}(t) \in S_i,\ \overline{x}(t+1) \in S_j,\ i,j \in \Im \right\}. \tag{2.16}$$

In the case of $(i, j) \in \Omega$ and $i = j$, the state trajectories evolve in the same region S_i at the time t. Otherwise, the state trajectories will transit from the region S_i to S_j at that time.

3. Robust Controller Design for SNNS

In this section, an LMI approach will be developed to solve the stabilization problem of the SNNS in (2.1) based on the generalized stochastic T-S fuzzy models in (2.4).

The following definitions are introduced first.

Definition 3.1. The closed-loop control system in (2.14) is said to be stochastically asymptotically stable in the mean square sense, if for any initial conditions $\overline{x}(0)$, the solution $\overline{x}(t)$ of (2.14) exists for all $t \geq 0$ and $\lim_{t\to\infty}\mathbb{E}\{\|\overline{x}(t)\|^2\} = 0$.

Definition 3.2. The closed-loop control system in (2.14) is said to be stochastically exponentially stable in the mean square sense, if there exist a set of positive constants C, $0 < \sigma < 1$ and $\lambda > 0$, such that given any initial states $\overline{x}(0)$, the solution $\overline{x}(t)$ of (2.14) exists for all $t \geq 0$ and $\mathbb{E}\{\|\overline{x}(t)\|\} \leq C\|\overline{x}(0)\|\sigma^t$.

Denote $\mathcal{M}_m = [(\mathcal{C}_{m1})^T, \ldots, (\mathcal{C}_{mq})^T]^T$ and $\Delta E_g(\overline{x}(t)) = [(R\Delta E_{g1}^T(\overline{x}(t))), \ldots, (R\Delta E_{gq}^T(\overline{x}(t)))]^T$. For the sake of simplicity, we denote $\mu_i(\overline{x}(t))$, $\Delta E_f(\overline{x}(t))$, $\Delta E_{gi}(\overline{x}(t))$, and $\Delta E_g(\overline{x}(t))$ as μ_i, ΔE_f, ΔE_{gi}, and ΔE_g, respectively.

Suppose that the upper bounds of the uncertainties ΔE_f and ΔE_g are given by

$$\Delta E_f^T \Delta E_f \leq \epsilon_1^2 \mathbf{I}_{(m+n)}, \qquad \Delta E_g^T \Delta E_g \leq \overline{\epsilon}_2^2 \mathbf{I}_{(m+n)}, \tag{3.1}$$

respectively.

Then the stochastic stability analysis result for the closed-loop control system (2.14) is provided in the following theorem.

Theorem 3.3. *The closed-loop stochastic fuzzy control system* (2.14) *is stochastically asymptotically stable in the mean square sense if there exist a set of positive definite matrices* P_i, $i \in \mathfrak{F}$, *two sets of positive constants* ε_{1i} *and* ε_{2i}, *and a positive constant* δ *such that the following matrix inequalities hold for all* $(i, j) \in \Omega$, $m \in \mathcal{I}(i)$,

$$\begin{bmatrix} \Xi_{ij} + \left(\dfrac{\overline{\epsilon}_1^2}{\varepsilon_{1i}} + \dfrac{\overline{\epsilon}_2^2}{\varepsilon_{2i}}\right)\mathbf{I} & \star & \star \\ R^T P_j \mathcal{A}_{mi} & R^T P_j R - \dfrac{1}{\varepsilon_{1i}}\mathbf{I} & \star \\ \mathbf{I}_q \otimes P_j \mathcal{M}_m & 0 & \mathbf{I}_q \otimes P_j - \dfrac{1}{\varepsilon_{2i}}\mathbf{I} \end{bmatrix} < -\delta \mathbf{I}, \tag{3.2}$$

where

$$\Xi_{ij} = \mathcal{A}_{mi}^T P_j \mathcal{A}_{mi} + \mathcal{M}_m^T (I_q \otimes P_j) \mathcal{M}_m - P_i. \tag{3.3}$$

Proof. Consider the following piecewise Lyapunov function candidate:

$$V(\overline{x}(t)) = \overline{x}^T(t)P_i\overline{x}(t), \quad \overline{x}(t) \in S_i. \tag{3.4}$$

For a given set of given positive definite matrices P_i, from (2.2) one has

$$\begin{aligned}
&\mathbb{E}\left\{\left(\sum_{m\in\mathcal{I}(i)} \mu_m(\mathcal{A}_{mi} + R\Delta E_f)\overline{x}(t)\right)^T P_j\left(\sum_{m\in\mathcal{I}(i)} \mu_m\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)\right\}\\
&\quad = \mathbb{E}\left\{\left(\sum_{m\in\mathcal{I}(i)} \mu_m\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)^T P_j\left(\sum_{m\in\mathcal{I}(i)} \mu_m(\mathcal{A}_{mi} + R\Delta E_f)\overline{x}(t)\right)\right\}\\
&\quad = 0.
\end{aligned} \tag{3.5}$$

Based on Lemmas A.1 and A.2, one has that

$$\begin{aligned}
&\left\{\sum_{m\in\mathcal{I}(i)} \mu_m(\mathcal{A}_{mi} + R\Delta E_f)\overline{x}(t)\right\}^T P_j\left\{\sum_{m\in\mathcal{I}(i)} \mu_m(\mathcal{A}_{mi} + R\Delta E_f)\overline{x}(t)\right\}\\
&\quad \leq \sum_{m\in\mathcal{I}(i)} \mu_m\overline{x}^T(t)(\mathcal{A}_{mi} + R\Delta E_f)^T P_j(\mathcal{A}_{mi} + R\Delta E_f)\overline{x}(t),\\
&\mathbb{E}\left\{\left(\sum_{m\in\mathcal{I}(i)} \mu_m\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)^T P_j\right.\\
&\quad \left.\times\left(\sum_{m\in\mathcal{I}(i)} \mu_m\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)\right\}\\
&\quad \leq \sum_{m\in\mathcal{I}(i)} \mu_m\mathbb{E}\left\{\left(\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)^T P_j \times \left(\sum_{k=1}^{q}(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)W_k(t)\right)\right\}\\
&\quad = \sum_{m\in\mathcal{I}(i)} \mu_m\sum_{k=1}^{q}\overline{x}^T(t)(\mathcal{C}_{mk} + R\Delta E_{gk})^T P_j(\mathcal{C}_{mk} + R\Delta E_{gk})\overline{x}(t)\\
&\quad = \sum_{m\in\mathcal{I}(i)} \mu_m\overline{x}^T(t)(\mathcal{M}_m + \Delta E_g)^T(\mathbf{I}_q \otimes P_j)(\mathcal{M}_m + \Delta E_g)\overline{x}(t).
\end{aligned} \tag{3.6}$$

Then one has that along the system trajectories of (2.14),

$$\begin{aligned}
&\mathbb{E}\{V(\overline{x}(t+1)\mid\overline{x}(t))\}-V(\overline{x}(t))\\
&\quad\le\sum_{m\in\mathcal{I}(i)}\mu_m\overline{x}^T(t)\Big\{(\mathcal{A}_{mi}+R\Delta E_f)^T P_j(\mathcal{A}_{mi}+R\Delta E_f)\\
&\qquad\qquad+(\mathcal{M}_m+\Delta E_g)^T(\mathbf{I}_q\otimes P_j)(\mathcal{M}_m+\Delta E_g)-P_i\Big\}\overline{x}(t).
\end{aligned}\tag{3.7}$$

Therefore one has that $\mathbb{E}\{V(\overline{x}(t+1)\mid\overline{x}(t))\}-V(\overline{x}(t))<-\delta\overline{x}^T(t)\overline{x}(t)$ if

$$\begin{aligned}
&(\mathcal{A}_{mi}+R\Delta E_f)^T P_j(\mathcal{A}_{mi}+R\Delta E_f)-P_i\\
&\quad+(\mathcal{M}_m+\Delta E_g)^T(\mathbf{I_q}\otimes P_j)(\mathcal{M}_m+\Delta E_g)<-\delta\mathbf{I}.
\end{aligned}\tag{3.8}$$

Denote $\zeta=[\mathbf{I}_{(m+n)},\Delta E_f^T,\Delta E_g^T]^T$. Then it can be seen that the inequality (3.8) is equivalent to

$$\zeta^T\begin{bmatrix}\Xi_{ij} & \star & \star\\ R^T P_j\mathcal{A}_{mi} & R^T P_j R & \star\\ \mathbf{I}_q\otimes P_j\mathcal{M}_m & 0 & \mathbf{I}_q\otimes P_j\end{bmatrix}\zeta<-\delta\mathbf{I},\tag{3.9}$$

where Ξ_{ij} is defined in (3.3).

The upper bounds defined in (3.1) can be rewritten, respectively, as

$$\zeta^T\begin{bmatrix}-\overline{\epsilon}_1^2\mathbf{I}_{(m+n)} & \star & \star\\ 0 & \mathbf{I}_{(m+n)} & \star\\ 0 & 0 & 0\end{bmatrix}\zeta<0,\qquad \zeta^T\begin{bmatrix}-\overline{\epsilon}_2^2\mathbf{I}_{(m+n)} & \star & \star\\ 0 & 0 & \star\\ 0 & 0 & \mathbf{I}_{(m+n)}\end{bmatrix}\zeta<0.\tag{3.10}$$

Then by applying Lemma A.3 (S-procedure) in the appendix, one can conclude that (3.8) holds if there exist two sets of positive constants ε_{1i} and ε_{2i} such that (3.2) holds.

Therefore, if (3.2) holds, one has that

$$\mathbb{E}\{V(\overline{x}(t+1)\mid\overline{x}(t))\}-V(\overline{x}(t))<-\delta\overline{x}^T(t)\overline{x}(t).\tag{3.11}$$

Taking expectation of both sides of (3.11) yields

$$\mathbb{E}\{V(\overline{x}(t+1))\}-\mathbb{E}\{V(\overline{x}(t))\}<-\delta\mathbb{E}\Big\{\|\overline{x}(t)\|^2\Big\},\tag{3.12}$$

which implies

$$\mathbb{E}\left\{\sum_{t=0}^{N}\|\overline{x}(t)\|^2\right\}<\frac{1}{\delta}(\mathbb{E}\{V(\overline{x}(0))\}-\mathbb{E}\{V(\overline{x}(N+1))\})\le\frac{1}{\delta}\mathbb{E}\{V(\overline{x}(0))\}<\infty.\tag{3.13}$$

Then one has that $\lim_{t\to\infty}\mathbb{E}\{\|\overline{x}(t)\|^2\} = 0$. From Definition 3.1, the closed-loop fuzzy system (2.14) is stochastically asymptotically stable in the mean square sense. The proof is thus completed. □

Corollary 3.4. *Under the conditions of Theorem 3.3, the closed-loop stochastic fuzzy control system* (2.14) *is also stochastically exponentially stable in the mean square sense.*

Proof. From (3.4) one has that

$$\lambda_1\|\overline{x}(t)\|^2 \le V(\overline{x}(t)) \le \lambda_2\|\overline{x}(t)\|^2, \tag{3.14}$$

where $\lambda_1 = \min_i \lambda\{P_i\}$ and $\lambda_2 = \max_i \lambda\{P_i\}$.

Then from (3.12), one has

$$\mathbb{E}\{V(\overline{x}(t+1))\} < \left(1-\frac{\delta}{\lambda_2}\right)\mathbb{E}\{V(\overline{x}(t))\}, \tag{3.15}$$

which implies

$$\lambda_1\mathbb{E}\left\{\|\overline{x}(t)\|^2\right\} \le V(\overline{x}(t)) \le \left(1-\frac{\delta}{\lambda_2}\right)^t\mathbb{E}\{V(\overline{x}(0))\}. \tag{3.16}$$

Thus one has that $\mathbb{E}\{\|\overline{x}(t)\|^2\} \le C\|\overline{x}(0)\|\sigma^t$, where $C = \lambda_2/\lambda_1 > 0$ and $0 < \sigma = 1-\delta/\lambda_2 < 1$. From Definition 3.2, one can conclude that the closed-loop fuzzy control system (2.14) is stochastically exponentially stable in the mean square sense. The proof is thus completed. □

Remark 3.5. It is noted that $\sigma = 1-\delta/\lambda_2$ represents the convergence rate of the closed-loop control system.

Based on Theorem 3.3, the following controller design results can be obtained.

Theorem 3.6. *The SNNS* (2.1) *can be semiglobally stochastically asymptotically stabilized in the mean square sense by the dynamic fuzzy controller in* (2.13), *if there exist a set of positive definite matrices X_i, $i \in \Im$, two sets of positive constants ε_{1i} and ε_{2i}, and a positive constant λ such that the following matrix inequalities hold for all $(i,j) \in \Omega$, $m \in \mathcal{O}(i)$,*

$$\begin{bmatrix} -X_i & \star & \star & \star & \star & \star \\ \overline{e}_1X_i & -\varepsilon_{1i}\mathbf{I} & \star & \star & \star & \star \\ \overline{e}_2X_i & 0 & -\varepsilon_{2i}\mathbf{I} & \star & \star & \star \\ X_i & 0 & 0 & -\lambda\mathbf{I} & \star & \star \\ \overline{A}_mX_i+\overline{B}\,\overline{Q}_{mi} & 0 & 0 & 0 & X_j-\varepsilon_{1i}\mathbf{I} & \star \\ \mathcal{M}_mX_i & 0 & 0 & 0 & 0 & \mathbf{I}_q\otimes X_j-\varepsilon_{2i}\mathbf{I}_{q(m+n)} \end{bmatrix} < 0. \tag{3.17}$$

Moreover, the controller gain matrices $\overline{K}_{mi}$ are given by $\overline{K}_{mi} = \overline{Q}_{mi}X_i^{-1}$.

Proof. It is noted that the SNNS (2.1) can be expressed by the generalized stochastic T-S fuzzy model in any compact set. It is also noted that the system (4.2) can be expressed by (2.12) in each local region of interest. Thus if the system (2.12) can be stochastically asymptotically stabilized in the mean square sense by the controller (2.13), with the bounded initial condition on the state $x(0)$ and the control $u(0)$, the original SNNS (2.1) can be shown to be semi-globally stochastically asymptotically stabilized in the mean square sense.

By using Schur's complement, (3.2) is equivalent to

$$\begin{aligned} &\mathcal{A}_{mi}^T P_j \mathcal{A}_{mi} - \mathcal{A}_{mi}^T P_j R \left(R^T P_j R - \frac{1}{\varepsilon_{1i}} \mathbf{I} \right)^{-1} R^T P_j \mathcal{A}_{mi} - P_i + \left(\frac{\bar{\varepsilon}_1^2}{\varepsilon_{1i}} + \frac{\bar{\varepsilon}_2^2}{\varepsilon_{2i}} \right) \mathbf{I} \\ &\quad + \mathcal{M}_m^T (\mathbf{I}_q \otimes P_j) \mathcal{M}_m - \mathcal{M}_m^T (\mathbf{I}_q \otimes P_j) \left(\mathbf{I}_q \otimes P_j - \frac{1}{\varepsilon_{2i}} \mathbf{I} \right)^{-1} (\mathbf{I}_q \otimes P_j) \mathcal{M}_m < -\delta \mathbf{I}. \end{aligned} \tag{3.18}$$

By matrix inverse lemma, (3.18) becomes

$$\mathcal{A}_{mi}^T \left(P_j^{-1} - \varepsilon_{1i} \mathbf{I} \right)^{-1} \mathcal{A}_{mi} - P_i + \left(\frac{\bar{\varepsilon}_1^2}{\varepsilon_{1i}} + \frac{\bar{\varepsilon}_2^2}{\varepsilon_{2i}} \right) \mathbf{I} + \mathcal{M}_m^T \left((\mathbf{I}_q \otimes P_j)^{-1} - \varepsilon_{2i} \mathbf{I} \right)^{-1} \mathcal{M}_m < -\delta \mathbf{I}. \tag{3.19}$$

Multiplying $X_i = P_i^{-1}$ from both sides to (3.19), one has

$$\begin{aligned} &X_i \mathcal{A}_{mi}^T (X_j - \varepsilon_{1i} \mathbf{I})^{-1} \mathcal{A}_{mi} X_i - X_i + \left(\frac{\bar{\varepsilon}_1^2}{\varepsilon_{1i}} + \frac{\bar{\varepsilon}_2^2}{\varepsilon_{2i}} \right) X_i X_i \\ &\quad + X_i \mathcal{M}_m^T (\mathbf{I}_q \otimes X_j - \varepsilon_{2i} \mathbf{I})^{-1} \mathcal{M}_m X_i < -\delta X_i X_i \end{aligned} \tag{3.20}$$

which is equivalent to (3.17) by using Schur's complement with the fact that $\overline{Q}_{mi} = \overline{K}_{mi} X_i$ and $\lambda = 1/\delta$.

Thus it follows from Theorem 3.3 that the closed-loop fuzzy system (2.14) is stochastically asymptotically stable in the mean square sense. Thus one has shown that the original SNNS (2.1) can be semi-globally stochastically asymptotically stabilized by the controller in (2.13). Thus the proof is completed. □

Remark 3.7. From (3.19), one can see that the LMIs in (3.17) are not easy to be satisfied if the upper bounds of the approximation errors, that is, $\bar{\varepsilon}_1$ and $\bar{\varepsilon}_2$, are too large. In order to achieve better approximation performance one has to use larger number of fuzzy rules, which, based on Theorems 3.3 and 3.6, leads to much higher computation cost of control design. However, this problem can be lessened to some extent due to the robustness of the proposed approach. In other words, the smaller number of fuzzy rules can be chosen since the robustness of the proposed approach allows larger approximation errors.

Remark 3.8. Theorems 3.3 and 3.6 are based on a piecewise quadratic Lyapunov function. When the positive definite matrices X_i are chosen as common ones, that is, $X_i = X$, $i \in \mathfrak{I}$, then the results of Theorems 3.3 and 3.6 reduce to those based on common Lyapunov functions. It will be shown in Section 4 that the results based on piecewise Lyapunov functions (FLPs) are less conservative than those based on common Lyapunov functions (CLFs).

4. Simulation Studies

In this section, to show the performance of the proposed controller design results, we consider the balancing problem of an inverted pendulum on a cart. The following discretized inverted pendulum plant with non-affine mathematical model [26] is used:

$$\begin{aligned} x_1(t+1) &= x_1(t) + Tx_2(t), \\ x_2(t+1) &= x_2(t) + T\left(f(x(t), v(t)) + [10x_1 + 10u, 15x_2 - 10u] \begin{bmatrix} W_1(t) \\ W_2(t) \end{bmatrix} \right), \end{aligned} \tag{4.1}$$

where $f(x,v) = (g\sin(x_1) - amlx_2^2\sin(2x_1)/2 - a\cos(x_1)v)/(4l/3 - aml\cos^2(x_1))$, $v(t) = (\arctan(u(t)) + 0.55u(t)) * 10^2$, x_1 denotes the angle of pendulum from the vertical, and x_2 is the angular velocity. $g = 9.8\,\mathrm{m/s^2}$ is the gravity constant, m is the mass of pendulum, M is the mass of the cart, $a = 1/(M+m)$, $T = 0.01\,\mathrm{s}$ is the sampling time in this study, and $2l$ is the length of the pendulum. Note that the input force is given by $\arctan(u) + 0.15u$ with an amplifier of gain 1000 connected. In this simulation, we choose $m = 2.0\,\mathrm{kg}$, $M = 8.0\,\mathrm{kg}$, and $2l = 1.0\,\mathrm{m}$.

We linearize the plant around the following operating points, $(x;u) = (0;0;0)$, $(0;0;3)$, $(\pm 88^\circ;0;0)$, and $(\pm 88^\circ;0;\pm 3)$, respectively, and consider the approximation errors between the linearized local model and the original nonlinear models as norm-bounded uncertainties. Then the following uncertain discrete-time dynamic T-S fuzzy model can be obtained:

$$\begin{aligned} x(t+1) = \sum_{l=1}^{4} \mu_l(x,u) \Bigg\{ &\left(A_l x(t) + B_l u(t) + \epsilon_f(x(t), u(t))\right) \\ &+ \sum_{k=1}^{2} \left(C_{lk}x(t) + D_{lk}u(t) + \epsilon_{gk}(x(t), u(t))\right) W_k(t) \Bigg\}, \end{aligned} \tag{4.2}$$

where the membership functions are shown in Figure 1,

$$\begin{gathered} A_1 = A_2 = \begin{bmatrix} 0 & 1 \\ 17.2941 & 0 \end{bmatrix}, \qquad A_3 = A_4 = \begin{bmatrix} 0 & 1 \\ 0.3593 & 0 \end{bmatrix}, \\ B_1 = \begin{bmatrix} 0 \\ -27.36 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ -15.56 \end{bmatrix}, \\ B_3 = \begin{bmatrix} 0 \\ -0.81 \end{bmatrix}, \qquad B_4 = \begin{bmatrix} 0 \\ -0.46 \end{bmatrix}, \\ C_{l1} = \begin{bmatrix} 0 & 0 \\ 10 & 0 \end{bmatrix}, \qquad D_{l1} = \begin{bmatrix} 0 \\ 10 \end{bmatrix}, \qquad C_{l2} = \begin{bmatrix} 0 & 0 \\ 0 & 15 \end{bmatrix}, \\ D_{l2} = \begin{bmatrix} 0 \\ -10 \end{bmatrix}, \quad l \in \mathfrak{L} := \{1, 2, \ldots, 4\}. \end{gathered} \tag{4.3}$$

As it has been defined in Section 2, the indices of fired rules in each local region are given in Table 1.

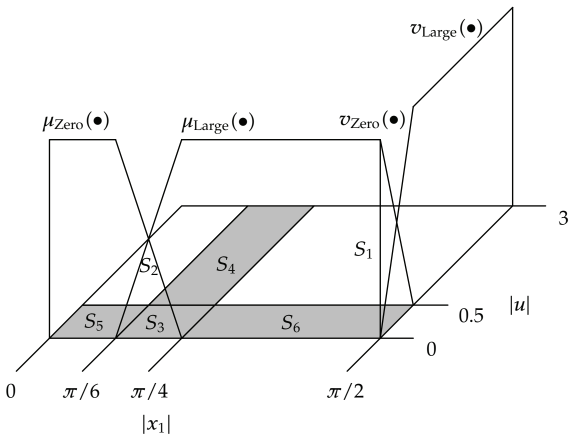

Figure 1: Membership functions.

Table 1: Indices of rules fired in each local region.

Local region S_i	Indices of rules fired $\mathcal{I}(i)$
S_1	$\{4\}$
S_2	$\{2\}$
S_3	$\{1,2,3,4\}$
S_4	$\{2,4\}$
S_5	$\{1,2\}$
S_6	$\{3,4\}$

It is noted that the exact or tightest upper bounds of the approximation errors are difficult to identify. However, one can apply the method shown in [26] to obtain the approximate upper bounds, which are $\overline{\epsilon}_1 = 0.3$ and $\overline{\epsilon}_2 = 0$, respectively. Then by applying Theorem 3.6, the controller gains with respect to each partitioned regions are obtained as

$$\begin{aligned}
\text{Region } S_1\text{: } \overline{K}_{41} &= \left[-0.0147 \quad 0.0615 \quad 0.0191\right], \\
\text{Region } S_2\text{: } \overline{K}_{22} &= \left[-0.0147 \quad 0.0615 \quad 0.0191\right], \\
\text{Region } S_3\text{: } \overline{K}_{13} &= \left[-0.3471 \quad 0.0333 \quad 0.5511\right], \\
\overline{K}_{23} &= \left[-0.3437 \quad 0.0296 \quad 0.3024\right], \\
\overline{K}_{33} &= \left[-0.0065 \quad 0.0279 \quad 0.0152\right], \\
\overline{K}_{43} &= \left[-0.0066 \quad 0.0279 \quad 0.0085\right], \\
\text{Region } S_4\text{: } \overline{K}_{24} &= \left[-0.3738 \quad 0.0341 \quad 0.3357\right], \\
\overline{K}_{44} &= \left[-0.0069 \quad 0.0300 \quad 0.0093\right], \\
\text{Region } S_5\text{: } \overline{K}_{15} &= \left[-0.0210 \quad -0.0010 \quad 0.0332\right], \\
\overline{K}_{25} &= \left[-0.0210 \quad -0.0010 \quad 0.0189\right], \\
\text{Region } S_6\text{: } \overline{K}_{36} &= \left[-0.0145 \quad 0.0611 \quad 0.0336\right], \\
\overline{K}_{46} &= \left[-0.0145 \quad 0.0608 \quad 0.0188\right].
\end{aligned} \tag{4.4}$$

Table 2: Comparison of the convergence rate for different cases.

Methods	The convergence rate $\sigma = 1 - (\delta/\lambda_2)$
Theorem 3.6	0.8317
Results based on PDCC and PLFs as indicated in Remark 2.3	0.9117
Results based on PDCC and CLFs as indicated in Remark 3.8	0.9991

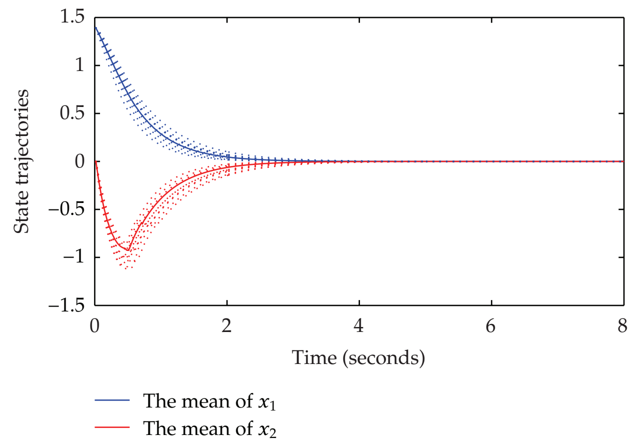

Figure 2: State trajectories.

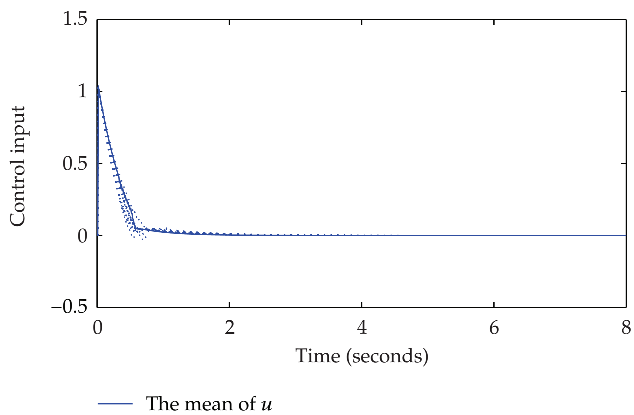

Figure 3: Control input.

To illustrate the performance of the approach proposed in this paper, the state trajectories and control input of the closed-loop system under initial condition $x(0) = (80°, 0)$ along 10 individual Wiener process paths are shown in Figures 2 and 3, respectively. One can observe that both the means of the system states and control input converge to zero as time approaches infinity.

To compare the proposed approach with results based on piecewise dynamic controller and piecewise/common Lyapunov functions, respectively, the convergence rates under different cases are presented in Table 2. It can be observed from Table 2 that the approaches based on piecewise quadratic Lyapunov functions are less conservative than those based on common Lyapunov functions, and the piecewise dynamic *fuzzy* controller (PDFC) has better performance than the piecewise dynamic *crisp* controller.

5. Conclusion

In this paper, T-S fuzzy model based control design of discrete-time stochastic non-affine nonlinear systems (SNNS) has been investigated. By using a piecewise Lyapunov function, it is shown that a discrete-time SNNS can be stochastically asymptotically stabilized in the mean square sense by solving a set of linear matrix equalities. Simulation results are provided to demonstrate the effectiveness of the approaches proposed in this paper. Some interesting future topics include filtering design and fault detection problems for complex discrete-time *SNNS* based on piecewise Lyapunov functions.

Appendix

Lemma A.1. *Given a set of independent stochastic processes* $W_1(t), \ldots, W_q(t)$ *satisfying*

$$\mathbb{E}\{W_i(t)\} = 0, \qquad \mathbb{E}\left\{W_i^2(t)\right\} = w_i, \tag{A.1}$$

a set of vectors $X_i \in \Re^{n\times 1}$, *and a symmetric matrix* $H \in \Re^{n\times n}$, *one has that*

$$\mathbb{E}\left\{\left(\sum_{i=1}^{q} W_i(t)X_i\right)^T H\left(\sum_{i=1}^{q} W_i(t)X_i\right)\right\} = \sum_{i=1}^{q} w_i X_i^T H X_i. \tag{A.2}$$

Proof. One has

$$\begin{aligned}
&\mathbb{E}\left\{\left(\sum_{i=1}^{q} W_i(t)X_i\right)^T H\left(\sum_{i=1}^{q} W_i(t)X_i\right)\right\} \\
&\quad = \mathbb{E}\left\{\sum_{i=1}^{q} W_i^2 X_i^T H X_i\right\} + \mathbb{E}\left\{\sum_{i\neq j} W_i W_j X_i^T H X_j\right\} = \sum_{i=1}^{q} w_i X_i^T H X_i.
\end{aligned} \tag{A.3}$$

□

Lemma A.2 (see [27]). *For any real matrices* X_i *and* $P > 0$ *with compatible dimensions, then*

$$\left\{\sum_{i=1}^{r} \gamma_i X_i\right\}^T P\left\{\sum_{i=1}^{r} \gamma_i X_i\right\} \leq \sum_{i=1}^{r} \gamma_i X_i^T P X_i, \tag{A.4}$$

where γ_i, $(i = 1, \ldots, r)$, *are nonnegative scalars with* $\sum_{i=1}^{r} \gamma_i = 1$.

Lemma A.3 (S-procedure [28]). *Let $T_0, \ldots, T_p \in \mathbb{R}^{n\times n}$ be symmetric matrices. Then the following condition on $T_0, \ldots, T_p$*

$$\xi^T T_0 \xi > 0, \quad \forall \xi \neq 0, \tag{A.5}$$

such that

$$\xi^T T_i \xi \geq 0, \quad i = 1, \ldots, p, \tag{A.6}$$

holds if there exists

$$\tau_1 \geq 0, \ldots, \tau_p \geq 0 \quad \text{such that } T_0 - \sum_{i=1}^{p} \tau_i T_i > 0. \tag{A.7}$$

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