Mathematical Problems in Engineering
Volume 5, pp. 379–396
Reprints available directly from the publisher
Photocopying permitted by license only

Multiple Objective Optimization of Hydro-Thermal Systems Using Ritz's Method

L. BAYÓN ARNÁU* and P. SUÁREZ RODRÍGUEZ

*Departamento de Matemáticas, Universidad de Oviedo,
Escuela Universitaria de Ingenieria Tecnica Industrial,
C/Manuel Llaneza 75, Gijón, Asturias, 33208 Spain*

(Received 11 June 1998)

This paper examines the applicability of the Ritz method to multi-objective optimization of hydro-thermal systems. The algorithm proposed is aimed to minimize an objective functional that incorporates the cost of energy losses, the conventional fuel cost and the production of atmospheric emissions such as NO_x and SO_2 caused by the operation of fossil-fueled thermal generation. The formulation includes a general layout of hydro-plants that may form multi-chains of reservoir network.

Time-delays are included and the electric network is considered by using the active power balance equation. The volume of water discharge for each hydro-plant is a given constant amount from the optimization interval. The generic minimization algorithm, which is not difficult to construct on the basis of the Ritz method, has certain advantages in comparison with the conventional methods.

Keywords: Multi-objective optimization; Economic dispatch; Environmental dispatch; NO_x and SO_2 control; Transmission losses; Ritz method

1. INTRODUCTION

The electric power systems are traditionally operated in such a way that the total fuel cost is minimized regardless of the emissions produced [1–4]. Recently, some papers present environmental dispatch algorithms [5] or combined optimizations for hydro-thermal

* Corresponding author. E-mail: bayon@sci.cpd.uniovi.es.

systems [6] but they use very simple models and great simplifications. The increasing requirements to the environmental protection give rise to the need for alternative strategies. This paper presents an algorithm [7] of multi-objective dispatch in a hydro-thermal system and discusses the problem of minimization of three objectives that include the usual minimum total quadratic fuel cost objective, the minimum SO_2 and NO_x emissions objective in quadratic form and the cost of energy losses. The loss objective and the reduction of emissions are expressed in monetary units to be compatible with the fuel objective of the dispatching problem. The algorithm is constructed with the use of the Ritz method.

The form of the emission function model depends (among other things) on the emission type. It is generally acknowledged that for SO_2 the emissions are proportional to the thermal unit's fuel consumption. As a result, the sulfuric emission function will be of the same form as that of the fuel cost function used in the economic dispatch. The NO_x emissions function is less straightforward to represent because these emissions are highly nonlinear in P. Several models have been used to represent the emission levels: the sum of a quadratic and an exponential term, a second order polynomial, a combination of a straight line and an exponential term, etc. We take for the function $E(P)$ the second order polynomial, $E(P) = \varepsilon P + \sigma P^2$, where the parameters are computed via the least square criteria from several tests. Also, the choice of a factor which converts transmission losses to equivalent thermal generation cost is a crucial aspect of the process and we use an approach to calculate the value of this factor.

In this work, we apply the Ritz method for the actual computer implementation of the optimal strategies in this hydro-thermal scheduling (HTS) problem of short range. The hydro-network is assumed to have several chains of hydro-plants on different streams. The volume of water discharge for each hydro-plant is a prespecified constant amount over the optimization interval, and we consider the river transport delay. Finally, the algorithm developed for this problem has been successfully applied to one real system; these results are presented below.

2. MATHEMATICAL OPTIMIZATION TECHNIQUES

In this Section, we discuss the problem of minimization of a quadratic functional [8]. Let F be a heterogeneous quadratic functional over a domain $D(F) \subseteq H$, where H is a Hilbert space. The functional $F(x)$ assumes its minimum value F_0 at x^*. We seek an element x to approximate F_0. A simple and efficient method of approximate solution of this problem was proposed by Ritz.

Let $g(u)$ be a homogeneous quadratic functional corresponding to the bilinear symmetric form $G(u, v)$, over the same domain $D(F)$ and such that $g(u) = G(u, u)$. Let $l(u)$ be a linear (that is, additive and homogeneous, but not necessarily bounded) functional such that its domain is $D(F)$. Let F be the functional: $F(x) = g(x) - 2lx$, and let us assume that the functional g is positive, that is: $g(x) \succ 0$ for $x \neq 0$, $x \in D(F)$.

LEMMA 1 *If the homogeneous quadratic functional g is* positive definite, *that is, there exists a constant $m_0 \succ 0$ such that: $g(x) \geq m_0 \|x\|^2$ and the functional l is bounded, then the functional F is bounded from below.*

In this case the problem of minimization of F makes sense. Let us introduce now the following concept.

A sequence $\{x^{(n)}\}_{n=1}^{\infty}$ of $D(F)$ is called *minimizing* for the functional F if: $F(x^{(n)}) \to F_0 = \inf F(x)$.

It is easy to prove that the Ritz approximate solutions form a minimizing sequence. Now we describe the Ritz process. We take the *coordinate elements* $\{u_i\}_{i=1}^{\infty}$ in $D(F)$, and will assume that every finite set of coordinate elements is linearly independent. Then we introduce the subspaces: $H^{(n)} = L(u_1, \ldots, u_n) \in D(F)$. We shall also use the following lemma.

LEMMA 2 *The functionals $g(x)$ and lx are continuous in each space $H^{(n)}$ and the functional $g(x)$ is positive definite in each space $H^{(n)}$, that is, there exist positive constants m_n such that:*

$$g(x) \geq m_n \|x\|^2 \quad (x \in H^{(n)}).$$

Let $F_n = \inf_{x \in H^{(n)}} F(x)$. Then $\forall n$ in $H^{(n)}$ there exists only one element where $F = F_n$.

The Ritz method consists in seeking the minimum of F not in H but in $H^{(n)}$, the n-dimensional subspace spanned by $\{u_i\}_{i=1}^{n}$. In other words, $x \in H^{(n)}$ can be written as $x = \sum_{i=1}^{n} a_i u_i$ where $a_1, \ldots, a_n$ are constants, and we thus seek the minimum of the function of n

variables, $F\left(\sum_{i=1}^{n} a_i u_i\right) = F(a_1, \ldots, a_n)$. In this case

$$F(a_1, \ldots, a_n) = \sum_{i,j=1}^{n} a_i a_j G(u_i, u_j) - 2\sum_{i=1}^{n} a_i l u_i.$$

As is easy to see, to minimize F amounts to solving the system of equations, $0 = \partial F/\partial a_k$. If we take for the $(a_i)_{i=1}^{n}$ the solution of the latter system, then the elements $x^{(n)}$, depending on the coordinate elements $(u_1, \ldots, u_n)$, will be called the *Ritz approximate solutions* of the basic optimization problem and $(a_i)_{i=1}^{n}$ will be called the *Ritz coefficients*. It is worth noting that the approximate solution does not change if the functional is of the general form $F(x) = g(x) - 2lx + c$, where c is a constant.

Let us now consider the problem of determining the minimum of the functional

$$F(x) = (Ax, x) - 2(y^*, x) \quad (x \in D(A)),$$

which is called the *energy functional*. Later on, it will be our cost functional. The next sections will be devoted to its construction. The following is supposed: (a) the operator A is symmetric, that is, $\forall u, v \in D(A)$ we have $(Au, v) = (u, Av)$; (b) the operator A is positive definite, that is, there exists a constant $m_0 \succ 0$ such that $\forall u \in D(A)$ $(Au, u) \succeq m_0 \|u\|^2$; (c) $y^* \in R(A)$. Let us assume that $D(F) = D(A)$, $G(u, v) = (Au, v)$, $lu = (u, y^*)$ for $u, v \in D(F)$, then the *energy functional* can be written as $F(x) = g(x) - 2lx$.

We now make use of the following construction [8]. Define a new scalar product (*the scalar energy product*) setting

$$[u, v] = (Au, v) \quad \text{for } u, v \in D(A),$$

and the new *energy norm* $|u| = [u, u]$.

Let us complete $D(A)$ with respect to this norm $|*|$. It is not difficult to verify that in the result of this completion $D(A)$ becomes a new Hilbert space H_A which is called the *energy space*. Moreover, if $u_i \in D(A)$, which is dense in H_A, we obtain $H^{(n)} \subset D(A) \subset H_A$. We thus arrive at the following theorem.

THEOREM *If the coordinate system is complete in H_A, then $x^{(n)}$ converges in H and in H_A to the element x^* which yields the minimum of F.*

3. HYDRO-THERMAL SYSTEM MODELS

The aim of this section is to present some fundamental aspects of modeling various parts of the electric power system. An electric power system with m thermal plants and $(n-m)$ hydro-plants is studied. The following models are considered [1,2,7].

(1) Cost fuel model: For economy operation problems fuel cost of the thermal plants is approximated by:

$$F(P_{si}(t)) = \alpha_i + \beta_i P_{si}(t) + \gamma_i P_{si}^2(t).$$

Here α_i, β_i and γ_i are known constants for each plant and $P_{si}(t)$ is the power generation of this plant.

(2) Emission function model: For the emissions we use a function $E(P)$ ($\$/h$) of total NO_x and SO_2 emissions which is assumed to be a second order polynomial:

$$E(P_{si}(t)) = \varepsilon_i P_{si}(t) + \sigma_i P_{si}^2(t),$$

where the parameters were calculated via the least square criterion from several tests [7] in thermal plant of Aboño (Spain) and with the use of European penalty cases for pollution emissions.

(3) The transmission losses: The transmission losses $P_L(t)$ are assumed to be represented by the model of Kirchmayer with the following loss equation:

$$P_L(t) = K_{LO} + \sum_{i=0}^{n} B_{i0} P_i(t) + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i(t) B_{ij} P_j(t),$$

where K_{LO}, B_{i0} and B_{ij} are known parameters of system and $P_i(t)$ is the generation power. The choice of a factor ϕ which converts transmission losses to equivalent thermal generation cost is a crucial aspect of the process [6]. We calculate ϕ as the incremental cost of generation given by:

$$\phi_i = \frac{\beta_i + 2\gamma P_i}{1 - 2B_{ii} P_i}; \quad i = 1, 2, \ldots, m.$$

The average value of ϕ is given by the average of ϕ_i over all time instants.

(4) Electric network model: The generation schedule must satisfy the active power balance equation:

$$\sum_{i=1}^{n} P_i(t) = P_{\mathrm{D}}(t) + P_{\mathrm{L}}(t).$$

denoted by APBE. Here $P_{\mathrm{D}}(t)$ is the system power demand.

(5) Hydro-network model: The hydro-network is assumed to have several chains of hydro-plants on different streams as well as hydraulically isolated plants. Let us denote by R_{h} the set of all hydro-plants in the system. We assume that the rate of discharge at the upstream plant affects the behavior at the downstream plants. We say that the hydraulic system has hydraulic coupling. The time delay of water discharge between two consecutive plants is assumed to be a constant τ.

The variables of the system are: $q_i(t)$, rate of water discharge by i plants; τ_i, time delay of water between i and downstream reservoir; $Q_i(t)$, volume of water discharge by i plants. The hydraulic power is

$$P_{\mathrm{h}m+i}(t) + A_{m+i}(t)q_{m+i}(t) + B_{m+i}q_{m+i}(t)Q_{m+i}(t) + C_{m+i}q_{m+i}^2(t) = 0; \quad (i \in R_{\mathrm{hCA}}).$$

$$\begin{aligned} P_{\mathrm{h}m+i}(t) &+ A_{m+i}(t)q_{m+i}(t) - B_{m+i}q_{m+i}(t)\sum_{j\in R_{\mathrm{h}i}} Y_j(t,\tau_j) \\ &+ B_{m+i}q_{m+i}(t)Q_{m+i}(t) + C_{m+i}q_{m+i}^2(t) = 0; \quad (i \in R_{\mathrm{hID}}). \end{aligned}$$

where R_{hCA} is the set of all upstream and isolated plants and R_{hID} is the set of intermediate, and downstream hydro-plants. The parameters $A_{m+i}(t)$, B_{m+i} and C_{m+i} are known for hydro-plants and the function $Y_j(t,\tau_j)$ is

$$Y_j(t,\tau_j) = \begin{cases} \Psi_j(t,\tau_j) & \text{for } 0 \le t \le \tau_j \\ \Psi_j(\tau_j,\tau_j) + Q_j(t-\tau_j) & \text{for } \tau_j \prec t \prec T_{\mathrm{f}} \end{cases} \quad j \in R_{\mathrm{h}i}$$

where

$$\Psi_j(t,\tau_j) = \int_{-\tau_j}^{t-\tau_j} q_j(x)\,\mathrm{d}x, \quad j \in R_{\mathrm{h}i}, \ t \le \tau_j.$$

Finally, it is assumed that the volume of water discharge b_i for each hydro-plant is a given constant amount from the optimization interval

$[0, T_f]$, and is given by:

$$\int_0^{T_f} q_i(t)\,\mathrm{d}t = b_i.$$

4. STATEMENT OF THE PROBLEM

An electric power system with m thermal plants and $(n-m)$ hydro-plants in w chains of hydro-plants on different streams is considered. In order to simplify the formulation, let us consider only one chain, and the results are generalized easily. The problem is to determine the power generation of each plant in order to minimize a combined objective in the optimization interval $[0, T_f]$:

$$J = \int_0^{T_f} \left\{ \kappa_1 \sum_{i=1}^{m} (\alpha_i + \beta_i P_{si}(t) + \gamma_i P_{si}^2(t)) + \kappa_2 \sum_{i=1}^{m} (\varepsilon_i P_{si}(t) + \sigma_i P_{si}^2(t)) + \kappa_3 \phi \sum_{i=1}^{n} B_{ii} P_i^2(t) \right\} \mathrm{d}t.$$

Here κ_1, κ_2, and κ_3, $(\kappa_i \leq 1,\ \kappa_1 + \kappa_2 + \kappa_3 = 1)$ are weighing constants assigning relative values to the three cost components. The problem is to be solved under the following conditions:

$$K_{\mathrm{LO}} + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i(t) B_{ij} P_j(t) + \sum_{i=0}^{n} (B_{i0} - 1) P_i(t) + P_{\mathrm{D}}(t) = 0.$$

$$q_{m+i}(t) - Q_{m+i}(t) = 0; \quad (i \in R_{\mathrm{h}}).$$

$$P_{\mathrm{h}m+i}(t) + A_{m+i}(t) q_{m+i}(t) + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) = 0; \quad (i \in R_{\mathrm{hCA}}).$$

$$P_{\mathrm{h}m+i}(t) + A_{m+i}(t) q_{m+i}(t) - B_{m+i} q_{m+i}(t) \sum_{j \in R_{\mathrm{h}i}} Y_j(t, \tau_j) + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) = 0; \quad (i \in R_{\mathrm{hID}}).$$

$$Y_j^2(t, \tau_j) = \begin{cases} \Psi_j^2(t, \tau_j) & \text{for } 0 \leq t \leq \tau_j, \\ \Psi_j^2(\tau_j, \tau_j) + Q_j^2(t - \tau_j) + 2\Psi_j(\tau_j, \tau_j) Q_j(t - \tau_j) & \text{for } \tau_j \prec t \prec T_f. \end{cases}$$

Furthermore, the water discharge at each hydro-plant has to satisfy the following constraint on the volume of water used, which is taken from the optimization interval:

$$\int_0^{T_f} q_i(t)\,\mathrm{d}t = b_i.$$

To satisfy these constraints, we introduce the unknown functions of time $l(t)$, $m_i(t)$, $n_i(t)$, and $r_i(t)$ which correspond to the previous constraints and the unknown constant factor μ_i for the constraint of the volume of water. These unknown functions, the constants μ_i, and some suitable conditions have to be included in the cost functional forming the *augmented cost functional*:

$$\begin{aligned}
J = \int_0^{T_f} & \left\{ \kappa_1 \sum_{i=1}^{m} (\alpha_i + \beta_i P_{si}(t) + \gamma_i P_{si}^2(t)) \right. \\
& + \kappa_2 \sum_{i=1}^{m} (\varepsilon_i P_{si}(t) + \sigma_i P_{si}^2(t)) + \kappa_3 \phi \sum_{i=1}^{n} B_{ii} P_i^2(t) \\
& + l(t) \left[K_{\mathrm{LO}} + \sum_{i=1}^{n} \sum_{j=1}^{n} P_i(t) B_{ij} P_j(t) + \sum_{i=0}^{n} (B_{i0} - 1) P_i(t) + P_{\mathrm{D}}(t) \right] \\
& + \sum_{i \in R_{\mathrm{h}}} m_{m+i}(t) \left[q_{m+i}(t) - Q_{m+i}(t) \right] \\
& + \sum_{i \in R_{\mathrm{hCA}}} n_{m+i}(t) [P_{\mathrm{h}m+i}(t) + A_{m+i}(t) q_{m+i}(t) \\
& + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t)] \\
& + \sum_{i \in R_{\mathrm{hID}}} n_{m+i}(t) \left[P_{\mathrm{h}m+i}(t) + A_{m+i}(t) q_{m+i}(t) \right. \\
& \left. - B_{m+i} q_{m+i}(t) \sum_{j \in R_{\mathrm{h}i}} Y_j(t, \tau_j) + B_{m+i} q_{m+i}(t) Q_{m+i}(t) + C_{m+i} q_{m+i}^2(t) \right] \\
& \left. + \sum_{i \in R_{\mathrm{hID}}} \sum_{i \in R_{\mathrm{h}i}} r_j(t) Y_j^2(t, \tau_j) \right\} \mathrm{d}t - \sum_{i \in R_{\mathrm{hID}}} \sum_{i \in R_{\mathrm{h}i}} \int_0^{\tau_3} r_j(t) \Psi_j^2(t, \tau_j)\,\mathrm{d}t \\
& - \sum_{i \in R_{\mathrm{hID}}} \sum_{i \in R_{\mathrm{h}i}} \int_{\tau_j}^{T_f} r_j(t) \left[\Psi_j^2(\tau_j, \tau_j) + Q_j^2(t - \tau_j) + 2\Psi_j(\tau_j, \tau_j) Q_j(t - \tau_j) \right] \\
& + \int_0^{T_f} \sum_{i \in R_{\mathrm{h}}} \mu_{m+i} q_{m+i}(t)\,\mathrm{d}t - \sum_{i \in R_{\mathrm{h}}} \mu_{m+i} b_{m+i}.
\end{aligned}$$

This cost functional can be written in matrix form. Let us define the control vector, $u(t) = col[P(t)\ \ W(t)]$, where $P(t)$ is a column vector of all active power generations, and $W(t) = col[W_i(t)\colon i \in R_{\mathrm{h}}]$.

Each of the hydro-plants offers a subvector $W_i(t)$, and each of the hydro-subvectors has a dimension and a definition that depend on the category of the plant. Thus we have:

$$W_i(t) = \begin{bmatrix} Q_i(t) \\ q_i(t) \end{bmatrix} \quad i \in R_{\mathrm{hCA}}; \qquad W_i(t) = \begin{bmatrix} Q_i(t) \\ q_i(t) \\ Y_{iw}(t) \end{bmatrix} \quad i \in R_{\mathrm{hID}}$$

with $Y_{iw}(t) = col[Y_j(t, \tau_j)\colon j \in R_{\mathrm{h}i}]$, $i \in R_{\mathrm{hID}}$.

With the above preliminary definitions, the problem is readily formulated as:

$$J[u(t)] = \int_0^{T_{\mathrm{f}}} [u^T(t)B(t)u(t)]\,\mathrm{d}t + \int_0^{T_{\mathrm{f}}} [L^T(t)u(t)]\,\mathrm{d}t + c,$$

where the matrix $B(t)$, the vector $L(t)$, and the constant c are defined and calculated in Bayón [7]. Let us define:

$$G(u,u) = (Au, u) = \int_0^{T_{\mathrm{f}}} [u^T(t)B(t)u(t)]\,\mathrm{d}t,$$

$$l \cdot u = \int_0^{T_{\mathrm{f}}} [L^T(t)u(t)]\,\mathrm{d}t; \quad c = -\sum_{i \in R_{\mathrm{h}}} \mu_i b_i.$$

The cost functional can be written as:

$$F(u) = (Au, u) - 2lu + c \quad (u \in D(A)).$$

With these preliminaries at hand, we conclude that the problem is ready for casting as a problem of the minimum of a quadratic functional. In considering the problem formulated, we will have the following assertion which is a simple consequence of the definitions given; see, for example [1]:

(i) $g(u)$: symmetric and positive definite functional.
(ii) $l(u)$: linear and bounded functional.

Under the above conditions every minimizing sequence for the functional $F(u)$ converges in H_A to the element which yields the minimum of $F(u)$. Let us choose the energy space: $L^n_{2B}[0, T_f] \equiv H_A$. The vector $u(t)$ is considered as an element of the Hilbert energy space of n vector-valued square-integrable functions defined on $[0, T_f]$, whose inner product is given by:

$$[v(t), u(t)] = (Av, u) = \int_0^{T_f} [v^T(t)B(t)u(t)]\,dt.$$

The question of a rational choice of the coordinate functions plays a fundamental role in the present work. For the sake of simplicity, we choose as the coordinate functions the system $\{t^i\}_{i=0}^{\infty}$, which satisfy the required boundary conditions. It is not difficult to verify [8] that the above-formulated conditions are satisfied. That is, the elements are linearly independent and form a complete system in the energy space H_A, which completes the study of the questions related to the construction of approximate solutions and verification of their convergence.

The approximate solutions are usually constructed as the linear combination of two or even one coordinate function.

5. OPTIMAL SOLUTION

In this section, we present a solution of the basic optimization problem under the above-formulated assumptions. The Ritz method consists in seeking the minimum of F not in H_A, but in $H^{(n)}$, the n-dimensional subspace spanned by $\{w_i\}_{i=1}^n$. In other words, $x \in H^{(n)}$ can be written as:

$$x = \sum_{i=1}^{n} a_i w_i.$$

Let us now consider the approximations to our control variables. We note here that the augmented and modified cost functional [7] depends only on two control variables: the thermal power generation $P_{si}(t)$, and the rate of water discharge by hydro-plants $q_i(t)$.

In general, with the coordinate functions $w_i(t)$, the approximations are given by:

$$P_{si}(t) = P_{si}(0) + \sum_{l=1}^{f} d_l^i w_l(t) \quad i = 1, 2, \ldots, m,$$
$$q_{m+i}(t) = q_{m+i}(0) + \sum_{l=1}^{f} c_l^{m+i} w_l(t) \quad i = 1, 2, \ldots, n-m,$$

where f is the number of coordinate functions and the dimension of the subspace $H^{(n)}$.

The results of computational experiments with a number of algorithms and samples show a very fast convergence of the minimizing sequence to the exact solution. In order to avoid instability of the Ritz process, and for simplicity on calculations, we choose the system of coordinate functions $w_i(t) \equiv \{t^i\}_{i=0}^{\infty}$ and take for the approximate Ritz representation (solution) only one coordinate function. Under this assumption, we have:

$$P_{si}(t) = P_{si}(0) + d_l^i(t) \quad i = 1, 2, \ldots, m,$$
$$q_{m+i}(t) = q_{m+i}(0) + c_l^{m+i}(t) \quad i = 1, 2, \ldots, n-m.$$

Now, we adopt a variational calculus approach that employs Lagrange multipliers $l(j)$ and μ_i. This approach simply transforms the constrained problem into a nonlinear unconstrained problem, where the function depends of the $n \times f \times q$ Ritz coefficients (f for each thermal or hydro-plant of the system and one for each node). We now consider the problem of the minimum of the function of n variables:

$$J = F(a_1, \ldots, a_{n \cdot f \cdot q}, \; l(1), \ldots, l(q), \; \mu_{m+1}, \ldots, \mu_n).$$

The set of equations defining the optimality can be obtained simply by setting the gradients of the augmented cost J with respect to the unknowns equal to zero. The nonlinear system of equations is:

$$\frac{\partial F}{\partial d_l^{i,j}} = 0; \quad i = 1, 2, \ldots, m; \; l = 1, 2, \ldots, f; \; j = 1, 2, \ldots, q,$$
$$\frac{\partial F}{\partial c_l^{m+i,j}} = 0; \quad i = 1, 2, \ldots, n-m; \; l = 1, 2, \ldots, f; \; j = 1, 2, \ldots, q,$$
$$\frac{\partial F}{\partial l(j)} = 0; \quad j = 1, 2, \ldots, q,$$
$$\frac{\partial F}{\partial \mu_{m+i}} = 0; \quad i = 1, 2, \ldots, n-m.$$

We know that Newton's method for solving nonlinear equations has an unfortunate tendency to wander off into the wild blue yonder if the initial guess is not sufficiently close to the root. A global method is the one that converges to a solution from almost any starting point.

To resolve this system we will develop an algorithm that combines the rapid local convergence of Newton's method with a globally convergent strategy that will guarantee some progress towards the solution at each iteration. The classic algorithm [9] is close to the quasi-Newton method of minimization and it is called the *line searches and backtracking (LSB) method.*

6. A NUMERICAL EXAMPLE

A computer program (Fortran 77) [7] was written to apply the results obtained in this paper to practical power system. The units for the coefficients of the thermal plants are: α in (\$/h); β and ε in (\$/h $\cdot$ MW); γ and σ in (\$/h $\cdot$ MW^2), the transmission loss coefficients B_{ii} are in (1/MW) and the initial power $P(0)$ in (MW). The data of the thermal plants and hydro-plants are summarized in Tables I and II respectively.

The units for the coefficients of the hydro-plants are: the efficiency G in (ft^4/h $\cdot$ MW), the restriction on the volume b in (ft^3), the loss coefficients B_{ii} in (1/MW), the natural inflow i in (ft^3/h), the initial volume $S(0)$ in (E9 $\cdot$ ft^3), the coefficients B_T in (E $-$ 8 $\cdot$ ft^{-2} $\cdot$ h), the coefficients B_Y in (E $-$ 12 $\cdot$ ft^{-2}), the time delay τ in (h) and $q(0)$ in (ft^3/h). The reader is referred to Suárez [2] for the relation between $S(0)$, i, B_T and B_Y with the coefficients A, B and C.

The system consists of three thermal plants and six hydro-plants and the hydro-network is as shown in Fig. 1. The system's power demand is shown in Fig. 2, and the variation of relative error with iterations is

TABLE I Thermal plant's coefficients

	α	β	γ	ε	σ	B	$P(0)$
Plant 1	0	4.2	0.0014	0.16	0.0004	0.00016	31.58
Plant 2	0	4.1	0.0013	0.17	0.0005	0.00010	66.98
Plant 3	0	4.0	0.0012	0.18	0.0006	0.00016	87.73

TABLE II Hydro-plant's coefficients

	G	b	B	i	$S(0)$	B_T	B_Y	τ	$q^{(0)}$
Plant 1	526 315	14 160 000	0.00022	10 195 200	203.9040	581.740	149.5	1	537 351
Plant 2	536 315	14 160 000	0.00020	10 195 200	203.9040	571.740	149.5	1	531 572
Plant 3	546 315	14 160 000	0.00024	10 195 200	203.9040	581.740	149.5	1	537 304
Plant 4	570 834	99 120 000	0.00016	30 195 200	40.7808	219.597	1495.0	2	3 815 119
Plant 5	580 834	99 120 000	0.00017	40 195 200	40.7808	239.597	1595.0	2	3 802 677
Plant 6	590 834	99 120 000	0.00016	50 195 200	40.7808	219.597	1495.0		3 751 587

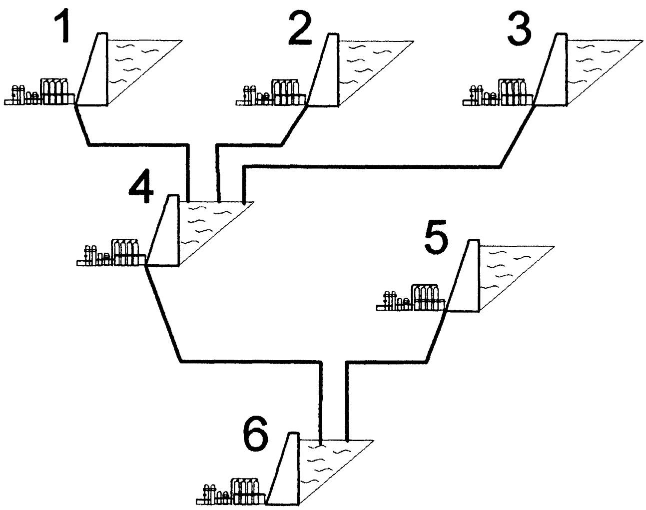

FIGURE 1 Sample hydro-system.

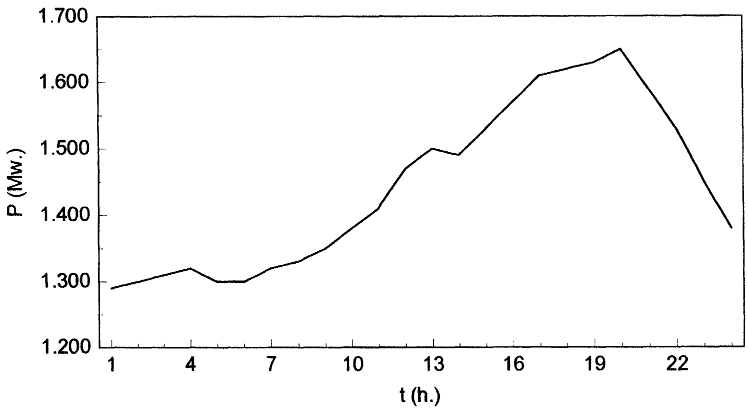

FIGURE 2 Power demand.

also shown in Fig. 3. Finally, the computed optimal active power generations for the sample system are shown only for two plants: in Fig. 4 for the thermal plant number 2 and in Fig. 5 for the hydro-plant number 4. In two cases, four studies are presented: minimization of fuel cost ($\kappa_1 = 1$), of pollution emissions ($\kappa_2 = 1$), of transmission losses

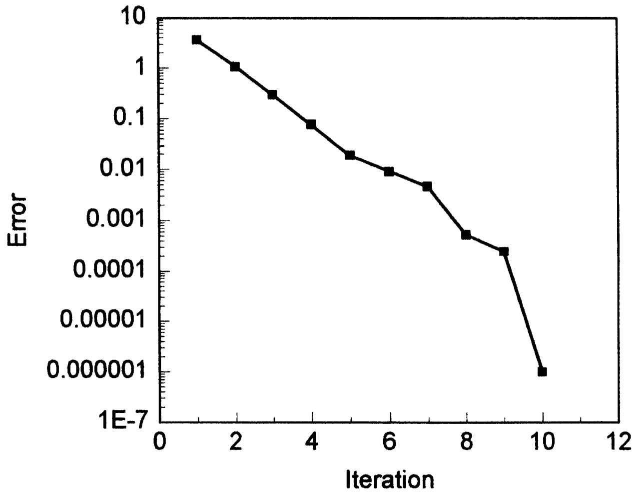

FIGURE 3 Variation of relative error.

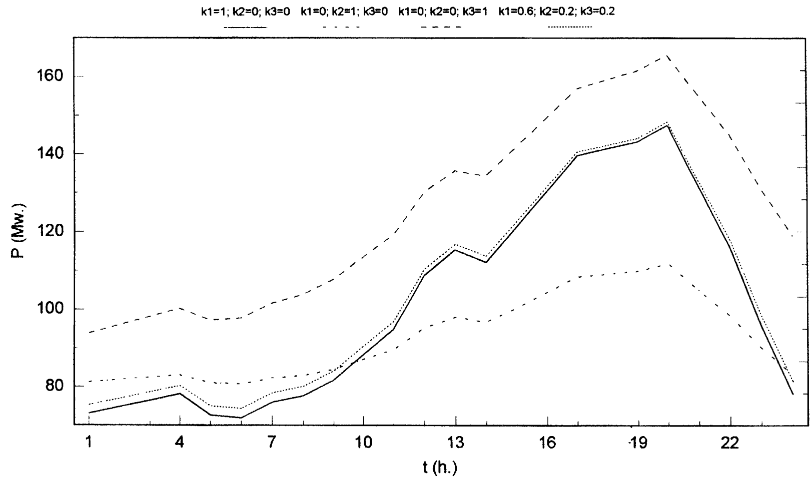

FIGURE 4 Optimal thermal power (plant 2).

($\kappa_3 = 1$), and multi-objective optimization ($\kappa_1 = 0.6$; $\kappa_2 = 0.2$; $\kappa_3 = 0.2$). The rest of plants is not presented, because it will be very extensive.

The algorithm of LSB accounts for many advantages. First of all, to run the method one does not have to start from specially selected initial values. The process leads to an optimal solution even if the

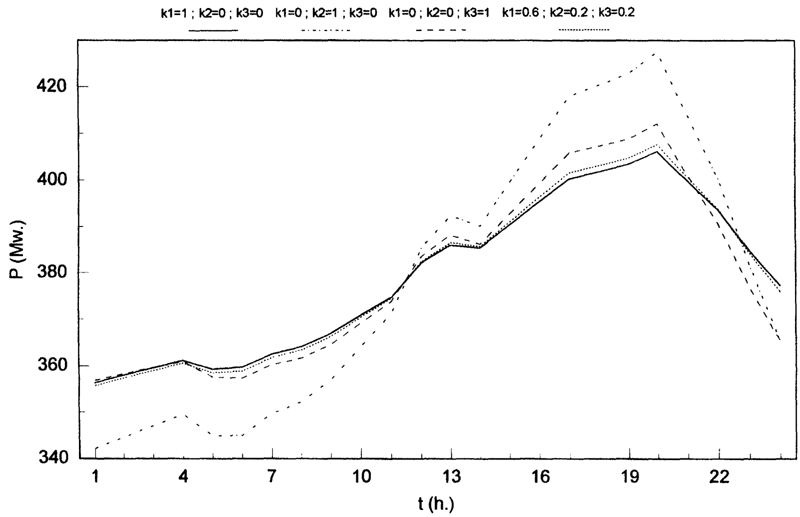

FIGURE 5 Optimal hydro-power (plant 4).

initial values were chosen as general as:

$$q_i^{(0)} = \frac{b_i}{q}; \qquad p_{si}^{(0)} = \frac{p_{si}^{\max} + p_{si}^{\min}}{2};$$
$$l^{(0)} \simeq \beta_i; \quad \mu^{(0)} \simeq 10^{-4}; \qquad \text{for all } j = 1, 2, \ldots, q.$$

Moreover, it shows a rapid convergence to the optimal solution (in the example, it happened to be sufficient to perform 10 iterations to acquire the prescribed error) and due to the simplicity of the operations which one has to perform in this method, the realization of the method does not take much time.

7. CONCLUSIONS AND FUTURE DIRECTIONS

In this paper, a new formulation of the economic operation of hydro-thermal electric power systems is given. The optimal solution for the problem is introduced with the use of the Ritz method. The formulation includes a general layout of hydro-plants that may form multi-chains of reservoir network. Time-delays are included and the electric network is considered by using the active power balance equation. The volume of water discharge for each hydro-plant is a given constant amount from the optimization interval. A computer program was written to solve the nonlinear system of optimal equations. The program used the above-mentioned LSB method.

The main merits of the method are the following: this is a general method which can be applied to various models of hydro-thermal systems; it permits one to eliminate the variables associated with the transport delay and it can be easily applied to combined optimizations. In conclusion, this iterative technique shows a rapid convergence for such complex problems and the LSB method is sure to converge to a solution from almost any initial approximation.

However the principal merit of this method consists in the fact that this is a general method which deals with any model of the hydraulic net without any substantial modification. Hence we could easily take into account restrictions formulated in the form of inequalities

$$p_i^{\max} \geq p_i \geq p_i^{\min}; \qquad q_i^{\max} \geq q_I \geq q_i^{\min}.$$

To this end we will make use of the Kuhn–Tucker conditions or penalty functions. And also, we will be able to substitute (to perform the same steps) the APBE by the exact equations of the load flow:

$$P_k = \sum_{j=1}^{N} e_k(e_j G_{kj} + f_j B_{kj}) + f_k(f_j G_{kj} - e_j B_{kj});$$

$$Q_k = \sum_{j=1}^{N} f_k(e_j G_{kj} + f_j B_{kj}) - e_k(f_j G_{kj} - e_j B_{kj}),$$

with the classic parameter defined in Suárez [2] or El-Hawary [1].

References

[1] M.E. El-Hawary, G.S. Chrystensen and S.A. Soliman, *Optimal Control Applications in Electric Power Systems*, Plenum Press, New York, 1987.

[2] P.M. Suárez, Optimización del coste de combustible en un sistema hidrotérmico, mediante técnicas del análisis funcional, Cuaderno 11, Dpto. de Matemáticas de la Universidad de Oviedo, 1989.

[3] R. Sridhara and P.S. Nayendra, A fixed point algorithm for the economic operation of variable head hydro-thermal systems using an optimal load flow model, *Proceedings of the Sixth National Power Systems Conference*, McGraw-Hill, 1990, pp. 500–4.

[4] M.E. El-Hawary and K.M. Ravindranath, Hydro-thermal power flow scheduling accounting for head variations, *IEEE Trans. on Power Systems*, **7**(3), Aug. 1992, 1232–8.

[5] T. Denise, M.E. El-Hawary and F. El-Hawary, Optimal environmental dispatching of electric power systems via an improved Hopfield neural network model, *IEEE Trans. on Power Systems*, **10**(3), Aug. 1995, 1559–65.

[6] M.E. El-Hawary and K.M. Ravindranath, Combining loss and cost objectives in daily hydro-thermal economic scheduling, *IEEE Trans. on Power Systems*, **6**(3), Aug. 1991, 1106–12.

[7] L. Bayón, Optimización combinada de Sistemas Hidrotérmicos mediante métodos aproximados, Cuaderno 31, Dpto. de Matemáticas, Universidad de Oviedo, 1998.

[8] S.G. Mikhlin, *The Numerical Performance of Variational Methods*, Wolters, Groningen, 1971.

[9] W.H. Press, *Numerical Recipes in Fortran*, Cambridge University Press, Cambridge, 1994.


Advances in
Operations Research
Advances in
Decision Sciences
Journal of
Applied Mathematics
Algebra
Journal of
Probability and Statistics
The Scientific
World Journal
International Journal of
Differential Equations
Hindawi
Submit your manuscripts at
http://www.hindawi.com
International Journal of
Combinatorics
Advances in
Mathematical Physics
Journal of
Complex Analysis
Journal of
Mathematics
Mathematical Problems
in Engineering
Abstract and
Applied Analysis
Discrete Dynamics in
Nature and Society
International
Journal of
Mathematics and
Mathematical
Sciences
Journal of
Discrete Mathematics
Journal of
Function Spaces
International Journal of
Stochastic Analysis
Journal of
Optimization