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Research Article

Analyzing the Dynamic Data Sponsoring in the Case of Competing Internet Service Providers and Content Providers

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With a sponsored content plan on the Internet market, a content provider (CP) negotiates with the Internet service providers (ISPs) on behalf of the end-users to remove the network subscription fees. In this work, we have studied the impact of data sponsoring plans on the decision-making strategies of the ISPs and the CPs in the telecommunications market. We develop game-theoretic models to study the interaction between providers (CPs and ISPs), where the CPs sponsor content. We formulate the interactions between the ISPs and between the CPs as a noncooperative game. We have shown the existence and uniqueness of the Nash equilibrium. We used the best response dynamic algorithm for learning the Nash equilibrium. Finally, extensive simulations show the convergence of a proposed schema to the Nash equilibrium and show the effect of the sponsoring content on providers' policies.

1. Introduction

One of the principal trends on the Internet in the last few years is the explosion in demand for cellular data usage. Therefore, one of the main challenges for CP is how to motivate end-users to access their content to achieve a higher profit. In addition, this increase in cellular data usage needs higher investments in wireless capacity. ISPs launched a new type of data pricing called data sponsoring to get additional revenue and to increase the capacity of their existing network architectures [1]. The critical idea of data sponsoring is to allow the CPs to pay the ISP on behalf of the end-users the network access fees. Data sponsoring plans benefit all entities in the network; the ISPs can generate more revenue with CP's subsidies, and end-users can enjoy free network access to the CPs, which increases the demand and attracts more traffic, resulting in higher income of the CP. As a real-world example, AT&T allows advertisers to sponsor video to attract more end-users to watch advertisements [2, 3], and GS Shop (Korea TV) has cooperated with SK Telecom to sponsor the traffic of its application [4].

Sponsoring data have recently been subject to modeling and analysis in the literature. The authors in [5] proposed a novel joint optimization approach of a Stackelberg contract game to characterize the market-oriented model for sponsored content market and to capture the interactions among the ISPs, CPs, and end-users. They have developed a Stackelberg game, where the ISP acts as the leader and the CPs and end-users act as the followers. In [6], the authors developed a new model to study the competition among CPs under sponsored data plans. The authors in [7] analyzed the interactions of three network entities, i.e., the end-users, the ISPs, and the CP, based on the game theory. The authors designed an effective data sponsoring control scheme using a novel dual-leader Stackelberg game model. The authors in [8, 9] investigated joint sponsored and caching content under the noncooperative game. The interactions among ISP, CP, and end-users are modeled as a three-stage Stackelberg game. In [10], the authors studied the sponsored data as the noncooperative game among ISPs. The authors derived the best response of the CP and the ISP and analyzed their implications for the sponsoring strategy. The authors in

[11, 12] analyzed content sponsoring data from an economic point of view. They examined the implications of sponsored data on the CPs and the end-users and identified how the sponsored data influence the CP inequality. In [13], the authors studied the sponsorship competition among CPs in the Internet content market and demonstrated that the competitions improve the welfare of the ISP and the CP. The authors in [3] considered a sponsored data market with one ISP, one CP, and a set of end-users. They have modeled the interactions among three entities as a two-stage Stackelberg game, where the ISP and the CP act as the leaders determining the pricing and sponsoring strategies, respectively, in the first stage and the end-users act as the followers deciding on their data demand in the second stage. In [14], the authors analyzed the interaction among ISP and CPs and proposed a pricing mechanism for sponsored content that is truthful in CP's valuation.

The sponsored content plan has been extensively investigated during the past few years; most papers focus on a simple model with a single ISP and a single CP interacting in a game-theoretic setting; few works study the completion between multiple CPs and multiple ISPs with content sponsoring plans. However, to the best of our knowledge, none of the current works includes the time constraint that makes sponsoring content more dynamic. The present paper moves toward this less explored direction. The main objective of this paper is to study a sponsored content market consists of multiple CPs, multiple ISPs, and end-users, with the time constraint using game theory.

Game theory has been used to solve many problems in communication networks [15–23]. It has been used to propose new pricing strategies for Internet services [24, 25]. Many other issues relating to wireless networks have been modeled and analyzed using game theory, such as resource allocation [26, 27], power control [28, 29], network routing [30], network caching [31], and security [32].

The contributions of this paper are as follows:

(i) We present new features in the mathematical modeling that include sponsoring content, CPs revenues, and ISPs revenues with the time constraint.

(ii) We model the interplay among ISPs as a function of two market parameters network access prices and quality of service; each ISP wants to maximize its utility. We formulate a competitive problem between ISP as a noncooperative game.

(iii) We model the interplay between CPs as a function of three market parameters content access price, the credibility of content, and the number of sponsored content. The number of sponsored content is modeled as a function of time. We formulate a competitive problem between CPs as a noncooperative game.

(iv) We analytically prove the existence and uniqueness of the Nash equilibrium in the noncooperative game between ISPs and between CPs, which means that there exists a stable state where all providers do not have an incentive to change their strategies. The best response algorithm is used to find the Nash equilibrium point.

(v) Numerical analysis shows the effect of sponsoring content on providers' policies.

This paper is organized as follows. Section 2 discusses the system model with temporality constraint. We prove the existence and uniqueness of a Nash equilibrium point in Section 3. Then, we present a numerical investigation in Section 4, and we conclude this paper in Section 5.

2. Problem Modeling

In our setting, we consider a telecommunication network with three types of actors: ISPs, CPs, and end-users. The ISP provides the network infrastructure to the end-users. The CPs provide N content for end-users and sponsor a fraction of content on behalf of the end-users to lower the network access price. The ISP_k sets two decision parameter network access price p_{s_k} and quality of service (QoS) q_{s_k}. Let p_{c_f} and c_f, respectively, be the content access price and the credibility of content decided by CP_f. End-users behavior is a function of CP and ISP policies (see (1)).

2.1. Demand Model. D_{fk} is the demand of end-users for the content provided by CP_f and transferred by ISP_k which is a function of content access price p_{c_f}, credibility of content c_f, network access price p_{s_k}, and QoS q_{s_k} (see [33, 34]). This demand function is also a function of prices $\mathbf{p}_{c_{-f}}$, credibilities $\mathbf{c}_{-f}$, prices $\mathbf{p}_{s_{-k}}$, and QoSs $\mathbf{q}_{s_{-k}}$ set by the opponents. The demand D_{fk} is decreasing with respect to p_{c_f} and p_{s_k} and increasing with respect to p_{c_g}, $g \neq f$ and p_{s_j}, $j \neq k$, whereas it is increasing with respect to c_f and q_{s_k} and decreasing with respect to c_g, $g \neq f$ and q_{s_j}, $j \neq k$. Then, the demand functions D_{ij} can be written as follows:

$$D_{fk} = d_{fk} - \sigma_f^f p_{c_f} + \varsigma_f^f c_f - \tau_k^k p_{s_k} + \varrho_k^k q_{s_k} + \sum_{g=1, f\neq g}^{F} \left(\sigma_f^g p_{c_g} - \varsigma_f^g c_g\right) + \sum_{j=1, k\neq j}^{K} \left(\tau_k^j p_{s_k} - \varrho_k^j q_{s_k}\right). \tag{1}$$

The parameter d_{fk} is the potential demand of end-users. σ_f^g and ς_f^g are two positive constants representing, respectively, the responsiveness of demand D_{fk} to content access price and credibility of CP_g. Moreover, τ_g^j and ϱ_g^j are two positive constants representing, respectively, the responsiveness of demand D_{fk} to network access price and QoS of ISP_j.

Assumption 1. The sensitivity σ verifies the following:

$$\sigma_f^f \geq \sum_{g=1,g\neq f}^{F} \rho_f^g, \quad \forall f = 1,\ldots,F. \tag{2}$$

The sensitivity ς verifies the following:

$$\varsigma_f^f \geq \sum_{g=1,g\neq f}^{F} \varsigma_f^g, \quad \forall f = 1,\ldots,F. \tag{3}$$

The sensitivity τ verifies the following:

$$\tau_k^k \geq \sum_{j=1,j\neq k}^{K} \tau_k^j, \quad \forall k = 1,\ldots,K. \tag{4}$$

Assumption 1 means that the effect of provider policies on its demand is greater than the effect of the policies of its opponent on its demand function [24, 35]. Assumption 1 will be used to prove the uniqueness of the Nash equilibrium.

2.2. Utility Model of the CP. The utility of CP_f can be modeled as follows:

$$\begin{aligned} U_{\mathrm{CP}_f} &= \sum_{k=1}^{K} p_{c_f} N_f \left(1+\chi_f S_{fk}\right) D_{fk} - \sum_{k=1}^{K} p_{u_f} S_{fk} N_f \left(1+\chi_f S_{fk}\right) D_{fk} \\ &\quad - \sum_{k=1}^{K} p_{t_k} c_f N_f \left(1+\chi_f S_{fk}\right) D_{fk} - \theta_f c_f, \end{aligned} \tag{5}$$

where S_{fk} is the fraction of content sponsored by CP_f for each ISP_k subscriber. Let $S_{fk} = 1$ if full sponsoring is decided and $S_{fk} = 0$ if CP_f decides not to sponsor any content. Recall that sponsoring could be an incentive to consume more CP_f content. $(1+\chi_f S_{fk})D_{fk}$ is the new demand for contents provided by CP_f and distributed by ISP_k, which is a function of the proportion of sponsored content S_{fk}. The quantity $\chi_f S_{fk}$ reflects the change in the demand for the contents of the CP. χ_f is a nonnegative constant. θ_i is the cost to produce a unit of the credibility of content c_i. p_{t_k} is the transmission price of ISP_k. p_{u_f} is the sponsoring price of ISP_f. The first term $\sum_{k=1}^{K} p_{c_f} N_f (1+\chi_f S_{fk}) D_{fk}$ is the revenue of CP_f. The second term $\sum_{k=1}^{K} p_{u_f} S_{fk} N_f (1+\chi_f S_{fk}) D_{fk}$ denotes the cost due to sponsorship. The third term $\sum_{k=1}^{K} p_{t_k} c_f N_f (1+\chi_f S_{fk}) D_{fk}$ is transmission fee results when the CP_f forwards to the end-users the demand with credibility of content c_f. The fourth term $\theta_f c_f$ is the cost to produce the credibility of content c_f.

Credibility of content c_f of CP_f is a linear function of the quality of service (QoS) q_{ss_f} and the quality content (QoC) q_{c_f}, which is written as follows [16, 34, 36, 37]:

$$c_f = \lambda q_{ss_f} + \mu q_{c_f}, \tag{6}$$

where λ and μ are nonnegative constants. The QoS is defined as the expected delay (see [17, 34]). The QoC can be specified for a specific domain of content (e.g., video streaming).

Then, the utility of CP_f is expressed as follows:

$$\begin{aligned} U_{\mathrm{CP}_f} &= \sum_{k=1}^{K} p_{c_f} N_f \left(1+\chi_f S_{fk}\right) D_{fk} - \sum_{k=1}^{K} p_{u_f} S_{fk} N_f \left(1+\chi_f S_{fk}\right) D_{fk} \\ &\quad - \sum_{k=1}^{K} p_{t_k} N_f \left(1+\chi_f S_{fk}\right) D_{fk} \left(\lambda q_{ss_f} + \mu q_{c_f}\right) \\ &\quad - \theta_f \left(\lambda q_{ss_f} + \mu q_{c_f}\right). \end{aligned} \tag{7}$$

2.3. Utility Model of the ISP. The utility function of ISP_k is the difference between the revenue and the fee:

$$\begin{aligned} U_{\mathrm{ISP}_k} &= \sum_{f=1}^{F} p_{s_k} \left(1-S_{fk}\right) N_f D_{fk} + \sum_{f=1}^{F} p_{u_k} S_{fk} N_f D_{fk} \\ &\quad + \sum_{f=1}^{F} p_{t_k} c_f N_f D_{fk} - v_k B_k. \end{aligned} \tag{8}$$

The first term $\sum_{f=1}^{F} p_{s_k} (1-S_{fk}) N_f D_{fk}$ is the revenue of network access. The second term $\sum_{f=1}^{F} p_{u_k} S_{fk} N_f D_{fk}$ is the revenue of sponsorship. The third term $\sum_{f=1}^{F} p_{t_k} c_f N_f D_{fk}$ is the revenue of ISP_f by forwarding the amount of content requests to the end-users. The fourth term $v_k B_k$ is the investment of ISP_k, where v_k is a cost per unit of requested bandwidth and B_k is the backhaul bandwidth. The QoS q_{s_f} is defined as the expected delay computed by the Kleinrock function (see [38, 39]):

$$q_{s_k} = \frac{1}{\sqrt{\mathrm{Delay}_k}} = \sqrt{B_k - \sum_{f=1}^{F} D_{fk}}, \tag{9}$$

this means that

$$B_f = \sum_{f=1}^{F} D_{fk} + q_{s_f}^2. \tag{10}$$

Then, the utility of ISP_f is given as follows:

$$\begin{aligned} U_{\mathrm{ISP}_k} &= \sum_{f=1}^{F} p_{s_k} \left(1-S_{fk}\right) N_f D_{fk} + \sum_{f=1}^{F} p_{u_k} S_{fk} N_f D_{fk} \\ &\quad + \sum_{f=1}^{F} p_{t_k} c_f N_f D_{fk} - v_k \left(\sum_{f=1}^{F} D_{fk} + q_{s_f}^2\right). \end{aligned} \tag{11}$$

2.4. Adding Temporality to the Model. We study in this section the impact of time on the number of sponsored content. We model the proportion of sponsored content by attaching it with the time parameter. This proportion is expressed in the following form:

$$S_{fk} = \left(1 - e^{-w_{fk}t}\right), \tag{12}$$

where w_{fk} represents the speed at which CP_f sponsors content for each ISP_k subscriber. We notice that when $t = 0$, $S_{fk} = 0$ and when $t = \infty$, $S_{fk} = 1$.

The temporal analysis of the number of sponsored content can be performed in the networks; we consider ξ as a discount factor, so that a monetary unit in t years is worth $e^{-\xi t}$ monetary units of today. CP_f with a profit U_{CP_f} at time t can predict this profit over a period ranging from $[0, T]$ as the average of the discounted revenue in this period as follows:

$$\begin{aligned}
\overline{U}_{\mathrm{CP}_f} &= \frac{1}{\int_0^T e^{-\xi t}} \int_0^T U_{\mathrm{CP}_f}(t) e^{-\xi t} \\
&= \frac{\xi}{1 - e^{-\xi T}} \int_0^T U_{\mathrm{CP}_f}(t) e^{-\xi t} \\
&= \sum_{k=1}^{K} p_{c_f} N_f D_{fk} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-(\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) - \sum_{k=1}^{K} p_{t_k} N_f c_f D_{fk} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-(\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) - \theta_f c_f \\
&\quad - \sum_{k=1}^{K} \left[\left(1 + \frac{\xi \left(e^{-(\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) + \chi_f \left(1 + \frac{2\xi \left(e^{-(\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) - \frac{\chi_f \xi \left(e^{-(2\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(2\omega + \xi)} \right] p_{u_k} D_{fk} N_f .
\end{aligned} \tag{13}$$

Similarly, we have

$$\begin{aligned}
\overline{U}_{\mathrm{ISP}_k} &= \frac{1}{\int_0^T e^{-\xi t}} \int_0^T U_{\mathrm{ISP}_k}(t) e^{-\xi t} \\
&= \frac{\xi}{1 - e^{-\xi T}} \int_0^T U_{\mathrm{ISP}_k}(t) e^{-\xi t} \\
&= \sum_{f=1}^{F} \frac{\xi \left(1 - e^{-(\omega_{fk}+\xi)T} \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} p_{s_k} N_f D_{fk} + \sum_{k=1}^{K} \left(1 + \frac{\xi \left(e^{-(\omega_{fk}+\xi)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) p_{u_k} N_f D_{fk} + \sum_{f=1}^{F} p_{t_k} N_f c_f D_{fk} - \nu_k B_k .
\end{aligned} \tag{14}$$

3. Game Analysis

Let $G_1 = [\mathcal{F}, \{P_{c_f}, Q_{ss_f}, Q_{c_f}\}, \{\overline{U}_{CP_f}(\mathbf{p}_c, \mathbf{q}_{ss}, \mathbf{q}_c)\}]$ denote the noncooperative QoC price QoS game (NQPQG), where $\mathcal{F} = \{1, \ldots, F\}$ is the index set identifying the CPs, P_{c_f} is the content access price strategy set of CP_f, Q_{ss_f} is the QoS strategy set of CP_f, and Q_{c_f} is the QoC strategy set of CP_f. We assume that the strategy spaces P_{c_f}, Q_{ss_f}, and Q_{c_f} of each CP_f are compact and convex sets with maximum and minimum constraints; for any given CP_f, we consider as strategy spaces the closed intervals $P_{c_f} = [\underline{p}_{c_f}, \overline{p}_{c_f}]$, $Q_{ss_f} = [\underline{q}_{ss_f}, \overline{q}_{ss_f}]$, and $Q_{c_f} = [\underline{q}_{c_f}, \overline{q}_{c_f}]$. Let the price vector $\mathbf{p}_c = (p_{c_1}, \ldots, p_{c_F})^T \in P_c^F = P_{c_1} \times P_{c_2} \times \cdots \times P_{c_F}$, QoS vector $\mathbf{q}_{ss} = (q_{ss_1}, \ldots, q_{ss_F})^T \in Q_{ss}^F = Q_{ss_1} \times Q_{ss_2} \times \cdots \times Q_{ss_F}$, and QoC vector $\mathbf{q}_c = (q_{c_1}, \ldots, q_{c_F})^T \in Q_c^F = Q_{c_1} \times Q_{c_2} \times \cdots \times Q_{c_F}$.

Let $G_2 = [\mathcal{K}, \{P_{s_k}, Q_{s_k}\}, \{\overline{U}_{ISP_k}(\mathbf{p}_s, \mathbf{q}_s)\}]$ denote the noncooperative price QoS game (NPQG), where $\mathcal{K} = \{1, \ldots, K\}$ is the set of the ISPs, P_{s_k} is the price strategy set of ISP_k, and Q_{s_k} is the QoS strategy set of ISP_k. We assume that the strategy spaces P_{s_k} and Q_{s_k} of each ISP_k are compact and convex sets with maximum and minimum constraints; for any given ISP_k, we consider as strategy spaces the closed intervals $P_{s_k} = [\underline{p}_{s_k}, \overline{p}_{s_k}]$ and $Q_{s_k} = [\underline{q}_{s_k}, \overline{q}_{s_k}]$. Let the price vector $\mathbf{p}_s = (p_{s_1}, \ldots, p_{s_K})^T \in P_s^K = P_{s_1} \times P_{s_2} \times \cdots \times P_{s_K}$ and QoS vector $\mathbf{q}_s = (q_{s_1}, \ldots, q_{s_K})^T \in Q_s^K = Q_{s_1} \times Q_{s_2} \times \cdots \times Q_{s_K}$.

3.1. Price P_s Game. A NPQG in network access price is defined for fixed $\mathbf{q}_s \in Q_s$ as $G_2(\mathbf{q}_s) = [\mathscr{K}, \{P_{s_k}\}, \{\overline{U}_{\mathrm{ISP}_k}(., \mathbf{q}_s)\}]$.

Definition 1. A price vector $\mathbf{p}_s^* = (p_{s_1}^*, \ldots, p_{s_K}^*)$ is a Nash equilibrium of the NPQPG $G_2(\mathbf{q}_s)$ if

$$\forall \left(k, p_{s_k}\right) \in \left(\mathscr{K}, P_{s_k}\right), \overline{U}_{\mathrm{ISP}_k}\left(p_{s_k}^*, \mathbf{p}_{s_{-k}}^*, \mathbf{q}_s\right) \geq \overline{U}_{\mathrm{ISP}_k}\left(p_{s_k}, \mathbf{p}_{s_{-k}}^*, \mathbf{q}_s\right). \tag{15}$$

Theorem 1. *For each $\mathbf{q}_s \in Q_s$, the game $[\mathscr{K}, \{P_{s_k}\}, \{\overline{U}_{ISP_k}(., \mathbf{q}_s)\}]$ admits a unique Nash equilibrium.*

Appendix A gives a proof of the above theorem.

3.2. QoS Q_s Game. A NPQG in QoS is defined for fixed $\mathbf{p}_s \in P_s$ as $G_2(\mathbf{p}_s) = [\mathscr{K}, \{Q_{s_k}\}, \{\overline{U}_{\mathrm{ISP}_k}(\mathbf{p}_s, .)\}]$.

Definition 2. A QoS vector $\mathbf{q}_s^* = (q_{s_1}^*, \ldots, q_{s_K}^*)$ is a Nash equilibrium of the NPQG $G_2(\mathbf{p}_s)$ if

$$\forall \left(k, q_{s_k}\right) \in \left(\mathscr{K}, Q_{s_k}\right), \overline{U}_{\mathrm{ISP}_k}\left(\mathbf{p}_s, q_{s_k}^*, \mathbf{q}_{s_{-k}}^*\right) \geq \overline{U}_{\mathrm{ISP}_k}\left(\mathbf{p}_s, q_{s_k}, \mathbf{q}_{s_{-k}}^*\right). \tag{16}$$

Theorem 2. *For each $\mathbf{p}_s \in P_s$, the game $[\mathscr{K}, \{Q_{s_k}\}, \{\overline{U}_{ISP_k}(\mathbf{p}_s, .)\}]$ admits a unique Nash equilibrium.*

Appendix B gives a proof of the above theorem.

3.3. Price P_c Game. A NQPQG in price p_c is defined for fixed $\mathbf{q}_{ss} \in Q_{ss}$ and $\mathbf{q}_c \in Q_c$ as $G_1(\mathbf{q}_{ss}, \mathbf{q}_c) = [\mathscr{F}, \{P_{c_f}\}, \{\overline{U}_{CP_f}(., \mathbf{q}_{ss}, \mathbf{q}_c)\}]$.

Definition 3. A price vector $\mathbf{p}_c^* = (p_{c_1}^*, \ldots, p_{c_F}^*)$ is a Nash equilibrium of the NQPQG $G_1(\mathbf{q}_{ss}, \mathbf{q}_c)$ if

$$\forall \left(f, p_{c_f}\right) \in \left(\mathscr{F}, P_{c_f}\right), \overline{U}_{\mathrm{CP}_f}\left(p_{c_f}^*, \mathbf{p}_{c_{-f}}^*, \mathbf{q}_s, \mathbf{q}_c\right) \geq \overline{U}_{\mathrm{CP}_f}\left(p_{c_f}, \mathbf{p}_{c_{-f}}^*, \mathbf{q}_s, \mathbf{q}_c\right). \tag{17}$$

Theorem 3. *For each $\mathbf{q}_{ss} \in Q_{ss}$ and $\mathbf{q}_c \in Q_c$, the game $[\mathscr{F}, \{P_{c_f}\}, \{\overline{U}_{CP_f}(., \mathbf{q}_{ss}, \mathbf{q}_c)\}]$ admits a unique Nash equilibrium.*

Appendix C gives a proof of the above theorem.

3.4. QoC Q_c Game. A NQPQG in QoC is defined for a fixed $\mathbf{p}_c \in P_c$ and $\mathbf{q}_{ss} \in Q_{ss}$ as $G_1(\mathbf{p}_c, \mathbf{q}_{ss}) = [\mathscr{F}, \{Q_{c_f}\}, \{\overline{U}_{\mathrm{CP}_f}(\mathbf{p}_c, \mathbf{q}_{ss}, .)\}]$.

Definition 4. A QoC vector $\mathbf{q}_c^* = (q_{c_1}^*, \ldots, q_{c_F}^*)$ is a Nash equilibrium of the NQPQG $G_1(\mathbf{p}_c, \mathbf{q}_{ss})$ if

$$\forall \left(f, q_{c_f}\right) \in \left(\mathscr{F}, Q_{c_f}\right), \overline{U}_{\mathrm{CP}_f}\left(\mathbf{p}_c, \mathbf{q}_{ss}, q_{c_f}^*, \mathbf{q}_{c_{-f}}^*\right) \geq \overline{U}_{\mathrm{CP}_f}\left(\mathbf{p}_c, \mathbf{q}_{ss}, q_{c_f}, \mathbf{q}_{c_{-f}}^*\right). \tag{18}$$

Theorem 4. *For each $\mathbf{p}_c \in P_c$ and $\mathbf{q}_{ss} \in Q_{ss}$, the game $[\mathscr{F}, \{Q_{c_f}\}, \{\overline{U}_{CP_f}(\mathbf{p}_c, \mathbf{q}_{ss}, .)\}]$ admits a unique Nash equilibrium.*

Appendix D gives a proof of the above theorem.

3.5. QoS Q_{ss} Game. A NQPQG in QoS is defined for a fixed $\mathbf{p}_c \in P_c$ and $\mathbf{q}_c \in Q_c$ as $G_1(\mathbf{p}_c, \mathbf{q}_c) = [\mathscr{F}, \{Q_{ss_f}\}, \{\overline{U}_{CP_f}(\mathbf{p}_c, ., \mathbf{q}_c)\}]$.

Definition 5. A QoS vector $\mathbf{q}_{ss}^* = (q_{ss_1}^*, \ldots, q_{ss_F}^*)$ is a Nash equilibrium of the NQPQG $G_1(\mathbf{p}_c, \mathbf{q}_c)$ if

$$\forall \left(f, q_{ss_f}\right) \in \left(\mathscr{F}, Q_{ss_f}\right), \overline{U}_{\mathrm{CP}_f}\left(\mathbf{p}_c, q_{ss_f}^*, \mathbf{q}_{ss_{-f}}^*, \mathbf{q}_c\right) \geq \overline{U}_{\mathrm{CP}_f}\left(\mathbf{p}_c, q_{ss_f}, \mathbf{q}_{ss_{-f}}^*, \mathbf{q}_c\right). \tag{19}$$

Theorem 5. *For each $\mathbf{p}_c \in P_c$ and $\mathbf{q}_c \in Q_c$, the game $[\mathscr{F}, \{Q_{ss_f}\}, \{\overline{U}_{CP_f}(\mathbf{p}_c, ., \mathbf{q}_c)\}]$ admits a unique Nash equilibrium.*

Appendix E gives a proof of the above theorem.

3.6. Learning Nash Equilibrium. In this section, based on our previous analysis, we introduce two distributed and iterative learning processes that convergence toward the Nash equilibrium point. In this algorithm, each provider observes the policy taken by its competitors in previous rounds and inputs them in its decision process to update its policy. Therefore, the best response and Nash seeking algorithms will converge to the unique equilibrium point.

The best response (BR) algorithm is known to reach equilibria for S-modular games, by exploiting the monotonicity of the best response functions. Each player fixes its desirable strategies to maximize its profit. Then, each player can observe the policy taken by its competitors in previous rounds and input them in its decision process to update its policy. Then, it becomes natural to accept the Nash equilibrium as the attractive point of the game. Yet, the best response algorithm requires perfect rationality and complete

(1) Initialize vectors $\mathbf{x}(0) = [x_1(0), \ldots, x_F(0)]$ randomly;
(2) For each E_f, $f \in \mathscr{L}$ at time instant t compute:
  (i) $x_f(t+1) = \text{argmax}_{x_f \in X_f}(U_{E_f}(\mathbf{x}(t)))$.
(3) If $\forall f \in \mathscr{L}$, $|x_f(t+1) - x_f(t)| < \varepsilon$, then STOP.
(4) Else, $t \longleftarrow t+1$ and go to step (2).

Algorithm 1: Best response algorithm.

(1) Data
  (i) $\varphi_f \in [0; 2\pi]$: perturbation phase;
  (ii) $b_f > 0$: perturbation amplitude;
  (iii) Ω_f: perturbation phase;
  (iv) z_f: the growth rate;
(2) *Result*. Equilibrium $\mathbf{x}_f(t)$
(3) Initialize vectors $\mathbf{x}^*(0) = [x_1^*(0), \ldots, x_F^*(0)]$ and $\tau^*(0) = [\tau_1^*(0), \ldots, \tau_F^*(0)]$ randomly;
(4) *Learning Pattern*. For each iteration t:
(5) Observes the payoff $U_{E_f}(\mathbf{x}(t))$ and estimates $\tau^*(t+1)$ using
  (v) $\tau_f^*(t+1) = \tau_f^*(t) + t^* z_f b_f \sin(\Omega_f t^* + \varphi_f) U_{E_f}(\mathbf{x}(t))$;
(6) Update $\mathbf{x}_f(t)$ using the following rules
  (vi) $x_f^*(t+1) = \tau_f^*(t+1) + b_f \sin(\Omega_f t^* + \varphi_f)$;

Algorithm 2: Nash seeking algorithm.

information, which is not practical for real-world applications and may increase the signaling load as well. Therefore, we propose an adaptive distributed learning framework to discover equilibria for the activation game based on the "Nash seeking algorithm" (NSA) with stochastic state-dependent payoffs for continuous actions.

The equilibrium-learning framework is an iterative process. At each iteration t, the player I chooses its policy and obtains from the environment the realization of its payoff. The improvement of the strategy is based on the current observation of the realized payoff and previously chosen strategies. Hence, we say players learn to play an equilibrium if after a given number of iterations; the strategy profile converges to an equilibrium strategy.

The proposed learning framework has the following parameters: φ_f is the perturbation phase, z_f is the growth rate, b_f is the perturbation amplitude, and Ω_f is the perturbation frequency. This procedure is repeated for the window T.

Algorithms 1 and 2 summarize the best response learning and Nash seeking algorithm steps that each player has to perform to discover its Nash equilibrium strategy.

such as

(i) E denotes a CP or ISP
(ii) $\mathscr{L}$ refers to $\mathscr{F}$ or $\mathscr{K}$
(iii) x refers to the vector price p_c, vector price p_s, vector q_s, vector q_c, or vector q_{ss}
(iv) X_f refers to the policy profile price, QoS, or QoC

4. Numerical Investigations

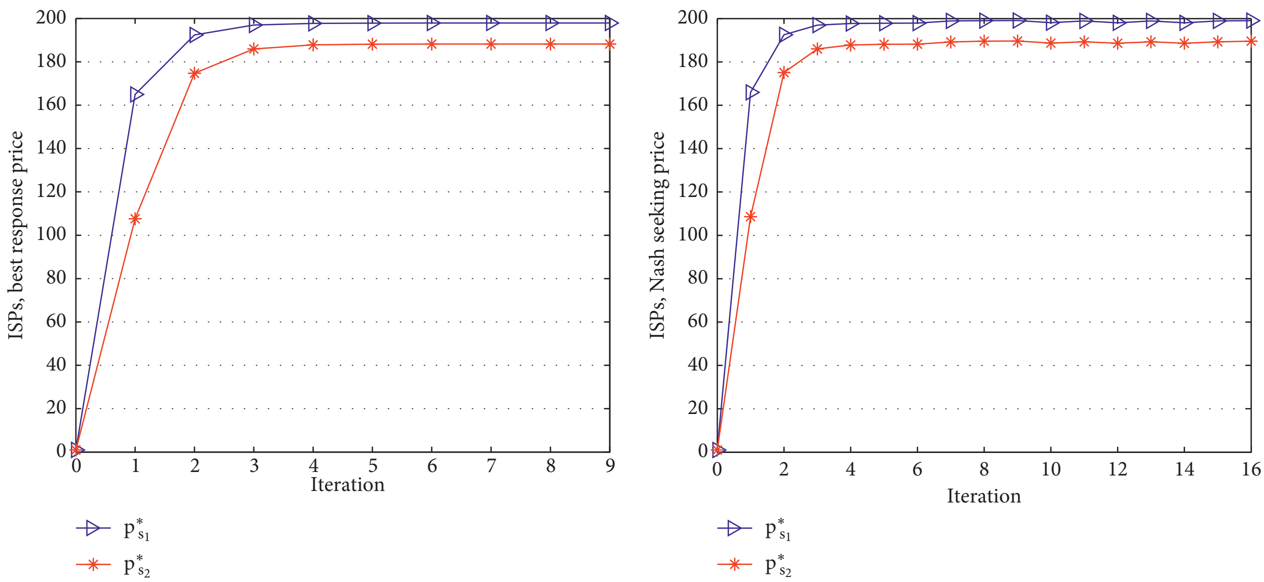
In this section, we study the telecommunication network numerically as a noncooperative game while considering the expressions of the utility functions and using the best response algorithm. We consider a network scenario that includes two ISPs and two CPs.

Figures 1–5 show the convergence toward Nash equilibrium price, Nash equilibrium QoS, and Nash equilibrium QoC of all providers. Figures 1–5 demonstrate the existence and uniqueness of a Nash equilibrium point at which no providers can profitably deviate given the strategies of another opponent. So, our model ensures the existence of an equilibrium for keeping the economy stable and achieving economic growth.

Table 1 gives a comparison between the two algorithms proposed for the learning of the numerical results; we notice that the algorithm of best response gives the same results as the Nash seeking algorithm but in fewer iterations and in a very small time compared with a Nash seeking algorithm.

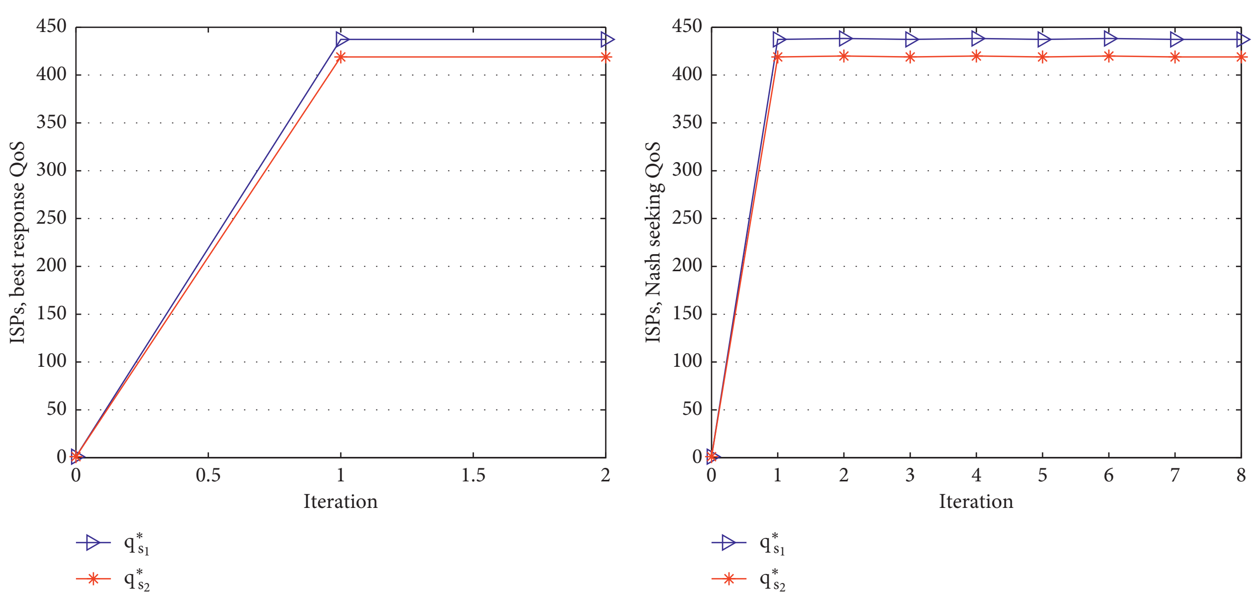
The effect of the parameter χ on the QoS and the QoC is shown in Figures 6 and 7. The QoS and the QoC of the proposed model are growing as χ increases. When χ increases, the demand of end-users increases, and then, the revenue of CPs increases. As a result, the CPs increase their QoS and QoC to attract more end-users.

Figures 8–10 represent the impact of sponsoring price p_u on the content access price, the QoS, and the QoC of the two CPs. As the sponsoring price increases, the content access price p_c increases, the QoC decreases, and the QoS decreases. When the sponsoring price is low, the CP invests



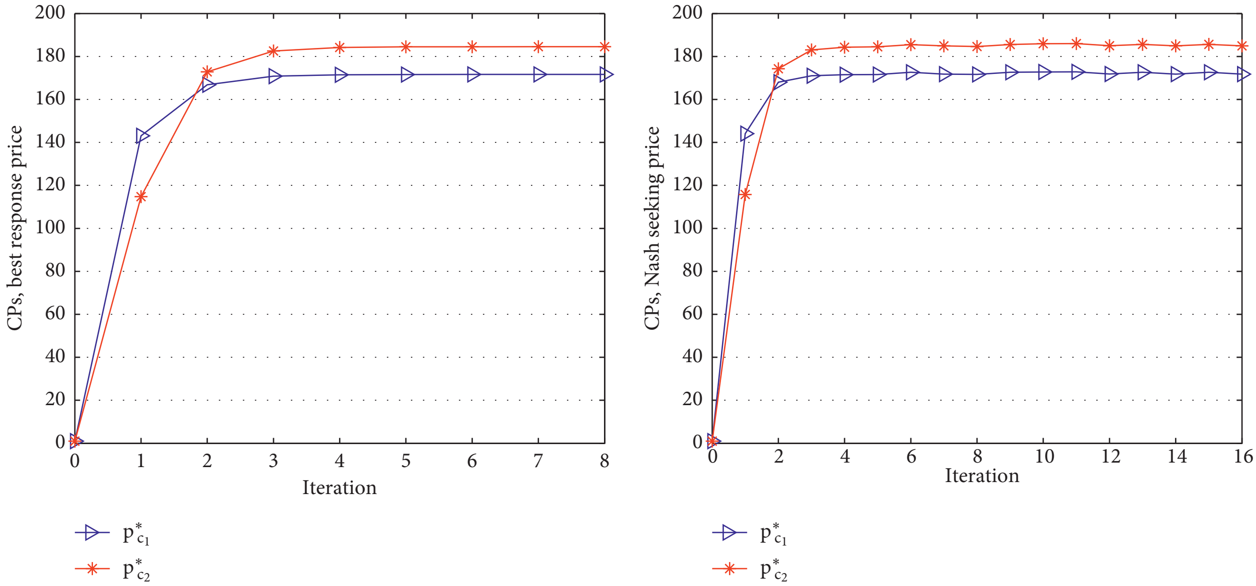
(a) (b)

Figure 1: Nash equilibrium price under the BR and NSA algorithms: (a) best response algorithm; (b) Nash seeking algorithm.



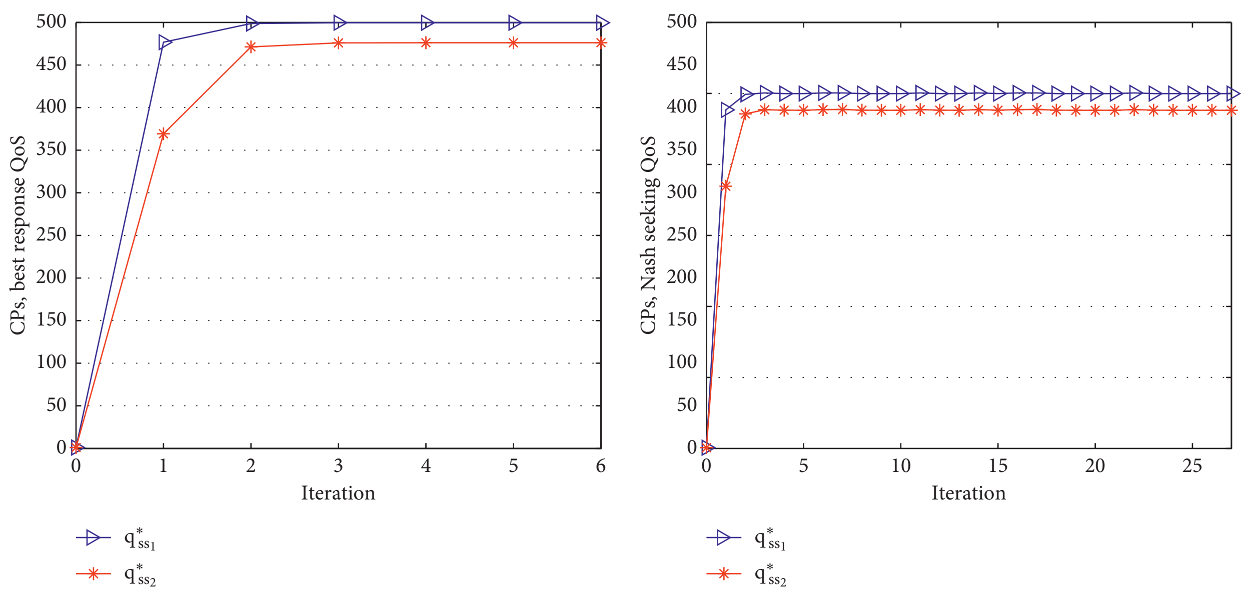
(a) (b)

Figure 2: Nash equilibrium QoS under the BR and NSA algorithms: (a) best response algorithm; (b) Nash seeking algorithm.



(a) (b)

Figure 3: Nash equilibrium price under the BR and NSA algorithms: (a) best response algorithm; (b) Nash seeking algorithm.



(a) (b)

Figure 4: Nash equilibrium QoS under the BR and NSA algorithms: (a) best response algorithm; (b) Nash seeking algorithm.

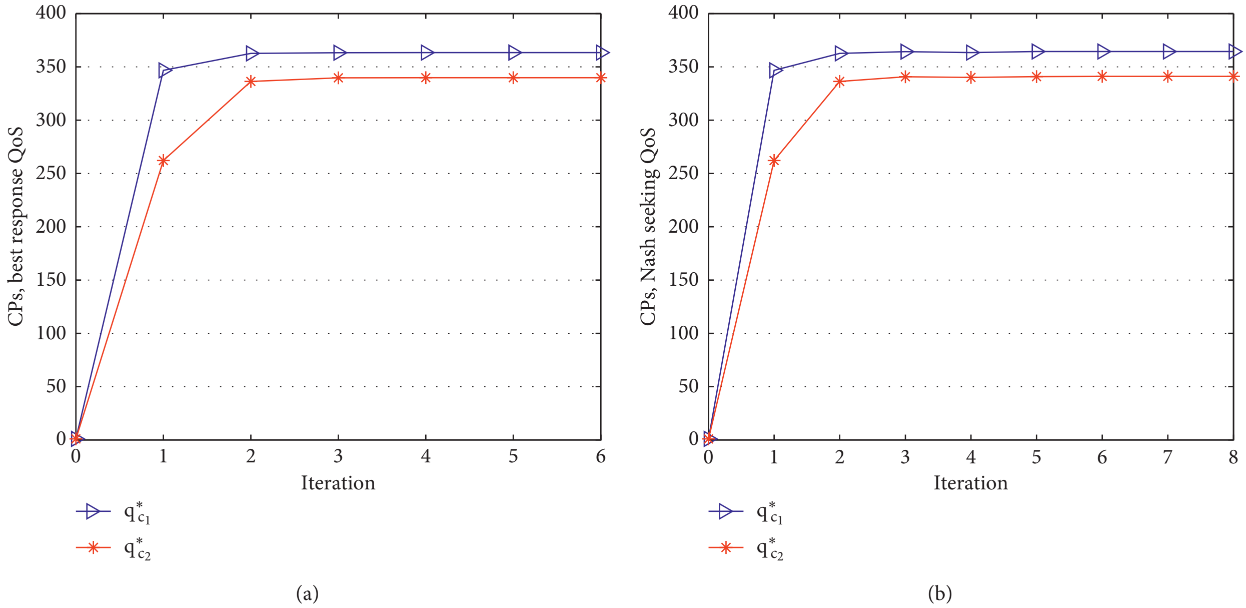


Figure 5: Nash equilibrium QoC under the BR and NSA algorithms: (a) best response algorithm; (b) Nash seeking algorithm.

Table 1: Best response algorithm versus Nash seeking algorithm.

	Time (s)		Iterations	
	Best response algorithm	Nash seeking algorithm	Best response algorithm	Nash seeking algorithm
ps *	28.7105	55.6239	2	16
pc *	2.8307	5.1134	8	16
Qc *	26.5229	32.7347	6	8
qss *	28.4598	122.8715	6	27
qs *	1.0003	4.4373	2	8

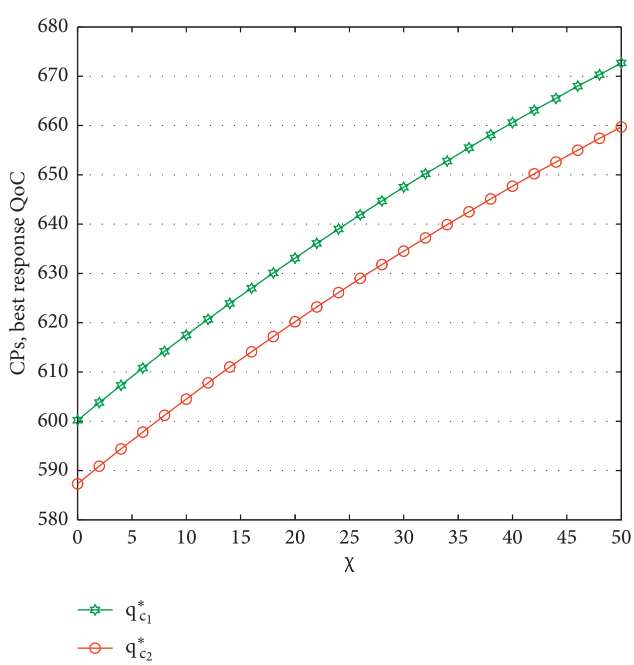


Figure 6: Nash equilibrium QoC q_c evolution with respect to parameter χ.

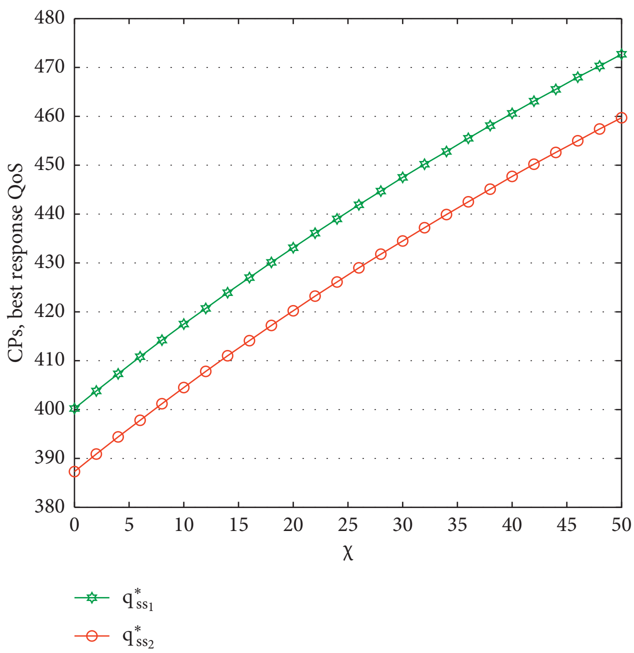


Figure 7: Nash equilibrium QoS q_{ss} evolution with respect to parameter χ.

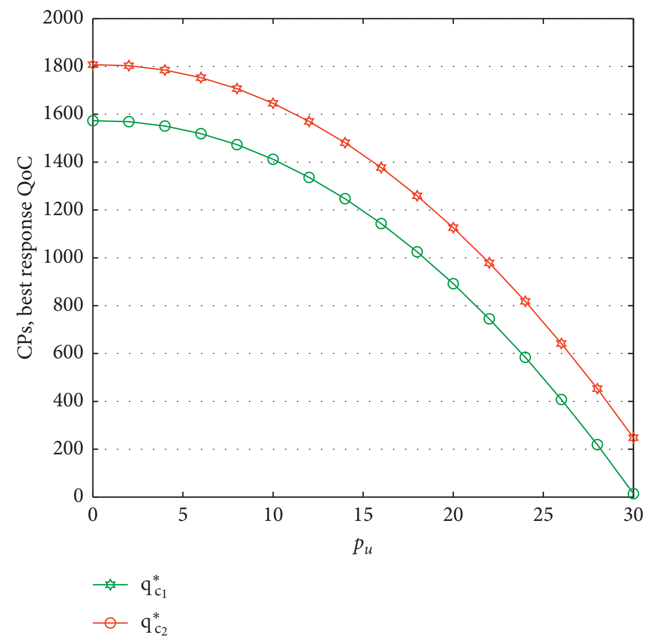


Figure 9: Nash equilibrium QoC q_c evolution as a function of sponsoring price p_u.

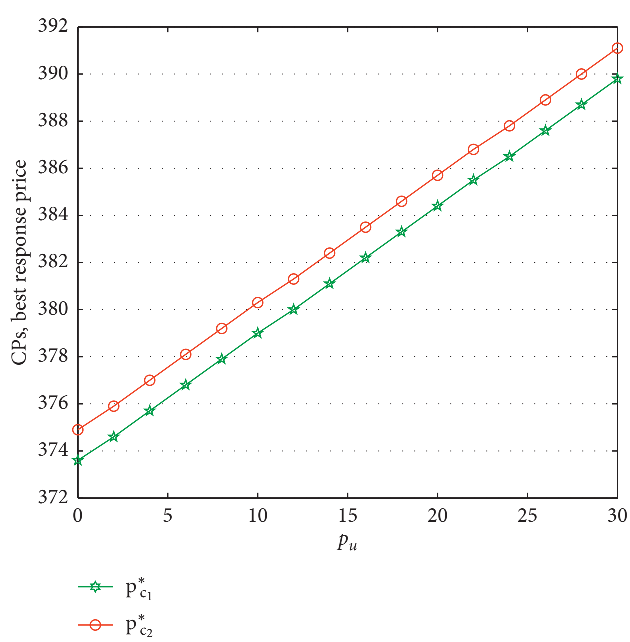


Figure 8: Nash equilibrium price p_c evolution as a function of sponsoring price p_u.

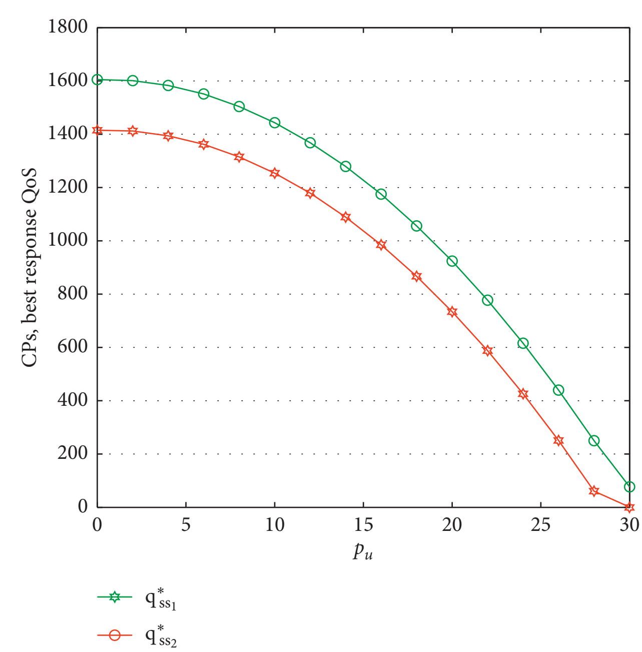


Figure 10: Nash equilibrium QoS q_{ss} evolution as a function of sponsoring price p_u.

more to offer better QoS, better QoC, and low content access price to induce increased demand from end-users. On the other hand, when the sponsoring price is high, the CP increases their content access price and decreases their QoS and QoC to compensate the increase in the sponsoring price.

The impact of sponsoring price p_u on the network access price and QoS of the two ISPs is illustrated in Figures 11 and 12. Figures 11 and 12 show that the network access price decreases and the QoS increases when sponsoring price increases. The reason is that as sponsoring price increases, the revenue of sponsoring increases, which leads to a rise in the income of the ISPs. Therefore, the ISPs decrease their network access price and invest for more bandwidth to increase their QoS to attract more end-users.

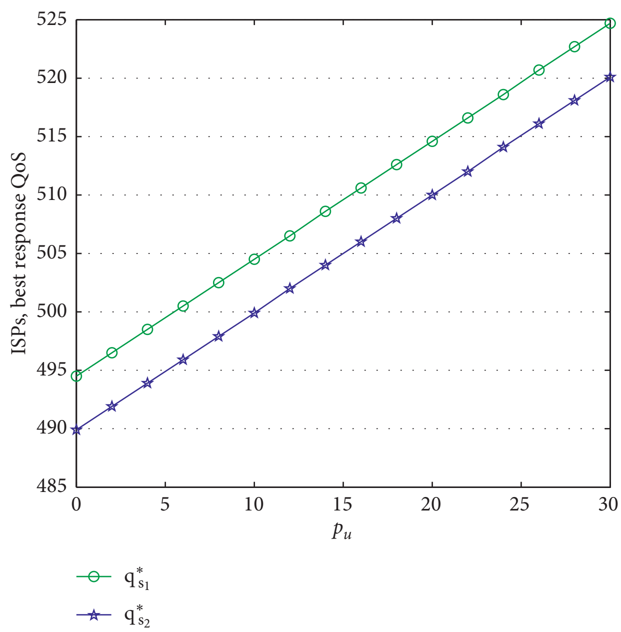


FIGURE 11: Nash equilibrium QoS q_s evolution as a function of sponsoring price p_u.

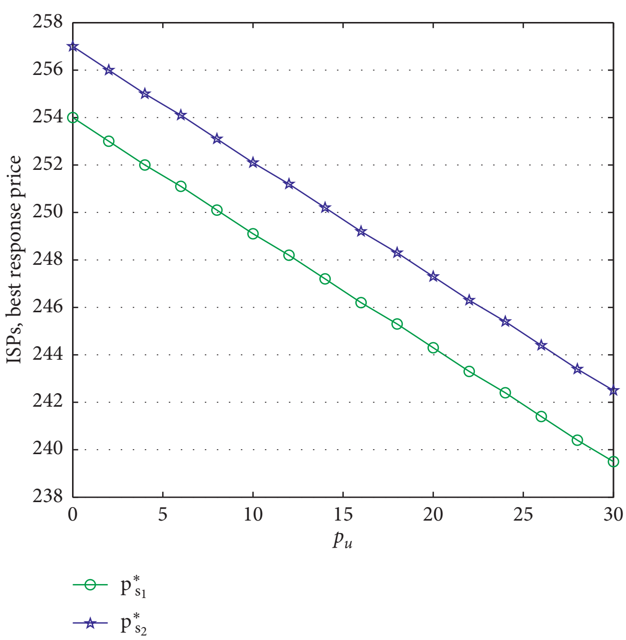


FIGURE 12: Nash equilibrium price p_s evolution as a function of sponsoring price p_u.

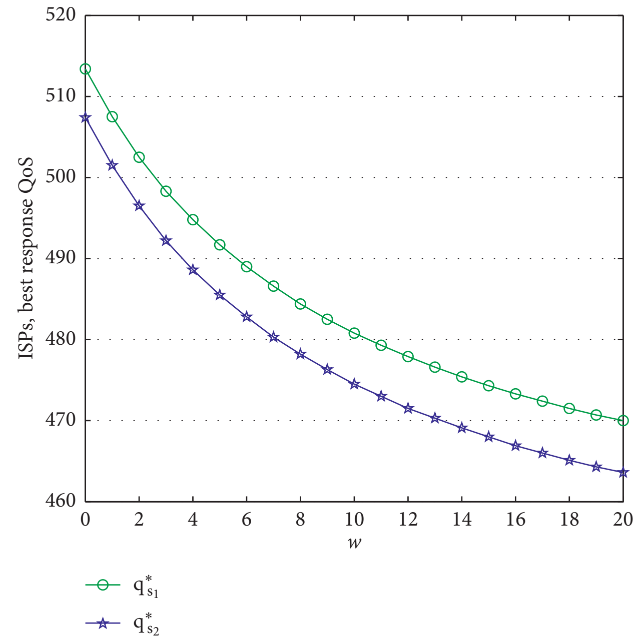


FIGURE 13: Nash equilibrium QoS q_s evolution as a function of sponsoring content speed w.

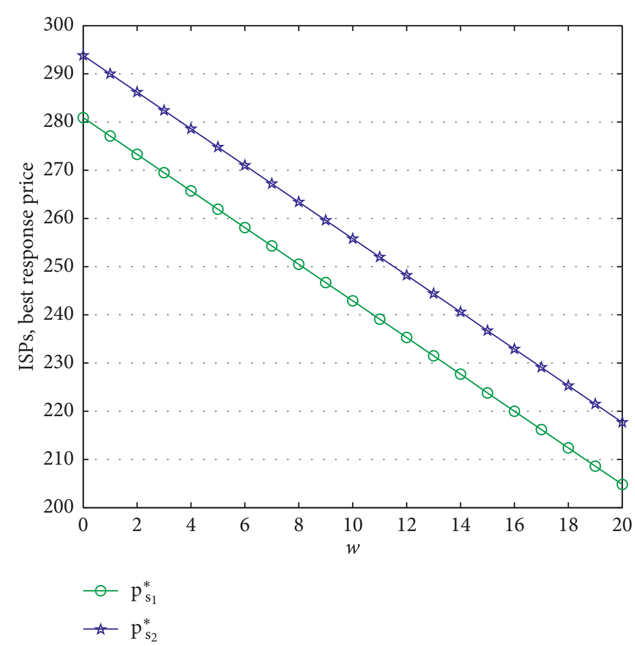


FIGURE 14: Nash equilibrium price p_s evolution as a function of sponsoring content speed w.

Figures 13 and 14 show the influence of sponsoring content speed w, respectively, on network access price and QoS equilibrium. From the two figures, we notice that when the speed of sponsoring content increases, the number of sponsored content increases and then the revenue of ISPs increases. Therefore, the ISP needs to lower the price and improve the QoS to induce increased demand from end-users.

We plot in Figures 15–17, respectively, the interplay of the speed of sponsoring content w on the content access price, the QoC, and the QoS at Nash equilibrium, for both CPs that we consider in this example. On the one hand, we note that the equilibrium content access price for both CPs is increasing with respect to the speed of sponsored content. On the other hand, we indicate that the equilibrium QoS and QoC for all CPs is decreasing with the speed of sponsored

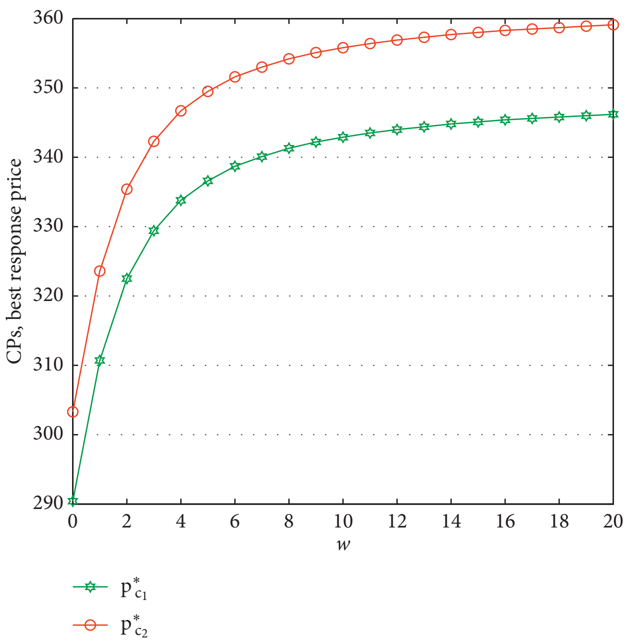


Figure 15: Nash equilibrium price p_c evolution as a function of sponsoring content speed w.

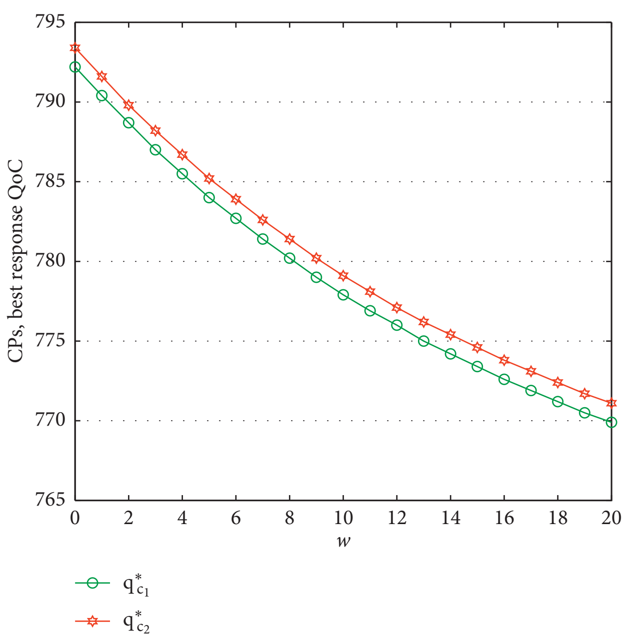


Figure 16: Nash equilibrium QoC q_c evolution as a function of sponsoring content speed w.

content. When the speed of sponsoring content is low, the CPs invest more to offer better QoS, QoC, and an attractive content access price. However, as the speed of sponsoring content increases, the CPs choose to raise their content access price and decrease their QoS and QoC to compensate the rise in the sponsoring price.

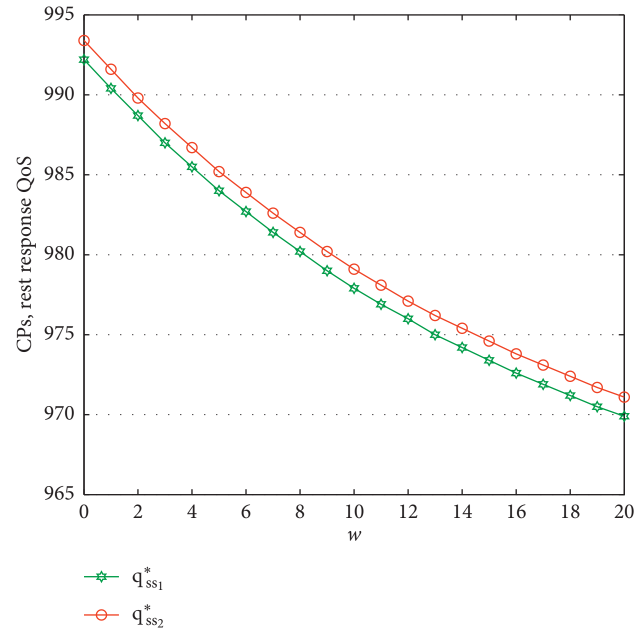


Figure 17: Nash equilibrium QoS q_{ss} evolution as a function of sponsoring content speed w.

5. Conclusion

In this paper, we study the data sponsoring problem with time constraint in the Internet market with multiple ISPs, multiple CPs, and a set of end-users. The interaction among ISPs and among CPs is investigated by using the noncooperative game. Then, we have proved the existence and uniqueness of the Nash equilibrium. This result is significant because it implies that a stable solution with suitable economic incentives in collaborative data sponsoring is feasible in the Internet paradigm. In addition, we describe a learning mechanism that allows each provider to discover accurately and rapidly its equilibrium policies. At last, we have presented a numerical investigation to validate the proposed approach, and we found that the sponsoring content has a negative effect on the strategies of CPs and positive one on the strategies of ISPs and to motivate the CPs to sponsor more content to reduce the cost of sponsorship in a long term.

Appendix

A. Proof of Theorem 1

The second derivative of the utility function is

$$\frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k}^2} = -2N_f \tau_k^k \sum_{f=1}^{F} \frac{\xi\left(1-e^{-(\omega_{fk}+\xi)T}\right)}{\left(1-e^{-\xi T}\right)(\omega+\xi)} \leq 0. \tag{A.1}$$

The second derivative of the utility function is negative, and then the utility function is thus concave, which ensures existence of a Nash equilibrium in the game $G_2(\mathbf{q}_s)$.

The authors use the following proposition that holds for a concave game [40]. If a concave game satisfies the dominance solvability condition

$$-\frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k}^2} \geq \sum_{j,j\neq k} \left| \frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k} \partial p_{s_j}} \right|, \tag{A.2}$$

then the game $G_2(\mathbf{q}_s)$ admits a unique Nash equilibrium point.

The mixed partial is written as follows:

$$\frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k} \partial p_{s_j}} = N_f \tau_k^j \sum_{f=1}^{F} \frac{\xi\left(1 - e^{-(\omega_{fk}+\xi)T}\right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)}. \tag{A.3}$$

Then,

$$\begin{aligned} &-\frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k}^2} - \sum_{j,j\neq k} \left| \frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial p_{s_k} \partial p_{s_j}} \right| \\ &= N_f \left(2\tau_k^k - \sum_{j,j\neq k} \tau_k^j \right) \sum_{f=0}^{F} \frac{\xi\left(1 - e^{-(\omega_{fk}+\xi)T}\right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \geq 0. \end{aligned} \tag{A.4}$$

Thus, the game $G_2(\mathbf{q}_s)$ admits a unique Nash equilibrium point.

B. Proof of Theorem 2

The second derivative of the utility function is

$$\frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial q_{s_k}^2} = -2v_k \leq 0. \tag{B.1}$$

The second derivative of the utility function is negative, and then the utility function is thus concave, which ensures existence of a Nash equilibrium in the game $G_2(\mathbf{p}_s)$.

In order to prove uniqueness, the authors follow [41] and define the weighted sum of user utility functions.

$$\psi(\mathbf{q}_s, \mathbf{x}) = \sum_{k=1}^{K} x_k \overline{U}_{ISP_k}\left(q_{s_k}, \mathbf{q}_{s_{-k}}\right). \tag{B.2}$$

The pseudogradient of (B.2) is given as follows:

$$\Theta(\mathbf{q}_s, \mathbf{x}) = \left[x_1 \nabla \overline{U}_{\mathrm{ISP}_1}\left(q_{s_1}, \mathbf{q}_{s_{-1}}\right), \ldots, x_k \nabla \overline{U}_{ISP_k}\left(q_{s_k}, \mathbf{q}_{s_{-k}}\right) \right]^T. \tag{B.3}$$

The Jacobian matrix J of the pseudogradient (with respect to q) is written as follows:

$$J = \begin{pmatrix} x_1 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_1}}{\partial q_{s_1}^2} & x_1 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_1}}{\partial q_{s_1} \partial q_{s_2}} & \cdots & x_1 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_1}}{\partial q_{s_1} \partial q_{s_k}} \\ x_2 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_2}}{\partial q_{s_2} \partial q_{s_1}} & x_2 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_2}}{\partial q_{s_2}^2} & \cdots & x_2 \frac{\partial^2 \overline{U}_{\mathrm{ISP}_2}}{\partial q_{s_2} \partial q_{s_k}} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ x_k \frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial q_{s_k} \partial q_{s_1}} & x_k \frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial q_{s_k} \partial q_{s_2}} & \cdots & x_k \frac{\partial^2 \overline{U}_{\mathrm{ISP}_k}}{\partial q_{s_k}^2} \end{pmatrix} = \begin{pmatrix} -2x_1 v_1 & 0 & \cdots & 0 \\ 0 & -2x_2 v_2 & \cdots & 0 \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ 0 & 0 & \cdots & -2x_k v_k \end{pmatrix}. \tag{B.4}$$

Thus, J is a diagonal matrix with negative diagonal elements. This implies that J is negative definite. Henceforth, $[J + J^T]$ is also negative definite, and according to Theorem 6 in [41], the weighted sum of the utility functions $\psi(q_s, x)$ is diagonally strictly concave. Thus, the game $G_2(\mathbf{p}_s)$ admits a unique Nash equilibrium point.

C. Proof of Theorem 3

The second derivative of the utility function $\overline{U}_{\mathrm{CP}_f}$ is

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f}^2} = -2N_f \sigma_f^f \sum_{k=1}^{K} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \leq 0. \tag{C.1}$$

Then, the second derivative of the utility function is negative, and then the utility function is thus concave, which ensures existence of a Nash equilibrium in the game $G_1(\mathbf{q}_{ss}, \mathbf{q}_c)$.

The authors use the following proposition that holds for a concave game [40]. If a concave game satisfies the dominance solvability condition

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f}^2} \geq \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f} \partial p_{c_g}} \right|, \tag{C.2}$$

then the game $G_1(\mathbf{q}_{ss}, \mathbf{q}_c)$ admits a unique Nash equilibrium point.

The mixed partial is written as follows:

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f} \partial p_{c_g}} = N_f \sigma_f^g \sum_{k=1}^{K} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \geq 0. \tag{C.3}$$

Then,

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f}^2} - \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial p_{c_f} \partial p_{c_g}} \right| = N_f \left(2\sigma_f^f - \sum_{g, g \neq f} \sigma_f^g \right)$$

$$\sum_{k=1}^{K} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \geq 0. \tag{C.4}$$

Thus, the game $G_1(\mathbf{q}_{ss}, \mathbf{q}_c)$ admits a unique Nash equilibrium point.

D. Proof of Theorem 4

The second derivative of the utility function $\overline{U}_{\mathrm{CP}_f}$ is

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f}^2} = -2\mu N_f \varsigma_f^f \sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \leq 0. \tag{D.1}$$

The second derivative of the utility function is negative, and then the utility function is thus concave, which ensures existence of a Nash equilibrium in the game $G_1(\mathbf{p}_c, \mathbf{q}_{ss})$.

The authors use the following proposition that holds for a concave game [40]. If a concave game satisfies the dominance solvability condition

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f}^2} \geq \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f} \partial q_{c_g}} \right|, \tag{D.2}$$

then the game $G_1(\mathbf{p}_c, \mathbf{q}_{ss})$ admits a unique Nash equilibrium point.

The mixed partial is written as follows:

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f} \partial q_{c_g}} = \mu N_f \varsigma_f^g \sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right). \tag{D.3}$$

Then,

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f}^2} - \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{c_f} \partial q_{c_g}} \right| = \mu N_f \left(\varsigma_f^f - \sum_{g, g \neq f} \varsigma_f^g \right)$$

$$\sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \geq 0. \tag{D.4}$$

Thus, the game $G_1(\mathbf{p}_c, \mathbf{q}_{ss})$ admits a unique Nash equilibrium point.

E. Proof of Theorem 5

The second derivative of the utility function $\overline{U}_{\mathrm{CP}_f}$ is

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f}^2} = -2\lambda N_f \varsigma_f^f \sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \leq 0. \tag{E.1}$$

The second derivative of the utility function is negative, and then the utility function is thus concave, which ensures existence of a Nash equilibrium in the game $G_1(\mathbf{p}_c, \mathbf{q}_c)$.

The authors use the following proposition that holds for a concave game [40]. If a concave game satisfies the dominance solvability condition

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f}^2} \geq \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f} \partial q_{ss_g}} \right|, \tag{E.2}$$

then the game $G_1(\mathbf{p}_c, \mathbf{q}_c)$ admits a unique Nash equilibrium point.

The mixed partial is written as follows:

$$\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f} \partial q_{ss_g}} = \lambda N_f \varsigma_f^g \sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right). \tag{E.3}$$

Then,

$$-\frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f}^2} - \sum_{g, g \neq f} \left| \frac{\partial^2 \overline{U}_{\mathrm{CP}_f}}{\partial q_{ss_f} \partial q_{ss_g}} \right| = \lambda N_f \left(\varsigma_f^f - \sum_{g, g \neq f} \varsigma_f^g \right)$$

$$\sum_{k=1}^{K} p_{t_k} \left(1 + \chi_f + \frac{\chi_f \xi \left(e^{-\left(\omega_{fk}+\xi\right)T} - 1 \right)}{\left(1 - e^{-\xi T}\right)(\omega + \xi)} \right) \geq 0. \tag{E.4}$$

Thus, the game $G_1(\mathbf{p}_c, \mathbf{q}_c)$ admits a unique Nash equilibrium point.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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