Hindawi
Mobile Information Systems
Volume 2020, Article ID 1347601, 16 pages
https://doi.org/10.1155/2020/1347601

Research Article

Performance Sensitivity to the High-Order Statistics of Time Interval Variables in Cellular Networks

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Received 23 February 2019; Accepted 4 January 2020; Published 30 January 2020

Academic Editor: Juraj Machaj

Cell dwell time (DT) and unencumbered interruption time (IT) are fundamental time interval variables in the teletraffic analysis for the performance evaluation of mobile cellular networks. Although a diverse set of general distributions has been proposed to model these time interval variables, the effect of their moments higher than the expected value on system performance has not been reported in the literature. In this paper, sensitivity of teletraffic performance metrics of mobile cellular networks to the first three standardized moments of both DT and IT is investigated in a comprehensive manner. Mathematical analysis is developed considering that both DT and IT are phase-type distributed random variables. This work includes substantial numerical results for quantifying the dependence of system level performance metrics to the values of the first three standardized moments of both DT and IT. For instance, for a high mobility scenario where DT is modeled by a hyper-Erlang distribution, we found that call forced termination probability decreases around 60% as the coefficient of variation (CoV) and skewness of DT simultaneously change from 1 to 20 and from 60 to 2, respectively. Also, numerical results confirm that as link unreliability increases the forced termination probability increases while both new call blocking and handoff failure probabilities decrease. Numerical results also indicate that for low values of skewness, performance metrics are highly sensitive to changes in the CoV of either the IT or DT. In general, it is observed that system performance is more sensitive to the statistics of the IT than to those of the DT. Such understanding of teletraffic engineering issues is vital for planning, designing, dimensioning, and optimizing mobile cellular networks.

1. Introduction

Cell residence/dwell time (DT), unencumbered interruption time (IT), and unencumbered call-holding/service time (ST) are fundamental time interval variables for the mathematical analysis of cellular networks (CNs). These telecommunication time variables allow us to compute key parameters of CNs (i.e., channel holding time, new session blocking, unsuccessful handoff, forced session termination probabilities, handoff rate, and Erlang capacity, among others). For analytical/computational tractability of CNs, the exponential-negative probability distribution function (pdf) was traditionally employed for modeling DT, IT, and ST telecommunication time variables. Under this pdf, only the expected value of these telecommunications time variables is needed for system performance evaluation [1]. Nonetheless, experimental studies have shown that these considerations are not valid for real cellular networks [2–6]. Recently, it has been found that the global effect of cellular size/shape, mobility characteristics of users, link unreliability, handoff mechanisms, and behavior of new type of services can be best captured if these time variables are modeled as random variables (RVs) with general probability distribution functions (pdfs) [2–9]. In this sense, some researchers have employed the gamma, log-normal, Pareto, and Weibull pdfs to model cell residence time [10]. Correspondingly, it has being shown that the Weibull pdf represents a good model for both multimedia applications [11] and session holding time in hierarchical CNs [12]. It has been also shown that the log-normal pdf represents an excellent model for data session holding time in real networks. Nonetheless, the authors in [11, 12] noticed that the Laplace transform of gamma, log-normal, Pareto, and Weibull pdfs cannot be written into a rational form, and thus the Markovian properties of the teletraffic model are lost when these pdfs are employed [3, 5]. It is well known that Markovian properties are indispensable in producing tractable teletraffic models for CNs [2, 13]. To overcome this problem, phase-type pdfs were proposed in [2] for modeling telecommunication time variables. The relevance of phase-type pdfs is two-fold: they offer accurate models for the different telecommunication time variables in CNs and at the same time, the Markovian properties of the teletraffic model are preserved. Furthermore, there exists relevant research work related to fit phase-type pdfs to data collected in real networks (see, for instance, [5] and the references therein). In these research directions, the hyper-Erlang pdf [2, 9] is of distinctive interest due to its universality property (i.e., it can be employed to approximate with an arbitrary degree of precision the comportment of any non-negative RV). Also, some important phase-type distributions (i.e., negative-exponential, Erlang, hyperexponential, and Coxian) are particular cases of the hyper-Erlang distribution. When experimental data (that represent certain telecommunication time variable) are best fitted with a general pdf, it is of paramount importance to comprehensively study the influence of moments higher than the expected value of this telecommunication time variable on the system performance [14]. This is the topic of research of the present work.

There exist many procedures proposed in the literature regarding fitting known pdfs (such as phase-type ones) to data collected from real networks [5]. Nevertheless, this is not the focus of our present work; instead, we are concerned in studying the impact on system performance of moments higher than the first one when general phase-type pdfs are employed to model DT and IT variables. In a related work [14], several experiments were performed by means of a G/G/1 simulation model to investigate the impact of moments of interarrival and service time RVs on queue waiting time. In [14], the gamma, log-normal, Weibull, and Person type 5 pdfs were employed to model service and interarrival time interval variables. The general conclusion obtained in [14] is that pdfs that are completed characterized with two parameters (i.e., mean value and coefficient of variation) are not suitable for modeling these time variables. They notice that even though moments of lower order dominate system performance, it is needed to capture moments of higher order to improve accuracy of numerical results. Under this context, we have identified that a comprehensive study of the impact of moments higher than the mean value on performance of CNs has not been carried out in the literature. The reason of this can be explained as follows: (1) the use of pdfs different to the negative-exponential one to model telecommunication time variables has been just recently proposed and (2) the related research work has been mainly concentrated on developing mathematical paradigms rather than performing numerical evaluations to investigate system performance. Exception of this is our previous work [15, 16]. In [15], it is investigated the effect of DT and IT statistics on forced call-termination probability while in [16], it is investigated the effect of mean value, coefficient of variation, and skewness of both DT and IT on new call blocking probability, forced call termination probability, and carried traffic of CNs. Contrary to [15, 16], in this paper, the impact of DT and IT statistics on the expected value of channel holding time (for handed off and new sessions) is also comprehensively analyzed. Also, in our related research work [10, 17], the functional relationship between DT statistics and channel holding time statistics is investigated. Nonetheless, the effect of both link unreliability and DT statistics on the performance of CNs is not addressed in [10, 17].

In this paper, using phase-type pdfs for modeling cell dwell time and unencumbered interruption time random variables, a comprehensive sensitivity study of performance of cellular networks to the first three standardized moments (i.e., mean value, coefficient of variation, and skewness) of both cell dwell time and unencumbered interruption time is carried out. In particular, in this work, hyper-Erlang and hyperexponential pdfs are employed to model cell dwell time and unencumbered interruption time. Even though the concepts presented in this work are essentially well known, the sensitivity analysis to moments higher than the first one is a novel contribution that gives interesting insights into the behavior of cellular networks. On the other hand, from the perspective of mathematical analysis, our contribution is to derive general expressions for several performance metrics (new call blocking probability, handoff failure probability, call forced termination probability, mean channel holding times, handoff rate, and carried traffic) as function of the first three standardized moments of cell dwell time and unencumbered interruption time in cellular networks. As explained in [18], blocking probability is still a useful performance metric in current cellular networks for real-time mobile applications (i.e., video streaming, mobile gaming, and video conferencing). The developed analysis in this paper can be extended for the performance evaluation of other cellular-based systems (i.e., green cellular networks (in [18], a robust and computationally efficient analytical approximation method is proposed to evaluate call blocking probability in green cellular networks considering different base station (BS) sleeping patterns; specifically, the Erlang fixed-point approximation technique is used to provide an accurate and computationally feasible analytical approximation to calculate call blocking probability in cellular networks with or without BS sleeping; contrary to this paper, in [18], both call service time and call sojourn (cell dwell) times are considered to be independent and exponentially distributed random variables) [18] and cognitive cellular networks [19]).

The rest of this article is structured as follows. Methods are addressed in Section 2, while the system model is described in Section 3. The queuing analysis for the performance evaluation of cellular networks is developed in Section 4. Also, expressions for forced session termination probability and mean channel holding time (for both new and handed off sessions) are obtained in Section 4. In Section 5, numerical results are presented and discussed. Finally, in Section 6, conclusions are exposed.

2. Methods

2.1. Analytical Method. In this paper, we followed the methods of [15, 16] to analyze sensitivity of DT and IT statistics on system performance of CNs. These methods are summarized as follows.

Cell dwell time, unencumbered interruption time, and unencumbered service time are modeled as phase-type distributed random variables. Residual cell dwell time is derived using the excess life theorem [1] (it is shown that the pdf of this time variable is phase type when the cell dwell time is hyper-Erlang distributed).

A multicellular system is considered. It is assumed that all cells in the service area are statistically identical; thus, the overall system performance can be analyzed by focusing on only one given cell. The servers represent the channels of the given cell, whereas the clients represent the mobile terminals. Each cell is modeled as a queuing system where new and handoff call arrivals correspond to service requests and the departures correspond to service termination due to either successful call termination, forced termination due to wireless link unreliability, or cell leaving. For this queuing system, the different transition rates and transitions rules are derived as functions of the different system parameters (new call arrival rate, hand-off call arrival rate, mean service time, mean inter-arrival time, and mean unencumbered interruption time, among others). Thus, the steady-state balance equations are obtained by equating rate out to rate in for each state of the system [20]. Finally, based on the steady-state balance equations, the different system performance metrics are derived; in particular, the handoff call rate is calculated using the fixed-point iteration method, as explained in [1].

On the other hand, call forced termination probability is derived using the total probability theorem and applying the residue theorem as explained in [15, 16]. Also, the channel holding time for new and for handed off calls is derived using elemental probabilistic functions such as the minimum of a set of independent random variables.

2.2. Simulation Method. The analytical teletraffic model developed in Section 4 is validated by a wide set of discrete-event computer simulation results for a variety of evaluation scenarios. To simulate a large cellular network in which the boundary effect is eliminated and each cell within the cellular-layout has exactly six neighboring cells, a wraparound hexagonal mesh (H-mesh) topology is used [13]. In this paper, a wrapped H-mesh topology with four cells per peripheral edge of the mesh is considered (i.e., the simulation model comprises 37 total cells) [13]. A macroscopic mobility model is used to capture user mobility. Under this mobility model, a user terminal stays in the coverage area of a base station for a period of time that has the selected phase-type distribution used to model DT (for ongoing calls) or RDT (for new call arrivals). It is assumed that a handed off terminal moves to any of the current cell's neighbors with equal probability. The developed discrete event-driven computer simulator considers the following basic events: arrival of a new call requests, handoff attempts of ongoing call to a neighbor cell, transition of the stage of the DT (or RDT), successful call completion, and call termination due to either resource insufficiency or link unreliability. Every event is associated with an event-time stamp representing the instant when the event occurs [21]. All unprocessed events are inserted in an ordered list of future events. Events are processed in chronological order according to their event-time stamps. Once an event is executed, it is removed from the list of pending events and system statistics are collected. For obtaining good estimates of the different performance metrics, each simulation study was run for more than one million of new calls. Simulation results have shown perfect agreement with analytical results.

3. System Model

Next, the system model assumptions and definitions of the different time interval variables involved in the teletraffic analysis are presented. Notations and symbols employed in the rest of the paper are summarized in Table 1.

3.1. Link Unreliability. Call forced termination probability is one of the most important quality of service (QoS) metrics for the performance evaluation of mobile communication networks (in packet switched mobile communication networks, call forced termination probability is especially important for the performance evaluation of real-time services (i.e., voice, audio, music, videophone, videoconference, etc.) [15, 22, 23]). In cellular networks, resource insufficiency and link unreliability are the two fundamental causes of call forced termination (link unreliability occurs when signal to interference ratio (SIR) is lower than a minimum required value for more than a certain time interval; for an ongoing call, the physical link between access point and user's equipment may suffer link unreliability due to propagation impairments such as multipath fading, shadowing, path loss, and interference [7]). Nevertheless, in cellular networks in operation, handoff failure can be an insignificant event [24]. In contrast, link unreliability has been shown to be the main reason of call forced termination [24–26]. The model proposed in [7] to characterize the effect of link unreliability on the system level performance is employed in this work. In [7], the effect of link unreliability is captured by the so called "*unencumbered call interruption time.*" This time variable is defined in the next section.

3.2. Assumptions and Definitions. A homogeneous multicellular system with omnidirectional antennas and a fixed number S of radio channels per cell is considered. Then, all cells are statistically identical (i.e., the underlying processes and parameters for all cells are the same). N channels in each cell are reserved to prioritize handoff call attempts over new call requests. Typical assumptions in the related literature [4] that both new call arrivals and handoff call attempts follow independent Poisson processes with arrival rate, respectively, λ_n and λ_h per cell, are here adopted. Some other relevant assumptions and definitions are specified below.

Unencumbered service time x_s (a.k.a. *requested call holding time* [5] or *call holding duration* [6]) is the amount of

Table 1: Symbols and notation used throughout the paper.

Variable	Description
$\alpha_i^{(h)}$	Probability of choosing the phase i (for $i = 1, 2, \ldots, m^{(h)}$) of the hyper-Erlang distribution of $m^{(n)}$ phases used to characterize the cell dwell time.
$\alpha_i^{(n)}$	Probability of choosing the phase i (for $i = 1, 2, \ldots, m^{(n)}$) of the hyper-Erlang distribution of $m^{(n)}$ phases used to characterize the residual cell dwell time.
$\eta_i^{(h)}$	Inverse of the mean sojourn time in every stage of the *i-th* phase of the hyper-Erlang distribution used to characterize the cell dwell time.
$\eta_i^{(n)}$	Inverse of the mean sojourn time in every stage of the *i-th* phase of the hyper-Erlang distribution used to characterize the residual cell dwell time.
$1/\eta$	Mean value of $\mathbf{X}_d^{(j)}$.
λ_n (λ_h)	Mean arrival rate for new calls (handoff attempts).
$1/\mu$	Mean value of unencumbered service time.
$\Sigma U\,(h)$	Total number of stages of the hyper-Erlang distribution used to characterize the cell dwell time.
$\Sigma U\,(n)$	Total number of stages of the hyper-Erlang distribution used to characterize the residual cell dwell time.
a_c	Carried traffic.
CDF	Cumulative distribution function.
DT	Cell dwell time.
CHTh	Channel holding time for handed off calls.
CHTn	Channel holding time for new calls.
$f_{\mathbf{X}_r}(t)$	Probability density function of $\mathbf{X}_r$.
$F_{\mathbf{X}_d}(t)$	Cumulative probability distribution function (CDF) of $\mathbf{X}_d$
$\mathbf{K} = [\mathbf{K}^{(n)}, \mathbf{K}^{(h)}]$	State variables vector.
pdf	Probability density function.
N	Total number of reserved channels per cell for handoff prioritization.
$P^{(h)}$	Handoff failure probability.
$P^{(n)}$	New call blocking probability.
$P_{ft}^{(j)}$	Forced call termination probability in cell j.
P_{ft}	Global forced call termination probability.
RDT	Residual cell dwell time.
S	Total number of channels per cell in the system.
$u_i^{(h)}$	Number of stages of the *i-th* phase of the hyper-Erlang distribution used to characterize the cell dwell time.
$u_i^{(n)}$	Number of stages of the *i-th* phase of the hyper-Erlang distribution used to characterize the residual cell dwell time.
IT	Unencumbered interruption time.
$x_c^{(j)}$	Channel holding time in the j-th handed off cell (for $j = 0, 1, \ldots$).
$x_d^{(j)}$	Cell dwell time in the j-th handed off cell (for $j = 0, 1, \ldots$) irrespective of whether it is engaged in a call or not.
$\mathbf{X}_d^{(j)}$	Random variables used to represent the cell dwell time.
$x_i^{(j)}$	Unencumbered call interruption time.
$\mathbf{X}_i^{(j)}$	Random variables used to represent the unencumbered call interruption time.
x_r	Residual cell dwell time.
$\mathbf{X}_r$	Random variable used to represent the residual cell dwell time.
x_s	Call holding time per call.
$\mathbf{X}_s$	The random variable used to represent the call holding time.

time that a call would remain in progress if it is not forced to terminate. In the literature, it has been widely accepted that the unencumbered service time for voice service is properly modeled by a random variable exponentially distributed [1]. The random variable used to represent the unencumbered service time is $\mathbf{X}_s$, and it has mean value $E\{\mathbf{X}_s\} = 1/\mu$.

On the other hand, *cell dwell time* or *cell residence time* $x_d^{(j)}$ is the time that a user is within the coverage area of the j-th (for $j = 0, 1, \ldots$) handed off cell irrespective of whether he has an ongoing call (or session) or not. Let $\mathbf{X}_d^{(j)}$ (for $j = 0, 1, \ldots$) be independent and identically distributed random variables used to represent these times. For homogeneous multicellular networks, this is a typical assumption in the literature [1, 4–6]. In this paper, the random variables $\mathbf{X}_d^{(j)}$ (for $j = 0, 1, \ldots$) are considered to be independent phase-type identically distributed with mean value $1/\eta$.

Moreover, *residual cell dwell time* x_r is the time between the moment that a new call of a user is initiated in a given cell and the moment that the user leaves the cell. Note that residual cell dwell time is associated only to users with new calls. The random variable $\mathbf{X}_r$ is used to represent the residual cell dwell time. According to the excess life theorem [1], the probability density function (pdf) of the residual cell dwell time $\mathbf{X}_r$, denoted by $f_{\mathbf{X}_r}(t)$ can be obtained in terms of the probability distribution of the cell dwell time $\mathbf{X}_d$ by

$$f_{\mathbf{X}_r}(t) = \frac{1}{E[\mathbf{X}_d]}\left[1 - F_{\mathbf{X}_d}(t)\right]. \tag{1}$$

The mean and cumulative probability distribution function (CDF) of the cell dwell time $\mathbf{X}_d$ are given by $E[\mathbf{X}_d]$ and $F_{\mathbf{X}_d}(t)$, respectively.

Besides, *unencumbered call interruption time* $x_i^{(j)}$ is the amount of time from the moment a user begins to be served by the base station of the j-th handed off cell (for $j = 0, 1, 2, \ldots$) to the moment his call would be dropped due to link unreliability if both the unencumbered service time and cell dwell time were large enough [7]. The random variables $\mathbf{X}_i^{(j)}$ (for $j = 0, 1, 2, \ldots$) are used to represent the unencumbered call interruption times, and they are assumed to be independent and phase-type distributed [7].

Lastly, *channel holding time* $x_c^{(j)}$ is the amount of time a user with an ongoing call occupies a radio channel in the j-th (for $j = 0, 1, \ldots$) handed off cell before his call is either successfully completed, successfully handed off to another cell, or interrupted due to either wireless channel unreliability or handoff failure.

The general architecture of the considered multicellular communication network is shown in Figure 1. Figure 1 also illustrates the relationship among the different time variables defined in this section.

3.3. Teletraffic Model. Teletraffic analysis in Section 4 is developed by following the approach proposed in [8, 9] to capture the general distributions of both DT and IT.

For the sake of space and to facilitate the understanding of the mathematical analysis, the teletraffic analyses developed in Section 4 consider the next case: (1) DT is hyper-Erlang distributed and IT is exponentially distributed. Nonetheless, teletraffic analyses are also developed considering the following cases: (2) DT is hyperexponentially distributed and IT is hyperexponentially distributed and (3) DT is exponentially distributed and IT is hyper-Erlang distributed. Notice that the evaluation scenario when DT is exponentially distributed and IT is hyperexponentially distributed represents a subcase of both the case (2) and case (3). Furthermore, the evaluation scenario when DT is hyperexponentially distributed and IT is exponentially distributed is a subcase of both the case (1) and case (2). Evidently, the evaluation scenario when both DT and IT are exponentially distributed is obtained from any of the previous cases. Nonetheless, to simplify interpretation of the numerical results and to evaluate the effect of each time variable (i.e., DT and IT) on the system performance, only some evaluation scenarios are considered. In particular, numerical results for the evaluation scenarios, (a) DT is exponentially distributed and IT is either hyperexponentially distributed or hyper-Erlang distributed and (b) DT is either hyperexponentially distributed or hyper-Erlang distributed and IT is exponentially distributed, are shown and analyzed.

3.4. Residual Cell Dwell Time Characterization. Due to the fact that DT is considered to be hyper-Erlang distributed, it is necessary to derive the distribution of the *residual cell dwell time* (RDT). This is obtained in this section.

Assume DT follows an $m^{(h)}$-th order hyper-Erlang distribution with shape and rate parameters $u_i^{(h)}$ and $\eta_i^{(h)}$ (for $i = 1, 2, \ldots, m^{(h)}$). The rate parameters $\eta_i^{(h)}$ are related to the inverse of the mean DT (η) as follows: $\eta_i^{(h)} = \eta u_i^{(h)}$ [2]. Considering that $\alpha_i^{(h)}$ represents the probability of choosing the phase i of the hyper-Erlang distribution (for $i = 1, 2, \ldots, m^{(h)}$), the pdf of $\mathbf{X}_d$ can be expressed as

$$f_{X_d}(t) = \sum_{i=1}^{m^{(h)}} \alpha_i^{(h)} \frac{\left(\eta_i^{(h)}\right)^{u_i^{(h)}} t^{u_i^{(h)}-1}}{\left(u_i^{(h)}-1\right)!} e^{-\eta_i^{(h)} t};$$
$$\eta_i^{(h)} > 0,\ t \geq 0,\ 0 \leq \alpha_i^{(h)} \leq 1,\ \sum_{i=1}^{m^{(h)}} \alpha_i^{(h)} = 1, \quad (2)$$

where $u_i^{(h)}$ is a positive integer and $\eta_i^{(h)}$ is a positive constant. Note that the hyper-Erlang distribution is a mixture of $m^{(h)}$ different Erlang distributions, and each of them has a shape parameter $u_i^{(h)}$ and a rate parameter $\eta_i^{(h)}$. The value $\alpha_i^{(h)}$ represents the weight of each Erlang distribution. Using (1), the pdf of RDT is given by

$$f_{X_r}(t) = \frac{1}{\sum_{i=1}^{m^{(h)}} \alpha_i^{(h)} u_i^{(h)} / \eta_i^{(h)}} \sum_{i=1}^{m^{(h)}} \sum_{j=0}^{u_i^{(h)}-1} \alpha_j^{(h)} \frac{\left(\eta_i^{(h)} t\right)^j}{j!} e^{-\eta_i^{(h)} t}. \quad (3)$$

This pdf can be rewritten as

$$f_{X_r}(t) = \sum_{i=1}^{m^{(n)}} \alpha_i^{(n)} \frac{\left(\eta_i^{(n)}\right)^{u_i^{(n)}} t^{u_i^{(n)}-1}}{\left(u_i^{(n)}-1\right)!} e^{-\eta_i^{(n)} t}, \quad (4)$$

where

$$\begin{aligned}
\alpha^{(n)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} &= \frac{\alpha_i^{(h)} \prod_{l=1 \cap l \neq i}^{m^{(h)}} \eta_l^{(h)}}{\sum_{k=1}^{m^{(h)}} \left(\alpha_k^{(h)} u_k^{(h)} \prod_{l=1 \cap l \neq k}^{m^{(h)}} \eta_l^{(h)}\right)}, \\
u^{(n)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} &= j, \\
\eta^{(n)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} &= \eta_i^{(h)}, \\
m^{(n)} &= \sum_{i=1}^{m^{(h)}} u_i^{(h)},
\end{aligned} \quad (5)$$

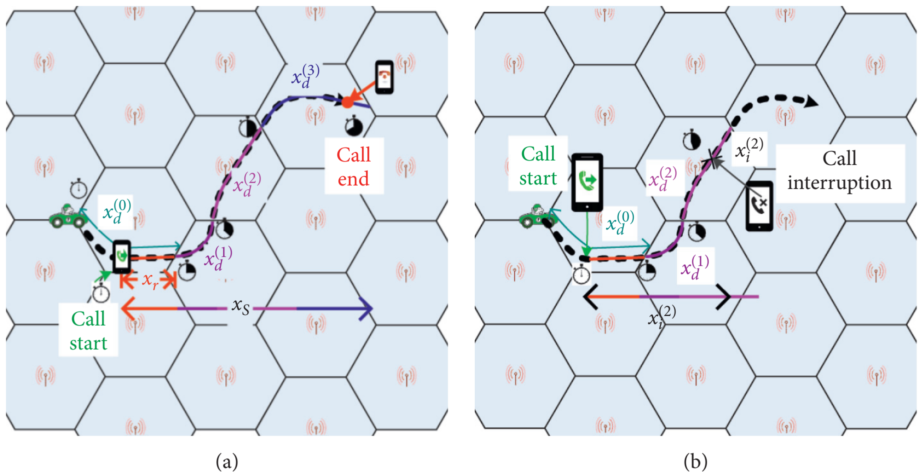


Figure 1: General architecture of the system and time variables involved in the teletraffic analysis: (a) scenario of a successfully terminated call; (b) scenario of a forced call termination due to link unreliability.

for $i = 1, 2, \ldots, m^{(h)}$, $j = 1, 2, \ldots, u_i^{(h)}$.

From (4), it is noticed that RDT has an $m^{(n)}$-th order hyper-Erlang distribution with shape and rate parameters, respectively, $u_i^{(n)}$ and $\eta_i^{(n)}$ (for $i = 1, 2, \ldots, m^{(n)}$). The probability of choosing the phase i of the hyper-Erlang distribution is represented by $\alpha_i^{(n)}$ (for $i = 1, 2, \ldots, m^{(n)}$). The diagrams of phases of the hyper-Erlang distributed random variables $\mathbf{X}_d$ and $\mathbf{X}_r$ are shown in Figure 2. In Figure 2, when $y = n$, it represents the diagram for $\mathbf{X}_r$, while when $y = h$, it represents the diagram for $\mathbf{X}_d$.

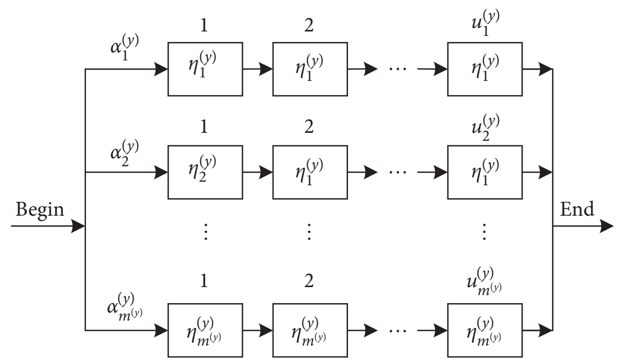


Figure 2: General phase and stage diagram for the probability distribution function of residual cell residence time ($y = n$) and cell residence time ($y = h$).

4. Teletraffic Analysis

In this section, the teletraffic analysis for the system-level performance evaluation of the multicellular cellular network described in Section 3 is presented. From the mathematical analysis point of view, our merit is to derive general expressions for a number of performance metrics as function of the first three standardized moments of DT and IT in mobile cellular networks. This in turn has allowed us to investigate in detail the sensitivity of system performance to statistics of both DT and IT. In this way, we found rather important insights into the behavior of mobile cellular networks not previously reported in the literature.

As stated in Section 3.3 and without loss of generality, in this section, the special case when IT is exponential distributed and DT is hyper-Erlang distributed is considered. The mean value of IT is denoted by $1/\gamma$. A total number of $\sum_{x=1}^{m^{(n)}} u_x^{(n)} + \sum_{x=1}^{m^{(h)}} u_x^{(h)}$ state variables are needed to model this system by means of a multidimensional birth and death process. Let us define $k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)} + j}$ as the number of users in stage i and phase j of residual cell dwell time ($y = n$) and cell dwell time ($y = h$).

To simplify mathematical notation, the following vectors and parameters are defined. Vector $\mathbf{K}^{(y)}$ is defined as follows:

$$K^{(y)} = \left[k_1^{(y)}, k_2^{(y)}, \ldots, k^{(y)}_{\sum_{i=1}^{m^{(y)}} u_i^{(y)}}\right]. \qquad (6)$$

Note that $y = n$ or $y = h$ when new calls or handoff calls are involved, respectively. Let $\mathbf{e}_i^{(y)}$ be a unit vector of size $m^{(y)}$ whose all entries are 0 except i-th entry which is 1 (for $i = 1, 2, \ldots, \sum_{i=1}^{m^{(y)}} u_i^{(y)}$). Finally, the parameters $N^{(n)}$ and $N^{(h)}$ are defined as follows: $N^{(n)} = N$ and $N^{(h)} = 0$.

Vector $\mathbf{K} = [\mathbf{K}^{(n)}, \mathbf{K}^{(h)}]$ represents the state of the analyzed cell. Table 2 shows the transition rates from the current reference state to the different successor states and the transition rules. As stated before, we assume that all the cells are probabilistically equivalent. That is, the new call arrival rate is the same in each cell, and the rate at which mobiles enter a given cell is equal to the rate at which they interrupt its connection (due to either a handed off call event or link unreliability) in that cell. Thus, equating rate out to rate in for each state, the steady-state balance equations are given by [20]

TABLE 2: Transition rates and transition rules for the case when the DT is hyper-Erlang distributed and IT is negative exponentially distributed.

Event	Successor state	Rate
A new call enters first stage of phase i of $\mathbf{X}_r$ ($i=1, 2, ..., m^{(n)}$)	$[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+1}, \mathbf{K}^{(h)}]$	$a_i^{(n)}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A new call leaves stage j of phase i and enters stage $j+1$ of phase i of $\mathbf{X}_r$ ($i=1, 2,..,m^{(n)}$), ($j=1,\ldots u_i^{(n)}-1$)	$[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j+1}, \mathbf{K}^{(h)}]$	$b^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A new call leaves last stage of phase i of $\mathbf{X}_r$ ($i=1, 2, ..., m^{(n)}$)	$[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}}, \mathbf{K}^{(h)}]$	$c_i^{(n)}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A new call leaves stage of phase i of $\mathbf{X}_r$ ($i=1, 2, ..., m^{(n)}$)	$[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}+j}, \mathbf{K}^{(h)}]$	$d^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A handed off call enters first stage of phase i of $\mathbf{X}_d$ ($i=1, 2, ..., m^{(h)}$)	$[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+1}]$	$a_i^{(h)}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A handed off call leaves stage j of phase i and enters stage $j+1$ of phase i of $\mathbf{X}_d$ ($i=1, 2, ..., m^{(h)}$) ($j=1,\ldots,u_i^{(h)}-1$)	$[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j+1}]$	$b^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A handed off call leaves last stage of phase i of $\mathbf{X}_d$ ($i=1, 2, ..., m^{(h)}$)	$[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}}]$	$c_i^{(h)}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$
A handed off call leaves stage of phase i of $\mathbf{X}_d$ ($i=1, 2, ..., m^{(n)}$)	$[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}+j}]$	$d^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j}([\mathbf{K}^{(n)}, \mathbf{K}^{(h)}])$

$$P(\mathbf{K}) = \frac{g_a^{(n)} + g_b^{(n)} + g_c^{(n)} + g_d^{(n)} + g_a^{(h)} + g_b^{(h)} + g_c^{(h)} + g_d^{(h)}}{\sum_{i=1}^{m^{(n)}} a_i^{(n)}(\mathbf{K}) + \sum_{i=1}^{m^{(n)}} \sum_{j=1}^{u_i^{(n)}-1} b^{(n)}_{\widehat{u}_x^{(n)}}(\mathbf{K}) + \sum_{i=1}^{m^{(n)}} c_i^{(n)}(\mathbf{K}) + \sum_{i=1}^{m^{(n)}} \sum_{j=1}^{u_i^{(n)}-1} d^{(n)}_{\widehat{u}_x^{(n)}}(\mathbf{K}) + \sum_{i=1}^{m^{(h)}} \sum_{j=1}^{u_i^{(h)}-1} b^{(h)}_{\widehat{u}_x^{(h)}}(\mathbf{K}) + \sum_{i=1}^{m^{(h)}} a_i^{(h)}(\mathbf{K}) + \sum_{i=1}^{m^{(h)}} c_i^{(h)}(\mathbf{K}) + \sum_{i=1}^{m^{(h)}} \sum_{j=1}^{u_i^{(h)}-1} d^{(h)}_{\widehat{u}_x^{(h)}}(\mathbf{K})}, \tag{7}$$

where

$$\widehat{u}_x^{(n)} = \sum_{x=1}^{i-1}\left(u_x^{(n)} - 1\right) + j,$$
$$\widehat{u}_x^{(h)} = \sum_{x=1}^{i-1}\left(u_x^{(h)} - 1\right) + j. \tag{8}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+1}, \mathbf{K}^{(h)}]$ (for $i=1, ..., m^{(n)}$) to the reference state $\mathbf{K}$, due to arrival of new call requests, is given by

$$g_a^{(n)} = \sum_{i=1}^{m^{(n)}} \left[a_i^{(n)}\left(\left[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+1}, \mathbf{K}^{(h)}\right]\right) \cdot P\left(\left[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+1}, \mathbf{K}^{(h)}\right]\right)\right]. \tag{9}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j+1}, \mathbf{K}^{(h)}]$ (for $i=1, ..., m^{(n)}$ and $j=1,\ldots u_i^{(n)}-1$) to the reference state $\mathbf{K}$, due to the interstage transitions in every phase of the hyper-Erlang distribution used to characterize the residual cell dwell time, is given by

$$g_b^{(n)} = \sum_{i=1}^{m^{(n)}} \sum_{j=1}^{u_i^{(n)}-1} \left[b^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j}\left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}-1+j} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j+1}, \mathbf{K}^{(h)}\right]\right) P\left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j+1}, \mathbf{K}^{(h)}\right]\right)\right]. \tag{10}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}}, \mathbf{K}^{(h)}]$ (for $i=1, ..., m^{(n)}$) to the reference state $\mathbf{K}$, due to channel release because of either call completion, call interruption due to link unreliability, or cell leaving, is given by

$$g_c^{(n)} = \sum_{i=1}^{m^{(n)}} \left[c_i^{(n)}\left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}}, \mathbf{K}^{(h)}\right]\right) \cdot P\left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}}, \mathbf{K}^{(h)}\right]\right)\right]. \tag{11}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j}, \mathbf{K}^{(h)}]$ (for $i=1, ..., m^{(n)}$ and $j=1,\ldots,u_i^{(n)}-1$) to the reference state $\mathbf{K}$, due to channel release because of either call completion or call interruption due to link unreliability, is given by

$$g_d^{(n)} = \sum_{i=1}^{m^{(n)}} \sum_{j=1}^{u_i^{(n)}-1} \left[d^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j} \left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j}, \mathbf{K}^{(h)} \right] \right) \cdot P\left(\left[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j}, \mathbf{K}^{(h)} \right] \right) \right]. \tag{12}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+1}]$ (for $i = 1, \ldots, m^{(h)}$) to the reference state $\mathbf{K}$, due to arrival of handoff call attempts, is given by

$$g_a^{(h)} = \sum_{i=1}^{m^{(h)}} \left[a_i^{(h)} \left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+1} \right] \right) \cdot P\left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+1} \right] \right) \right]. \tag{13}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j+1}]$ (for $i = 1, \ldots, m^{(h)}$ and $j = 1, \ldots, u_i^{(h)} - 1$) to the reference state $\mathbf{K}$, due to the interstage transitions in every phase of the hyper-Erlang distribution used to characterize the cell dwell time, is given by

$$g_b^{(h)} = \sum_{i=1}^{m^{(h)}} \sum_{j=1}^{u_i^{(h)}-1} \left[b^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j} \left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j+1} \right] \right) P\left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j+1} \right] \right) \right]. \tag{14}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}}]$ (for $i = 1, \ldots, m^{(h)}$) to the reference state $\mathbf{K}$, due to channel release because of either call completion, call interruption due to link unreliability, or cell leaving, is given by

$$g_c^{(h)} = \sum_{i=1}^{m^{(h)}} \left[c_i^{(h)} \left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}} \right] \right) \cdot P\left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}} \right] \right) \right]. \tag{15}$$

The conditional incoming state transition rate from the states $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j}]$ (for $i = 1, \ldots, m^{(h)}$ and $j = 1, \ldots, u_i^{(h)} - 1$) to the reference state $\mathbf{K}$, due to channel release because of either call completion or call interruption due to link unreliability, is given by

$$g_d^{(h)} = \sum_{i=1}^{m^{(h)}} \sum_{j=1}^{u_i^{(h)}-1} \left[d^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j} \left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} \right] \right) P\left(\left[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} \right] \right) \right]. \tag{16}$$

The state transition rates $a_i^{(n)}$ and $c_i^{(n)}$ ($i = 1, \ldots, m^{(n)}$), $a_i^{(h)}$ and $c_i^{(h)}$ ($i = 1, \ldots, m^{(h)}$), $b^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j}$ and $d^{(n)}_{\sum_{x=1}^{i-1}(u_x^{(n)}-1)+j}$ (for $i = 1, \ldots, m^{(n)}$ and $j = 1, \ldots, u_i^{(n)} - 1$), and $b^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j}$ and $d^{(h)}_{\sum_{x=1}^{i-1}(u_x^{(h)}-1)+j}$ (for $i = 1, \ldots, m^{(h)}$ and $j = 1, \ldots, u_i^{(h)} - 1$) are detailed below.

Of course, the state probabilities must satisfy the normalization equation given by

$$\underbrace{\sum_{k_1^{(n)}=0}^{S-N} \cdots \sum_{k_{\Sigma U(n)}^{(n)}=0}^{S-N} \sum_{k_1^{(h)}=0}^{S} \cdots \sum_{k_{\Sigma U(h)}^{(h)}=0}^{S}}_{\{\mathbf{K} \mid \mathbf{K} \in \Omega\}} P(\mathbf{K}) = 1, \tag{17}$$

where $\Sigma U(n) = \sum_{x=1}^{m^{(n)}} u_x^{(n)}$ and $\Sigma U(h) = \sum_{x=1}^{m^{(h)}} u_x^{(h)}$ are, respectively, the total number of stages of the hyper-Erlang distributions used to characterize the residual cell dwell time and the cell dwell time and Ω is the valid states space given by

$$\Omega = \left\{ \mathbf{K} \left| \left(\sum_{i=1}^{\Sigma U(n)} k_i^{(n)} + \sum_{i=1}^{\Sigma U(h)} k_i^{(h)} \le S \right) \cap \left(\sum_{i=1}^{\Sigma U(n)} k_i^{(n)} \le S - N \right) \right. \right\}. \tag{18}$$

All states involved in (7) and (17) must be valid states. The corresponding steady-state probabilities are calculated by means of the well-known Gauss–Seidel relaxation method [20].

It is straightforward to show that the transition rates defined in Table 2 can be mathematically expressed as follows (notice that $y = n$ when new calls are involved and $y = h$ when handed off calls are involved). The state transition rate from the reference state $\mathbf{K}$ to the state $[\mathbf{K}^{(n)} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+1}, \mathbf{K}^{(h)}]$ (or $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+1}]$) (for $i = 1, \ldots, m^{(y)}$), due to arrival of new (or handoff) call requests in the i-th phase of the hyper-Erlang distribution used to characterize the residual cell dwell time (or cell dwell time), is given by

$$a_i^{(y)}(\mathbf{K}) = \begin{cases} \alpha_i^{(y)} \lambda^{(y)}; & \sum_{l=1}^{\sum_{x=1}^{m^{(n)}} u_x^{(n)}} k_l^{(n)} + \sum_{l=1}^{\sum_{x=1}^{m^{(h)}} u_x^{(h)}} k_l^{(h)} < S - N^{(y)} \cap k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+1} \ge 0, \\ 0; & \text{otherwise}. \end{cases} \tag{19}$$

The state transition rate from the reference state $\mathbf{K}$ to the state $[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j} + \mathbf{e}^{(n)}_{\sum_{x=1}^{i-1} u_x^{(n)}+j+1}, \mathbf{K}^{(h)}]$ (or $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j} + \mathbf{e}^{(h)}_{\sum_{x=1}^{i-1} u_x^{(h)}+j+1}]$) (for $i = 1, \ldots, m^{(y)}$ and $j = 1, \ldots, u_i^{(y)}$-1), due to the transition from the stage j to the stage j+1 of the *i-th* phase of the hyper-Erlang distribution used to characterize the residual cell dwell time (or cell dwell time), is given by

$$b^{(y)}_{\sum_{x=1}^{i-1}\left(u_x^{(y)}-1\right)+j}(\mathbf{K}) = \begin{cases} k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j}\, \eta_i^{(y)}; & \sum_{l=1}^{\sum_{x=1}^{m^{(n)}} u_x^{(n)}} k_l^{(n)} + \sum_{l=1}^{\sum_{x=1}^{m^{(h)}} u_x^{(h)}} k_l^{(h)} \le S \cap k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j} > 0 \cap k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j+1} \ge 0, \\ 0; & \text{otherwise.} \end{cases} \tag{20}$$

The state transition rate from the reference state $\mathbf{K}$ to the state $[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}}, \mathbf{K}^{(h)}]$ (or $[\mathbf{K}^{(n)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}}, \mathbf{K}^{(h)}]$ (for $i = 1, \ldots, m^{(y)}$), due to channel release because of either call completion, call interruption due to link unreliability, or cell leaving with the *i-th* phase of the hyper-Erlang distribution used to characterize the residual cell dwell time (or cell dwell time), is given by

$$c_i^{(y)}(\mathbf{K}) = \begin{cases} k^{(y)}_{\sum_{x=1}^{i} u_x^{(y)}}\left(\mu + \eta_i^{(y)} + \gamma\right); & \sum_{l=1}^{\sum_{x=1}^{m^{(n)}} u_x^{(n)}} k_l^{(n)} + \sum_{l=1}^{\sum_{x=1}^{m^{(h)}} u_x^{(h)}} k_l^{(h)} \le S \cap k^{(y)}_{\sum_{x=1}^{i} u_x^{(y)}} > 0, \\ 0; & \text{otherwise.} \end{cases} \tag{21}$$

The transition rate from the reference state $\mathbf{K}$ to the state $[\mathbf{K}^{(n)} - \mathbf{e}^{(n)}_{\sum_{x=1}^{i} u_x^{(n)}+j}, \mathbf{K}^{(h)}]$ (or $[\mathbf{K}^{(n)}, \mathbf{K}^{(h)} - \mathbf{e}^{(h)}_{\sum_{x=1}^{i} u_x^{(h)}+j}]$) (for $i = 1, \ldots, m^{(y)}$ and $j = 1, \ldots, u_i^{(y)} - 1$), due to channel release because of either call completion or call interruption due to link unreliability when the user is in the *j-th* stage of the *i-th* phase of the hyper-Erlang distribution used to characterize the residual cell dwell time (or cell dwell time), is given by

$$d^{(y)}_{\sum_{x=1}^{i-1}\left(u_x^{(y)}-1\right)+j}(\mathbf{K}) = \begin{cases} k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j}(\mu + \gamma); & \sum_{l=1}^{\sum_{x=1}^{m^{(n)}} u_x^{(n)}} k_l^{(n)} + \sum_{l=1}^{\sum_{x=1}^{m^{(h)}} u_x^{(h)}} k_l^{(h)} \le S \cap k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j} > 0 \cap k^{(y)}_{\sum_{x=1}^{i-1} u_x^{(y)}+j+1} \ge 0, \\ 0; & \text{otherwise.} \end{cases} \tag{22}$$

New call blocking probability, $P^{(n)}$, is the sum of the probabilities of states that cannot accommodate more new arrival requests, and handoff failure probability, $P^{(h)}$, is the sum of the probabilities of states that cannot accommodate more handed off call requests. These probabilities can be computed using

$$P^{(y)} = \sum_{k_1^{(n)}=0}^{S-N^{(y)}} \cdots \sum_{k_{\Sigma U(n)}^{(n)}=0}^{S-N^{(y)}} \sum_{k_1^{(h)}=0}^{S} \cdots \sum_{k_{\Sigma U(h)}^{(h)}=0}^{S} P(\mathbf{K}); \quad \text{for } y = \{n, h\}, \cdot \left\{ \mathbf{K} \,\middle|\, S - N^{(y)} \le \sum_{i=1}^{\Sigma U(n)} k_i^{(n)} + \sum_{i=1}^{\Sigma U(h)} k_i^{(h)} \le S \right\}. \tag{23}$$

Carried traffic (a_c) is given by

$$a_c = \sum_{k_1^{(n)}=0}^{S-N^{(y)}} \cdots \sum_{k_{\Sigma U(n)}^{(n)}=0}^{S-N^{(y)}} \sum_{k_1^{(h)}=0}^{S} \cdots \sum_{k_{\Sigma U(h)}^{(h)}=0}^{S} \cdot \left\{ \left[\sum_{l=1}^{\Sigma U(n)} k_l^{(n)} + \sum_{l=1}^{\Sigma U(h)} k_l^{(h)} \right] P(\mathbf{K}) \right\}, \cdot \left\{ \mathbf{K} \,\middle|\, \sum_{i=1}^{\Sigma U(n)} k_i^{(n)} + \sum_{i=1}^{\Sigma U(h)} k_i^{(h)} \le S \right\}. \tag{24}$$

The handoff call rate is calculated using the fixed point iteration method, as explained in [1]. Call forced termination probability and channel holding time are derived in Sections 4.1 and 4.2, respectively.

4.1. Forced Termination Probability. Call forced termination may result due to either wireless link unreliability or handoff failure (resource insufficiency). In general, a dropped call experiences j ($j = 0, 1, 2, \ldots$) successful handoffs previously to be interrupted (due to either a handoff failure or link unreliability). Thus, call forced termination probability after j successful handoffs is given by

$$P_{ft}^{(j)} = \begin{cases} P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_r)) + P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_i))P^{(h)}; & j = 0, \\ P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_i))(1 - P^{(h)}) \times \left[P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_d)) + P(\mathbf{X}_d \le \min(\mathbf{X}_s, \mathbf{X}_i))P^{(h)}\right]; & j = 1, \\ P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_r))P(\mathbf{X}_d \le \min(\mathbf{X}_s, \mathbf{X}_i))^{j-1}(1 - P^{(h)})^j \times P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_d)) + P(\mathbf{X}_d \le \min(\mathbf{X}_s, \mathbf{X}_i)); & j < 1, \end{cases} \tag{25}$$

where $P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_r))$ and $P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_d))$ represent interruption probabilities due to link unreliability for new and handed off calls, respectively. Function $\min(\cdot, \cdot)$ returns the smallest random variable.

Call forced termination occurs after a random number of successful handoffs. Then, by employing the total probability theorem, call forced termination probability can be expressed as follows:

$$\begin{aligned} P_{ft} = \sum_{j=0}^{\infty} P_{ft}^{(j)} &= P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_r)) \\ &+ P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_i))P^{(h)} + \\ P(\mathbf{X}_r \le \min(\mathbf{X}_s, \mathbf{X}_i))&\left(1 - P^{(h)}\right) \\ &\cdot \left[\frac{P(\mathbf{X}_i \le \min(\mathbf{X}_s, \mathbf{X}_d)) + P(\mathbf{X}_d \le \min(\mathbf{X}_s, \mathbf{X}_i))P^{(h)}}{1 - P(\mathbf{X}_d \le \min(\mathbf{X}_s, \mathbf{X}_i))(1 - P^{(h)})}\right]. \end{aligned} \tag{26}$$

By letting $\mathbf{Z}_s^{(w)} = \min(\mathbf{X}_s, \mathbf{X}_w)$ with $w = \{r, i, d\}$ and using the residue theorem, the different probabilities in (26) can be calculated by using the following relationship for the nonnegative independent random variables $\mathbf{X}_w$ and $\mathbf{Z}_s^{(w)}$ (see [3]):

$$\begin{aligned} P\left(\mathbf{X}_w \le \mathbf{Z}_s^{(w)}\right) &= \frac{1}{2\pi i} \int_{\sigma - i\infty}^{\sigma + i\infty} \frac{f_{\mathbf{X}_w}^*(s)}{s} f_{\mathbf{Z}_s^{(w)}}^*(-s) \mathrm{d}s \\ &= -\sum_{p \in \sigma_P} \operatorname*{Res}_{s=p}\left[\frac{f_X^*(s)}{s} f_{\mathbf{Z}_s^{(w)}}^*(-s)\right], \end{aligned} \tag{27}$$

where $f_{\mathbf{X}_w}^*(s)$ and $f_{\mathbf{Z}_s^{(w)}}^*(s)$ represent the Laplace transform of $\mathbf{X}_w$ and $\mathbf{Z}_s^{(w)}$, respectively, with $w = \{r, i, d\}$. σ_P is the set of poles of $f_{\mathbf{Z}_s^{(w)}}^*(-s)$. Equation (27) is valid when the probability density functions of $\mathbf{X}_w$ and $\mathbf{Z}_s^{(w)}$ are proper rational functions [3]. For the different considered evaluation scenarios in this paper, the probability density functions of $\mathbf{X}_w$ and $\mathbf{Z}_s^{(w)}$ are proper rational functions.

4.2. Channel Holding Time. In order to model channel holding time, it is necessary to express it as a function of the unencumbered service, cell dwell, and unencumbered call interruption time variables. The random variable $\mathbf{X}_c^{(j)}$, which models channel holding time in the j-th handed off cell, is defined as follows:

$$\mathbf{X}_c^{(j)} = \begin{cases} \min\left(\mathbf{X}_i^{(j)}, \mathbf{X}_s, \mathbf{X}_r\right); & j = 0, \\ \min\left(\mathbf{X}_i^{(j)}, \mathbf{X}_s, \mathbf{X}_d^{(j)}\right); & j > 0. \end{cases} \tag{28}$$

Thus, channel holding time should be defined for the following type of calls: new calls [$j = 0$ in (28)] and handed off calls [$j > 0$ in (28)]. Let us denote by $\mathbf{X}_c^{(n)}$ and $\mathbf{X}_c^{(h)}$ the channel holding time for new calls and handed off calls, respectively.

Assuming that the different involved time variables are independent, the cumulative distribution function (CDF) of the channel holding time for new and handed off calls is, respectively, given by

$$\begin{aligned} F_{\mathbf{X}_c^{(n)}}(t) &= 1 - \left[1 - F_{\mathbf{X}_i}(t)\right]\left[1 - F_{\mathbf{X}_s}(t)\right]\left[1 - F_{\mathbf{X}_r}(t)\right]; \\ F_{\mathbf{X}_c^{(h)}}(t) &= 1 - \left[1 - F_{\mathbf{X}_i}(t)\right]\left[1 - F_{\mathbf{X}_s}(t)\right]\left[1 - F_{\mathbf{X}_d}(t)\right]. \end{aligned} \tag{29}$$

Table 3 provides the CDF of channel holding time for new and handed off calls for the cases when both DT and IT are negative-exponential, hyperexponential, or hyper-Erlang distributed. Notice that all the cases considered in this paper are special cases of the third entry of Table 3 (that is, when both DT and IT are modeled by a hyper-Erlang distribution). In [17], we perform a comprehensive study on the functional relationship between cell dwell time and channel holding time statistics. Nonetheless, the impact of the wireless channel unreliability is ignored in [17]. Finally, given the mathematical expressions for the CDF of channel holding time for new and handed off calls, it is straightforward to obtain the mean value of these time variables.

5. Results and Discussion

In this section, the impact of mean value, coefficient of variation, and skewness of cell dwell/residence time (DT) and unencumbered interruption time (IT) on performance of a cellular network (CN) is presented. Numerical results presented in this section demonstrate the applicability, robustness, and accuracy of the mathematical paradigm developed in Section 4. In order to attain good understandings on the sensitivity study carried out in this section, a hyper-Erlang pdf with two phases and two stages and a 2nd order

Table 3: Examples of corresponding distributions for $x_c^{(n)}$ and $x_c^{(h)}$.

Cell dwell/unencumbered call interruption time probability density	pdf of channel holding time for new and handed off calls
Both DT and IT are negative exponential distributed: $f_{\mathbf{X}_d}(t) = \eta e^{-\eta t}$ $f_{\mathbf{X}_i}(t) = \gamma e^{-\gamma t}$	$F_{\mathbf{X}_c^{(n)}}(t) = 1 - e^{-(\mu+\eta+\gamma)t}$ $F_{\mathbf{X}_c^{(h)}}(t) = 1 - e^{-(\mu+\eta+\gamma)t}$
Both DT and IT are hyperexponentially distributed: $f_{\mathbf{X}_d}(t) = \sum_{k=1}^{m} \alpha_k \eta_k e^{-\eta_k t}$ $f_{\mathbf{X}_i}(t) = \sum_{l=1}^{n} \beta_l \gamma_l e^{-\gamma_l t}$	$F_{\mathbf{X}_c^{(n)}}(t) = 1 - (1/\sum_{k=1}^{m} \alpha_k/\eta_k)$ $\cdot \sum_{k=1}^{m} \sum_{l=1}^{n} (\alpha_k/\eta_k) \beta_l e^{-(\mu+\eta_k+\gamma_l)t}$ $F_{\mathbf{X}_c^{(h)}}(t) = 1 - \sum_{k=1}^{m} \sum_{l=1}^{n} \alpha_k \beta_l e^{-(\mu+\eta_k+\gamma_l)t}$
Both DT and IT are hyper-Erlang distributed: $f_{\mathbf{X}_d}(t) = \sum_{k=1}^{m} \alpha_k (\eta_k^{u_k} t^{u_k-1}/(u_k-1)!) e^{-\eta_k t}$ $f_{\mathbf{X}_i}(t) = \sum_{l=1}^{n} \beta_l (\gamma_l^{v_l} t^{v_l-1}/(v_l-1)!) e^{-\gamma_l t}$	$F_{\mathbf{X}_c^{(n)}}(t) = 1 - (1/\sum_{k=1}^{m} (\alpha_k u_k/\eta_k))$ $\cdot \sum_{k=1}^{m} \sum_{l=1}^{n} \sum_{i=0}^{u_k-1} \sum_{j=0}^{v_l-1} \sum_{q=0}^{i} (\alpha_k/\eta_k) \beta_l$ $((\eta_k t)^q/q!)((\gamma_l t)^j/j!) e^{-(\mu+\eta_k+\gamma_l)t}$ $F_{\mathbf{X}_c^{(n)}}(t) = 1 - \sum_{k=1}^{m} \sum_{l=1}^{n} \sum_{i=0}^{u_k-1}$ $\sum_{j=0}^{v_l-1} (\alpha_k \beta_l (\eta_k t)^q/i! (\gamma_l t)^j/j! e^{-(\mu+\eta_k+\gamma_l)t})$

hyperexponential pdf are employed. Note that the label "HErl-2" denotes the hyper-Erlang pdf with two phases and two stages and the label "HExp-2" represents the 2nd order hyperexponential pdf. The following parameters are considered in this section: mean service time ($1/\mu$) is equal to 180 s, total number of channels per cell (S) is equal to 8, offered traffic per cell is equal to 4.4 Erlangs, and total number of channels reserved for handoff prioritization ($N^{(n)}$) is equal to 1.

Figures 3–7 illustrate, respectively, forced termination, new session blocking, handoff failure probabilities, handoff rate, and carried traffic versus coefficient of variation and skewness of IT. On the other hand, Figures 8–12 plot, respectively, forced call termination, new session blocking probabilities, carried traffic, mean channel holding time for new calls, and mean channel holding time for handoff calls versus coefficient of variation and skewness of DT. A particular case is presented in Figures 3–7 when IT is modeled with either hyper-Erlang pdf with two phases and two stages or 2nd order hyperexponentially pdf and DT is negative-exponentially distributed. In contrast, Figures 8–12 present numerical results for the particular case when DT is modeled with either hyper-Erlang pdf with two phases and two stages or 2nd order hyperexponential pdf and IT is negative-exponentially distributed. In Figures 3–7, two different values of the mean IT time are considered and compared, say 1500 s (which represents a low reliability scenario) and 5000 s (which represents a high reliability scenario) (note that both considered values of the mean IT are significantly greater than the mean DT; this is due to the fact that telecommunication systems are designed to be reliable; to this end, mean IT should be typically greater than both mean unencumbered service time and mean cell residence/dwell time). Moreover, in Figures 8–12, two different values of the mean DT are evaluated, say 100 s (which represents a high mobility scenario) and 900 s (which represents a low mobility scenario).

5.1. Influence of Unencumbered Interruption Time Statistics. The influence of the expected value, coefficient of variation, and skewness of unencumbered interruption time on the performance of a CN is studied in this section.

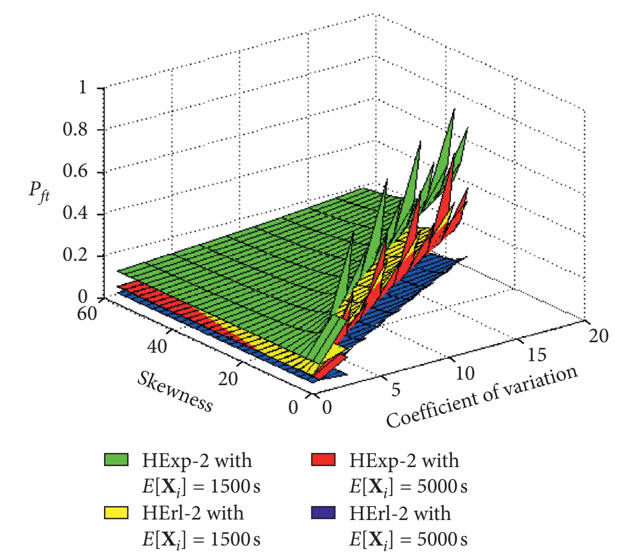


Figure 3: Forced session termination probability against skewness and coefficient of variation of unencumbered interruption time, with the probability density function type and expected value of unencumbered interruption time as parameters.

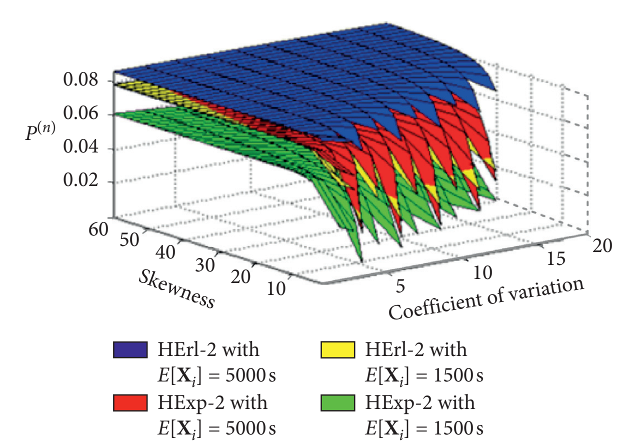


Figure 4: New session blocking probability against skewness and coefficient of variation of unencumbered interruption time, with the probability density function type and expected value of unencumbered interruption time as parameters.

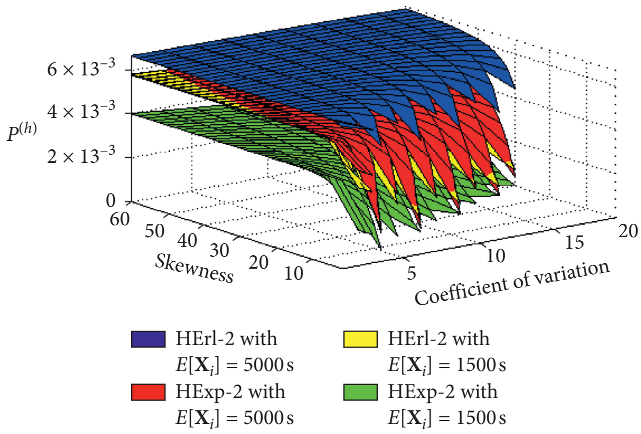


Figure 5: Handoff failure probability against skewness and coefficient of variation of unencumbered interruption time, with the probability density function type and expected value of unencumbered interruption time as parameters.

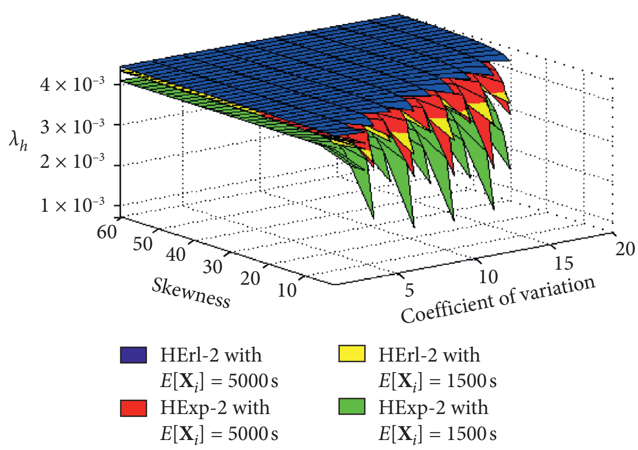


Figure 6: Handoff rate against skewness and coefficient of variation of unencumbered interruption time, with the probability density function type and expected value of unencumbered interruption time as parameters.

Figure 3 illustrates that while the mean value of the IT decreases, the forced session termination probability increases. This fact indicates an adverse effect of link unreliability on the performance of CNs (remember that physically, the mean value of IT represents a direct measure of link reliability). In contrast, from Figures 4 and 5, it is perceived that as link unreliability grows (i.e., when the mean unencumbered call interruption time decreases), both new session blocking and handoff failure probabilities decrease, indicating a positive impact of channel unreliability (from the perspective of radio resource insufficiency) on the performance of CNs. This is due to the fact that as link unreliability increases, more ongoing calls are forced to terminate due to wireless channel impairments. This fact contributes to reducing the handoff rate as it is illustrated in Figure 6. Consequently, more radio resources are available for new and handed off sessions, reducing in this way both new session blocking and handoff failure probabilities.

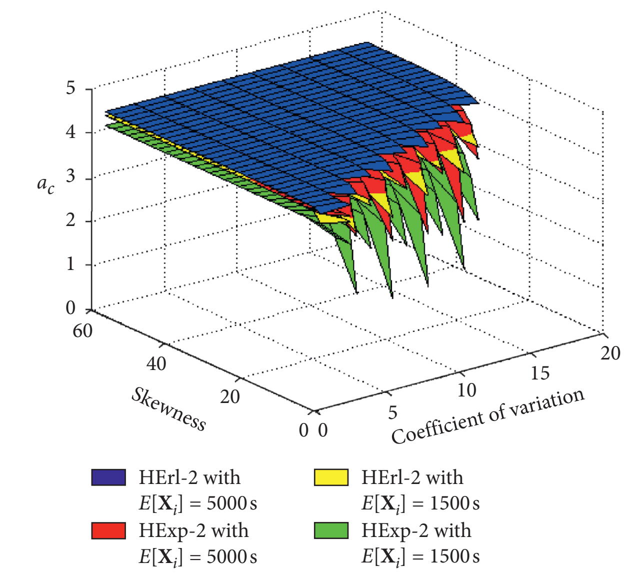


Figure 7: Carried traffic against skewness and coefficient of variation of unencumbered interruption time, with the probability density function type and expected value of unencumbered interruption time as parameters.

Figures 3–5 also show that the forced session termination probability increases and both new call blocking and handoff failure probabilities decrease as CoV of IT augments. Let us explain this comportment. Firstly, observe that as the CoV augments, the probability that IT takes smaller values augments. Consequently, more sessions are forced to terminate due to link unreliability. Simultaneously, as more ongoing sessions are forced to terminate due to link unreliability, the handoff rate diminishes and, as a consequence, the handoff failure probability decreases as well. The overall effect of these facts is to increase the forced call termination probability and, at the same time, both new call blocking and handoff failure probabilities decrease. Furthermore, from Figures 3 and 4, the following interesting result can be extracted: for the scenarios where skewness takes values less than 20, it is observed that forced session termination probability due to resource insufficiency is significantly augmented and new session blocking probability is decreased as the CoV is raised. For instance, Figures 3 and 4 show that for scenarios of low reliability where skewness is equal to 2 and the IT is hyper-Erlang distributed, forced termination probability due to resource insufficiency increases around 700% and blocking probability decreases 67% as the CoV of IT moves from 1 to 20. The scenario where skewness is equal to 2 and CoV is equal to 1 corresponds to the case when IT is negative-exponentially distributed. Figures 3 and 4 show that forced call termination probability due to resource insufficiency is a monotonically decreasing function of skewness, while the opposite behavior is observed for the new call blocking probability.

Alternatively, Figure 7 shows that the carried traffic is an increasing function of both the skewness and mean value of IT. Also, Figure 7 shows that for values of skewness smaller than around 30, carried traffic decreases as CoV of IT

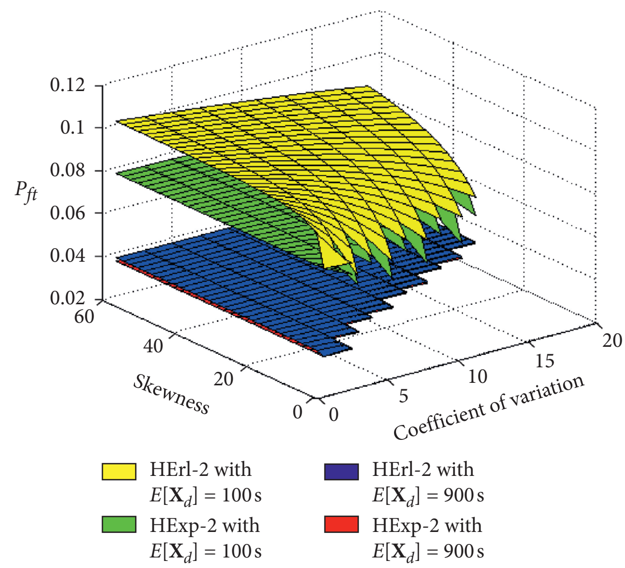
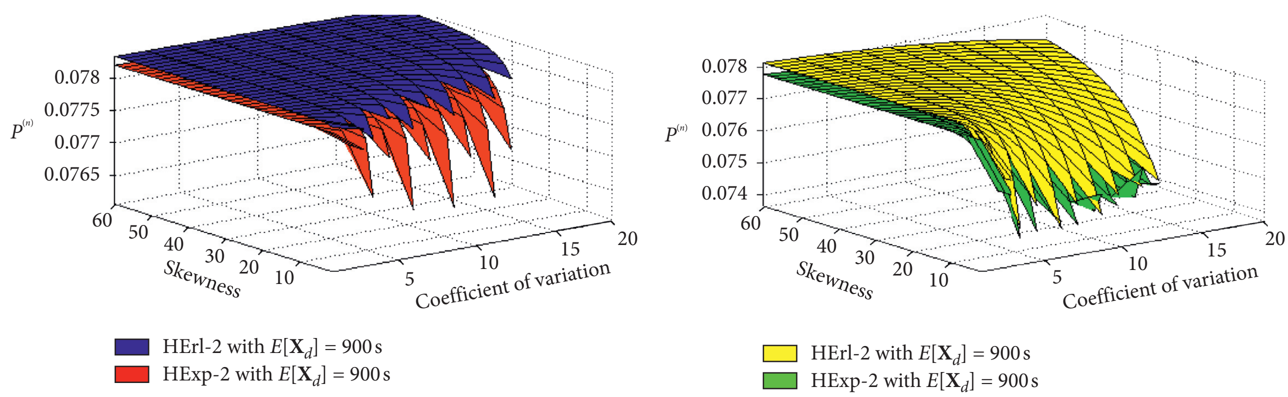


Figure 8: Forced session termination probability against skewness and coefficient of variation of cell residence time, with the probability density function type and mean value of cell residence time as parameters.



(a) (b)

Figure 9: New session blocking probability against skewness and coefficient of variation of cell residence time, with the probability density function type and mean value of cell residence time as parameters.

increases. These observations indicate a detrimental effect of link unreliability on carried traffic. Moreover, it is noted that for values of skewness greater than around 30 and for the same mean value and type of distribution of IT, the carried traffic is almost insensitive to the CoV of IT. The reason is as follows. Consider that the mean value, CoV, and distribution type of IT remain without change. Then, as the skewness of IT increases, the tail on the right side of the IT distribution function becomes longer (that is, the probability that IT takes higher values increases and, consequently, less calls are interrupted due to link unreliability). That is, the influence of skewness on forced termination probability becomes negligible. At the same time, because link unreliability is not considered to accept a call, the blocking probability is not sensitive to changes on neither skewness nor CoV of IT statistics. As the carried traffic directly depends on both blocking and forced call termination probabilities, the combined effect of these two facts is observed in Figure 7.

An interesting study of the results illustrated in Figures 3–7 is that for a given scenario with the same skewness and CoV, a nonnegligible difference is observed between the values taken by the different performance metrics when IT is modeled by the hyper-Erlang pdf and the corresponding values when IT is modeled by the hyper-exponential pdf. Thus, it is apparent that moments higher than the first one used to characterize IT have a relevant impact on the performance of CNs.

5.2. Influence of Cell Dwell Time Statistics. In this section, it is investigated the influence of the expected value, coefficient of variation, and skewness of cell dwell time on system performance. Figure 8 illustrates that the forced session termination probability is augmented as the mean value of the DT is decreased, indicating a negative impact of user mobility on the performance of CNs. This fact is explained as follows. When the mean value of DT is diminished, the average number of handoffs per call is augmented; consequently, the forced termination probability due to resource insufficiency is increased. On the other hand, Figure 9 shows that the blocking probability increases as the mean value of DT increases. This is because as the mean DT becomes larger, users with ongoing calls remain for a longer period of time inside the cell. Consequently, the rate at which radio resources are released decreases, causing a detrimental effect on blocking probability.

Figure 8 also shows that forced session termination probability is practically insensitive to skewness and CoV of DT for the low mobility scenario. The reason of this conduct is explained by the fact that under scenarios with low mobility, most of the ongoing sessions are successfully completed in the cell where those sessions were originated; consequently, the expected number of handoffs per call is small in benefit of forced session termination probability. Additionally, from Figure 8 it is noted that for scenarios with high mobility and regardless of skewness, forced session termination probability diminished as CoV of DT augments. This behavior can be explained by the fact that as the CoV of DT rises, the variability of the DT augments, that is, the values of cell residence time spread out over a larger range. The overall effect is controlled by larger values of DT, that is, a diminution of mobility takes place. Consequently, the expected number of handoffs per call is reduced, which has a beneficial impact on forced session termination probability.

Additionally, from Figure 8, it is interesting to note that under scenarios with high mobility and values of skewness less than 20, forced session termination probability is considerably reduced as CoV of DT rises. For example, Figure 8 shows that under the scenario with high mobility and cell dwell time hyper-Erlang distributed, forced session termination probability is reduced 60% as the CoV and skewness of DT move, respectively, from 1 to 20 and from 60 to 2. Observe that the scenario where CoV is equal to 1 and skewness is equal to 2 resembles the case when DT is negative-exponentially distributed.

On the other hand, Figures 8 and 9 reveal that for scenarios with high mobility, both forced session termination and blocking probabilities are monotonically increasing functions of skewness of DT. The reason is as follows. Consider that the mean value, CoV, and distribution type of DT remain without change. Then, as the skewness of DT rises, the tail on the right side of the DT pdf becomes longer (that is, the probability that cell dwell time takes higher values augments and, consequently, less active sessions move to another cell). In this manner, the rate at which channels are used by handed off calls diminishes in benefit of both blocking and handoff failure probabilities. This behavior contributes to augment carried traffic in concordance with the results presented in Figure 10. Figure 10 reveals that carried traffic is a decreasing function of skewness of DT. Figure 10 also shows that carried traffic rises as CoV of DT augments. These remarks indicate a beneficial effect of the variability of DT on carried traffic.

Figures 11 and 12 reveal that the qualitative behavior of the mean value of both channel holding time for new calls

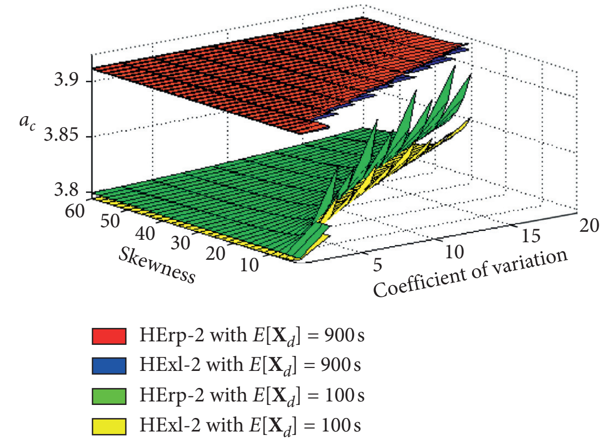


Figure 10: Carried traffic against skewness and coefficient of variation of cell residence time, with the probability density function type and expected value of cell residence time as parameters.

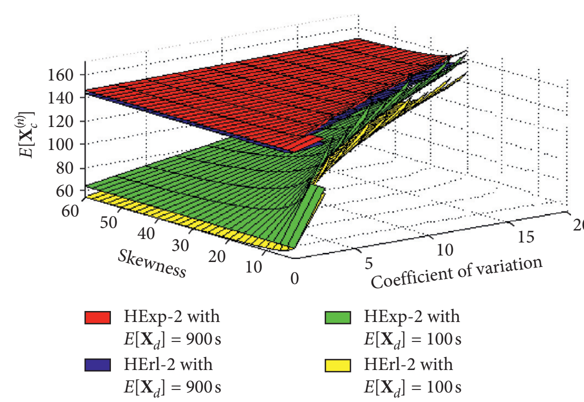


Figure 11: Mean channel holding time for new calls against skewness and coefficient of variation of cell residence time, with the probability density function type and expected value of cell residence time as parameters.

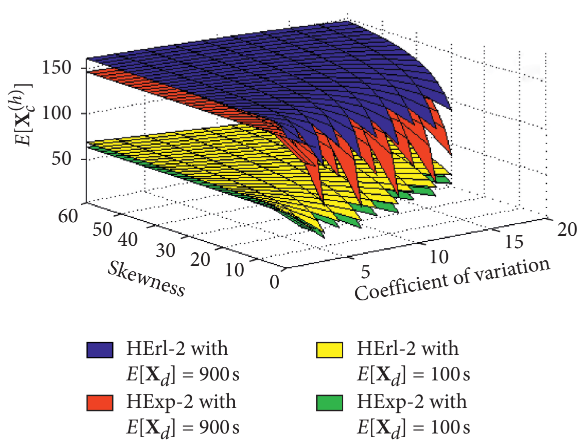


Figure 12: Mean channel holding time for handed off calls against skewness and coefficient of variation of cell residence time, with the probability density function type and expected value of cell residence time as parameters.

(CHTn) and channel holding time for handed off calls (CHTh) is very similar for the cases where cell dwell time is modeled by either hyper-Erlang or hyperexponential distributions. The small quantitative difference among them is attributed to moments higher than the third one.

It is important to remark that the situation when cell residence time is negative-exponential distributed is represented for a single point in all the plots presented in Section 4. That is, the situation when the coefficient of variation equals 1 and skewness equals 2 corresponds to the case when either IT or DT is modeled by the negative-exponential pdf. Thus, as far as real distributions of telecommunication time interval variables are best modeled by general distributions, numerical results presented in Section 4 evidence the inefficiency of the negative-exponential model and emphasize the relevance of considering the impact of moments higher than the expected value.

Finally, the results illustrated in Figures 8–12 reveal that for the same scenario and same mean value, skewness, and CoV of DT, there exists a small but nonnegligible difference between the values taken by the different performance metrics when DT is modeled by the hyper-Erlang pdf and the corresponding values when DT is modeled by the hyperexponential one. Evidently, these differences are due to the certainly dissimilar values between moments higher than the third one of hyper-Erlang and hyperexponential distribution models. Therefore, it is important to investigate the influence of moments higher than the third one (of both DT and IT) on the performance of mobile cellular networks. This important task represents a topic of our current research.

6. Conclusions

Relevant insights into the performance sensitivity of mobile cellular systems to the first three standardized moments of both cell dwell time (DT) and unencumbered interruption time (IT) were found in this paper. Despite the fact that the numerical results presented in this work were obtained from particular situations with certain set of system parameter values, these results reveal that there exist important sensitive questions concerning moments higher than the expected value of both DT and IT. It was found that cellular system performance is more sensitive to the statistics of the IT than to those of the DT. Numerical results demonstrate that performance metrics are greatly influenced by smaller values of the skewness of either the IT or DT. Also of importance, it was found that, for the same mobility/unreliability scenario and matching the first three standardized moments, there exists significant differences among the values taken by the different system performance metrics when IT is modeled by the hyperexponential distribution and the corresponding values when IT is modeled by the hyper-Erlang distribution (a similar behavior was observed when DT was considered.) The small quantitative difference between the hyper-Erlang and hyperexponential cases is due to moments higher than the third one. Analyzing the impact of moments of both DT and IT distributions higher than the third one on system performance metrics represents a topic of our current research.

Data Availability

The numerical results data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

We thank UAM-A under project no. EL001-17 for providing us resources to acquire computing equipment for the numerical evaluation of the studied systems.

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