

# Improving sensing accuracy in cognitive PANs through modulation of sensing probability<sup>1</sup>

Vojislav B. Mišić\* and Jelena Mišić  
*University of Manitoba, Winnipeg, MB, Canada*

**Abstract.** Cognitive radio technology necessitates accurate and timely sensing of primary users' activity on the chosen set of channels. The simplest selection procedure is a simple random choice of channels to be sensed, but the impact of sensing errors with respect to primary user activity or inactivity differs considerably. In order to improve sensing accuracy and increase the likelihood of finding channels which are free from primary user activity, the selection procedure is modified by assigning different sensing probabilities to active and inactive channels. The paper presents a probabilistic analysis of this policy and investigates the range of values in which the modulation of sensing probability is capable of maintaining an accurate view of the status of the working channel set. We also present a modification of the probability modulation algorithm that allows for even greater reduction of sensing error in a limited range of the duty cycle of primary users' activity. Finally, we give some guidelines as to the optimum application ranges for the original and modified algorithm, respectively.

Keywords: Opportunistic spectrum access, cooperative spectrum sensing, wireless personal area networks

## 1. Introduction

Cognitive communications technology [1] is predominantly envisaged for use in the area of local and wide-range networks [3,6], but it can provide benefits for wireless personal area networks as well [4]. Co-existence of several WPANs in the same physical area may be achieved by combining opportunistic spectrum access with frequency hopping [8], similar to Bluetooth [2] except that the hopping sequence of a PAN with cognitive access (cognitive PAN, or CPAN) should dynamically adapt to the activity patterns of primary users. Those patterns must be obtained by sensing, which should be performed repetitively and cooperatively [15,17], and in such manner as to minimize the sensing error.

Random selection of channels with equal probability (hereafter referred to as uniform random sensing) ensures that all channels are sensed with equal frequency and, thus, with equal error in statistical sense [12]. This error may be reduced by increasing the ratio of sensed channels vs. total channels, either by increasing the number of channels to be sensed in each cycle or, if that is not possible, by reducing the total number of working channels.

---

<sup>1</sup>The paper is a substantially revised and expanded version of the paper "Reducing Sensing Error in Cognitive PANs Through Modulation of Sensing Probability" that has been presented at the GlobeCom'2008 conference in New Orleans, LA.

\*Corresponding author. E-mail: vmisic@cs.umanitoba.ca.

However, sensing error can be reduced in another manner. Namely, not all errors affect the operation of a CPAN in the same way: a channel that becomes inactive while considered active is but a missed opportunity, whereas a channel that becomes active during CPAN operation may cause interference and, in extreme cases, complete failure of communication in the CPAN. Therefore, the onset of primary user activity on an inactive channel is more critical for the CPAN operation, and should be sensed with smaller error than the preceding (or succeeding) end of such activity.

Smaller error for inactive channels requires shorter sensing intervals, which may be accomplished by selecting inactive channels with higher probability than the active ones, as will be explained below. The performance of this technique, hereafter referred to as probability modulation, is modeled and analyzed through a queueing theoretic approach. The probability modulation approach is shown to be capable of reducing the error in both magnitude and duration, compared to that of uniform random sensing.

Since the proposed algorithm selects the channels to be sensed in a random manner, although with different selection probabilities, it may be advantageous to further modify those probabilities according to the duration of the interval between two consecutive sensing events. A simple modification of the original algorithm is presented that achieves this, increasing the fairness in sensing and equalizing the duration of sensing intervals, whilst preserving the random manner of selecting the channels to be sensed.

The paper is organized as follows: Section 2 gives more details about the operation of a frequency hopping cognitive personal area network and the sensing process in particular, together with a detailed description of the channel sensing policies under consideration. Section 3 presents the probabilistic model of the sensing activity, while Section 3.3 models probabilities of incorrect channel information. Section 4 presents and discusses performance of these policies. Section 5 presents the rationale for the modified version of the probability modulation algorithm and demonstrates that such modification is indeed capable of further reduction in sensing error, albeit with some constraints. Finally, Section 6 concludes the paper and highlights some avenues for future research.

## 2. CPAN operation and sensing

Let us now describe the operation of a cognitive PAN in more detail. Consider a generic piconet that consists of a dedicated coordinator and a number of nodes. All communications within the piconet are performed under the control of the piconet coordinator, which is responsible for administrative tasks such as starting the piconet, admitting nodes to join the piconet, and monitoring and controlling the operation of the piconet.

We assume that the available time is partitioned into slots or superframes of fixed size. Each superframe is marked by a beacon frame emitted by the coordinator, and each superframe uses a different RF channel from the working frequency band. The sequence of channels (i.e., the hopping sequence) should be random, insofar as technically possible, but also adaptive, in the sense that any channel currently occupied by a primary user ought to be avoided. To that end, the piconet coordinator must maintain an accurate and timely channel map, i.e., the map of primary user activity on each of the  $N$  working channels, which is accomplished by instructing a certain number of secondary users to sense and report back the status of  $X$  distinct channels in each superframe. Upon receiving this information, the coordinator updates the channel map, selects the best channel for a subsequent frequency hop, and announces it in the beacon frame.

The sensing activity may be performed in parallel with the regular data communications, provided that the superframe is further subdivided into a number of specialized subframes, similar to the HRMA protocol described in [16] or the more recent IEEE 802.15.3 MAC protocol [7]. During the reservation

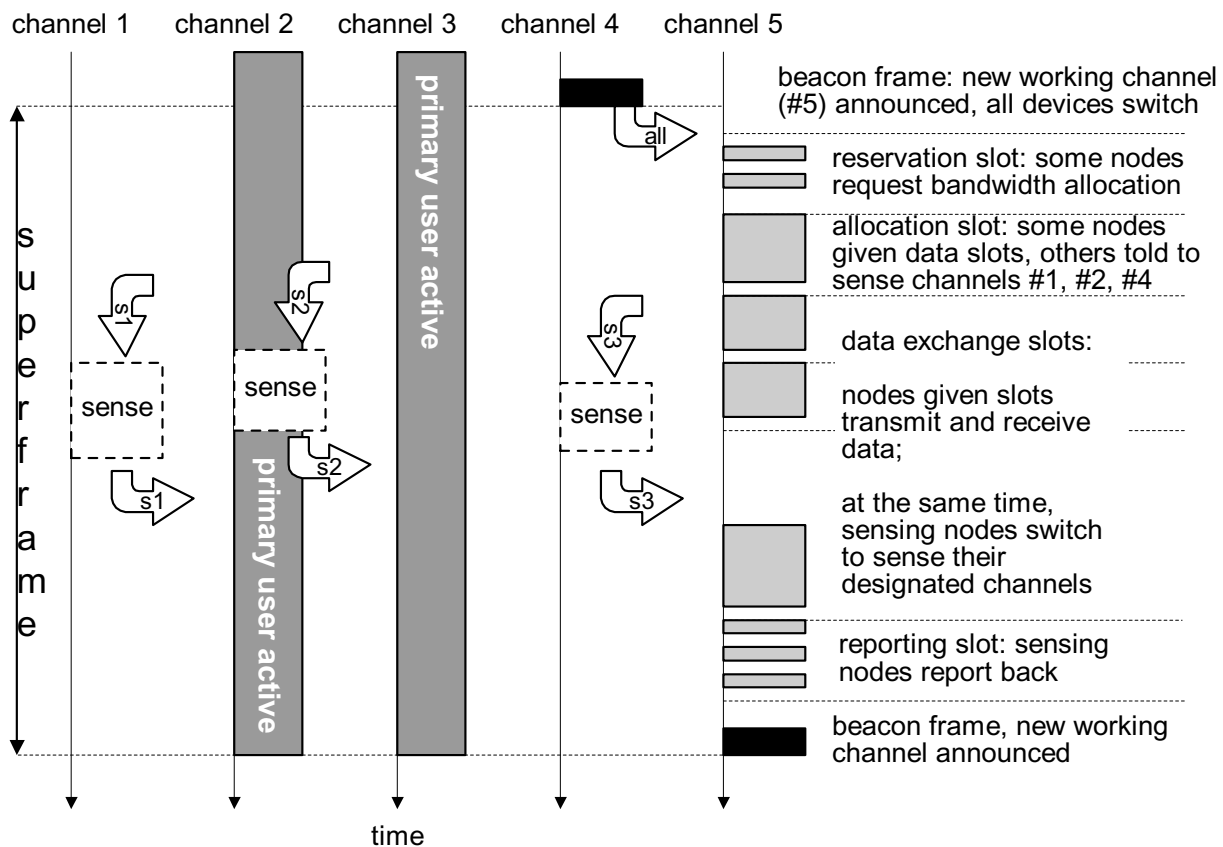


Fig. 1. Activities during a CPAN superframe.

sub-frame, any device that wants to transmit a packet (or packets) to another device will request a dedicated time slot. The coordinator will grant (or reject) this request, taking into account the amount of available resources, the traffic in the piconet, and other criteria such as fairness, performance, and the like; this information is sent to the interested parties during the allocation sub-frame, together with the sensing instructions. Actual packet transmissions take place in data exchange slots, during which the sensing nodes perform sensing and report back their findings in a separate slot that immediately precedes the next beacon frame. The structure of the superframe and various activities in the CPAN are schematically depicted in Fig. 1, while Fig. 2 shows how the results of sensing affect frequency hopping of a CPAN; in both cases, we assumed that the RF band has  $N = 5$  working channels, and that  $X = 3$  channels are sensed in each sensing cycle.

Ideally, the channel map should accurately reflect the state of the channels at any given time. However, the sensing process incurs errors due to the following. First, sensing is performed in discrete intervals and any change in the state of a channel can be detected only upon the next sensing event. Therefore, state changes in the channel map are always delayed with respect to the actual changes. Second, a sensing cycle may include some but not all channels, i.e.,  $N > X$ , which means that the information for those channels that were not sensed in the last cycle is by default obsolete.

As a result, the information in the channel map is only partially correct at any given time. The magnitude of this error will determine the success rate of CPAN transmissions and, ultimately, its quality

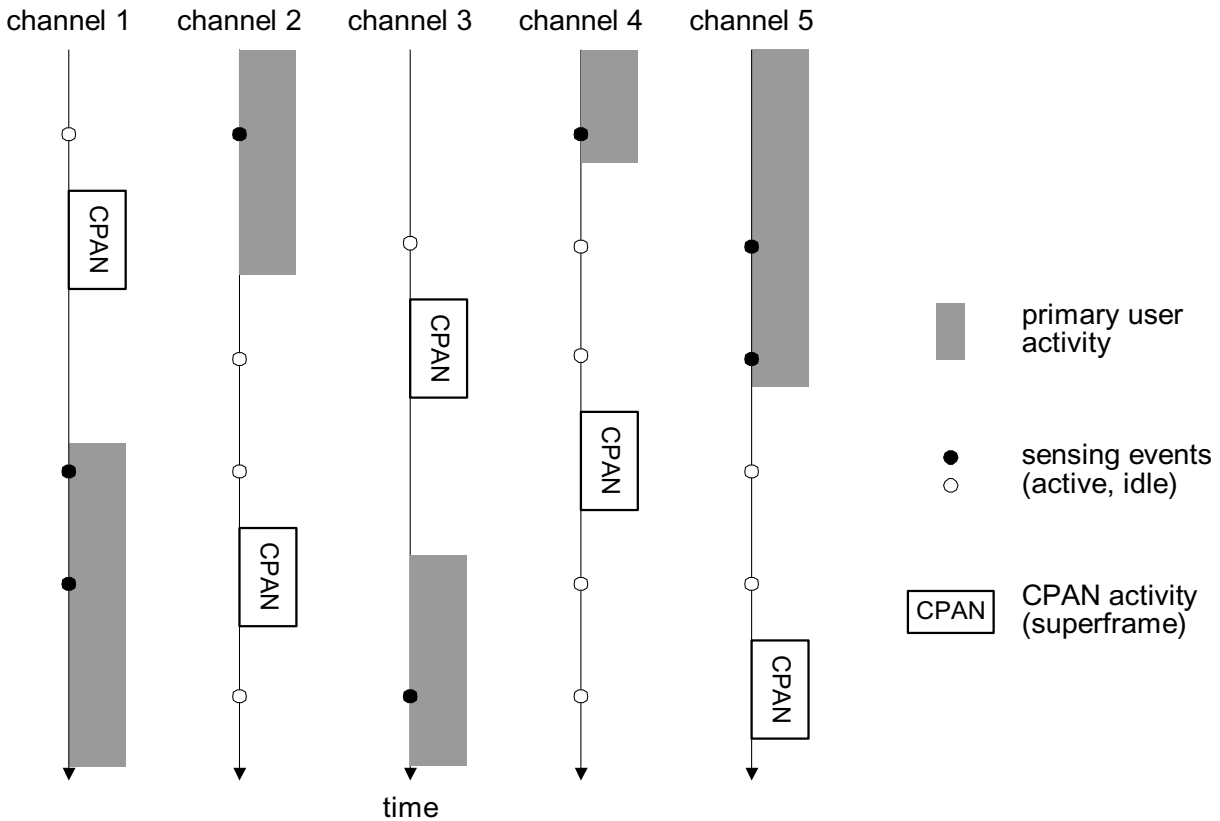


Fig. 2. CPAN frequency hopping.

of service. The error may be expressed through the mean number of channels with incorrect information and the delay in detecting changes in channel status. Error reduction may be accomplished by increasing the number of sensors or by reducing the number of working channels, whichever is more convenient.

The number of sensed channels  $X$  should ideally be equal to, or larger than, the number of channels  $N$  in the working set, so that the channel map may be kept accurate. As sensing is performed in discrete intervals, any change in the state of a channel can be detected only upon the next sensing event; therefore, state changes in the channel map are always delayed with respect to the actual changes.

When there are more channels than sensing nodes, i.e.,  $N > X$ , not every channel can be sensed in every sensing cycle, and the information in the channel map is only partially correct at any given time. The magnitude of this error will determine the success rate of CPAN transmissions and, ultimately, its quality of service. The error may be expressed through the mean number of channels with incorrect information and the delay in detecting changes in channel status. Error reduction may be accomplished by increasing the number of sensors (i.e., the number of channels which are sensed in each cycle) or by reducing the number of working channels, whichever is more convenient.

Not all errors have the same impact, though. A missed inactive channel is just a missed opportunity which is not too critical as long as there are other inactive channels available; a missed activity on a channel, on the other hand, can harm the CPAN operation if that channel has been selected for data transmission. In order to reduce the probability of errors of the latter kind, the delay in detecting the beginning of primary user activity (i.e., the end of a spectral opportunity) should be lower than the

corresponding delay in detecting its end (i.e., the beginning of a spectral opportunity). This may be accomplished by sensing inactive channels more frequently than the active ones. Note that, in this context, ‘inactive’ refers to channels which are marked as inactive in the channel map, since this is the only information available to a CPAN piconet and its coordinator.

Preferential treatment of inactive channels may be accomplished by modulating the sensing probability, i.e., by choosing the probability that an inactive channel is selected for sensing to be exactly  $w$  times higher than the corresponding probability for an active channel. By adjusting the value of  $w$  we can reduce or increase the mean sensing interval within certain limits and, thus, reduce the sensing error for inactive channels. The corresponding error for active channels will be changed in inverse proportion. Pseudocode for this algorithm is shown in Algorithm 1.

---

**Algorithm 1:** Probability modulation algorithm.

---

**Data:** number of channels  $N$  ;

probability modulation factor  $w$

**Result:** set of channel selection probabilities for the current sensing cycle  $p_s(i)$

1 **forall** *channels recorded as inactive* **do**

2 | assign selection probability of  $sp(i) = w$

3 **end**

4 **forall** *channels recorded as active* **do**

5 | assign selection probability of  $sp(i) = 1$

6 **end**

7 **forall** *channels in the channel map* **do**

8 | set selection probability for channel  $i$  to  $p_s(i) = sp(i) / \sum_1^N sp(i)$

9 **end**

10 randomly select  $X$  channels to be sensed amongst the  $N$  channels with probabilities  $p_s$ ;

---

We note that a number of papers deal with sensing issues in cognitive networks [10,15,17]. However, most of them discuss sensing from the perspective of the physical (PHY) layer of the protocol stack, and thus take ‘cooperation’ to mean the combination of sensing results from several sensing nodes in order to improve the confidence in those results, rather than in the sense used in this paper.

### 3. Analytical model of sensing with probability modulation

Let us now present a probabilistic model of the sensing process. Let the working spectrum band be partitioned into  $N$  narrowband RF channels (which we will refer to as the working channel set), and let each primary user be active on a distinct channel from that group. (The common control channel is excluded from this group.) Let us also denote relevant variables in the active and inactive periods of the channel state, with subscripts  $a$  and  $i$ , respectively, while subscripts  $r$  and  $o$  refer to real and observed values of the network parameters, respectively. In the analysis that follows, we assume that active and inactive times on each channel follow probability distributions with cumulative density functions  $T_{a,r}(x)$

and  $T_{i,r}(x)$ , and mean values of  $\overline{T_{a,r}}$  and  $\overline{T_{i,r}}$ , respectively. Actual mean value of the cycle time will be denoted as  $\overline{T_{cyc,r}} = \overline{T_{a,r}} + \overline{T_{i,r}}$ .

Each sensing node performs sensing in a frequency hopping manner with the normalized sensing period of  $T_s$ ; for convenience, we set  $T_s = 1$ . Assuming that  $T_s \ll \overline{T_a}, \overline{T_i}$ , the probability distributions of active and inactive times of primary users may be taken to be discrete, and their state changes can occur at the boundaries of the sensing period  $T_s$ . Let  $a$  and  $i$  be discrete random variables taking values on some subset of the non-negative integers. Then, the Probability Generating Functions (PGFs) for  $a$  and  $i$  can be defined as  $T_a(z) = \sum_{k=0}^{\infty} p_a(k)z^k$  and  $T_i(z) = \sum_{k=0}^{\infty} p_i(k)z^k$ , where  $p_a$  and  $p_i$  are the probability mass functions of  $a$  and  $i$ , respectively [14]. For example, if  $T_{a,r}$  and  $T_{i,r}$  are geometrically distributed with parameters  $\alpha$  and  $\beta$ , respectively, then  $p_a(k) = \alpha(1 - \alpha)^{k-1}$  and  $p_i(k) = \beta(1 - \beta)^{k-1}$ .

### 3.1. Sensing periods and residual sensing times

Let the mean observed durations of active and inactive period be  $\overline{T_{i,o}}$  and  $\overline{T_{a,o}}$ , respectively. The probability that a channel is inactive, as observed via the channel map, is  $p_{i,o} = \overline{T_{i,o}} / (\overline{T_{a,o}} + \overline{T_{i,o}})$ , while the observed probability that channel is occupied by the primary user is  $p_{a,o} = 1 - p_{i,o}$ . The mean observed numbers of inactive and active channels are  $\overline{N_{i,o}} = p_{i,o}N$  and  $\overline{N_{a,o}} = p_{a,o}N = N - \overline{N_{i,o}}$ , respectively. The probability that a channel is actually inactive is  $p_{i,r} = \overline{T_{i,r}} / (\overline{T_{a,r}} + \overline{T_{i,r}}) \neq p_{i,o}$ ; by the same token,  $p_{a,r} \neq p_{a,o}$ .

To model the probability modulation policy, let us introduce additional  $w - 1$  virtual channels for each inactive channel, to give a pool with a total of  $N_w$  channels with equal priority. The probability of selecting a channel from this pool is  $p_i = (w\overline{N_{i,o}}) / N_w$  and  $p_a = \overline{N_{a,o}} / N_w$  for inactive and active channels, respectively. The number of inactive and active channels to be sensed in one cycle are

$$\begin{aligned} X_i &= \sum_{k=1}^{\min(\overline{N_{i,o}}, X)} \binom{X}{k} k p_i^k (1 - p_i)^{X-k} \\ X_a &= \sum_{k=1}^{\min(\overline{N_{a,o}}, X)} \binom{X}{k} k p_a^k (1 - p_a)^{X-k} \end{aligned} \quad (1)$$

Since the number of sensors should not exceed the number of channels in each group, e.g.,  $X_i \leq \overline{N_{i,o}}$ , inactive channels will be given priority until all of them are selected for sensing. (No channel is assigned to two or more sensing nodes in any given sensing cycle.) The sensing probabilities become

$$\begin{aligned} P_{w,i} &= \begin{cases} \frac{X_i}{\overline{N_{i,o}}}, & X_i < \overline{N_{i,o}} \\ 1, & X_i \geq \overline{N_{i,o}} \end{cases} \\ P_{w,a} &= \begin{cases} \frac{X_a}{\overline{N_{a,o}}}, & X_a < \overline{N_{a,o}} \\ 1, & X_a \geq \overline{N_{a,o}} \end{cases} \end{aligned} \quad (2)$$

The last line is given for completeness only, since under normal circumstances  $X < N$  and  $P_{w,a}$  should not reach 1.

The time between two consecutive sensing events has a geometric distribution. As probabilities of selecting inactive and active channels differ, so do the appropriate PGFs, which are

$$\begin{aligned}
 B_i(z) &= \begin{cases} \sum_{i=1}^{\infty} P_{w,i}(1 - P_{w,i})^{i-1} z^i = \sum_{j=1}^{\infty} b_{i,j} z^j, & X_i < \overline{N_{i,o}} \\ z, & X_i \geq \overline{N_{i,o}} \end{cases} \\
 B_a(z) &= \begin{cases} \sum_{i=1}^{\infty} P_{w,a}(1 - P_{w,a})^{i-1} z^i = \sum_{j=1}^{\infty} b_{a,j} z^j, & X_a < \overline{N_{a,o}} \\ z, & X_a \geq \overline{N_{a,o}} \end{cases}
 \end{aligned} \tag{3}$$

for inactive and active channels, respectively. The mean values of these times are  $\overline{b}_i = \lim_{z \rightarrow 1} \partial B_i(z) / \partial z = B'_i(1)$ , for inactive channels, and  $\overline{b}_a = \lim_{z \rightarrow 1} \partial B_a(z) / \partial z = B'_a(1)$ , for active ones.

The time between the actual change of channel state and the moment when it is detected through sensing can be found as follows. The sensing process for a single channel may be considered as a discrete-time renewal process where sensing events correspond to renewal points, while the renewal time corresponds to the period between two consecutive sensing events [5]. In terms of renewal theory, the time between the channel state change and the next sensing event is denoted as the residual life (time) or forward recurrence time; note that the channel state change can occur anywhere between two sensing points (which take place on the basic sensing slot boundary). This residual time defines the time period in which the where information about the channel state might differ from the actual channel state. Let us consider a transition from inactive to active state, and a random edge  $j$  of the sensing slot within a sensing period. The probability that the next sensing event will occur exactly  $k$  basic sensing slots later is  $b_{i,j+k}/b_i$ . By summing over all basic sensing slots within the sensing period, the probability that the residual sensing time is  $k$  basic sensing slots is obtained as

$$R_{i,k} = \sum_{j=1}^{\infty} \frac{b_{i,j+k}}{b_i} = \sum_{j=k+1}^{\infty} \frac{b_{i,j}}{b_i} \tag{4}$$

Note that the first subscript of  $R_{i,k}$  denotes that the channel is inactive, whereas the second one corresponds to the index within the sequence. Then, PGF for the residual sensing time and its mean values are

$$\begin{aligned}
 R_i(z) &= \sum_{k=0}^{\infty} R_{i,k} z^k = \frac{1 - B_i(z)}{\overline{b}_i(1 - z)} \\
 \overline{R}_i &= B''_i(1) / (2B'_i(1))
 \end{aligned} \tag{5}$$

where  $B''_i(1) = \lim_{z \rightarrow 1} \frac{\partial^2 B_i(z)}{\partial z^2}$ . In a similar fashion, PGF for the distribution of the residual sensing time for active-to-inactive transitions and its mean value are obtained as

$$\begin{aligned}
 R_a(z) &= \sum_{k=0}^{\infty} R_{a,k} z^k = \frac{1 - B_a(z)}{\overline{b}_a(1 - z)} \\
 \overline{R}_a &= B''_a(1) / (2B'_a(1))
 \end{aligned} \tag{6}$$

### 3.2. Observed durations of inactive and active periods

Let us now find the probability distribution of the observed duration of active and inactive period of a channel. As noted above, the observed durations of inactive and active channel periods will be affected by delays in detecting the channel state changes; if the residual sensing time is larger than the period under observation (inactive or active), an entire (in)activity period may be skipped. Given the PGFs for residual sensing times, the probability that an entire period is skipped is

$$p_{s,i} = \sum_{k=2}^{\infty} R_{a,k} \sum_{n=1}^{k-1} p_i(n) = \sum_{n=1}^{\infty} p_i(n) \sum_{k=n+1}^{\infty} R_{a,k} \quad (7)$$

$$p_{s,a} = \sum_{k=2}^{\infty} R_{i,k} \sum_{n=1}^{k-1} p_a(n) = \sum_{n=1}^{\infty} p_a(n) \sum_{k=n+1}^{\infty} R_{i,k} \quad (8)$$

where the second form in both equations has been obtained from the first by exchanging the order of summation of series.

The probability distributions of the duration of inactive and active periods, conditioned on the event that they are not skipped by the sensing, i.e., that at least one sensing event occurs during the inactive or active period, are

$$\begin{aligned} \Theta_i(z)(1 - p_{s,i}) &= \sum_{n=1}^{\infty} p_i(n) z^n \sum_{k=0}^n R_{a,k} \\ \Theta_a(z)(1 - p_{s,a}) &= \sum_{n=1}^{\infty} p_a(n) z^n \sum_{k=0}^n R_{i,k} \end{aligned} \quad (9)$$

From expressions (7) and (8) it follows that  $1 - p_{s,i} = \sum_{n=1}^{\infty} p_i(n) \sum_{k=0}^n R_{a,k}$  and  $1 - p_{s,a} = \sum_{n=1}^{\infty} p_a(n) \sum_{k=0}^n R_{i,k}$ . Therefore,  $\Theta_i(1) = 1$  and  $\Theta_a(1) = 1$ , which shows that  $\Theta_i(z)$  and  $\Theta_a(z)$  are indeed probability generating functions.

Finally, we can derive the expressions for the probability distribution of observed values of inactive and active periods on the channel. The PGF for the duration of inactive period is

$$T_{i,o}(z) = \Theta_i(z) \left( (1 - p_{s,a}) + (1 - p_{sa}) p_{s,a} z^{\overline{T_{cyc,r}}} + (1 - p_{sa}) (p_{s,a} z^{\overline{T_{cyc,r}}})^2 + \dots \right) R_i(z) \quad (10)$$

After summing the infinite series we obtain

$$T_{i,o}(z) = \frac{\Theta_i(z)(1 - p_{s,a})}{1 - p_{s,a} z^{\overline{T_{cyc,r}}}} R_i(z) \quad (11)$$

Since  $\Theta_i(1) = 1$ , it follows that  $T_{i,o}(1) = 1$ , which shows that  $T_{i,o}(z)$  is indeed the probability generating function and its mean value is

$$\overline{T_{i,o}} = T'_{i,o}(1) \quad (12)$$



In a similar way, we obtain

$$T_{i,a}(z) = \frac{\Theta_a(z)(1 - p_{s,i})}{1 - p_{s,i}z^{\overline{T}_{cyc,r}}} R_a(z) \quad (13)$$

with mean value of  $\overline{T_{a,o}} = T'_{a,o}(1) + \overline{R_a}$ .

Equations (2), (5), (6), (7), (8), (9), (11) and (13), allow us to find the values of  $p_{i,o}$ ,  $p_{i,a}$ ,  $P_{w,i}$ ,  $P_{w,a}$ ,  $\overline{T_{i,o}}$ ,  $\overline{T_{a,o}}$ , and  $\overline{R_i}$ ,  $\overline{R_a}$ . Furthermore, mean detection delays are  $(1 - p_{s,i})\overline{R_a}$  and  $(1 - p_{s,a})\overline{R_i}$ , for detection of inactive and active channel state, respectively.

### 3.3. Probability of incorrect information about channel activity

All sensing points within a single active period of a primary user will find the channel active, and this information will be recorded in the channel map. Once the primary user ends its activity, this information becomes incorrect; it will become correct again upon the next sensing point which occurs, on the average,  $\overline{R_a}$  basic sensing slots after the first state transition of the primary channel. If the probability that the primary user will have a single transition from active to inactive state between two successive sensing events is  $P_a^{(1)} = \sum_{n=0}^{\infty} R_{a,n} \sum_{k=n+1}^{\infty} p_i(k)$ , the joint PGF for the duration of residual sensing time and inactive time of the primary user may be expressed as

$$G_a^{(1)}(z, y) = \frac{1}{P_a^{(1)}} \sum_{n=0}^{\infty} R_{a,n} z^n \sum_{k=n+1}^{\infty} p_i(k) y^k \quad (14)$$

and the time interval during which the channel state information in the channel map is incorrectly considered active is

$$L_a = P_a^{(1)} \lim_{z \rightarrow 1} \lim_{y \rightarrow 1} \frac{\partial G_a^{(1)}(z, y)}{\partial z} + \epsilon \quad (15)$$

where the error  $\epsilon$  reflects the possibility of incorrect information caused by multiple state transitions. The difference may easily be obtained but the detailed derivation, which can be found in [11], is not shown here due to limited space.

The probability that the channel is incorrectly observed to be in the active (inactive) state is  $a_1 = L_a/\overline{T_{a,o}}$  and  $b_1 = L_i/\overline{T_{i,o}}$ , respectively, with the complementary probabilities (of correct observed state) of  $a_0 = 1 - a_1$  and  $b_0 = 1 - b_1$ , respectively.

Given that the primary user is inactive or active with the probabilities  $p_{i,r} = \overline{T_{i,r}} / (\overline{T_{a,r}} + \overline{T_{i,r}})$  and  $p_{a,r} = 1 - p_{i,r}$ , respectively, the number of channels with obsolete information can be characterized with the PGF of

$$E(w, z, y) = \sum_{k=0}^N \sum_{i=0}^k \sum_{j=0}^{N-k} \binom{N}{k} p_{a,r}^k (1 - p_{a,r})^{N-k} w^k \cdot \binom{k}{i} a_1^i (1 - a_1)^{k-i} z^i \binom{N-k}{j} b_1^j (1 - b_1)^{N-k-j} y^j \quad (16)$$

Mean number of channels for which the information of being in the active or inactive state (obtained earlier) is obsolete, is  $E_a = \frac{\partial E(1, 1, 1)}{\partial z}$  and  $E_i = \frac{\partial E(1, 1, 1)}{\partial y}$ , for active and inactive channels, respectively. Finally, mean number of observed active and inactive channels can, then, be obtained as  $N_{a,o} = p_{a,o}N$  and  $N_{a,i} = p_{i,o}N$ , respectively.

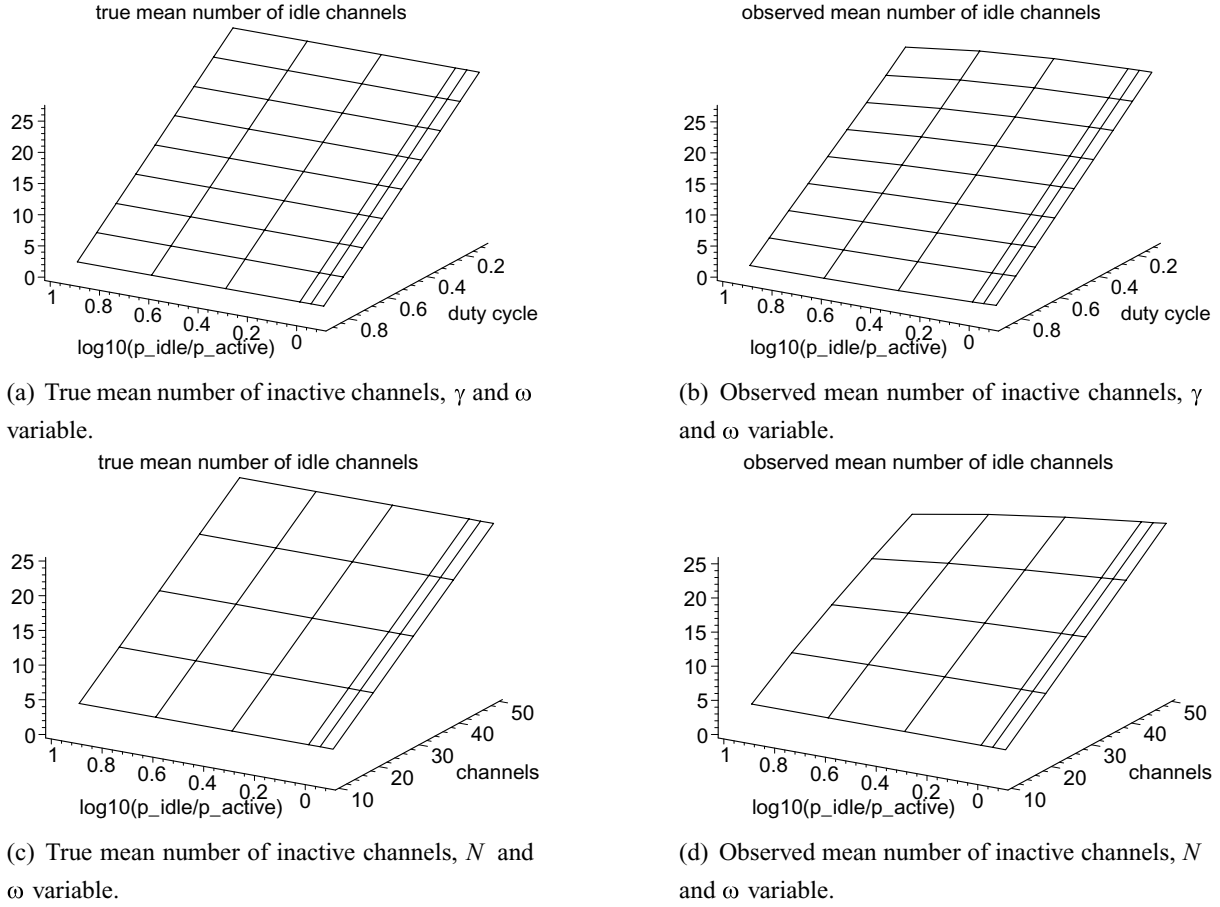


Fig. 3. Accuracy of the sensing process. Top row: results under fixed  $N = 30$ ; bottom row: results under fixed  $\gamma = 0.5$ .

#### 4. Performance evaluation

Let us now investigate the practical meaning of the formulae derived above. To that end, let us assume that the distribution of active and inactive periods of primary user activity is geometric, i.e.,  $T_{a,r}(z) = \sum_{k=1}^{\infty} \alpha(1 - \alpha)^{k-1} z^k$  and  $T_{i,r}(z) = \sum_{k=0}^{\infty} \beta(1 - \beta)^{k-1} z^k$ , with parameters  $\alpha = 1/\overline{T_{a,r}}$  and  $\beta = 1/\overline{T_{i,r}}$ . With these values, we have solved the equations shown above to calculate a number of performance measures, including the mean sensing interval, mean number of channels with incorrect (obsolete) information in the channel map, and mean duration (lifetime) of that information, for both inactive and active channels. Our calculations were done using Maple 11 from Maplesoft, Inc. [9], truncating the infinite series for various PGFs after 500 members. All time variables are expressed in units of basic sensing period  $T_s$ . The default parameter values were  $X = 10$  channels sensed in each sensing cycle,  $N = 30$  channels with mean activity/inactivity periods of  $T_{on} = T_{off} = 50$  basic sensing periods (resulting in mean activity period of 100 basic sensing periods, and duty cycle of  $\gamma = 0.5$ ). Different experiments were performed in which the independent variables were the number of channels and duty cycle, together with the modulation factor  $w$ .

The performance of the sensing process is presented in Fig. 3 where the true mean number of inactive channels is compared to the mean number of such channels as observed through sensing and recorded

in the channel map. The modulation factor values were 0.8, 0.9, 1.0, 2.0, 4.0, and 8.0, with the value of 1.0 corresponding to uniform random sensing; for clearer presentation, we used a logarithmic scale for  $w$ . As can be seen, the sensing mechanism that uses probability modulation is capable of maintaining an accurate record of inactive channels in a wide range of values of  $W$ , although the accuracy at large values of  $w$  deteriorates somewhat with the increase of the number of channels.

Further insight into the performance of the sensing process may be obtained from the diagrams in Fig. 4. As can be seen from Fig. 4(a) and 4(b), the increase in the modulation factor  $w$  leads to a sharp decrease in sensing error related to active channels mistakenly thought to be inactive; this was the initial motivation for introducing the probability modulation in the first place. The price to pay for this improvement is a slight increase in the complementary error, i.e., the number of active channels mistakenly thought to be inactive, as shown in Fig. 4(c) and 4(d). This increase contributes to an overall increase in the total error, Fig. 4(e) and 4(f).

As could be expected, all errors increase when the number of working channels increases, and, consequently, the ratio of the number of channels sensed to the total number of channels decreases. This pattern can be countered by increasing the number of channels sensed in each sensing cycle or, if that is not possible, by reducing the number of working channels. The diagrams in Fig. 4(b), (4d), 5(d), and 5(b), vividly demonstrate the feasibility of the latter approach.

Another measure of sensing accuracy is the delay in detecting a change in activity on the channel, the results of which are shown in Fig. 5. Not unexpectedly, these diagrams closely resemble their counterparts in Fig. 4, since the main source of errors in channel sensing is the delay in detecting the change in channel activity. However, the main effect of increasing the modulation factor  $w$  – the improved accuracy of detecting the beginning of primary user activity on a channel – is clearly achieved, as can be seen from Fig. 5(a) and 5(b).

## 5. Should modulation account for aging of sensing information?

The probability modulation mechanism outlined above changes the relative probability that a channel will be selected according to the channel status recorded in the channel map. This selection process does not take into account the age of that information. As a result, an inactive (active) channel that has been sensed in the previous sensing cycle will have the same probability of being selected as another inactive (active) channel that has not been sensed for many sensing cycles. However, the longer a channel is not sensed, the higher is the probability that it has changed state since; therefore, channels that have not been sensed for longer time should have higher probability of being selected.

To this end, we have built a simulator of a cognitive CPAN that uses the modified probability modulation algorithm, the pseudocode of which is given in Algorithm 2. The simulator was built using the object-oriented Petri Net simulation engine Artifex by RSoft Design, Inc. [13]. The results obtained in this manner are shown in Figs 6 and 7 as ratios of respective error metrics with and without sorting, as was the case with the original algorithm. For clarity, all values below one are shown in shades of gray while values above one are shown in purple/black.

In the experiment with fixed duty cycle of primary sources of  $\gamma = 0.5$ , shown in Fig. 6, the improvement obtained by the modified algorithm is not too big and rarely goes below 0.8 of the original error (i.e., the modified algorithm does not offer more than 20% improvement over the original one). On the bright side, however, improvement may be observed at virtually all values of modulation factor  $w$ , except in a limited range where the number of channels is between  $N = 10$  and 20, and the modulation factor is above  $w = 4$ .

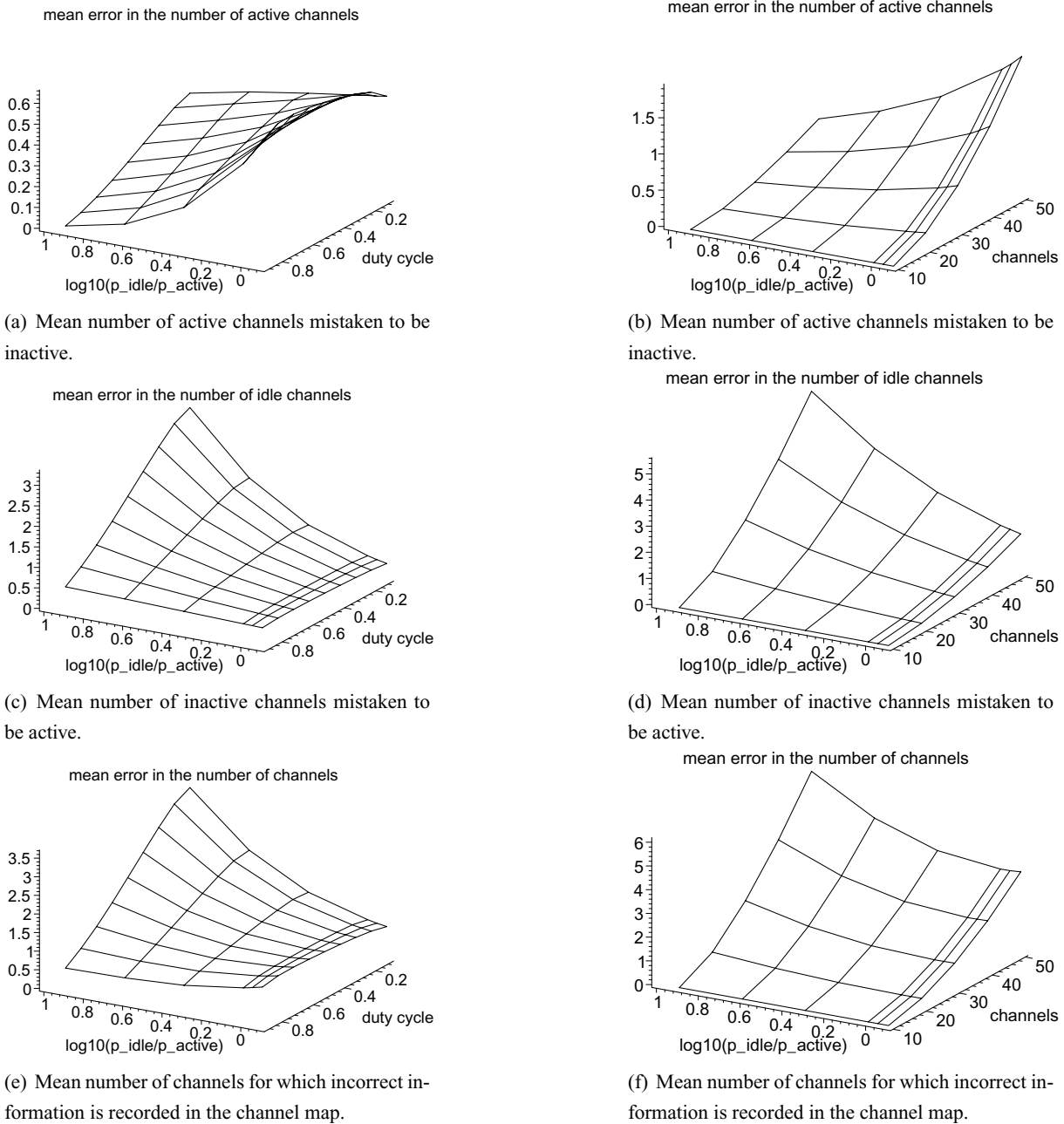
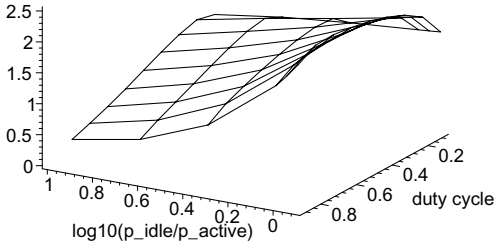


Fig. 4. Sensing error expressed through the number of channels for which the information in the channel map is incorrect. On the left, results under fixed  $N = 30$ ; on the right, results under fixed  $\gamma = 0.5$ .

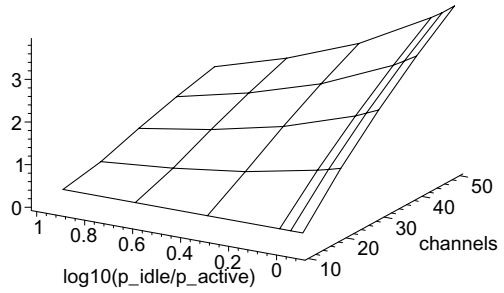
Under fixed number of channels and variable primary duty cycle, Fig. 7, results are more varied. (Note that diagrams on the left and on the right have their coordinate axes oriented in different ways, for reasons of clarity.) In general, we may conclude that the modified probability modulation algorithm does lead to improvements in the detection delay for the beginning of channel activity, as was our original objective,

mean delay in detecting beginning of channel activity



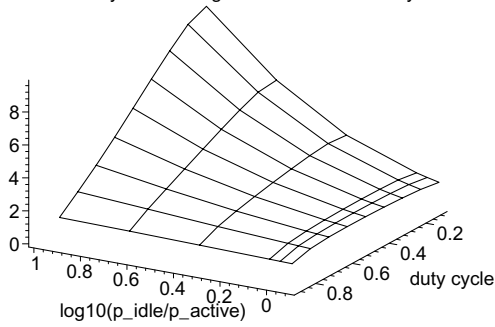
(a) Mean detection delay for beginning of activity on a channel.

mean delay in detecting beginning of channel activity



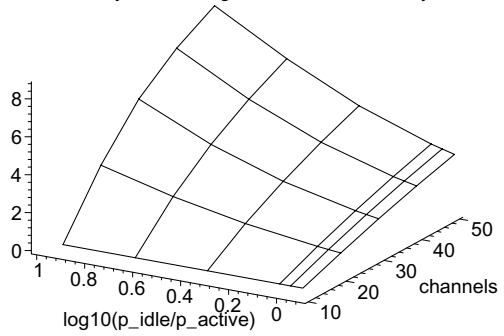
(b) Mean detection delay for beginning of activity on a channel.

mean delay in detecting end of channel activity



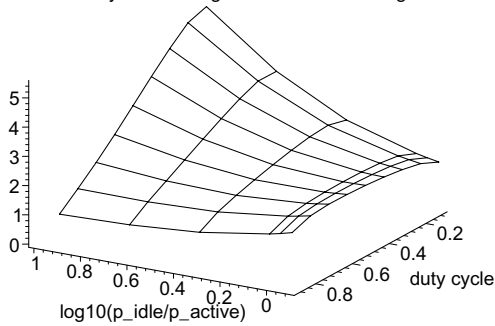
(c) Mean detection delay for end of activity on a channel.

mean delay in detecting end of channel activity



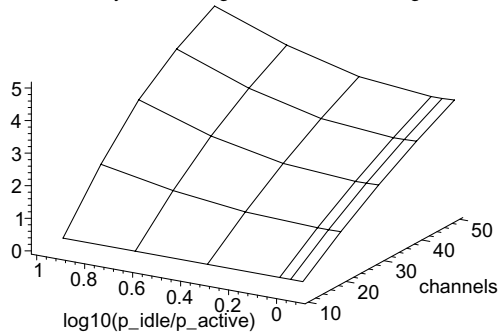
(d) Mean detection delay for end of activity on a channel.

mean delay in detecting channel status change



(e) Mean detection delay for change of activity on a channel.

mean delay in detecting channel status change



(f) Mean detection delay for change of activity on a channel.

Fig. 5. Sensing error expressed through the delay in detecting a change in channel status. On the left, results under fixed  $N = 30$ ; on the right, results under fixed  $\gamma = 0.5$ .

at values of  $\gamma$  below 0.7 or so. Similar to the original algorithm, this improvement is achieved at the expense of the detection delay for the end of the channel activity, which increases by as much as four times at low duty cycles below 0.3 to 0.35 range. The range in which the modified algorithm offers improvement in terms of the number of channels erroneously thought to be active is somewhat smaller,

---

**Algorithm 2:** Modified probability modulation algorithm.

---

**Data:** number of channels  $N$  ;  
number of inactive channels in the channel map  $n_{i,o}$  ;  
number of active channels in the channel map  $n_{a,o} = N - n_{i,o}$  ;  
probability modulation factor  $w$

**Result:** set of channel selection probabilities for the current sensing cycle  $p_s(i)$

- 1 re-order inactive channels by ascending time of last sensing ;
- 2 **for**  $i=1$  **to**  $n_{i,o}$  **do** set  $sp(i) = w * (n_{i,o} - i + 1)$  ;
- 3 re-order active channels by ascending time of last sensing ;
- 4 **for**  $j=1$  **to**  $n_{a,o}$  **do** set  $sp(j) = n_{a,o} - j + 1$  ;
- 5 **forall** channels in the channel map **do**
- 6 | set selection probability for channel  $i$  to  $p_s(i) = sp(i) / \sum_1^N sp(i)$
- 7 **end**
- 8 randomly select  $X$  channels to be sensed amongst the  $N$  channels with probabilities  $p_s$  ;

---

approximately at values of  $\gamma$  below 0.6 or so. However, at the high end of the observed range of duty cycle values, say, at  $\gamma = 0.85$  and above, the error of the modified algorithm is more than twice the corresponding value incurred by the original algorithm.

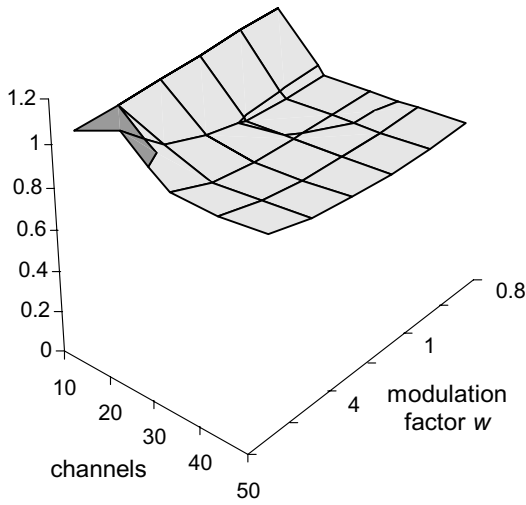
Conclusions that can be made from these results could be summarized as follows. At low values of the duty cycle  $\gamma$ , where the transmission opportunities run aplenty, one should use the modified algorithm. As the duty cycle approaches the value of  $\gamma = 0.6$  to  $0.65$ , it is best to revert to the original probability modulation algorithm where the errors will be more balanced. As the difference between the algorithms consist of dynamically changing the relative probabilities of channel selection, an adaptive procedure to incorporate both the original and the modified algorithm should not be difficult to implement.

## 6. Conclusion

In this paper, we have presented a simple algorithm that dynamically alters the relative probability of channel sensing in order to reduce the sensing error in cognitive CPANs. We have presented an analytical framework that allows accurate analysis of the performance of this algorithm, and we have shown that it is indeed effective in reducing the sensing error. We have also presented a simple modification that allows for even greater reduction in sensing error, albeit in a limited range of values of the duty cycle of primary users' activity.

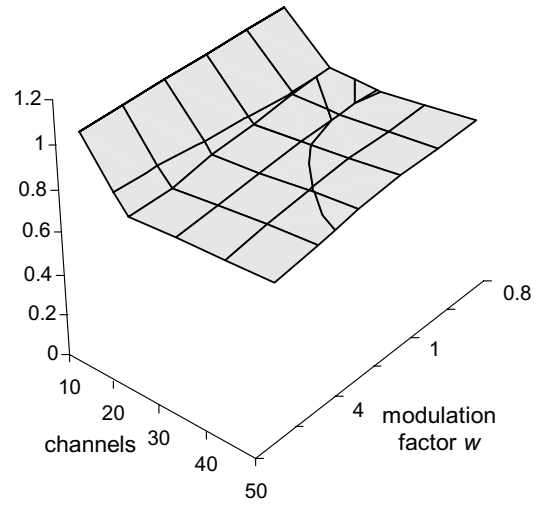
The analytical framework presented in this paper may be used to design cognitive piconets with a given threshold for sensing error, be it through the number of channels or the detection delay, or maybe even both. Our results confirm that accurate channel sensing is feasible even when the number of sensing nodes is well below the number of available channels, and that modulation of sensing probability may help reduce the sensing error with respect to inactive channels to almost any desired value; the price to pay for this is the decrease in accuracy with respect to active channels.

error ratio re: number of active channels



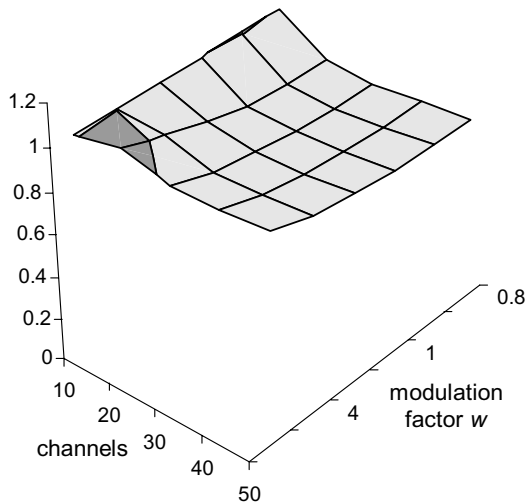
(a) Error ratio re: number of active channels mistaken to be inactive.

error ratio re: number of inactive channels



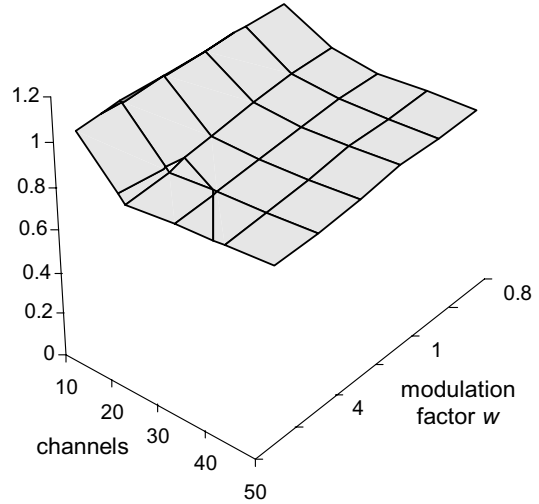
(b) Error ratio re: number of inactive channels mistaken to be active.

delay ratio re: beginning of channel activity



(c) Delay ration re: detecting the beginning of channel activity.

delay ratio re: end of channel activity



(d) Delay ration re: detecting the end of channel activity.

Fig. 6. Improvements obtained by the modified probability modulation algorithm. Duty cycle of primary sources is fixed at  $\gamma = 0.5$ .

In future work, we plan to address imperfect transmission of sensing results, learning and adapting to the dynamics of primary users, and the design, analysis, and optimization of a MAC protocol suitable for frequency-hopping CPANs.

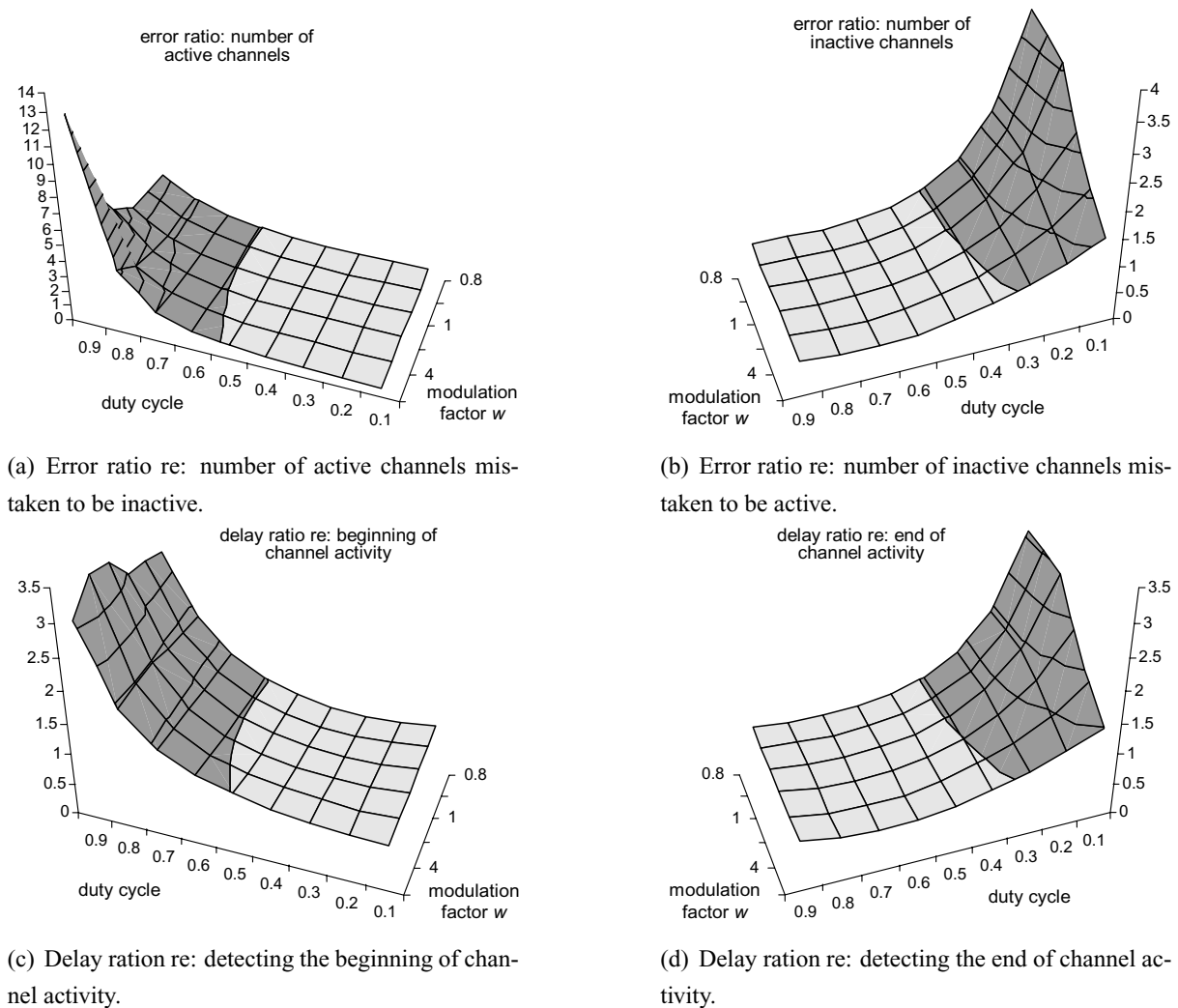


Fig. 7. Improvements obtained by the modified probability modulation algorithm. Number of working channels is fixed at  $N = 30$ .

## References

- [1] I.F. Akyildiz, W.-Y. Lee, M.C. Vuran and S. Mohanty, Next generation/dynamic spectrum access/cognitive radio wireless networks: A survey, *Computer Networks* **50** (2006), 2127–2159.
- [2] Bluetooth SIG, *Core Specification of the Bluetooth System*, Version 2.0 + EDR, November 2004.
- [3] C.-T. Chou, N. Sai Shankar, H. Kim and K.G. Shin, What and how much to gain by spectrum agility? *IEEE Journal on Special Areas in Communications – Wireless Series* **25**(3) (April 2007), 576–588.
- [4] S. Geirhofer, L. Tong and B.M. Sadler, Cognitive medium access: A protocol for enhancing coexistence in WLAN bands, in: *Proceedings Global Telecommunications Conference GLOBECOM'07*, Washington, DC, November 2007.
- [5] D.P. Heyman and M.J. Sobel, *Stochastic Models in Operations Research, Volume I: Stochastic Processes and Operating Characteristics*, McGraw-Hill, New York, 1982.
- [6] W. Hu and E. Sofer, IEEE 802.22 wireless RAN standard PHY and MAC proposal, technical proposal submitted to IEEE 802.22 WG, 22-05-0098-00-0000, IEEE 802.22 WG, 2005.
- [7] IEEE, Wireless MAC and PHY specifications for high rate WPAN. IEEE Std 802.15.3, IEEE, New York, NY, 2003.



- [8] P.K. Lee, Joint frequency hopping and adaptive spectrum exploitation, in: *IEEE Military Communications Conference MILCOM2001*, (Vol. 1), October 2001, pp. 566–570.
- [9] Maplesoft, Inc. *Maple 11*, Waterloo, ON, Canada, 2007.
- [10] S. Mishra, A. Sahai and R. Brodersen, Cooperative sensing among cognitive radios, in: *Proceedings of IEEE International Conference on Communications ICC 2006*, (Vol. 4), Istanbul, Turkey, June 2006, pp. 1658–1663.
- [11] J. Mišić and V.B. Mišić, The impact of inaccurate sensing information in cognitive wireless personal area networks, in: *IEEE CogNet 2008*, Beijing, China, May 2008.
- [12] V.B. Mišić and J. Mišić, Performance of incomplete channel sensing for frequency hopping cognitive personal area networks, in: *WWASN2008*, Beijing, China, June 2008.
- [13] RSoft Design, *Artifex v.4.4.2*. RSoft Design Group, Inc., San Jose, CA, 2003.
- [14] H. Takagi, *Queueing Analysis*, volume 1: Vacation and Priority Systems. North-Holland, Amsterdam, The Netherlands, 1991.
- [15] D. Čabrić, S.M. Mishra, D. Willkomm, R. Brodersen and A. Wolisz, A cognitive radio approach for usage of virtual unlicensed spectrum, in: *Proc 14th IST Mobile Wireless Communications Summit*, Dresden, Germany, June 2005.
- [16] Z. Yang and J. Garcia-Luna-Aceves. Hop-reservation multiple access (HRMA) for ad-hoc networks. In *Eighteenth Annual Joint Conference of the IEEE Computer and Communications Societies INFOCOM'99*, volume 1, pages 194–201, New York, NY, March 1999.
- [17] J. Zhao, H. Zheng and G.-H. Yang. Spectrum sharing through distributed coordination in dynamic spectrum access networks. *Wireless Communications and Mobile Computing*, in press.

---

**Jelena Mišić** received her PhD degree in Computer Engineering from University of Belgrade, Serbia, in 1993. She is currently Associate Professor of Computer Science at the University of Manitoba in Winnipeg, Manitoba, Canada. Her current research interests include wireless networks and security in wireless networks. She is a Senior Member of IEEE and Member of ACM.

**Vojislav B. Mišić** received his PhD in Computer Science from University of Belgrade, Serbia, in 1993. He is currently Associate Professor of Computer Science at the University of Manitoba in Winnipeg, Manitoba, Canada. His research interests include modeling and performance evaluation of wireless networks, and systems and software engineering. He is a Senior Member of IEEE, and member of ACM and AIS.



**Hindawi**

Submit your manuscripts at  
<http://www.hindawi.com>

