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Research Article

Some Theorems about $(\psi, \varphi, \epsilon, \lambda)$-Contraction in Fuzzy Metric Spaces

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We previously proved a number of fixed point theorems for some kinds of contractions like b_n-contraction and $\varphi - H$ contraction in fuzzy metric spaces. In this paper, we discuss the problem of existence of fixed point for $(\psi, \varphi, \epsilon, \lambda)$-contraction in fuzzy metric spaces in sense of George and Veeramani.

1. Introduction

Probabilistic metric spaces are generalizations of metric spaces which have been introduced by Menger [1]. George and Veeramani [2] modified the concept of the fuzzy metric spaces, which were introduced by Kramosil and Michalek [3]. Fixed point theory for contraction type mappings in fuzzy metric spaces is closely related to the fixed point theory for the same type of mappings in probabilistic metric spaces. Hicks introduced C-contraction [4]. Radu in [5] extended C-contraction to the generalized C-contraction. Mihet in [6] presented the notion of a q-contraction of (ϵ, λ)-type. He also introduced the class of $(\psi, \varphi, \epsilon, \lambda)$-contraction in fuzzy metric spaces [7] which is a generalization of the (ϵ, λ)-contraction [8]. Ciric also proved some new results for Banach contractions and Edelstin contractive mappings on fuzzy metric spaces [9]. We obtained the fixed point for $(\psi, \varphi, \epsilon, \lambda)$-contraction in probabilistic metric spaces and we introduced the generalized $(\psi, \varphi, \epsilon, \lambda)$-contraction too [10]. The outline of the paper is as follows. In Section 2, we briefly recall some basic concepts. In Section 3, two fixed point theorems about $(\psi, \varphi, \epsilon, \lambda)$-contractions are proved.

2. Preliminary Notes

Since the definitions of the notion of fuzzy metric space are closely related to the definition of generalized Menger spaces, we review a number of definitions from probabilistic metric space theory used in this paper as an example and Schweizer and Sklar's definition can be considered. For more details, we refer the reader to [11, 12].

A mapping $F : (-\infty, \infty) \to [0, 1]$ is called a distance distribution function if it is nondecreasing and left continuous with $F(0) = 0$. The class of all distance distribution functions is denoted by Δ_+. A probabilistic metric space is an ordered pair (X, F) where X is a nonempty set and F is a mapping from $X \times X$ to Δ_+. The value of F at $(p, q) \in X \times X$ is denoted by $F_{pq}(\cdot)$ satisfying the following conditions:

(1) $\forall x \geq 0,\ F_{pq}(x) = 1$ iff $p = q$.

(2) $\forall p, q \in X,\ \forall x \geq 0,\ F_{pq}(x) = F_{qp}(x)$.

(3) If $F_{pq}(x) = 1$, $F_{qr}(y) = 1$ then $F_{pr}(x + y) = 1$ for all $p, q, r \in X$ and for all $x, y \geq 0$.

A binary operation $T : [0, 1] \times [0, 1] \to [0, 1]$ is called a triangular norm (abbreviated t-norm) if the following conditions are satisfied:

(1) $T(a, 1) = a$ for all $a \in [0, 1]$.

(2) $T(a, b) = T(b, a)$ for every $a, b \in [0, 1]$.

(3) $a \geq b,\ c \geq d \Rightarrow T(a, c) \geq T(b, d)$ for all $a, b, c \in [0, 1]$.

(4) $T(T(a, b), c) = T(a, T(b, c))$ for every $a, b, c \in [0, 1]$.

Definition 1. A Menger space is a triple (X, F, T) where (X, F) is a probabilistic metric space; T is a t-norm and the following inequality holds:

$$F_{pr}(x+y) \geq T\left(F_{pq}(x), F_{qr}(y)\right) \quad \forall p, q, r \in X,\ \forall x, y \geq 0. \tag{1}$$

If (X, F, T) is a Menger space with T satisfying $\sup_{0 \leq a < 1} T(a, a) = 1$, then the family $\{U_{\epsilon,\lambda}\ \epsilon > 0,\ \lambda \in (0,1)\}$, where $U_{\epsilon,\lambda} = \{(x, y) \in X \times X,\ F_{x,y}(\epsilon) > 1 - \lambda\}$, is a base for a uniformity on X and is called the F-uniformity on X. A topology on X is determined by this F-uniformity, $(\epsilon - \lambda)$-topology.

Definition 2. A sequence $(x_n)_{n\in\mathbf{N}}$ is called an F-convergent sequence to $x \in X$ if, for all $\epsilon > 0$ and $\lambda \in (0, 1)$, there exists $N = N(\epsilon, \lambda) \in \mathbf{N}$ such that $F_{x_n x}(\epsilon) > 1 - \lambda$, for all $n \geq N$.

Definition 3. A sequence $(x_n)_{n\in\mathbf{N}}$ is called an F-Cauchy sequence if, for all $\epsilon > 0$ and $\lambda \in (0, 1)$, there exists $N = N(\epsilon, \lambda) \in \mathbf{N}$ such that $F_{x_n x_{n+m}}(\epsilon) > 1 - \lambda$, for all $n \geq N$, for all $m \in \mathbf{N}$.

A probabilistic metric space (X, F, T) is called F-sequentially complete if every F-Cauchy sequence is F-convergent.

It is helpful to mention that George and Veeramani [2] have extended fuzzy metric spaces (GV-fuzzy metrics spaces).

Definition 4. The 3-tuple (X, M, T) is said to be a fuzzy metric space if X is a continuous t-norm and M is a fuzzy set on $X^2 \times (0, \infty)$ satisfying the following conditions:

(1) $\forall t > 0,\ \forall p, q \in X,\ M(p, q, t) > 0$.

(2) $\forall t > 0,\ \forall p, q \in X,\ M(p, q, t) = 1$ iff $p = q$.

(3) $\forall t > 0,\ \forall p, q \in X,\ M(p, q, t) = M(q, p, t)$.

(4) $\forall t, s > 0,\ \forall p, q, r \in X,\ T(M(p, q, t), M(q, r, s)) \leq M(p, r, t + s)$.

(5) $\forall p, q \in X,\ M(p, q, \cdot) : [0, \infty] \to [0, 1]$ is continuous.

Gregori and Romaguera introduced the next definition [13].

Definition 5. Let (X, M, T) be a fuzzy metric space. A sequence $\{x_n\}$ in X is said to be point convergent to $x \in X$ (shown as $x_n \xrightarrow{p} x$) if there exists $t > 0$ such that

$$\lim_{n\to\infty} M(x_n, x, t) = 1. \tag{2}$$

Now, we recall the definition of the *generalized* C-contraction [5]; let $\mathbf{M}$ be the family of all the mappings $m : \overline{R} \to \overline{R}$ such that the following conditions are satisfied:

(1) $\forall t, s \geq 0 : m(t + s) \geq m(t) + m(s)$.

(2) $m(t) = 0 \Leftrightarrow t = 0$.

(3) m is continuous.

Definition 6. Let (X, F) be a probabilistic metric space and $f : X \to X$. A mapping f is a *generalized* C-contraction if there exist a continuous, decreasing function $h : [0, 1] \to [0, \infty]$ such that $h(1) = 0$, $m_1, m_2 \in \mathbf{M}$, and $k \in (0, 1)$ such that the following implication holds for every $p, q \in X$ and for every $t > 0$:

$$h \circ F_{p,q}(m_2(t)) < m_1(t) \Longrightarrow h \circ F_{f(p),f(q)}(m_2(kt)) < m_1(kt). \tag{3}$$

If $m_1(s) = m_2(s) = s$ and $h(s) = 1 - s$ for every $s \in [0, 1]$, we obtain Hicks's definition.

Mihet in [7] showed that if $f : X \to X$ is a $(\psi, \varphi, \epsilon, \lambda)$-contraction and (X, M, T) is a complete fuzzy metric space, then f has a unique fixed point, and Ćirić et al. presented the theorem of fixed and periodic points for nonexpansive mappings in fuzzy metric spaces [14].

The comparison functions from the class ϕ of all mappings $\varphi : (0, 1) \to (0, 1)$ have the following properties:

(1) φ is an increasing bijection.

(2) $\forall \lambda \in (0, 1),\ \varphi(\lambda) < \lambda$.

Since every comparison mapping of this type is continuous, if $\varphi \in \phi$, then, for every $\lambda \in (0, 1)$, $\lim_{n\to\infty} \varphi^n(\lambda) = 0$.

Definition 7. Let (X, F) be a probabilistic space, $\varphi \in \phi$, and let ψ be a map from $(0, \infty)$ to $(0, \infty)$. A mapping $f : X \to X$ is called a $(\psi, \varphi, \epsilon, \lambda)$-contraction on X if it satisfies the following condition:

$$F_{x,y}(\epsilon) > 1 - \lambda \Longrightarrow F_{f(x),f(y)}(\psi(\epsilon)) > 1 - \varphi(\lambda), \quad x, y \in X,\ \epsilon > 0,\ \lambda \in (0, 1). \tag{4}$$

In the rest of the paper, we suppose that ψ is an increasing bijection.

Example 8. Let $X = \{0, 1, 2, \ldots\}$ and (for $x \neq y$)

$$M(x, y, t) = \begin{cases} 0, & \text{if } t \leq 2^{-\min(x,y)}, \\ 1 - 2^{-\min(x,y)}, & \text{if } 2^{-\min(x,y)} < t \leq 1, \\ 1, & \text{if } t > 1. \end{cases} \tag{5}$$

Suppose that $A : X \to X$, $A(r) = r + 1$, and the Łukasiewicz t-norm defined by $T_L(a, b) = \max\{a + b - 1, 0\}$. Then (X, M, T_L) is a fuzzy metric space [15].

Assume x, y, ϵ, λ are such that $M(x, y, \epsilon) > 1 - \lambda$:

(i) If $2^{-\min(x,y)} < \epsilon \leq 1$, then $1 - 2^{-\min(x,y)} > 1 - \lambda$. This indicates that $1 - 2^{-\min(x+1,y+1)} > 1 - (1/2)\lambda$; that is,

$$M(Ax, Ay, \epsilon) > 1 - \frac{1}{2}\lambda. \tag{6}$$

(ii) If $\epsilon > 1$, then $M(Ax, Ay, \epsilon) = 1$; hence again $M(Ax, Ay, \epsilon) > 1 - (1/2)\lambda$. Thus, the mapping A is a $(\psi, \varphi, \epsilon, \lambda)$-contraction on X with $\psi(\epsilon) = \epsilon$ and $\varphi(\lambda) = (1/2)\lambda$.

3. Main Results

In this section, we recall some contraction through the several definitions and then we prove the existence of fixed point for these contractions. It would be interesting to compare different types of contraction mapping in fuzzy metric spaces. It is useful to note that the concept of b_n-contraction has been introduced by Mihet [6].

As we mentioned, existence of convergent sequence is sometimes a difficult condition. Gregori and Romaguera presented another type of convergence that is called p-convergence [13]. A GV-fuzzy metric space (X, M, T) with the point convergence is a space with convergence in sense of Fréchet too.

In [16] we showed the existence of fixed point on B-contraction and C-contraction in the case of p-convergence subsequence. Furthermore, we have proved a theorem for b_n-contraction [17].

First, we introduce $\varphi - H$ contraction; then we review the fixed point theorem by p-convergent subsequence.

Let (X, M, T) be a fuzzy metric space and $\varphi \in \phi$. A mapping $f : X \to X$ is called a $\varphi - H$ contraction on X if the following condition holds:

$$M(x, y, \lambda) > 1 - \lambda \Longrightarrow M(f(x), f(y), \varphi(\lambda)) > 1 - \varphi(\lambda), \quad x, y \in X,\ \lambda \in (0,1). \tag{7}$$

Consider the mapping $\varphi : (0,1) \to (0,1)$; we say that the t-norm T is φ-convergent if, $\forall \delta \in (0,1)$, $\forall \lambda \in (0,1)$, $\exists s = s(\delta, \lambda) \in N$;

$$\top_{i=1}^{n} \left(1 - \varphi^{s+i}(\delta)\right) > 1 - \lambda, \quad \forall n \geq 1. \tag{8}$$

A theorem for $\varphi - H$ contraction on a GV-fuzzy metric space is as follows [18].

Theorem 9. *Let (X, M, T) be a GV-fuzzy metric space and $\sup_{0\leq a<1} T(a,a) = 1$ and let $A : X \to X$ be a $\varphi - H$ contraction. Suppose that, for some $x \in X$ and $\delta > 0$, $M(x, A(x), \delta) > 1 - \delta$ and the sequence $A^n(x)$ has a convergent subsequence. If T is φ-convergent and $\sum_{n=1}^{\infty} \varphi^n(\delta)$ is convergent, then A will have a fixed point.*

In this paper, due to the next theorem, the existence of fixed point for $(\psi, \varphi, \epsilon, \lambda)$-contraction is proved under the new conditions.

Theorem 10. *Let (X, M, T) be a GV-fuzzy metric space and $\sup_{0\leq a<1} T(a,a) = 1$ and let $A : X \to X$ be a $(\psi, \varphi, \epsilon, \lambda)$-contraction. Suppose that, for some $x \in X$ and $\epsilon > 0$ and $\delta \in (0,1)$, $M(x, A(x), \epsilon) > 1 - \delta$ and the sequence $A^n(x)$ has a convergent subsequence. If T is φ-convergent and $\sum_{n=1}^{\infty} \psi^n(\epsilon)$ is convergent, then A will have a fixed point.*

Proof. Let, for every $n \in \mathbf{N}$, $x_n = A^n(x) = A(x_{n-1})$ and $x = A^0(x)$.

By the assumption $M(x, A(x), \epsilon) > 1 - \delta$, so

$$M\left(A(x), A^2(x), \psi(\epsilon)\right) > 1 - \varphi(\delta) \tag{9}$$

and by induction for every $n \in \mathbf{N}$

$$M\left(A^n(x), A^{n+1}(x), \psi^n(\epsilon)\right) > 1 - \varphi^n(\delta). \tag{10}$$

We show that the sequence $A^n(x)$ is a Cauchy sequence; that is, for every $\zeta > 0$ and $\lambda \in (0,1)$ there exists an integer $n_0 = n_0(\zeta, \lambda) \in \mathbf{N}$ such that, for all $x, y \in X$, $n \geq n_0$, $m \in \mathbf{N}$, $M(A^n(x), A^{n+m}(x), \zeta) > 1 - \lambda$.

Let $\zeta > 0$ and $\lambda \in (0,1)$ be given; since the series $\sum_{n=1}^{\infty} \psi^n(\epsilon)$ converges, there exists $n_0(\epsilon)$ such that $\sum_{n=n_0}^{\infty} \psi^n(\epsilon) < \zeta$. Then, for every $n \geq n_0$,

$$\begin{aligned} M\left(A^n(x), A^{n+m}(x), \zeta\right) &\geq M\left(A^n(x), A^{n+m}(x), \sum_{i=n}^{\infty} \psi^i(\epsilon)\right) \geq M\left(A^n(x), A^{n+m}(x), \sum_{i=n}^{n+m-1} \psi^i(\epsilon)\right) \geq T \\ &\cdots T\left(MA^n(x), A^{n+1}(x), \psi^n(\epsilon)\right), \\ &M\left(A^{n+1}(x), A^{n+2}(x), \psi^{n+1}(\epsilon)\right), \ldots, \\ &M\left(A^{n+m-1}(x), A^{n+m}(x), \psi^{n+m-1}(\epsilon)\right)). \end{aligned} \tag{11}$$

Let $n_1 = n_1(\lambda)$ be such that $\top_{i=n_1}^{\infty}(1 - \varphi^i(\delta)) > 1 - \lambda$. Since T is φ-convergent, such a number n_1 exists. By using (10), we obtain for every $n \geq \max(n_0, n_1)$ and $m \in \mathbf{N}$

$$\begin{aligned} M\left(A^n(x), A^{n+m}(x), \zeta\right) &\geq \top_{i=n}^{n+m-1} \left(1 - \varphi^i(\delta)\right) \\ &\geq \top_{i=n}^{\infty} \left(1 - \varphi^i(\delta)\right) \geq 1 - \lambda. \end{aligned} \tag{12}$$

Suppose $\{x_{n_j}\}$ is a convergent subsequence of $\{x_n\}$ which converges to y_0. Then, for every $t > 0$, $\lim_{j\to\infty} M(x_{n_j}, y_0, t) = 1$.

Let $\lambda > 0$ be given. Since $\sup_{0\leq a<1} T(a,a) = 1$, there is a $\lambda_1 > 0$ such that $T((1 - \lambda_1), (1 - \lambda_1)) > 1 - \lambda$. Since $\{x_n\}$ and then $\{x_{n_j}\}$ are Cauchy sequence, we can take n_2 large enough such that $M(x_{n_j}, x_{n_j+1}, t) > 1 - \lambda_1$ and $M(x_{n_j}, y_0, t) > 1 - \lambda_1$, for every $j \geq n_2$; then $M(x_{n_j+1}, y_0, 2t) \geq T((1 - \lambda_1), (1 - \lambda_1)) > 1 - \lambda$ which implies that $x_{n_j+1} \to y_0$. By $(\psi, \varphi, \epsilon, \lambda)$-contraction condition from $M(x_{n_j}, y_0, \epsilon) > 1 - \lambda$, we have

$$M\left(A\left(x_{n_j}\right), A\left(y_0\right), \psi(\epsilon)\right) > 1 - \varphi(\lambda) > 1 - \lambda \quad \text{for every } \epsilon > 0. \tag{13}$$

It means $A(x_{n_j}) = x_{n_j+1} \to A(y_0)$. Since the convergence is F-convergence in a GV-fuzzy metric space, we get $A(y_0) = y_0$ which means y_0 is a fixed point. □

We mention an example for Theorem 13.

Example 11. Let (X, M, T) be a complete fuzzy metric space where $X = \{x_1, x_2, x_3, x_4\}$, $T(a,b) = \min\{a,b\}$, and $M(x,y,t)$ is defined as

$$M(x_1,x_2,t) = M(x_2,x_1,t) = \begin{cases} 0, & \text{if } t \le 0, \\ 0.9, & \text{if } 0 < t \le 3, \\ 1, & \text{if } t > 3. \end{cases}$$

$$\begin{aligned} M(x_1,x_3,t) &= M(x_3,x_1,t) = M(x_1,x_4,t) \\ &= M(x_4,x_1,t) = M(x_2,x_3,t) \\ &= M(x_3,x_2,t) = M(x_2,x_4,t) \\ &= M(x_4,x_2,t) = M(x_3,x_4,t) \\ &= M(x_4,x_3,t) = \begin{cases} 0, & \text{if } t \le 0, \\ 0.7, & \text{if } 0 < t \le 6, \\ 1, & \text{if } 6 < t \end{cases} \end{aligned} \tag{14}$$

$A : X \to X$ is given by $A(x_1) = A(x_2) = x_2$ and $A(x_3) = A(x_4) = x_1$. If we take $\varphi(\lambda) = \lambda/2$, $\psi(\epsilon) = \epsilon/2$, then A is a $(\psi, \varphi, \epsilon, \lambda)$-contraction if we take $\epsilon = 0.1, \delta = 0.2$, and $x = x_1$ as $A(x) = A(x_1) = x_2$. Therefore,

$$\begin{aligned} M(x, A(x), 0.1) &= M(x_1, x_2, 0.1) = 0.9 > 1 - 0.2 \\ &= 0.8. \end{aligned} \tag{15}$$

On the other hand, $A(x_1) = x_2$, $A(x_2) = x_2$, $A^2(x_3) = A(x_1) = x_2$, and $A^2(x_4) = A(x_1) = x_2$. So $A^n(x)$ is a convergent sequence and hence has a convergent subsequence to x_2. It is clear that T is φ-convergent and $\sum_{n=1}^{\infty} \psi^n(\epsilon) = \sum_{n=1}^{\infty} (\epsilon/2)^n = \epsilon/(2-\epsilon)$. It means x_2 is a unique fixed point for A.

For more information, reader can refer to [10].

Definition 12. Let (X, M, T) be a GV-fuzzy metric space and $A : X \to X$. The mapping A is a *generalized* $(\psi, \varphi, \epsilon, \lambda)$-contraction if there exist a continuous, decreasing function $h : [0,1] \to [0,\infty]$ such that $h(1) = 0$, $m_1, m_2 \in \mathbf{M}$, and $\lambda \in (0,1)$ such that the following implication holds for every $u, v \in X$ and for every $\epsilon > 0$:

$$\begin{aligned} & h \circ M(u, v, m_2(\epsilon)) < m_1(\lambda) \\ & \quad \Longrightarrow h \circ M(A(u), A(v), m_2(\psi(\epsilon))) < m_1(\varphi(\lambda)). \end{aligned} \tag{16}$$

If $m_1(a) = m_2(a) = a$, and $h(a) = 1 - a$ for every $a \in [0,1]$, we obtain the Mihet definition.

In the next theorem, we will prove a theorem for existing fixed point in the generalized $(\psi, \varphi, \epsilon, \lambda)$-contraction.

Now, we prove the new theorem for this kind of contraction.

Theorem 13. *Let (X, M, T) be a GV-fuzzy metric space with t-norm T such that $\sup_{0 \le a < 1} T(a,a) = 1$ and let $A : X \to X$ be a generalized $(\psi, \varphi, \epsilon, \lambda)$-contraction such that ψ is continuous on $(0,\infty)$ and $\lim_{n\to\infty} \psi^n(\delta) = 0$ for every $\delta \in (0,\infty)$. Suppose that there exists $\lambda \in (0,1)$ such that $h(0) < m_1(\lambda)$ and φ, ψ satisfied $\varphi(0) = \psi(0) = 0$. Suppose that for some $x \in X$ the sequence $A^n(x)$ has a convergent subsequence. Then A has a fixed point.*

Proof. A satisfies the following condition:

$$\begin{aligned} & h \circ M(u, v, m_2(\epsilon)) < m_1(\lambda) \\ & \quad \Longrightarrow h \circ M(A(u), A(v), m_2(\psi(\epsilon))) < m_1(\varphi(\lambda)), \end{aligned} \tag{17}$$

for all $u, v \in X$ and for all $\epsilon > 0$, and h, m_1, m_2 are given as in Definition 6. Let $x_n = A^n(x) = A(x_{n-1})$ for every $n \in \mathbf{N}$ and $(A^0(x) = x)$. First, we show that the sequence $A^n(x)$ is a Cauchy sequence. We prove that for every $\alpha > 0$ and $\beta \in (0,1)$ there exists an integer $N = N(\epsilon, \lambda) \in \mathbf{N}$ such that, for every $x, y \in X$ and $n \ge N$, $M(A^n(x), A^n(y), \alpha) > 1 - \beta$.

By the assumption, there is a $\lambda \in (0,1)$ such that $h(0) < m_1(\lambda)$.

From $M(x, y, m_2(\epsilon)) \ge 0$ it follows that

$$h \circ M(x, y, m_2(\epsilon)) \le h(0) < m_1(s), \tag{18}$$

which implies that $h \circ M(A(x), A(y), m_2(\psi(\epsilon))) < m_1(\varphi(\lambda))$, and by continuing in this way we obtain that for every $n \in \mathbf{N}$

$$h \circ M(A^n(x), A^n(y), m_2(\psi^n(\epsilon))) < m_1(\varphi(\lambda)). \tag{19}$$

Suppose $n_0(\alpha, \beta)$ is a natural number such that $m_2(\psi^n(\epsilon)) < \alpha$ *and* $m_1(\varphi^n(\lambda)) < h(1-\beta)$ for every $n \ge n_0(\alpha,\beta)$. Then $n > n_0(\alpha, \beta)$ implies that

$$\begin{aligned} & M(A^n(x), A^n(y), \alpha) \\ & \quad \ge M(A^n(x), A^n(y), m_2(\psi^n(\epsilon))) > 1 - \beta. \end{aligned} \tag{20}$$

Now let $y = A^m(x)$; then we obtain

$$\begin{aligned} & M(A^n(x), A^{n+m}(x), \alpha) > 1 - \beta, \\ & \qquad \forall n \ge n_0, \ \forall m \in \mathbf{N}, \end{aligned} \tag{21}$$

which means that $A^n(x)$ is a Cauchy sequence.

Suppose $\{x_n\}$ has a convergent subsequence $\{x_{n_j}\}$ which is convergent to y_0. Then, for every $t_0 > 0$,

$$\lim_{j\to\infty} M(x_{n_j}, y_0, t_0) = 1. \tag{22}$$

Let $\epsilon > 0$ be given since $\sup_{0 \le a < 1} T(a,a) = 1$; there is a $\delta > 0$ such that $T((1-\delta),(1-\delta)) > 1 - \epsilon$. Since $\{x_n\}$ and then $\{x_{n_j}\}$ are Cauchy sequences, we can take n_0 large enough such that $M(x_{n_j}, x_{n_j+1}, t_0) > 1 - \delta$, and $M(x_{n_j}, y_0, t_0) > 1 - \delta$, $\forall j \ge n_0$; then $M(x_{n_j+1}, y_0, 2t_0) \ge T((1-\delta),(1-\delta)) > 1 - \epsilon$ which implies that

$$x_{n_j+1} \longrightarrow y_0. \tag{23}$$

Let $\epsilon > 0$ be such that $m_2(\psi(\epsilon)) < \zeta$ and $\lambda \in (0,1)$ such that $m_1(\varphi(\lambda)) < h(1-\eta)$. Since m_1 and m_2 are continuous at zero and $m_1(0) = m_2(0) = 0$, such numbers, ϵ and λ, exist.

If we take $t_0 = m_2(\epsilon)$ in relation (22), then we have $M(x_{n_j}, y_0, m_2(\epsilon)) \geq 0$ so

$$h \circ M\left(x_{n_j}, y_0, m_2(\epsilon)\right) \leq h(0) < m_1(\lambda), \tag{24}$$

and this implies that

$$h \circ M\left(A\left(x_{n_j}\right), A(y_0), m_2(\psi(\epsilon))\right) < m_1(\varphi(\lambda)) < h(1-\eta). \tag{25}$$

Thus

$$M\left(A\left(x_{n_j}\right), A(y_0), \zeta\right) > M\left(A\left(x_{n_j}\right), A(y_0), m_2(\psi(\epsilon))\right) > 1-\eta; \tag{26}$$

then $A(x_{n_j}) = x_{n_j+1} \to A(y_0)$, and since convergence is F-convergence in a GV-fuzzy metric space, then $A(y_0) = y_0$ which means y_0 is a fixed point. □

By an example, we describe Theorem 13 more.

Example 14. Let (X, M, T) and the mappings A, ψ, and φ be the same as in Example 11. Set $h(a) = e^{-a} - e^{-1}$ for every $a \in [0,1]$ and $m_1(a) = m_2(a) = a$. It is obvious that ψ is continuous on $(0,\infty)$. The mapping A is *generalized* $(\psi, \varphi, \epsilon, \lambda)$-contraction and $\lim_{n\to\infty}\psi^n(\delta) = \lim_{n\to\infty}(\delta/2^n) = 0$ for every $\delta \in (0,\infty)$. On the other hand, there exists $\lambda \in (0,1)$ such that $h(0) = 1 - 1/e < \lambda$ and $\psi(0) = \varphi(0) = 0$. By the previous example, $A^n(x)$ has a convergent subsequence. So x_2 is the unique fixed point for A.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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