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Research Article

New Approaches for Solving Fokker Planck Equation on Cantor Sets within Local Fractional Operators

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We discuss new approaches to handling Fokker Planck equation on Cantor sets within local fractional operators by using the local fractional Laplace decomposition and Laplace variational iteration methods based on the local fractional calculus. The new approaches maintain the efficiency and accuracy of the analytical methods for solving local fractional differential equations. Illustrative examples are given to show the accuracy and reliable results.

1. Introduction

The Fokker Planck equation arises in various fields in natural science, including solid-state physics, quantum optics, chemical physics, theoretical biology, and circuit theory. The Fokker Planck equation was first used by Fokker and Plank [1] to describe the Brownian motion of particles. A FPE describes the change of probability of a random function in space and time; hence it is naturally used to describe solute transport.

The local fractional calculus was developed and applied to the fractal phenomenon in science and engineering [2–13]. Local fractional Fokker Planck equation, which was an analog of a diffusion equation with local fractional derivative, was suggested in [5] as follows:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = \frac{\Gamma(1+\alpha)}{4}\chi_{C}(t) + \frac{\partial^{2} u(x,t)}{\partial x^{2}}. \tag{1}$$

The Fokker Planck equation on a Cantor set with local fractional derivative was presented in [6, 7] as follows:

$$\frac{\partial^{\alpha} u(x,t)}{\partial t^{\alpha}} = -\frac{\partial^{\alpha} u(x,t)}{\partial x^{\alpha}} + \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}, \tag{2}$$

subject to the initial condition

$$u(x,0) = f(x). \tag{3}$$

In recent years, a variety of numerical and analytical methods have been applied to solve the Fokker Planck equation on Cantor sets such as local fractional variational iteration method [6] and local fractional Adomian decomposition method [7]. Our main purpose of the paper is to apply the local fractional Laplace decomposition method and local fractional variational iteration method to solve the Fokker Planck equations on a Cantor set. The paper has been organized as follows. In Section 2, the basic mathematical tools are reviewed. In Section 3, we give analysis of the methods used. In Section 4, we consider several illustrative examples. Finally, in Section 5, we present our conclusions.

2. Mathematical Fundamentals

Definition 1. Setting $f(x) \in C_{\alpha}(a,b)$, the local fractional derivative of $f(x)$ of order α at the point $x = x_0$ is defined as [2, 3, 9–12]

$$\left.\frac{d^{\alpha}}{dx^{\alpha}} f(x)\right|_{x=x_0} = f^{(\alpha)}(x) = \lim_{x \to x_0} \frac{\Delta^{\alpha}(f(x) - f(x_0))}{(x - x_0)^{\alpha}}, \quad 0 < \alpha \le 1, \tag{4}$$

where $\Delta^{\alpha}(f(x) - f(x_0)) \cong \Gamma(\alpha+1)(f(x) - f(x_0))$.

Definition 2. Setting $f(x) \in C_\alpha(a,b)$, local fractional integral of $f(x)$ of order α in the interval $[a,b]$ is defined through [2, 3, 9–12]

$$\begin{aligned} {}_aI_b^{(\alpha)} f(x) &= \frac{1}{\Gamma(1+\alpha)} \int_a^b f(t)(dt)^\alpha \\ &= \frac{1}{\Gamma(1+\alpha)} \lim_{\Delta t \to 0} \sum_{j=0}^{N-1} f(t_j)(\Delta t_j)^\alpha, \\ & \qquad 0 < \alpha \le 1, \end{aligned} \tag{5}$$

where the partitions of the interval $[a,b]$ are denoted as (t_j, t_{j+1}), with $\Delta t_j = t_{j+1} - t_j$, $t_0 = a$, $t_N = b$, and $\Delta t = \max\{\Delta t_0, \Delta t_1, \ldots\}$, $j = 0, \ldots, N-1$.

Definition 3. Let $(1/\Gamma(1+\alpha))\int_0^\infty |f(x)|(dx)^\alpha < k < \infty$. The Yang-Laplace transform of $f(x)$ is given by [2, 3]

$$\begin{aligned} L_\alpha\{f(x)\} &= f_s^{L,\alpha}(s) \\ &= \frac{1}{\Gamma(1+\alpha)} \int_0^\infty E_\alpha(-s^\alpha x^\alpha) f(x)(dx)^\alpha, \\ & \qquad 0 < \alpha \le 1, \end{aligned} \tag{6}$$

where the latter integral converges and $s^\alpha \in R^\alpha$.

Definition 4. The inverse formula of the Yang-Laplace transforms of $f(x)$ is given by [2, 3]

$$\begin{aligned} L_\alpha^{-1}\left\{f_s^{L,\alpha}(s)\right\} &= f(t) \\ &= \frac{1}{(2\pi)^\alpha} \int_{\beta - i\omega}^{\beta + i\omega} E_\alpha(s^\alpha x^\alpha) f_s^{L,\alpha}(s)(ds)^\alpha, \\ & \qquad 0 < \alpha \le 1, \end{aligned} \tag{7}$$

where $s^\alpha = \beta^\alpha + i^\alpha \omega^\alpha$, fractal imaginary unit i^α, and $\mathrm{Re}(s) = \beta > 0$.

3. Analytical Methods

In order to illustrate two analytical methods, we investigate the local fractional partial differential equation as follows:

$$L_\alpha u(x,t) + R_\alpha u(x,t) = g(x,t), \tag{8}$$

where $L_\alpha = \partial^\alpha/\partial t^\alpha$ denotes the linear local fractional differential operator, R_α is the remaining linear operators, and $g(x,t)$ is a source term of the nondifferential functions.

3.1. Local Fractional Laplace Decomposition Method (LFLDM). Taking Yang-Laplace transform on (8), we obtain

$$\text{Ł}_\alpha\{L_\alpha u(x,t)\} + \text{Ł}_\alpha\{R_\alpha u(x,t)\} = \text{Ł}_\alpha\{g(x,t)\}. \tag{9}$$

By applying the local fractional Laplace transform differentiation property, we have

$$\begin{aligned} s^\alpha \text{Ł}_\alpha\{u(x,t)\} - u(x,0) + \text{Ł}_\alpha\{R_\alpha u(x,t)\} \\ = \text{Ł}_\alpha\{g(x,t)\}, \end{aligned} \tag{10}$$

or

$$\begin{aligned} \text{Ł}_\alpha\{u(x,t)\} = \frac{1}{s^\alpha} u(x,0) + \frac{1}{s^\alpha} \text{Ł}_\alpha\{g(x,t)\} \\ - \frac{1}{s^\alpha} \text{Ł}\{R_\alpha(x,t)\}. \end{aligned} \tag{11}$$

Taking the inverse of local fractional Laplace transform on (11), we obtain

$$\begin{aligned} u(x,t) = u(x,0) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{g(x,t)\}\right) \\ - \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{R_\alpha u(x,t)\}\right). \end{aligned} \tag{12}$$

We are going to represent the solution in an infinite series given below:

$$u(x,t) = \sum_{n=0}^\infty u_n(x,t). \tag{13}$$

Substitute (13) into (12), which gives us this result

$$\begin{aligned} \sum_{n=0}^\infty u_n(x,t) = u(x,0) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{g(x,t)\}\right) \\ - \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\left\{R_\alpha \sum_{n=0}^\infty u_n(x,t)\right\}\right). \end{aligned} \tag{14}$$

When we compare the left- and right-hand sides of (14), we obtain

$$\begin{aligned} u_0(x,t) &= u(x,0) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{g(x,t)\}\right), \\ u_1(x,t) &= -\text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{R_\alpha u_0(x,t)\}\right), \\ u_2(x,t) &= -\text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{R_\alpha u_1(x,t)\}\right) \\ &\vdots \end{aligned} \tag{15}$$

The recursive relation, in general form, is

$$\begin{aligned} u_0(x,t) &= u(x,0) + \text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{g(x,t)\}\right), \\ u_{n+1}(x,t) &= -\text{Ł}_\alpha^{-1}\left(\frac{1}{s^\alpha}\text{Ł}_\alpha\{R_\alpha u_n(x,t)\}\right), \quad n \ge 0. \end{aligned} \tag{16}$$

3.2. Local Fractional Laplace Variational Iteration Method (LFLDM). According to the rule of local fractional variational iteration method, the correction local fractional functional for (8) is constructed as [13]

$$\begin{aligned} u_{n+1}(t) &= u_n(t) \\ &+ {}_0I_t^{(\alpha)}\left(\frac{\lambda(t-\xi)^\alpha}{\Gamma(1+\alpha)}\left[L_\alpha u_n(\xi) + R_\alpha \tilde{u}_n(\xi) - g(\xi)\right]\right), \end{aligned} \tag{17}$$

where $\lambda(t-\xi)^\alpha/\Gamma(1+\alpha)$ is a fractal Lagrange multiplier.

We now take Yang-Laplace transform of (17); namely,

$$\begin{aligned} \text{Ł}_\alpha \{u_{n+1}(t)\} &= \text{Ł}_\alpha \{u_n(t)\} \\ &+ \text{Ł}_\alpha \left\{ {}_0I_t^{(\alpha)} \left(\frac{\lambda (t-\xi)^\alpha}{\Gamma(1+\alpha)} \left[L_\alpha u_n(\xi) + R_\alpha \tilde{u}_n(\xi) \right. \right. \right. \\ &\left. \left. \left. - g(\xi) \right] \right) \right\}, \end{aligned} \tag{18}$$

or

$$\begin{aligned} &\text{Ł}_\alpha \{u_{n+1}(t)\} \\ &= \text{Ł}_\alpha \{u_n(t)\} \\ &+ \text{Ł}_\alpha \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} \right\} \text{Ł}_\alpha \{L_\alpha u_n(t) + R_\alpha \tilde{u}_n(t) - g(t)\}. \end{aligned} \tag{19}$$

Take the local fractional variation of (19), which is given by

$$\begin{aligned} \delta^\alpha (\text{Ł}_\alpha \{u_{n+1}(t)\}) &= \delta^\alpha (\text{Ł}_\alpha \{u_n(t)\}) \\ &+ \delta^\alpha \left(\text{Ł}_\alpha \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} \right\} \right. \\ &\left. \cdot \text{Ł}_\alpha \{L_\alpha u_n(t) - R_\alpha \tilde{u}_n(t) - g(t)\} \right). \end{aligned} \tag{20}$$

By using computation of (20), we get

$$\begin{aligned} &\delta^\alpha (\text{Ł}_\alpha \{u_{n+1}(t)\}) \\ &= \delta^\alpha (\text{Ł}_\alpha \{u_n(t)\}) \\ &+ \text{Ł}_\alpha \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} \right\} \delta^\alpha (\text{Ł}_\alpha \{L_\alpha u_n(t)\}) = 0. \end{aligned} \tag{21}$$

Hence, from (21) we get

$$1 + \text{Ł}_\alpha \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} \right\} s^\alpha = 0, \tag{22}$$

where

$$\begin{aligned} \delta^\alpha (\text{Ł}_\alpha \{L_\alpha u_n(t)\}) &= \delta^\alpha (s^\alpha \text{Ł}_\alpha \{u_n(t)\} - u_n(0)) \\ &= s^\alpha \delta^\alpha (\text{Ł}_\alpha \{u_n(t)\}). \end{aligned} \tag{23}$$

Therefore, we get

$$\text{Ł}_\alpha \left\{ \frac{\lambda(t)^\alpha}{\Gamma(1+\alpha)} \right\} = -\frac{1}{s^\alpha}. \tag{24}$$

Therefore, we have the following iteration algorithm:

$$\begin{aligned} \text{Ł}_\alpha \{u_{n+1}(t)\} &= \text{Ł}_\alpha \{u_n(t)\} \\ &- \frac{1}{s^\alpha} \text{Ł}_\alpha \{L_\alpha u_n(t) + R_\alpha u_n(t) - g(t)\} \\ &= \text{Ł}_\alpha \{u_n(t)\} - \frac{1}{s^\alpha} \text{Ł}_\alpha \{L_\alpha u_n(t)\} \\ &- \frac{1}{s^\alpha} \text{Ł}_\alpha \{R_\alpha u_n(t) - g(t)\} \\ &= \text{Ł}_\alpha \{u_n(t)\} - \frac{1}{s^\alpha} \text{Ł}_\alpha \{s^\alpha u_n(t) - u(0)\} \\ &- \frac{1}{s^\alpha} \text{Ł}_\alpha \{R_\alpha u_n(t) - g(t)\} \\ &= \frac{1}{s^\alpha} u(0) - \frac{1}{s^\alpha} \text{Ł}_\alpha \{R_\alpha u_n(t) - g(t)\}, \end{aligned} \tag{25}$$

where the initial value reads as

$$\text{Ł}_\alpha \{u_0(t)\} = \frac{1}{s^\alpha} u(0). \tag{26}$$

Thus, the local fractional series solution of (8) is

$$u(x,t) = \lim_{n \to \infty} \text{Ł}_\alpha^{-1} (\text{Ł}_\alpha \{u_n(x,t)\}). \tag{27}$$

4. Illustrative Examples

In this section three examples for Fokker Planck equation are presented in order to demonstrate the simplicity and the efficiency of the above methods.

Example 1. Let us consider the following Fokker Planck equation on Cantor sets with local fractional derivative in the form

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -\frac{\partial^\alpha u(x,t)}{\partial x^\alpha} + \frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}, \tag{28}$$

subject to the initial value

$$u(x,0) = E_\alpha(-x^\alpha). \tag{29}$$

(I) By Using LFLDM. In view of (16) and (28) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} &u_0(x,t) = E_\alpha(-x^\alpha), \\ &u_{n+1}(x,t) \\ &\quad = \text{Ł}_\alpha^{-1} \left(\frac{1}{s^\alpha} \text{Ł}_\alpha \left\{ -\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha} + \frac{\partial^{2\alpha} u_n(x,t)}{\partial x^{2\alpha}} \right\} \right), \\ &\qquad n \geq 0. \end{aligned} \tag{30}$$

Therefore, from (30) we give the components as follows:

$$\begin{aligned} u_0(x,t) &= E_\alpha(-x^\alpha), \\ u_1(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_0(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_0(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(\frac{2}{s^{2\alpha}}E_\alpha(-x^\alpha)\right) = \frac{2t^\alpha}{\Gamma(1+\alpha)}E_\alpha(-x^\alpha), \\ u_2(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(\frac{4}{s^{3\alpha}}E_\alpha(-x^\alpha)\right) = \frac{4t^{2\alpha}}{\Gamma(1+2\alpha)}E_\alpha(-x^\alpha), \\ u_3(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(\frac{8}{s^{4\alpha}}E_\alpha(-x^\alpha)\right) = \frac{8t^{3\alpha}}{\Gamma(1+3\alpha)}E_\alpha(-x^\alpha) \\ &\vdots \end{aligned} \tag{31}$$

Finally, we can present the solution in local fractional series form as

$$\begin{aligned} u(x,t) &= E_\alpha(-x^\alpha) \\ &\cdot\left(1+\frac{2t^\alpha}{\Gamma(1+\alpha)}+\frac{4t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{8t^{3\alpha}}{\Gamma(1+3\alpha)}+\cdots\right) \\ &= E_\alpha(-x^\alpha)E_\alpha(2t^\alpha) = E_\alpha(2t^\alpha-x^\alpha). \end{aligned} \tag{32}$$

(II) By Using LFLVIM. Using relation (25) we structure the iterative relation as

$$\begin{aligned} Ł_\alpha\{u_{n+1}(x,t)\} &= Ł_\alpha\{u_n(x,t)\} \\ &\quad - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial t^\alpha}+\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\} \\ &= Ł_\alpha\{u_n(x,t)\} - \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_n(x,t)\}-u_n(x,0)\right) \\ &\quad - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\}, \end{aligned} \tag{33}$$

where the initial value is given by

$$Ł_\alpha\{u_0(x,t)\} = Ł_\alpha\{E_\alpha(-x^\alpha)\} = \frac{1}{s^\alpha}E_\alpha(-x^\alpha). \tag{34}$$

Therefore, the successive approximations are

$$\begin{aligned} Ł_\alpha\{u_1(x,t)\} &= Ł_\alpha\{u_n(x,t)\} \\ &\quad - \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_n(x,t)\}-u_n(x,0)\right) \\ &\quad - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\} \\ &= \frac{1}{s^\alpha}E_\alpha(-x^\alpha)+\frac{2}{s^{2\alpha}}E_\alpha(-x^\alpha), \\ Ł_\alpha\{u_2(x,t)\} &= Ł_\alpha\{u_1(x,t)\} \\ &\quad - \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_1(x,t)\}-u_1(x,0)\right) \\ &\quad - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}\right\} \\ &= \frac{1}{s^\alpha}E_\alpha(-x^\alpha)+\frac{2}{s^{2\alpha}}E_\alpha(-x^\alpha)+\frac{4}{s^{3\alpha}}E_\alpha(-x^\alpha) \\ &= E_\alpha(-x^\alpha)\left(\frac{1}{s^\alpha}+\frac{2}{s^{2\alpha}}+\frac{4}{s^{3\alpha}}\right), \\ Ł_\alpha\{u_3(x,t)\} &= Ł_\alpha\{u_2(x,t)\} \\ &\quad - \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_2(x,t)\}-u_2(x,0)\right) \\ &\quad - \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}\right\} \\ &= \frac{1}{s^\alpha}E_\alpha(-x^\alpha)+\frac{2}{s^{2\alpha}}E_\alpha(-x^\alpha)+\frac{4}{s^{3\alpha}}E_\alpha(-x^\alpha) \\ &\quad + \frac{8}{s^{4\alpha}}E_\alpha(-x^\alpha) \\ &= E_\alpha(-x^\alpha)\left(\frac{1}{s^\alpha}+\frac{2}{s^{2\alpha}}+\frac{4}{s^{3\alpha}}+\frac{8}{s^{4\alpha}}\right) \\ &\vdots \end{aligned} \tag{35}$$

Hence, the local fractional series solution is

$$\begin{aligned} u(x,t) &= \lim_{n\to\infty}Ł_\alpha^{-1}\left(Ł_\alpha\{u_n(x,t)\}\right) \\ &= \lim_{n\to\infty}Ł_\alpha^{-1}\left(E_\alpha(-x^\alpha)\sum_{k=0}^{n}\frac{2^k}{s^{(k+1)\alpha}}\right) \\ &= E_\alpha(-x^\alpha)\sum_{k=0}^{\infty}\frac{2^k t^{k\alpha}}{\Gamma(1+k\alpha)} = E_\alpha(2t^\alpha-x^\alpha). \end{aligned} \tag{36}$$

Example 2. We present the Fokker Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -\frac{\partial^\alpha u(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u(x,t)}{\partial x^{2\alpha}}, \tag{37}$$

and the initial condition is

$$u(x,0) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{38}$$

(I) By Using LFLDM. In view of (16) and (37) the local fractional iteration algorithm can be written as follows:

$$u_0(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)},$$
$$u_{n+1}(x,t) = \L_\alpha^{-1}\left(\frac{1}{s^\alpha}\L_\alpha\left\{-\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\}\right),\quad n \geq 0. \tag{39}$$

Therefore, from (39) we give the components as follows:

$$\begin{aligned}
u_0(x,t) &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)},\\
u_1(x,t) &= \L_\alpha^{-1}\left(\frac{1}{s^\alpha}\L_\alpha\left\{-\frac{\partial^\alpha u_0(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_0(x,t)}{\partial x^{2\alpha}}\right\}\right)\\
&= \L_\alpha^{-1}\left(-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{2\alpha}}\right)\\
&= -\frac{t^\alpha}{\Gamma(1+\alpha)}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)},\\
u_2(x,t) &= \L_\alpha^{-1}\left(\frac{1}{s^\alpha}\L_\alpha\left\{-\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}\right\}\right)\\
&= \L_\alpha^{-1}\left(\frac{1}{s^{3\alpha}}\right) = \frac{t^{2\alpha}}{\Gamma(1+2\alpha)},\\
u_3(x,t) &= \L_\alpha^{-1}\left(\frac{1}{s^\alpha}\L_\alpha\left\{-\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}\right\}\right)\\
&= 0,\\
u_4(x,t) &= \L_\alpha^{-1}\left(\frac{1}{s^\alpha}\L_\alpha\left\{-\frac{\partial^\alpha u_3(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha}u_3(x,t)}{\partial x^{2\alpha}}\right\}\right) = 0\\
&\vdots
\end{aligned} \tag{40}$$

Finally, we can present the solution in local fractional series form as

$$u(x,t) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)}. \tag{41}$$

(II) By Using LFLVIM. Using relation (25) we structure the iterative relation as

$$\begin{aligned}
\L_\alpha\{u_{n+1}(x,t)\} &= \L_\alpha\{u_n(x,t)\}\\
&\quad - \frac{1}{s^\alpha}\L_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial t^\alpha}+\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\}\\
&= \L_\alpha\{u_n(x,t)\} - \frac{1}{s^\alpha}\left(s^\alpha\L_\alpha\{u_n(x,t)\}-u_n(x,0)\right)\\
&\quad - \frac{1}{s^\alpha}\L_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_n(x,t)}{\partial x^{2\alpha}}\right\},
\end{aligned} \tag{42}$$

where the initial value is given by

$$\L_\alpha\{u_0(x,t)\} = \L_\alpha\left\{\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\right\} = \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}. \tag{43}$$

Therefore, the successive approximations are

$$\begin{aligned}
\L_\alpha\{u_1(x,t)\} &= \L_\alpha\{u_0(x,t)\}\\
&\quad - \frac{1}{s^\alpha}\left(s^\alpha\L_\alpha\{u_0(x,t)\}-u_0(x,0)\right)\\
&\quad - \frac{1}{s^\alpha}\L_\alpha\left\{\frac{\partial^\alpha u_0(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_0(x,t)}{\partial x^{2\alpha}}\right\}\\
&= \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{2\alpha}},\\
\L_\alpha\{u_2(x,t)\} &= \L_\alpha\{u_1(x,t)\}\\
&\quad - \frac{1}{s^\alpha}\left(s^\alpha\L_\alpha\{u_1(x,t)\}-u_1(x,0)\right)\\
&\quad - \frac{1}{s^\alpha}\L_\alpha\left\{\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}\right\}\\
&= \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{3\alpha}}+\frac{1}{s^{2\alpha}},\\
\L_\alpha\{u_3(x,t)\} &= \L_\alpha\{u_2(x,t)\}\\
&\quad - \frac{1}{s^\alpha}\left(s^\alpha\L_\alpha\{u_2(x,t)\}-u_2(x,0)\right)\\
&\quad - \frac{1}{s^\alpha}\L_\alpha\left\{\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}\right\}\\
&= \frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{1}{s^{3\alpha}}+\frac{1}{s^{2\alpha}}\\
&\vdots
\end{aligned} \tag{44}$$

Hence, the local fractional series solution is

$$\begin{aligned} u(x,t) &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(Ł_\alpha\{u_n(x,t)\}\right) \\ &= \lim_{n\to\infty} Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{1}{s^{3\alpha}}-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)} + \frac{1}{s^{2\alpha}}\right) = \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{x^\alpha}{\Gamma(1+\alpha)} \\ &\cdot \frac{t^\alpha}{\Gamma(1+\alpha)}+\frac{t^\alpha}{\Gamma(1+\alpha)}. \end{aligned} \quad (45)$$

Example 3. We consider the Fokker Planck equation on a Cantor set with local fractional derivative

$$\frac{\partial^\alpha u(x,t)}{\partial t^\alpha} = -\frac{\partial^\alpha u(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u(x,t)}{\partial x^{2\alpha}}, \quad (46)$$

and the initial condition is

$$u(x,0) = -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \quad (47)$$

(I) By Using LFLDM. In view of (16) and (46) the local fractional iteration algorithm can be written as follows:

$$\begin{aligned} u_0(x,t) &= -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}, \\ u_{n+1}(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u_n(x,t)}{\partial x^{2\alpha}}\right\}\right), \\ & n\ge 0. \end{aligned} \quad (48)$$

Therefore, from (30) we give the components as follows:

$$\begin{aligned} u_0(x,t) &= -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}, \\ u_1(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_0(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u_0(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(\frac{1}{s^{2\alpha}}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}-\frac{1}{s^{2\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}\right) \\ &= \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}-\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)}, \\ u_2(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_1(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u_1(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(-\frac{1}{s^{3\alpha}}\frac{x^\alpha}{\Gamma(1+\alpha)}+\frac{2}{s^{3\alpha}}\right) \\ &= -\frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{2t^{2\alpha}}{\Gamma(1+2\alpha)}, \\ u_3(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_2(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u_2(x,t)}{\partial x^{2\alpha}}\right\}\right) \\ &= Ł_\alpha^{-1}\left(\frac{1}{s^{4\alpha}}\right) = \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}, \\ u_4(x,t) &= Ł_\alpha^{-1}\left(\frac{1}{s^\alpha}Ł_\alpha\left\{-\frac{\partial^\alpha u_3(x,t)}{\partial x^\alpha}+\frac{\partial^{2\alpha} u_3(x,t)}{\partial x^{2\alpha}}\right\}\right) = 0 \\ &\vdots \end{aligned} \quad (49)$$

Finally, we can present the solution in local fractional series form as

$$\begin{aligned} u(x,t) &= -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}+\frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)} \\ &- \frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^\alpha}{\Gamma(1+\alpha)} \\ &- \frac{x^\alpha}{\Gamma(1+\alpha)}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)}+\frac{2t^{2\alpha}}{\Gamma(1+2\alpha)} \\ &+ \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}. \end{aligned} \quad (50)$$

(II) By Using LFLVIM. Using relation (18) we structure the iterative relation as

$$\begin{aligned} Ł_\alpha\{u_{n+1}(x,t)\} &= Ł_\alpha\{u_n(x,t)\} \\ &- \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial t^\alpha}+\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha} u_n(x,t)}{\partial x^{2\alpha}}\right\} \\ &= Ł_\alpha\{u_n(x,t)\}-\frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_n(x,t)\}-u_n(x,0)\right) \\ &- \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_n(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha} u_n(x,t)}{\partial x^{2\alpha}}\right\}, \end{aligned} \quad (51)$$

where the initial value is given by

$$Ł_\alpha\{u_0(x,t)\} = Ł_\alpha\left\{-\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}\right\} = -\frac{1}{s^\alpha}\frac{x^{3\alpha}}{\Gamma(1+3\alpha)}. \quad (52)$$

Therefore, the successive approximations are

$$\begin{aligned} Ł_\alpha\{u_1(x,t)\} &= Ł_\alpha\{u_0(x,t)\} \\ &- \frac{1}{s^\alpha}\left(s^\alpha Ł_\alpha\{u_0(x,t)\}-u_n(x,0)\right) \\ &- \frac{1}{s^\alpha}Ł_\alpha\left\{\frac{\partial^\alpha u_0(x,t)}{\partial x^\alpha}-\frac{\partial^{2\alpha} u_0(x,t)}{\partial x^{2\alpha}}\right\} \end{aligned}$$

$$
\begin{aligned}
&= -\frac{1}{s^{\alpha}}\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{1}{s^{2\alpha}}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{1}{s^{2\alpha}}\frac{x^{\alpha}}{\Gamma(1+\alpha)},\\
\text{Ł}_{\alpha}\{u_2(x,t)\} &= \text{Ł}_{\alpha}\{u_1(x,t)\}\\
&\quad - \frac{1}{s^{\alpha}}\left(s^{\alpha}\text{Ł}_{\alpha}\{u_1(x,t)\} - u_1(x,0)\right)\\
&\quad - \frac{1}{s^{\alpha}}\text{Ł}_{\alpha}\left\{\frac{\partial^{\alpha}u_1(x,t)}{\partial x^{\alpha}} - \frac{\partial^{2\alpha}u_1(x,t)}{\partial x^{2\alpha}}\right\}\\
&= -\frac{1}{s^{\alpha}}\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{1}{s^{2\alpha}}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{1}{s^{2\alpha}}\frac{x^{\alpha}}{\Gamma(1+\alpha)}\\
&\quad - \frac{1}{s^{3\alpha}}\frac{x^{\alpha}}{\Gamma(1+3\alpha)} + \frac{2}{s^{3\alpha}},\\
\text{Ł}_{\alpha}\{u_3(x,t)\} &= \text{Ł}_{\alpha}\{u_2(x,t)\}\\
&\quad - \frac{1}{s^{\alpha}}\left(s^{\alpha}\text{Ł}_{\alpha}\{u_2(x,t)\} - u_2(x,0)\right)\\
&\quad - \frac{1}{s^{\alpha}}\text{Ł}_{\alpha}\left\{\frac{\partial^{\alpha}u_2(x,t)}{\partial x^{\alpha}} - \frac{\partial^{2\alpha}u_2(x,t)}{\partial x^{2\alpha}}\right\}\\
&= -\frac{1}{s^{\alpha}}\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{1}{s^{2\alpha}}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{1}{s^{2\alpha}}\frac{x^{\alpha}}{\Gamma(1+\alpha)}\\
&\quad - \frac{1}{s^{3\alpha}}\frac{x^{\alpha}}{\Gamma(1+3\alpha)} + \frac{2}{s^{3\alpha}} + \frac{1}{s^{4\alpha}},\\
\text{Ł}_{\alpha}\{u_4(x,t)\} &= \text{Ł}_{\alpha}\{u_3(x,t)\}\\
&\quad - \frac{1}{s^{\alpha}}\left(s^{\alpha}\text{Ł}_{\alpha}\{u_3(x,t)\} - u_3(x,0)\right)\\
&\quad - \frac{1}{s^{\alpha}}\text{Ł}_{\alpha}\left\{\frac{\partial^{\alpha}u_3(x,t)}{\partial x^{\alpha}} - \frac{\partial^{2\alpha}u_3(x,t)}{\partial x^{2\alpha}}\right\}\\
&= -\frac{1}{s^{\alpha}}\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{1}{s^{2\alpha}}\frac{x^{2\alpha}}{\Gamma(1+2\alpha)} - \frac{1}{s^{2\alpha}}\frac{x^{\alpha}}{\Gamma(1+\alpha)}\\
&\quad - \frac{1}{s^{3\alpha}}\frac{x^{\alpha}}{\Gamma(1+3\alpha)} + \frac{2}{s^{3\alpha}} + \frac{1}{s^{4\alpha}}\\
&\quad\vdots
\end{aligned}
\tag{53}
$$

Hence, the local fractional series solution is

$$
\begin{aligned}
u(x,t) &= \lim_{n\to\infty}\text{Ł}_{\alpha}^{-1}\left(\text{Ł}_{\alpha}\{u_n(x,t)\}\right)\\
&= -\frac{x^{3\alpha}}{\Gamma(1+3\alpha)} + \frac{x^{2\alpha}}{\Gamma(1+2\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}\\
&\quad - \frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{t^{\alpha}}{\Gamma(1+\alpha)}\\
&\quad - \frac{x^{\alpha}}{\Gamma(1+\alpha)}\frac{t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{2t^{2\alpha}}{\Gamma(1+2\alpha)}\\
&\quad + \frac{t^{3\alpha}}{\Gamma(1+3\alpha)}.
\end{aligned}
\tag{54}
$$

5. Conclusions

In this work solving Fokker Planck equation by using the local fractional Laplace decomposition method and local fractional Laplace variational iteration method with local fractional operators is discussed in detail. Three examples of applications of these methods are investigated. The reliable obtained results are complementary to the ones presented in the literature.

Conflict of Interests

The author declares that there is no conflict of interests regarding the publication of this paper.

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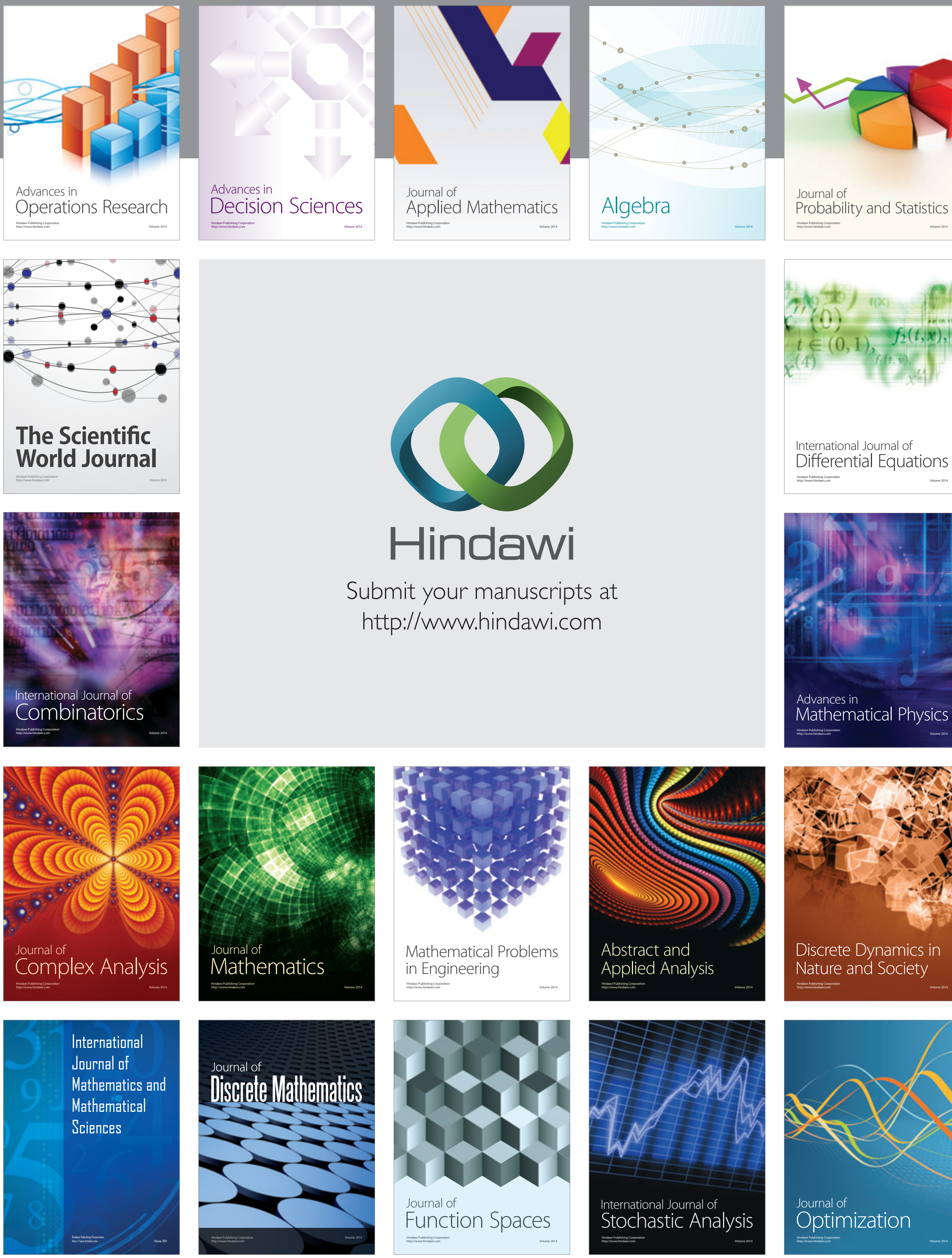
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