Hindawi Publishing Corporation
Journal of Electrical and Computer Engineering
Volume 2016, Article ID 9304597, 8 pages
http://dx.doi.org/10.1155/2016/9304597

Research Article

On the Capacity of MIMO Weibull-Gamma Fading Channels in Low SNR Regime

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Received 19 July 2016; Accepted 25 October 2016

Academic Editor: George Tsoulos

We present capacity analysis for multiple-input multiple-output (MIMO) system under a low signal-to-noise ratio (SNR) regime. We have selected a composite fading channel that considers Weibull fading for multipath and gamma fading for shadowing. We have presented the analysis in a detailed form for three different techniques, namely, spatial multiplexing (SM) with optimal detection, SM with minimum mean square error (MMSE) detection, and orthogonal space-time block codes (OSTBC). Because capacity analysis at arbitrary signal-to-noise (SNR) is stringent, a low SNR regime is considered to achieve a positive rate and wideband slope. The improvement is the result of minimizing energy per information bit (E_b). For the first time, closed-form expressions are evaluated for the capacity of MIMO systems at low SNR under WG fading channels which facilitate the performance comparison for proposed techniques.

1. Introduction

MIMO wireless systems have been employed to overcome the disruptive effects of multipath fading and shadowing. These systems gradually became mature which enabled high reliability and increased capacity in bandwidth limited channels. The varying nature of wireless radio environment is responsible for some well-known multipath fading such as Rayleigh [1], Rician [2], Weibull [3], and Nakagami-m [4]. In addition, shadowing conditions can also be formed and determined in terms of gamma and lognormal distribution [5]. The shadowing effects resulting from the slow variations are incorporated in the rapid fluctuations arising from multipath propagation. Subsequently, several models have been proposed for analyzing the performance of the wireless system. Table 1 presents the existing models which define both the multipath fading and shadowing conditions, collectively termed as composite channel models.

Several generic distributions have been proposed using gamma distribution instead of lognormal distribution. The problem is that the lognormal distribution is not mathematically tractable [9]. In [16], author has proposed a composite Weibull-gamma (WG) distribution for modeling multipath along with shadowing. In the recent work [17–21], WG fading channel is used to analyze the MIMO system performance due to its less computational complexity. Consequently, under such fading channel conditions, the ergodic capacity has been evaluated in [19] and the error rate performance of MIMO systems has been evaluated using efficient detection technique in [20].

Since many locations in the geometry lie on the edge of their cells, apart from operating at high SNR, users frequently operate at low SNR. According to [22], 40% of geographical area receives signal as low as 0 dB and less than 10% area has a SNR greater than 10 dB. In such conditions, the system performance evaluation at low SNR is imperative to the designers. In MIMO systems, figure of merit depends on normalized energy per information bit (E_b/N_0) instead of SNR. In low SNR regime, the channel capacity analysis in terms of per symbol gives misguiding outcomes and hence

Table 1: Composite channel models.

Proposed models	Channel modeling/generation	Remarks
Suzuki model [6]	A product of Rayleigh processes with the lognormal process	For nondirect line of sight communication. However, it is not suitable for direct line of sight communication
Extended Suzuki models [7]	A product of a Rice process and a lognormal process	Suitable to describe the behavior of large classes of frequency nonselective land mobile satellite channels. However, it is unable to model nonuniform scattering
Generalized modified Suzuki model [8]	A product of Weibull process and lognormal process	To model shadow fading along with inhomogeneous and anisotropic diffuse scattering.
K-fading model [9]	A product of Rayleigh process and lognormal process	To model non-line-of-sight and shadowing
Weibull-lognormal fading model [10, 11]	A product of Weibull process and lognormal process	Composite model but due to lognormal distribution it faces mathematical complexity
Generalized K-fading model [12, 13]	A product of a gamma random variable and a channel matrix with i.i.d. Nakagami-m entries	An accurate approximation of most of the fading and shadowing effects
Extended generalized K-fading model [14, 15]	The combination of two independent Rayleigh distributed RVs with the average unit powers such that each pair of these RVs are independent, and one RV represents shadowing, and another one represents multipath fading components. Both components are distributed according to generalized Nakagami-m PDFs	For modeling wireless and optical communication channels. Superior model but mathematically complex
Weibull-gamma fading model [16–21]	A product of Weibull and gamma random process	Simpler and relatively new one which is used for outdoor, indoor, communication, and radar clutters including shadowing effects

capacity is defined in terms of minimum energy per information bit ($E_b/N_{0_{\min}}$) [23]. Henceforth, channel bandwidth, power, and rate of transmission have been analyzed with arbitrary number of transmitting and receiving antennas by minimizing E_b [24].

In [12, 13], the ergodic capacity of spatially multiplexed (SM) and OSTBC MIMO system has been computed over GK fading channel. This channel model decreases energy levels to a prominent range [25]. MIMO systems, namely, SM with optimal detector, SM with MMSE detector, and OSTBC, have been developed to evaluate performance at low SNR over Weibull fading channel [26]; however, the shadowing effect is not considered.

To the best of the authors' knowledge, statistical characterization of MIMO WG fading channel is not fully investigated in existing literature. Hence, the capacity performance is measured with this generic distribution in this paper. This generic channel model is analytically better than other composite channel models which depict the linear approximation of the multipath and shadow fading conditions. $E_b/N_{0_{\min}}$ and wideband slope (S_0) are important parameters to convey any positive rate of data reliably and to analyze the capacity of MIMO system at low SNR. The superiority of SM against OSTBC in giving better capacity is established elsewhere. However, SM scheme is not optimum at low SNRs and, apart from eigenstatistics, it permits working with channel matrix trace which is given in [23]. Optimal detectors offer the maximum capacity of MIMO system; however, MMSE detectors are less complex than optimal detectors and offer improved capacity performance. In addition, OSTBC is diversity-oriented technique. Thus, we have explored these three techniques under WG fading in our work. Also, we have evaluated the capacity of MIMO system at low SNR in this scenario.

This paper is organized as follows. In Section 2, we present MIMO system models used in this work. In Section 3, closed form expressions are evaluated for capacity analysis of MIMO systems in low SNR regime using three models: (a) SM with the optimal detector, (b) SM with MMSE detector, and (c) OSTBC. Section 4 concludes the paper.

2. MIMO System Models

Consider a MIMO system with N_t transmit and N_r receive antennas and complex channel matrix $H \in \mathbb{C}^{N_t \times N_r}$. Each channel element is independent and identically distributed (i.i.d.) Gaussian random variables with zero mean and σ^2 variance. Let P denote the total transmit energy attainable at each time interval. The received signal Y is a complex matrix with size $N_r \times T$ over T successive symbol durations and is given by

$$Y = \sqrt{\frac{P}{N_t}} HX + \mathbb{N}, \tag{1}$$

where $\mathbb{N}$ is the receiver noise having matrix size $N_r \times T$ with constituents modeled as i.i.d. random variables and X is transmit signal with the size $N_t \times T$.

The effects of both large-scale fading and small-scale fading are considered to model H, which is determined as WG fading channel. The channel matrix is given by $H = \sqrt{g/d^{\upsilon}}H_w$. Here, υ is path loss exponent, d is the distance between transmitter and receiver; for average fading power x, $\Omega = E[x^2]$; g represents i.i.d. gamma random variable with probability density function (PDF), given by

$$P(x) = \frac{1}{\Gamma m}\left(\frac{m}{\Omega}\right)^m x^{m-1}e^{-mx/\Omega}, \quad x, \Omega, m \geq 0. \quad (2)$$

The elements of matrix H_w follow an i.i.d Weibull PDF, given in (3). Here, phase distribution is uniform in the interval $[0, 2\pi]$ and amplitude $x = |h_{i,j}|$ is Weibull distributed for each entry of H_w:

$$P(x) = \beta\lambda^{-\beta}x^{\beta-1}e^{-(x/\lambda)^{\beta}}, \quad x, \lambda, \beta \geq 0. \quad (3)$$

In (3), λ represents scaling parameter; m and β are gamma and Weibull fading parameters, respectively. It has been reported in [16] that a conditional PDF called the WG composite PDF is determined by combining (2) and (3). The WG distribution is approximated to Weibull distribution for $m \to \infty$, Rayleigh distribution for $\beta = 2$, $m \to \infty$, and additive white Gaussian noise (AWGN) channel for $\beta \to \infty$, $m \to \infty$. It follows K or Rayleigh-lognormal distribution for $\beta = 2$ using [27, Equation (9.34/3)].

It is desirable to estimate $(\text{trace}({H_w}^{\dagger}H_w))$ and g to compute $(\text{trace}(H^{\dagger}H))$, as both the Weibull and gamma random variables are independent of each other. Here, $(\cdot)^{\dagger}$ is Hermitian transposition and trace$(\cdot)$ is the summation of diagonal elements of matrix or matrix trace. First and second moments of gamma random variable are computed as

$$E[g] = \Omega,$$
$$E[g^2] = \Omega^2\left(1 + \frac{1}{m}\right). \quad (4)$$

Similarly, the nth moment is also calculated for Weibull random variables [24] and given by

$$E(x^n) = \lambda^n\Gamma\left(1 + \frac{n}{\beta}\right), \quad (5)$$

where $\text{trace}(H^{\dagger}H) = \|H\|^2 = g\,\text{trace}({H_w}^{\dagger}H_w)$, $\rho = P/\sigma^2$, $\|\cdot\|$ is Frobenius norm of matrix, and $\Gamma(\cdot)$ denotes gamma function.

The proposed three configurations are as follows.

2.1. Optimal Detection for SM MIMO Systems. When all data vectors are identical, optimal detectors are used to minimize the probability of error. Although the implementation complexity is very high, power distribution is uniform across the transmit antennas with an average SNR (ρ). The SNR and ergodic capacity [1] are expressed, respectively, as

$$\gamma_{\text{optimal}} = \frac{\rho}{N_t}\left[\text{trace}\left(H^{\dagger}H\right)\right], \quad (6)$$

$$C_{\text{ergodic}}(\rho) \triangleq E_H\left[\log_2\left\{\det\left(I_{N_t} + \gamma_{\text{optimal}}\right)\right\}\right]. \quad (7)$$

2.2. MMSE Signal Detection for SM MIMO Systems. Optimal detectors are complex in nature which makes them unemployable in cost-effective communication systems. Therefore, linear detectors like MMSE detectors are designed to reduce computational complexity [28]. Thus, at the kth receiver output, the postprocessing SNR when $N_r \geq N_t$ is represented as

$$\gamma_k^{\text{MMSE}} \triangleq \frac{1}{\left[\left(I_{N_t} + (\rho/N_t)\left(H^{\dagger}H\right)\right)^{-1}\right]_{k,k}} - 1, \quad k = 1, 2, \ldots, N_t, \quad (8)$$

where $[\cdot]_{k,k}$ returns the kth diagonal element of a matrix. The achievable sum rate can be determined by assuming the independent decoding at the receiver side, which is given by

$$\mathscr{R}^{\text{MMSE}}(\rho) \triangleq \sum_{k=1}^{N_t} E_{\gamma_k^{\text{MMSE}}}\left[\log_2\left(1 + \gamma_k^{\text{MMSE}}\right)\right]. \quad (9)$$

2.3. OSTBC MIMO Systems. OSTBC scheme is preferred due to its simplicity and reliability. This scheme is used to achieve the maximum diversity order of N_tN_r and is computationally efficient for per symbol detection. MIMO channel can be converted into identical scalar channel by taking the response similar to that of Frobenius norm of channel matrix [29]. For the OSTBC MIMO systems, the SNR and Shannon's capacity with rate R_c can be expressed, respectively, as

$$\gamma_{\text{OSTBC}} = \frac{\rho}{R_cN_t}\left[\text{trace}\left(H^{\dagger}H\right)\right], \quad (10)$$

$$C_{\text{STBC}}(\rho) \triangleq R_cE_H\left[\log_2\left(1 + \gamma_{\text{OSTBC}}\right)\right]. \quad (11)$$

3. Low SNR Analysis

According to [13], minimization of E_b governs a trade-off between bandwidth and power of the communication channels in wideband regime which is desirable for efficient signal communication. Also, E_b/N_0 is preferred over per symbol SNR in the measurement of MIMO system performance precisely at low SNR over distinct fading channels. Therefore, the capacity C is characterized in [23] as

$$C\left(\frac{E_b}{N_0}\right) \approx S_0\log_2\left(\frac{E_b/N_0}{E_b/N_{0_{\min}}}\right), \quad (12)$$

where the parameters $E_b/N_{0_{\min}}$ and S_0 prompt the low SNR nature desired for efficient transmission of positive rate and wideband slope. First-order derivative $C'(\cdot)$ and second-order derivative $C''(\cdot)$ of ergodic capacity are derived from (7) at $\rho = 0$. They are used to determine the following two figures of merits:

$$\frac{E_b}{N_{0_{\min}}} = \frac{1}{C'(0)},$$
$$S_0 = -2\ln 2\frac{\left(C'(0)\right)^2}{C''(0)}. \quad (13)$$

Theorem 1. *The following properties are represented by $N_t \times N_r$ MIMO systems over i.i.d. WG fading channels in low SNR regime:*

$$E\left[\text{trace}\left(H^{\dagger}H\right)\right] = \frac{\Omega\lambda^2 N_t N_r}{d^{\upsilon}}\Gamma\left(1+\frac{2}{\beta}\right), \quad (14)$$

$$E\left[\text{trace}\left(\left(H^{\dagger}H\right)^2\right)\right] = \frac{\Omega^2\lambda^4 N_t N_r}{d^{2\upsilon}}\left[\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{4}{\beta}\right) + \left(N_t+N_r-2\right)\Gamma\left(1+\frac{2}{\beta}\right)^2\right], \quad (15)$$

$$E\left[\left(\text{trace}\left(H^{\dagger}H\right)\right)^2\right] = \frac{\Omega^2\lambda^4 N_t N_r}{d^{2\upsilon}}\left[\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{4}{\beta}\right) + \left(N_t N_r-1\right)\Gamma\left(1+\frac{2}{\beta}\right)^2\right]. \quad (16)$$

Proof. Refer to the Appendix. □

Using (12), we evaluated the capacity performance of SM MIMO with the optimal detector and SM MIMO with MMSE detector and OSTBC MIMO systems at low SNR.

Proposition 2. *Respective $E_b/N_{0_{\min}}$ and S_0 for SM MIMO systems with optimal detectors using $N_t \times N_r$ antennas are represented as*

$$\frac{E_b^{\text{optimal}}}{N_{0_{\min}}} = \frac{d^{\upsilon}\ln 2}{\Omega\lambda^2 N_r \Gamma\left(1+2/\beta\right)}, \quad (17)$$

$$S_0^{\text{optimal}} = \frac{2N_t N_r}{\left(1+1/m\right)\Phi\left(\beta\right)+\left(N_t+N_r-2\right)}, \quad (18)$$

where

$$\Phi\left(\beta\right) = \frac{\Gamma\left(1+4/\beta\right)}{\Gamma\left(1+2/\beta\right)^2}. \quad (19)$$

Proof. In [22], the matrices for low SNR are rearranged to form

$$\frac{E_b}{N_{0_{\min}}} = \frac{N_t \ln 2}{E\left\{\text{trace}\left(H^{\dagger}H\right)\right\}}, \qquad S_0 = \frac{\left(E\left[\text{trace}\left(H^{\dagger}H\right)\right]\right)^2}{E\left[\text{trace}\left(\left(H^{\dagger}H\right)^2\right)\right]}. \quad (20)$$

Substituting (14)-(15) into (20), we get (17) and (18) using simple mathematical formulation. Equation (17) is the increasing function of β and the decreasing function of λ. As $\Phi(\beta)$ is a function of β in (19), S_0^{optimal} increases with the increase in β. In (17), $E_b^{\text{optimal}}/N_{0_{\min}}$ do not depend on m and β. When extra receive antennas get additional power, $E_b/N_{0_{\min}}$ decreases monotonically with the increase in N_r [12, 13]. Equation (17) remains unchanged with the increase in N_t; however, capacity increases due to higher value of S_0^{optimal}. The wideband slope S_0^{optimal} is lower bounded ($\beta \to 0$) and upper bounded ($\beta \to \infty$) as

$$0 \le S_0^{\text{optimal}} \le \frac{2N_t N_r}{N_t+N_r-1}. \quad (21)$$

Nevertheless, $E_b/N_{0_{\min}}$ is reduced by increasing N_r because a higher number of receive antennas need more power as mentioned in [12, 23]. The parameters chosen for K-fading and Rayleigh fading are $\Omega=\beta=2$, $\lambda=1$ and $\Omega=\beta=2$, $\lambda=1$, $m\to\infty$, respectively. Equations (18)-(19) are simplified to form Rayleigh fading as

$$\frac{E_b^{\text{optimal}}}{N_{0_{\min}}} = \frac{\ln 2}{N_r\Omega}, \qquad S_0^{\text{optimal}} = \frac{2N_t N_r}{N_t+N_r}. \quad (22)$$

□

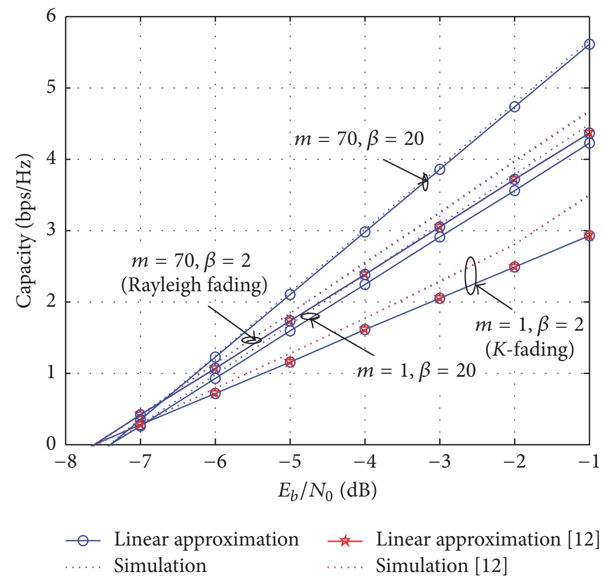

Figure 1: Capacity of SM MIMO system with optimal detector over WG fading channels for different m and β when $N_t = N_r = 2$.

In Figure 1, the capacity results are evaluated at $\lambda = 1$. In this case, the effects of low and high multipath fading are considered for $\beta = 20$ and 2, respectively, in the presence of light shadowing ($m = 70$) and heavy shadowing ($m = 1$). In light shadowing environment, that is, for $\beta = 2$, Rayleigh fading is observed. In addition, the capacity for the special cases (i.e., ≈Rayleigh and K-fading) of GK fading in [12] approaches both the special cases of WG fading. Thus, it is observed that WG is an alternative to GK fading channel. The range of low SNR takes numerically negative values from −8 dB to −1 dB. Simulation results show that capacity increases in low multipath and light shadowing environment.

As $1 + 1/m \approx 1$ for light shadowing (when m is large), the impact of shadowing on wideband slope is negligible.

The generation of WG MIMO fading channels occurs as the product of a gamma random variable and H_w with i.i.d. Weibull entries. Moreover, the simulation results of capacity at low SNR for SM MIMO systems with optimal detectors follow analytical approximations of Proposition 2.

Proposition 3. *$E_b/N_{0_{\min}}$ and S_0 for SM MIMO systems with MMSE detectors using $N_t \times N_r (N_r \geq N_t)$ are given by*

$$\frac{E_b^{\text{MMSE}}}{N_{0_{\min}}} = \frac{d^{\upsilon} \ln 2}{\Omega \lambda^2 N_r \Gamma(1 + 2/\beta)}, \tag{23}$$

$$S_0^{\text{MMSE}} = \frac{2N_t N_r}{(1 + 1/m)\,\Phi(\beta) + (2N_t + N_r - 3)}. \tag{24}$$

Proof. In [28], the following expressions are derived for the derivatives above:

$$\begin{aligned}\mathscr{R}'^{\text{MMSE}}(0) &= \frac{1}{\ln 2}\Bigg(E_H\left[\operatorname{trace}\left(H^{\dagger}H\right)\right] \\ &\quad - \frac{1}{N_t}\sum_{k=1}^{N_t} E_H\left[\operatorname{trace}\left(H_k^{\dagger}H_k\right)\right]\Bigg) \\ &= \frac{\Omega\lambda^2 N_r \Gamma(1+2/\beta)}{d^{\upsilon}\ln 2},\end{aligned} \tag{25}$$

$$\begin{aligned}\mathscr{R}''^{\text{MMSE}}(0) &= -\frac{1}{N_t^{2}\ln 2}\Bigg(N_t E_H\left[\operatorname{trace}\left(\left(H^{\dagger}H\right)^2\right)\right] \\ &\quad - \sum_{k=1}^{N_t} E_H\left[\operatorname{trace}\left(\left(H_k^{\dagger}H_k\right)^2\right)\right]\Bigg) \\ &= -\frac{\Omega^2\lambda^4}{d^{2\upsilon}N_t \ln 2}\left[N_r\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{4}{\beta}\right)\right. \\ &\quad \left. + \Gamma\left(1+\frac{2}{\beta}\right)^2\left(2N_tN_r + N_r^{2} - 3N_r\right)\right].\end{aligned} \tag{26}$$

H is replaced by H_k after removal of kth column. The elements of H follow i.i.d WG fading. Using (14)-(15), the expectations in (25)-(26) can be evaluated. The desired results are obtained using (13). It is observed that (17) and (23) represent the same mathematical expression. Therefore, due to $E_b/N_{0_{\min}}$, optimal detection is realizable through MMSE detectors and S_0 gives suboptimal detection. It can be shown that

$$\frac{S_0^{\text{optimal}}}{S_0^{\text{MMSE}}} = 1 + \frac{(N_t - 1)}{(1 + 1/m)\,\Phi(\beta) + N_t + N_r - 2}. \tag{27}$$

Equation (27) decreases with the increase in N_r and increases with the increase in N_t. Subsequently, MMSE detector has degraded interference cancellation capability for larger N_t which increases number of data streams. In case $N_t = 1$, $S_0^{\text{optimal}} = S_0^{\text{MMSE}}$ and, therefore, interfering data streams cancellation is not possible. For Rayleigh fading conditions, (23)-(24) are simplified and resemble [28, Eq. (33) and Eq. (52)], which is given by

$$\begin{aligned}\frac{E_b^{\text{MMSE}}}{N_{0_{\min}}} &= \frac{\ln 2}{N_r \Omega}, \\ S_0^{\text{MMSE}} &= \frac{2N_t N_r}{2N_t + N_r - 1}.\end{aligned} \tag{28}$$

□

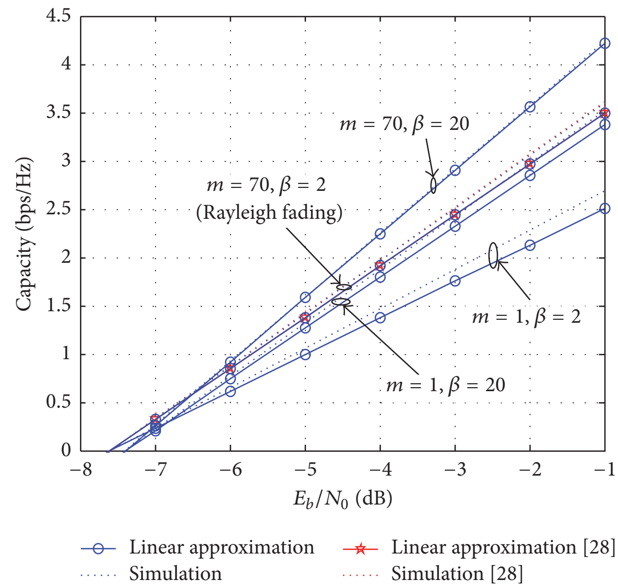

Figure 2: Capacity of SM MIMO system with MMSE detector over WG fading channels for different m and β when $N_t = N_r = 2$.

Also, S_0 decreases consistently due to the influence of shadowing on wideband slope, which reflects the divergence from small scale fading. Here, $m = 1$ shows the severe shadowing effect and consequent reduction in S_0 which is depicted in Figure 2. For a fair comparison, we have compared our result with the results of [28] under a special case (i.e., Rayleigh fading) of WG fading except we considered shadowing effect. SM MMSE offers performance with a high data rate keeping the complexity low. However, it is disadvantageous to system reliability. Therefore, we have considered OSTBC in Proposition 4.

Proposition 4. *The respective $E_b/N_{0_{\min}}$ and S_0 for OSTBC MIMO systems using $N_t \times N_r$ are expressed as*

$$\begin{aligned}\frac{E_b^{\text{STBC}}}{N_{0_{\min}}} &= \frac{d^{\upsilon}\ln 2}{\Omega\lambda^2 N_r \Gamma(1+2/\beta)}, \\ S_0^{\text{STBC}} &= \frac{2R_c N_t N_r}{(1+1/m)\,\Phi(\beta) + (N_t N_r - 1)}.\end{aligned} \tag{29}$$

Proof. It can be seen from (11) that

$$C'_{\text{STBC}}(0) = \frac{E\left[\operatorname{trace}\left(H^{\dagger}H\right)\right]}{N_t \ln 2},$$

$$C''_{\text{STBC}}(0) = -\frac{E\left[\left(\text{trace}\left(H^{\dagger}H\right)\right)^2\right]}{R_c N_t^2 \ln 2}. \tag{30}$$

If we combine (14) and (16) with (30) and then substitute into (13), desired results (29) are obtained after some simple algebraic calculation:

$$\begin{aligned}\frac{S_0^{\text{optimal}}}{S_0^{\text{STBC}}} &= \frac{(1+1/m)\,\Phi(\beta) + (N_t N_r - 1)}{\left[R_c(1+1/m)\,\Phi(\beta) + (N_t + N_r - 2)\right]},\\ \frac{S_0^{\text{MMSE}}}{S_0^{\text{STBC}}} &= \frac{(1+1/m)\,\Phi(\beta) + (N_t N_r - 1)}{\left[R_c(1+1/m)\,\Phi(\beta) + (2N_t + N_r - 3)\right]}.\end{aligned} \tag{31}$$

Since $(N_tN_r - 1) \geq (N_t + N_r - 2)$, $(N_tN_r - 1) \geq (2N_t + N_r - 3)$ and $R_c \leq 1$; therefore $S_0^{\text{optimal}} \geq S_0^{\text{MMSE}} \geq S_0^{\text{STBC}}$; this shows that SM MIMO systems with optimal/MMSE detectors have higher wideband slope than OSTBC system. For i.i.d. Rayleigh fading conditions, (29) is simplified as

$$\begin{aligned}\frac{E_b^{\text{STBC}}}{N_{0_{\min}}} &= \frac{\ln 2}{N_r \Omega},\\ S_0^{\text{STBC}} &= \frac{2R_c N_t N_r}{N_t N_r + 1}.\end{aligned} \tag{32}$$

In Proposition 4, it is discussed that $E_b/N_{0_{\min}}$ is affected by code rate, transmission distance, and average channel gains. In Figure 3, spectral efficiency of OSTBC systems in MIMO WG fading channels for $\beta = 2$ and 20 and $m = 1$ and 70 is shown for $N_t = N_r = 2$, $\lambda = 1$, and $R_c = 1$ bps. It is shown that S_0 is reduced by 50% when $m = 1$ instead of $m = 70$. $E_b/N_{0_{\min}}$ increases with the increase in fading parameter β, and an elevated wideband slope is observed which compensates the increased maximum energy per bit. OSTBC diversity leads to a reduction in capacity, which we examined by comparing it with the average capacity obtained from i.i.d. Gaussian inputs and by assigning equal transmit power. □

After some simple mathematical formulation, it is observed that $E_b/N_{0_{\min}}$ for all three configurations is identical, while S_0 is different. We have determined that the approximations are precise for the proposed cases at adequately low SNR values, particularly for high values of fading parameters. However, they become inappropriate for small values of β and m ($0 \leq \beta \leq 1.5$; $0 \leq m \leq 1$) due to the unreliable nature of $\Phi(\beta)$. As shown in Figure 3, the capacity does not show significant improvement for less fading and high shadowing under *GK* fading [13] compared to WG fading condition. In addition, for $\beta = 20$, $m = 1$, MIMO system can achieve more capacity in WG fading than that of *GK* fading; however, WG fading degrades the capacity at high values of both parameters. Our results are approximately same as obtained and compared in [13] for special cases such as Rayleigh fading and Weibull fading. Also, analytical results are well suited to simulation results.

Figure 4 shows the comparison of optimal and MMSE detection with OSTBC diversity technique on the basis

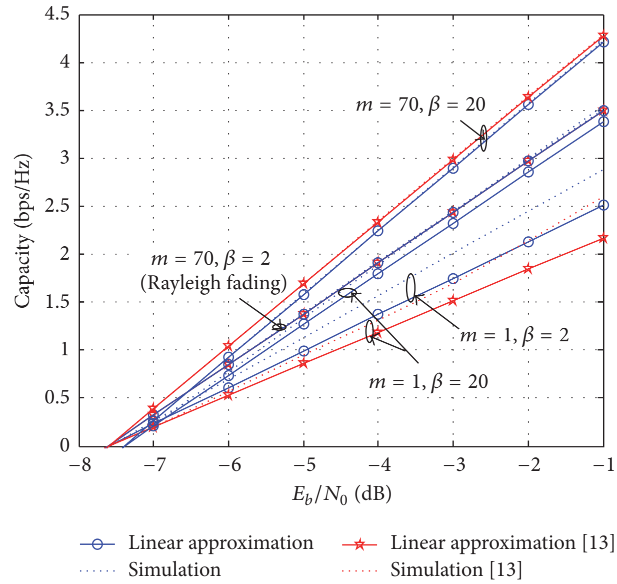

FIGURE 3: Capacity of OSTBC MIMO system over WG fading channels for different m and β when $N_t = N_r = 2$ (m and β are shadowing and fading parameters for both WG fading and *GK* fading [13]).

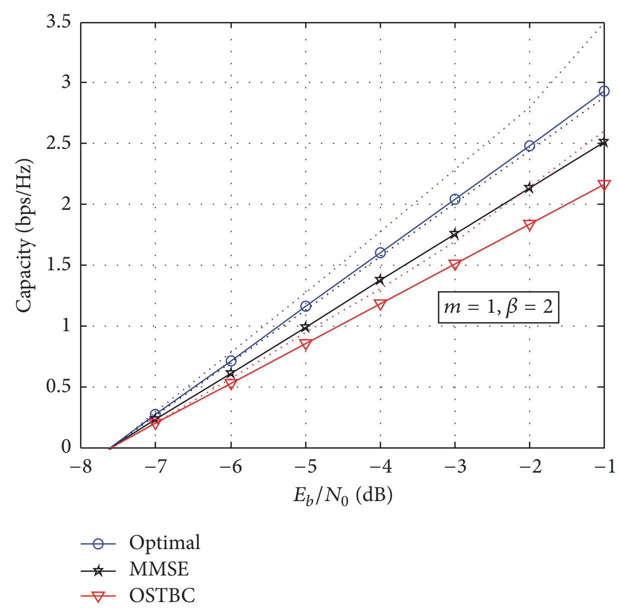

FIGURE 4: Capacity comparison of optimal and MMSE detection with OSTBC over WG fading channel for $m = 1$, $\beta = 2$ when $N_t = N_r = 2$ (dotted lines illustrate simulation results using optimal, MMSE, and OSTBC techniques).

of capacity performance. At −3 dB SNR, approximately 0.5 bps/Hz and 0.24 bps/Hz improved capacity is achieved using optimal and MMSE detection, respectively, compared to that of OSTBC for $m = 1$, $\beta = 2$. However, MMSE detector reduces 0.25 bps/Hz capacity compared to that of optimal detector for the same parameters. It is noted that optimal detector experiences more computation complexity than MMSE detector.

4. Conclusion

The WG fading model has shown a wide range of agreement with measured data under various environment conditions. The performance estimation of MIMO systems in WG fading channels is bounded, generally due to the adversity to deal with the non-Gaussian nature of the fading coefficients. A specified low SNR analysis of different MIMO systems operating in WG fading channels is presented in this paper. Novel tractable expressions for $E_b/N_{0_{\min}}$ and S_0 are deduced. These expressions are verified by previous results of Rayleigh and *K*/*GK*-fading. The proposed analysis of MIMO system deals with three techniques, namely, SM with optimal detection, SM with MMSE detection, and OSTBC. The average capacity of OSTBC systems is usually subsidiary to that of capacity-oriented SM MIMO techniques, whereas OSTBC system is a diversity-oriented technique. Further, this work can be extended using the same channel model with other performance measures.

Appendix

The proof depends on the moments of WG variates, which can be evaluated by combining (4)-(5). The procedure to evaluate (15) is given by (A.1). Firstly, ith ($i = 1, 2, \ldots, N_t$) diagonal element of $(H^\dagger H)^2$ is augmented, which is computed by

$$\begin{aligned}\left[\left(H^\dagger H\right)^2\right]_{i,i} &= \left(\sum_{i=1}^{N_r} \left|h_{k,i}\right|^2\right) \\ &+ \sum_{n=1,n\neq i}^{N_t} \left(\sum_{k=1}^{N_r} h_{k,i} h_{k,n}^*\right)\left(\sum_{k=1}^{N_r} h_{k,i}^* h_{k,n}\right).\end{aligned} \tag{A.1}$$

After some simple algebraic calculation, the expected value of (A.1) can be obtained as

$$\begin{aligned}E\left(\left[\left(H^\dagger H\right)^2\right]_{i,i}\right) &= \frac{\Omega^2 \lambda^4 N_r}{d^{2\upsilon}} \left[\left(1+\frac{1}{m}\right)\Gamma\left(1+\frac{4}{\beta}\right)\right. \\ &\left. + \left(N_t + N_r - 2\right)\Gamma\left(1+\frac{2}{\beta}\right)^2\right].\end{aligned} \tag{A.2}$$

All diagonal elements of (A.2) are summed up to get (15). Concerning (16), it is noted that all the elements of H are i.i.d. random variables to obtain

$$\begin{aligned}E&\left[\left(\operatorname{trace}\left(H^\dagger H\right)\right)^2\right] \\ &= N_t N_r E\left(\left|h_{11}\right|^4\right) \\ &\quad + N_t N_r \left(N_t N_r - 1\right)\left(E\left(\left|h_{11}\right|^2\right)\right)^2.\end{aligned} \tag{A.3}$$

Equation (16) is obtained by employing (4) and (5) in (A.3) after some simplifications.

Competing Interests

The authors declare that they have no competing interests.

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