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Research Article

Performance Analysis of Precoded MIMO PLC System Based on Two-Sided Jacobi SVD

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This paper evaluates the performance of closed loop multiple input multiple output power line communication (CL MIMO PLC) system based on enhanced zero-forcing (ZF) equalizer. In this work, the two-sided Jacobi (TSJ) algorithm has been investigated for the computation of singular value decomposition of the channel matrix. Quantized parameters are feedback from the receiver to the transmitter for precoding process. Numerous simplifications are introduced for the reduction of the algorithm complexity. The performance of the CL MIMO PLC is evaluated in terms of bit error rate (BER), constellation error vector magnitude (EVM), and mean square error (MSE) between the constructed SVD matrices and Matlab computed ones.

1. Introduction

The PLC systems present the new trend for high level communication. The application of MIMO scenarios on PLC systems enhances the data throughput significantly. In MIMO wireline systems [1], the data streams can be demultiplexed into several substreams transmitted by different ports to improve the throughput performance of the overall communication system by utilizing the transmit diversity [2, 3].

Various architectures of receivers has been proposed in literature such as the zero-forcing (ZF) receiver, the minimum mean square error (MMSE) receiver, and the successive interference canceller (SIC) receiver. These techniques are investigated to decode the spatially multiplexed signals over MIMO systems [4–6]. Generally, the performance improvement from one type of the MIMO receiver to another comes at the price of higher implementation cost. For example, despite its reduced complexity of implementation, the ZF receiver is known to suffer from the effect of noise enhancement.

One can enhance the performance of the ZF receiver, by splitting the equalization algorithm among the transmitter and the receiver. The TSJ-SVD is used for the computing of precoding and decoding matrix. Several simplifications are introduced to reduce the hardware implementation of the precoding/decoding processes.

The remainder of the paper is organized as follows. In Section 2, the CL MIMO system is described. The precoding design scheme is then presented in Section 3 and introduced simplifications making computing complexity lower. Subsequently, the performance analysis of TSJ-SVD algorithm is given in Section 4. Finally, the paper is concluded in Section 5.

The superscript $(\cdot)^H$ denotes the conjugate transpose. In addition, $(\cdot)^+$ and $(\cdot)^T$ represent the pseudoinverse and transpose operations, respectively. $\mathbb{C}^{N_r \times N_t}$ denotes the set of $N_r \times N_t$ matrices over complex field.

2. System Description

For a MIMO PLC system composed of N_t transmission ports and N_r reception ports, the MIMO channel can be described by a complex matrix $H \in \mathbb{C}^{N_r \times N_t}$. The MIMO PLC model is then given by

$$r = Hx + n, \tag{1}$$

where $x = [x_1, x_2, \ldots, x_{Nt}]^T$ is the transmit signal, $r = [r_1, r_2, \ldots, r_{Nr}]^T$ is the received signal vector, and $n = [n_1, n_2, \ldots, n_{Nr}]^T$ is the noise at the receiver. In the remainder of the paper

$N_t = 2$ and $N_r = 4$. The channel matrix H has the following formula:

$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \\ h_{31} & h_{32} \\ h_{41} & h_{42} \end{bmatrix}, \tag{2}$$

where $h_{ij} \in \mathbb{C}$ is the complex coefficient, $i \in [1, \ldots, 4]$ and $j \in [1, 2]$.

In conventional MIMO communication system based on zero-forcing (ZF) equalizer, the equalization matrix is given by

$$W_{\mathrm{ZF}} = H^{+} = \left(H^{H} H\right)^{-1} H^{H}. \tag{3}$$

The main idea of the closed loop MIMO system is based on decomposing the channel matrix H into precoding and decoding parts. The precoding matrix is then sent by the receiver to the transmitter in a feedback in order to precode the signal before being sent.

Here the decomposition is carried out by the SVD method largely used in MIMO. The channel matrix H can be decomposed into 2 parallel and independent SISO branches by SVD (see Figure 1):

$$H = UDV^{H}, \tag{4}$$

where $V \in \mathbb{C}^{2\times 2}$ is the right-hand unitary matrix of the SVD, $U \in \mathbb{C}^{4\times 2}$ is the left-hand unitary matrix, and $D \in \mathbb{R}^{2\times 2}$ is a diagonal matrix containing the singular values of the channel matrix H.

In order to improve the MIMO equalization, rather than using simple ZF equalizer (2), the precoding matrix is incorporated in (1) by replacing the transmit symbol vector x by

$$s = Vx. \tag{5}$$

Equation (1) becomes

$$r = HVx + n = \left(UDV'\right) Vx + n = UDx + n. \tag{6}$$

If we denote W as the reception first-stage matrix, the received signal crossing W is

$$y = Wr = WUDx + Wn. \tag{7}$$

If $W = U^{H}$ then

$$y = Wr = Dx + U^{H} n. \tag{8}$$

The combination of precoding, channel, and reception first-stage matrices decomposes the equivalent channel into parallel streams reduced to the diagonal matrix D (see Figure 1). Then the ZF equalizer matrix becomes

$$W_{\mathrm{ZF}} = W = U^{H} D^{-1}. \tag{9}$$

The precoding matrix V mixes the symbols of x, and thus the transmit symbol vector s contains all symbols of x; that is, each symbol is transmitted via each MIMO path. Thus, the full spatial diversity is achieved. The receiver side model is depicted in Figure 2. In this work, the 2×4 MIMO system is considered.

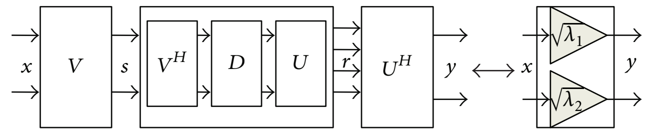

Figure 1: SVD based MIMO transmitter and receiver.

3. The Precoding Process

The proposed MIMO scheme is mainly based on the precoding process. After estimating the channel matrix H, its SVD is carried out using the TSJ method.

In this section, we introduce the TSJ-SVD algorithm. Then, we describe the main steps of the proposed simplified algorithm.

3.1. Basic Transformation on the Channel Matrix. In the literature, the TSJ algorithm is only used for real symmetric matrix [7]. In our case, the channel matrix H is complex. So to apply the TSJ algorithm we should transform the complex matrix into real symmetric matrix. We then firstly proceed by transforming the matrix $H \in \mathbb{C}^{2\times 2}$ into Hermitian matrix A:

$$A = H^{H} H = \begin{bmatrix} a & be^{j\beta} \\ be^{-j\beta} & d \end{bmatrix}, \tag{10}$$

where a, b, and d are real values, β is an angle, and A is a Hermitian matrix such that

(i) $A^{H} = A$;

(ii) The diagonal elements of A are real; only the off-diagonals are complex conjugates:

$$C = \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix}^{H} \begin{bmatrix} a & be^{j\beta} \\ be^{-j\beta} & d \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix} = \begin{bmatrix} a & b \\ b & d \end{bmatrix}, \tag{11}$$

where $\theta = -\beta$.

The obtained matrix C is real and symmetric and the Jacobi algorithm can be applied to decompose it into SVD form.

3.2. The Two-Sided Jacobi Transformation. As the MIMO system needs to feed back the precoding matrix to the transmitter, an optimization of the transferred parameters is fundamental to make the transferred parameters transfer overhead as little as possible. The transformation of H to C is carried out for this aim as will be detailed below.

The TSJ algorithm iteratively minimizes the off-diagonal elements of the matrix C expressed in

$$\mathrm{Off}(C) = \sqrt{\sum_{m=1}^{2} \sum_{n=1, n\neq m}^{2} c_{mn}^{2}}. \tag{12}$$

The Jacobi transformations work by performing a sequence of orthogonal updates of C matrix described by

$$C_{i+1} \longleftarrow Q_i^{T} C_i Q_i, \tag{13}$$

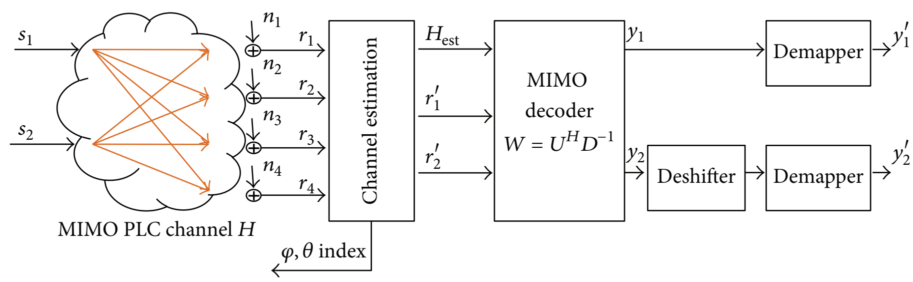

Figure 2: Receiver side of precoded MIMO PLC system.

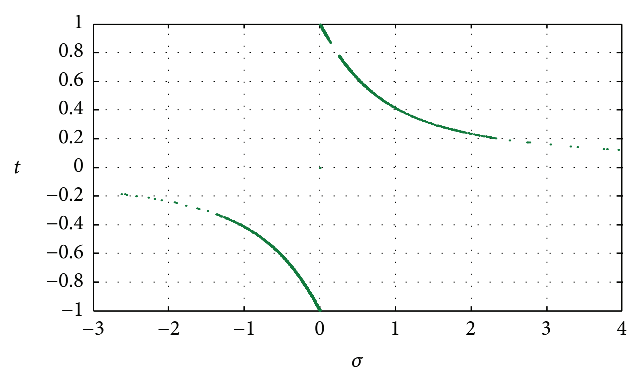

Figure 3: Variation of t as function of σ.

where C_i is the processed matrix at iteration i and Q_i is a Jacobi matrix at iteration i.

With C_{i+1} being "more diagonal" than its predecessor C_i, the off-diagonal values are compared to a threshold in order to decide whether the Jacobi algorithm converged or not. This threshold is proportional to the on-diagonal values of C and is defined by

$$\tau = \varepsilon \left(\frac{1}{2} \sqrt{\sum_{m=n=1,2} c_{mn}^2} \right), \tag{14}$$

where ε is a constant value.

The Jacobi matrix (named also given rotation) is defined by the following formula:

$$Q = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}. \tag{15}$$

The rotation angle φ is chosen so that the off-diagonal elements of C are equal to zero [7]. φ should then satisfy the following:

$$\begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix}^T \begin{bmatrix} a & b \\ b & d \end{bmatrix} \begin{bmatrix} \cos\varphi & \sin\varphi \\ -\sin\varphi & \cos\varphi \end{bmatrix} = \begin{bmatrix} a_1 & 0 \\ 0 & d_1 \end{bmatrix}. \tag{16}$$

Equation (16) can be simplified to

$$b\left(\cos\varphi^2 - \sin\varphi^2\right) + (a-d)\cos\varphi\sin\varphi = 0. \tag{17}$$

Two cases arise:

(i) If $b = 0$ then C is already diagonal. Considering in this case $\cos\varphi = 1$ and $\sin\varphi = 0$ (rather than $\cos\varphi = 0$ and $\sin\varphi = 1$) makes no change to C. We have then $\varphi = 0$.

(ii) If $b \neq 0$ then (17) remains to be solved.

If $b \neq 0$, we define the parameters $\sigma = (a-d)/2b$ and $t = \cos\varphi/\sin\varphi$. Equation (17) becomes

$$t^2 + 2\sigma t - 1 = 0. \tag{18}$$

Equation (17) has the two roots:

$$t = \sigma \pm \sqrt{1+\sigma^2}. \tag{19}$$

As a result, the cosine and sine values of the Jacobi matrix (see (15)) can be calculated as

$$\cos\varphi = \frac{1}{\sqrt{t^2+1}}$$
$$\sin\varphi = \cos\varphi \cdot t. \tag{20}$$

With C matrix being real symmetric and 2×2 sized, the theoretical maximum number of iterations i is thus equal to 1. In practice, the expression of t is quite complex to implement; it involves a division and calculation of square root, which is resource consuming when implemented on embedded platform. t should then be approximated, and the off-diagonal values of C are consequently no longer equal to 0 from the first iteration.

According to [8], the t parameter can be approximated to make the algorithm faster. The variation of t as function of σ is given in Figure 3.

It is easy to see that the error between t and its approximated value is limited when σ is close to 0. The value converges to zero when σ goes to infinity. The approximation of t will then develop for positive values of σ.

Using Taylor expansion the approximation of t is given by

$$\lim_{\sigma \to 0} \frac{1}{\sigma + \sqrt{1+\sigma^2}} \approx 1 - \sigma + \frac{\sigma^2}{2},$$
$$\lim_{\sigma \to +\infty} \frac{1}{\sigma + \sqrt{1+\sigma^2}} \approx \frac{1}{2\sigma}. \tag{21}$$

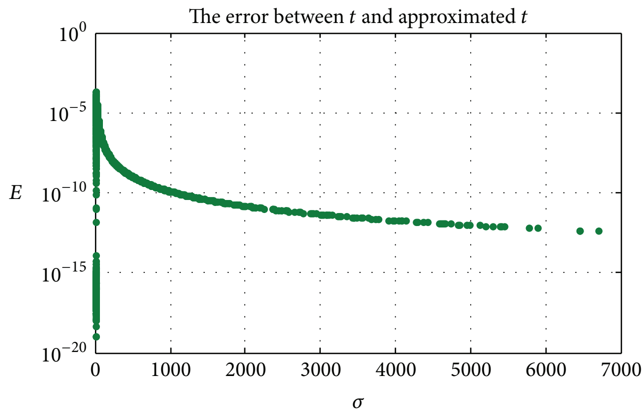

FIGURE 4: Approximation error of t.

Based on the piecewise approximation we get the following result:

$$\tilde{t} = \begin{cases} 1 - \sigma + \dfrac{\sigma^2}{2}, & 0 \leq \sigma < 1.25 \\ -\sigma + \sqrt{1 + \sigma^2}, & 1.25 \leq \sigma < 5 \\ \dfrac{1}{2\sigma}, & \sigma \geq 5. \end{cases} \quad (22)$$

The approximation error $E = |t - \tilde{t}|$ is reported in Figure 4. It is obvious that the approximation errors have been controlled within a precision almost equal to 10^{-4}, knowing that around $0t$ is very close to ± 1.

The Jacobi process is finished when the off-diagonal elements of the matrix become close to zero. Finally, the Jacobi transformation process determines the following:

(1) The right-hand singular vector matrix V is equal to the multiplication of successive matrix rotations Q by the transformation matrix applied to A (see (11)):

$$V = \begin{bmatrix} 1 & 0 \\ 0 & e^{j\theta} \end{bmatrix} \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix}, \quad (23)$$

where

$$Q = \begin{bmatrix} \cos(\varphi) & \sin(\varphi) \\ -\sin(\varphi) & \cos(\varphi) \end{bmatrix} = \prod_i \begin{bmatrix} \cos\varphi_i & \sin\varphi_i \\ -\sin\varphi_i & \cos\varphi_i \end{bmatrix}, \quad (24)$$

$$\varphi = \sum_i \varphi_i \quad (25)$$

with φ_i being the rotation angle of the Jacobi matrix Q_i at iteration i.

(2) The singular values matrix $D = \sqrt{C_{i+1}}$.

(3) The left-hand U matrix is deduced by

$$U = HVD^{-1}. \quad (26)$$

In Figure 6 is reported the flowchart of the precoding process.

3.3. Quantization of the Precoding Matrix Fed Back to the Transmitter. As described above, performing the Jacobi based SVD on the channel matrix leads to 3 matrices: the diagonal matrix D, which contains the square root of the singular values of the channel matrix, the left-hand decoding matrix U, and the right singular vector matrix V also called precoding matrix.

In the transmitter, the precoding operation is performed on each carrier and consists of multiplying the output signals by the precoding matrix V. V is constructed using the two parameters θ and φ fed back from the receiver. To do this, the two parameters are uniformly quantized as follows:

$$\begin{aligned} \varphi &= \frac{k\pi}{2^{b_\varphi+1}}, \quad k = 0, 1, \ldots, 2^{b_\varphi} - 1, \\ \theta &= \frac{k\pi}{2^{b_\theta-1}} - \pi, \quad k = 0, 1, \ldots, 2^{b_\theta} - 1, \end{aligned} \quad (27)$$

where b_φ is the number of b used to quantize φ and b_θ is the number of bits used to quantize θ.

4. Simulation Results

To evaluate the TSJ based SVD precoding/decoding process, we developed a MIMO communication system composed of the following:

(i) A transmitter chain that contains a random source delivering random streams containing $N = 13820$ bits, a splitter module to split the data into two MIMO signals, a mapping module using 16-QAM modulation, and a phase shifter which introduces a $\pi/2$ angle to the signal transmitted via the second antenna; the transmitter chain is shown in Figure 5.

(ii) A receiver chain that contains a channel estimation module, a MIMO decoder, a deshifter, and demapping, as reported in Figure 2.

The frequency band of the transmitted signal is [10 MHz, 86 MHz]. Simulations are carried out for 2×4 MIMO channel generated and based on the MIMO PLC model detailed in [9] and based on the Multiconductor Transmission Line theory (MTL) [10].

4.1. Validation of the Proposed Precoding Algorithm. The performance of the Jacobi transformation algorithm is evaluated according to the mean square error (MSE) between the computed eigenvalues denoted as λ^l_{svd}, $l = 1, 2$, obtained from the proposed Jacobi algorithm and the eigenvalues λ^l, $l = 1, 2$, obtained from the SVD function of Matlab (see Figure 7) where l is the antenna index. The MSE is defined by

$$\text{MSE}^l_{\text{jacobi}} = \frac{1}{2}\left(\lambda^l_{\text{svd}} - \lambda^l\right)^2, \quad l = 1, 2. \quad (28)$$

The transmitted streams constellations before and after precoding are presented in Figure 8.

The 16-QAM constellations undergo a specific rotation due to precoding matrix multiplication.

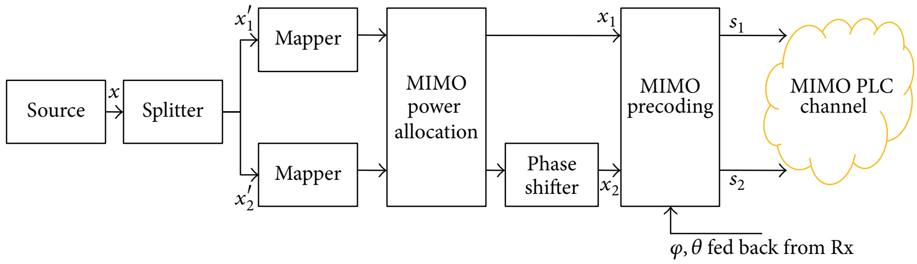

Figure 5: Transmitter chain of the precoded MIMO PLC system.

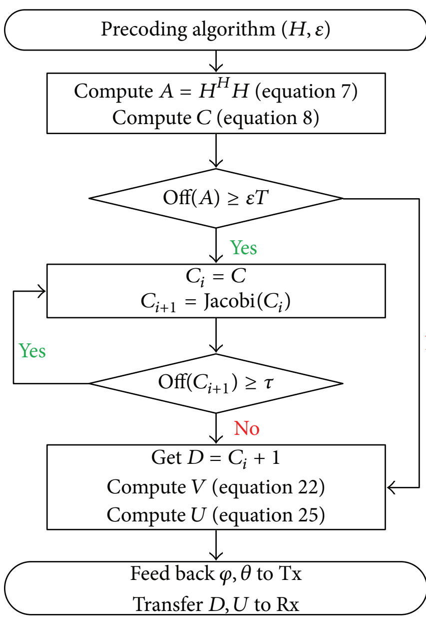

Figure 6: The precoding process flowchart.

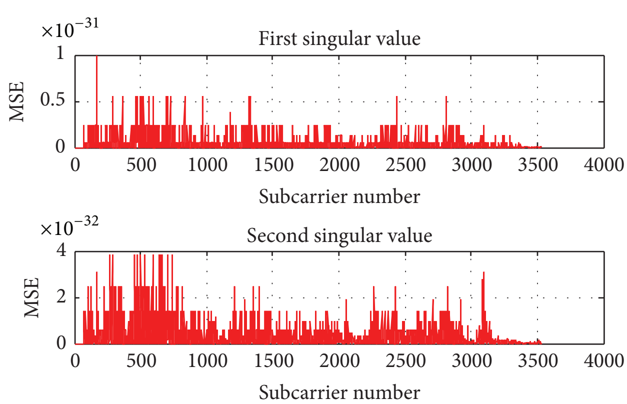

Figure 7: MSE of the two eigenvalues λ^1 and λ^2, respectively.

4.2. Evaluation of the MIMO System Performance. In this section, the MIMO system performance evaluation is carried out in terms of energy conservation, Bit error rate (BER) variation, and the effect of the number of φ and θ quantization bits.

4.2.1. Energy Conservation of the System. The aim here is to verify the correctness of the TSJ-SVD algorithm in the decomposition of the estimated channel matrix H. The MSE is calculated between the coefficients computed using the proposed SVD algorithm denoted by $h_{12,\mathrm{SVD}}$ and the real values $h_{12,\mathrm{real}}$ for 3 generated MIMO channels. The comparison results are shown in Figure 9.

According to Figure 9, the maximum value of MSE for the 3 different MIMO channels is below $6 \cdot 10^{-6}$. The same results are found for other matrix coefficients. This undeniably proves that the SVD algorithm based on Jacobi transform is energy conservative. This is explained by the fact that all matrix transformations applied to channel matrix are unitary.

4.2.2. Bit Error Rate as Function of SNR. In this section is evaluated the BER for different values of SNR, for bit streams coming from both MIMO antennas. Simulations are carried out without any error correcting encoding/decoding. BER is compared to theoretical BER for uncoded 16-QAM.

According to Figure 10, BER behavior of our MIMO system is close to theoretical BER. This proves that no additional noise is brought by our MIMO precoding/decoding processing.

4.2.3. The Error Vector Magnitude (EVM) as a Function of Quantization Bits. As described above, the receiver feeds back the two quantized parameters θ and φ to the transmitter in order to make the latter construct the precoding matrix V. In this section we study the effect of the number of quantization bits on the system performance. Simulations are carried out without adding any additive noise in order to only see the quantization noise effect. For evaluation purpose, we considered the constellation error vector magnitude (EVM) criteria. The EVM reflects the error between the received constellation after equalization and the original normalized 16-QAM constellation. In Figure 11 is reported the EVM (95th percentile point, which is the value where 95% of the individual symbol EVM values are below that point) as a function of the number of quantization bits, calculated on the two MIMO streams. Each EVM point is calculated for 3455 constellations.

We observe that the EVM converges to 0 when the number of quantization bits becomes high. In this case, the received constellation becomes closer to the original one. The EVM decreases below 0.1% from a quantization number equal to 16.

5. Conclusion

In this paper is described a method for linear precoded 2×4 MIMO PLC system based on TSJ-SVD algorithm.

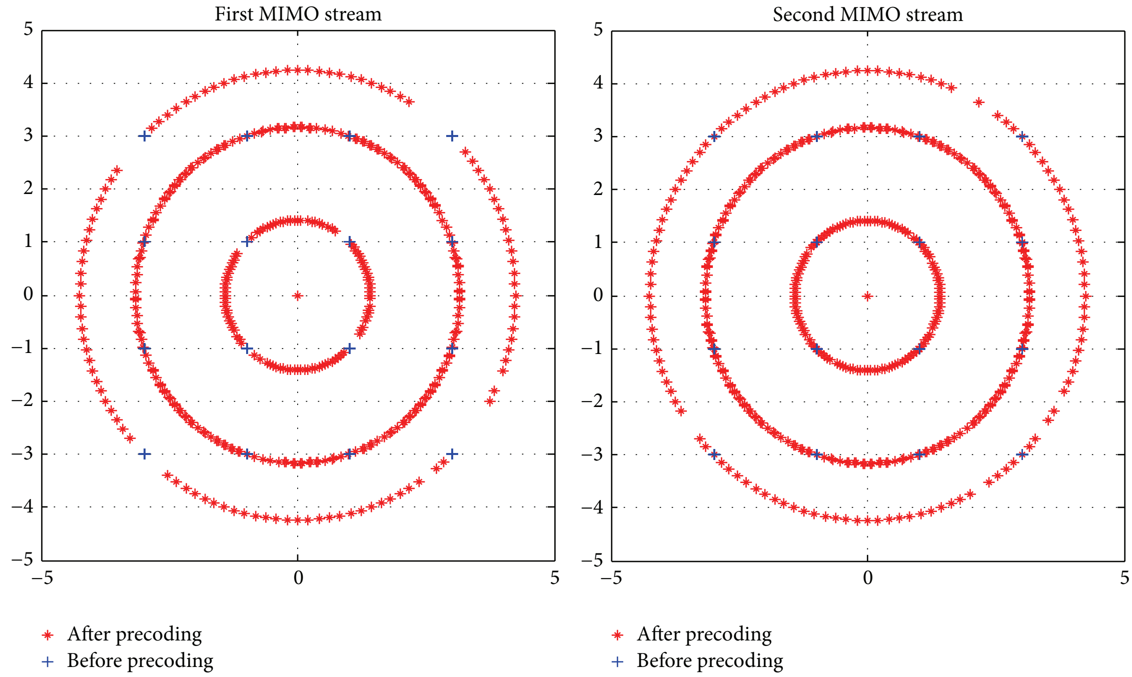

Figure 8: Transmitter chain of the precoded MIMO PLC system.

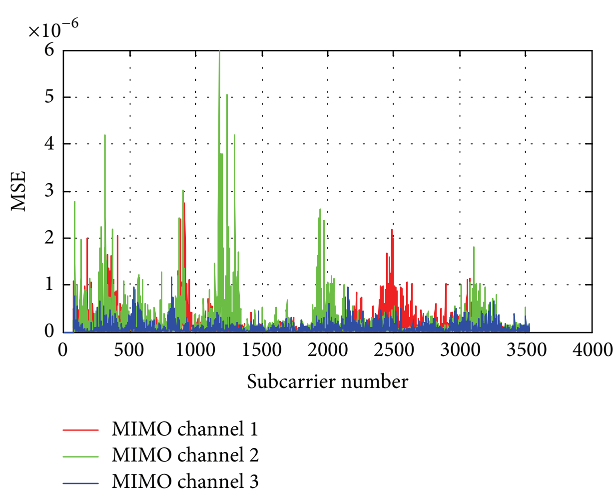

Figure 9: MSE between $h_{12,\mathrm{SVD}}$ and $h_{12,\mathrm{real}}$.

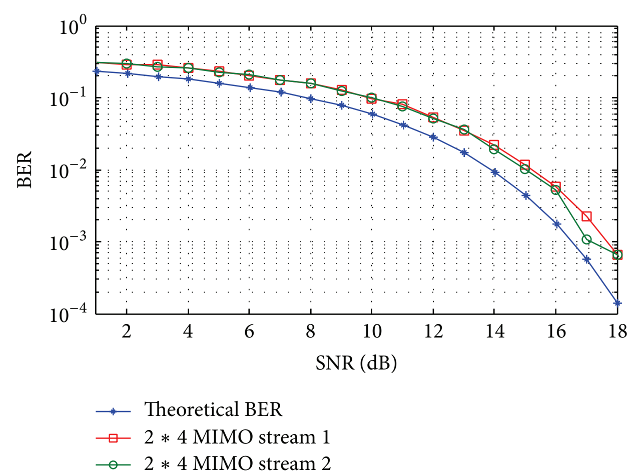

Figure 10: BER as function of SNR for bit streams from MIMO ports 1 and 2.

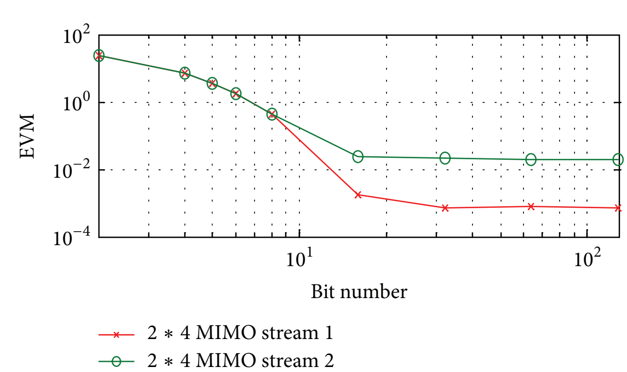

Figure 11: Effect of the number of quantization bits on EVM.

A simplified ZF equalizer is presented. The precoding matrix is compressed into 2 phase values which are quantified before being fed back from the receiver to the transmitter.

The proposed TSJ-SVD outputs were revalidated through comparison to Matlab SVD. Singular values λ^l, $l = 1, 2$, calculated from SVD and Matlab were juxtaposed.

The MIMO system performance was finally evaluated in terms of precoding matrix energy conservation, BER as function of SNR, and effect of quantization bits on EVM. It is proven that without error correcting code the proposed system BER is close to theoretical BER meaning that no additional noise is brought by our precoding/decoding processing. Also, based on the EVM 95th percentile point criteria, it was shown that the performance of the MIMO system is better for large number of quantization bits and that the EVM decreases below 0.1% from a quantization bits number equal to 16.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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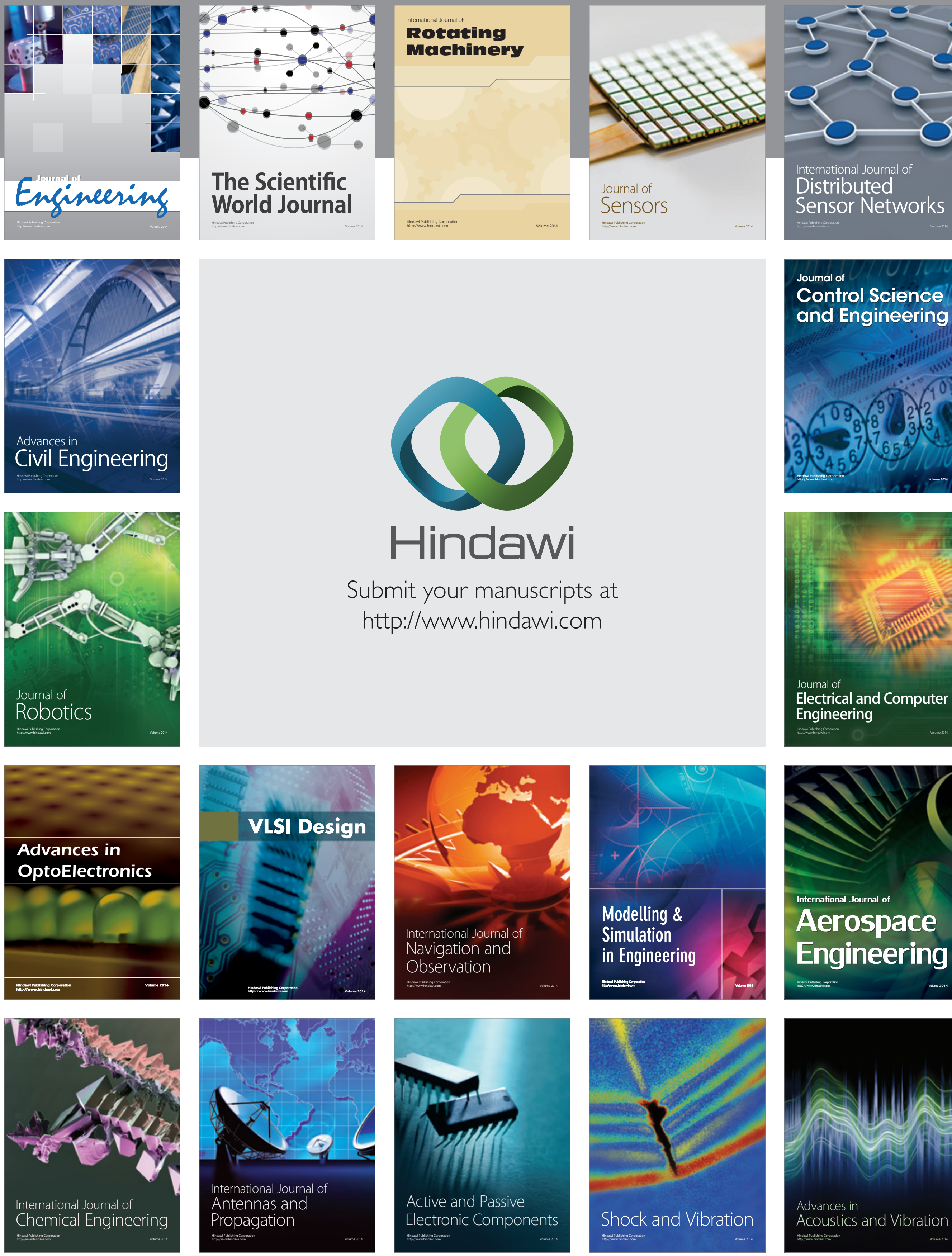
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