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*Research Article*

# Performance Analysis of Different Types of Sensor Networks for Cognitive Radios

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We consider the problem of using multiple sensors to detect whether a certain spectrum is occupied or not. Each sensor sends its spectrum sensing result to a data fusion center, which combines all the results for an overall decision. With the existence of wireless fading on the channels from sensors to data fusion center, we examine three different mechanisms on the transmissions from sensors to data fusion center: (1) direct transmissions; (2) transmissions with the assistance of relays and (3) transmissions with the assistance of an intermediate fusion helper which fuses the sensing results from the sensors and sends the fused result to the data fusion center. For each mechanism, we analyze the correct probability of the overall decision made by the data fusion center. Our evaluation establishes that a sensor network with an intermediate fusion helper performs almost as good as a sensor network with relays, but providing energy and spectral advantages. Both networks significantly outperform the sensor network without relay or intermediate fusion helper. Such analysis facilitates the design of sensor networks. Furthermore, we generalize this evaluation to sensor networks with an arbitrary number of sensors and to sensor networks applying various information combining rules. Our simulations validate the analytical results.

## 1. Introduction

Cognitive radio (CR) is a potential technology for increasing spectral efficiency in wireless communications systems. In a cognitive radio system, secondary users temporarily use spectrum that is not utilized, as long as negligible impact is caused to primary licensed users. In order to opportunistically access temporarily unused spectrum, the spectrum in an area needs to be sensed from time to time. In a simple scenario, a secondary user acts as a sensory node; it senses and uses the available spectrum. The spectrum sensing techniques include energy detector-based sensing, waveform-based sensing, cyclostationarity-based sensing, radio identification-based sensing, and matched-filtering, and so forth [1].

Due to noise uncertainty and wireless channel fading, the sensing decision made by a single sensor is sometimes unreliable. Cooperative sensing among multiple sensors is an efficient approach to addressing this issue, because it provides multiple measurements and, hence, increases the diversity. Additionally, having sensors cooperating over a wide area also provides a possible solution to the hidden-terminal problem. This is because sensors, separated by a distance larger than the correlation distance of shadow fading, are unlikely to be shadowed simultaneously from the primary user.

In cooperative sensing, after performing the spectrum sensing operations, each sensor sends its sensing results to a data fusion center, which makes an overall decision about the spectrum occupancy. The process of making an overall decision based on multiple sensing results is called *data fusion* or *information combining*. Depending on the type of sensing results sent from the sensors to the data fusion center, the information combining can be classified into three categories: hard combining (cf., e.g., [2]), hard combining with side information (cf., e.g., [2–4]), and soft combining (cf., e.g., [4–6]).

In the above work, the sensing results from all the sensors are assumed to be delivered to the data fusion center without error. In other words, the fusion channels, namely, the channels from sensors to the data fusion center, are error-free and bandwidth unlimited. There are many applications (e.g., [7]) for large-scale sensor networks, with power limited sensors wirelessly connected to the data fusion center. The fusion channels are noisy and experience wireless fading. Much work [8–11] has been devoted to examine the information combining rules under the condition of rate-constrained fusion channels. The optimal information combining rules were extensively studied in [12–17], when the fusion channels are noisy channels or wireless fading channels. Furthermore, it was proposed in [14, 15] to use relays for reliable transmissions on the noisy fusion channels. It should be mentioned that most of the efforts, in the presence of the noisy or rate-constrained fusion channels, were focused on the optimal information combining rules.

It was recently proposed in [18] to reduce the traffic load at the data fusion center by using an *intermediate fusion helper* in a sensor network. Specifically, the intermediate fusion helper combines the decisions it receives from several sensors and transmits the (combined) intermediate decision to the data fusion center. Although the spectral advantage of the sensor network with an intermediate fusion helper is obvious, its detection performance, especially in the noisy fusion channel environment, is unclear.

The contribution of this paper is fourfold.

(i) We establish a system model to incorporate the practical situations of wireless fading fusion channels. Within this model, we analyze the performance of a basic sensor network, a sensor network with relays, and a sensor network with an intermediate fusion helper. Our analysis shows that the correct probability of the overall decision in the sensor network with an intermediate fusion helper is almost as good as that in the sensor network with relays and is much higher than that in the basic sensor network. On the other hand, the sensor network with an intermediate fusion helper has the energy and spectral advantage over the sensor network with relays.

(ii) In the sensor networks with an intermediate fusion helper, we study the locations of the intermediate fusion helper for the optimal network performance. Such examination facilitates the design of sensor networks.

(iii) We investigate the performance of different networks under multiple information combining rules, which include the majority rule, the AND rule, the OR rule, a hard combining with side information rule, and the Maximum Ratio Combining (MRC) rule.

(iv) We generalize the above performance evaluation to a sensor network with an arbitrary number of sensors. In this setting, we propose to use multiple intermediate fusion helpers in the network.

The rest of this paper is organized as follows. The problem formulation is given in Section 2. Section 3 discusses the sensor network with relays. The sensor network with an intermediate fusion helper is introduced in Section 4. The performance of all these sensor networks is analyzed in the separate sections. Section 5 generalizes the discussions to sensor networks with an arbitrary number of sensors, as well as to sensor networks with different information combining rules. Section 6 examines the case of Rayleigh faded fusion channels. Simulation results are provided in Section 7. Section 8 concludes this paper and discusses the future work.

## 2. Problem Formulation

Consider a wireless sensor network (cf., Figure 1) deployed with three sensors to detect whether a spectrum is occupied or not. The detection problem can be stated in terms of a binary hypothesis test: hypothesis $H_0$ is the signal absence or spectrum unoccupied, and hypothesis $H_1$ is the signal presence or spectrum occupied. The *a priori* probabilities of the two hypotheses are $\Pr(H_0) = \pi_0$ and $\Pr(H_1) = \pi_1$. Suppose each sensor listens to a certain spectrum and applies some spectrum sensing technique. Let $S_i$, $1 \leq i \leq 3$, denote the decision made by the $i$th sensor, where

$$S_i = \begin{cases} -1 & \text{if } H_0 \text{ is declared,} \\ 1 & \text{if } H_1 \text{ is declared.} \end{cases} \tag{1}$$

The probability $A_i$ that the decision $S_i$ is true is given by

$$A_i = \Pr(S_i = -1 \mid H_0)\pi_0 + \Pr(S_i = 1 \mid H_1)\pi_1. \tag{2}$$

The observations and decisions made by the three sensors are assumed to be statistically independent conditioned on either hypothesis, that is,

$$\Pr\Big(S_1, S_2, S_3 \mid H_j\Big) = \prod_{i=1}^{3} \Pr\Big(S_i \mid H_j\Big), \quad j = 0, 1. \tag{3}$$

After the spectrum sensing operations, each sensor sends its decision to the data fusion center through its own fusion channel. The three fusion channels are mutually independent wireless fading channels. Let $X_i$ and $Y_i$ be the input and the output of the $i$th fusion channel. Then,

$$Y_i = h_i\sqrt{G_i}X_i + N_i, \tag{4}$$

where $h_i$ is the channel fading, $G_i$ is the path loss, and $N_i$ is the additive white Gaussian noise with distribution $\mathcal{N}(0, \sigma_i^2)$. Before transmission, the $i$th sensor modulates its decision $S_i$ to $X_i$, using the BPSK scheme with transmission power $P_i$. Hence, we have $X_i = \sqrt{P_i}S_i$. Throughout this paper, we omit error-correction coding as it would have the same effects in all of the discussions.

The data fusion center demodulates the received signal $Y_i$ to $T_i \in \{-1, 1\}$. It then applies the majority combining rule to make an overall decision. Specifically, if at least two of the demodulated decisions are 1, then the data fusion center declares the presence of the signal. Otherwise, it declares the absence of the signal. The overall decision at the data fusion center can be expressed as

$$U = \begin{cases} -1 & \text{if} \sum_{i=1}^{3} T_i < 0, \\ 1 & \text{if} \sum_{i=1}^{3} T_i \geq 0, \end{cases} \tag{5}$$

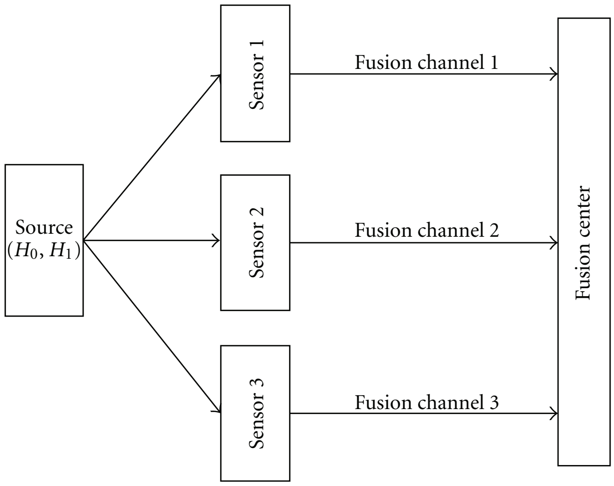


Figure 1: Block diagram for the basic sensor network.

The probability $P_c$ that the overall decision $U$ matches the true hypothesis is defined as

$$P_c = \pi_0 \Pr(U = -1 \mid H_0) + \pi_1 \Pr(U = 1 \mid H_1). \tag{6}$$

Next, we shall characterize this probability. To simplify our calculations, we make the following symmetry assumptions in the rest of this paper.

(i) For each sensor, $\Pr(S_i = -1 \mid H_0) = \Pr(S_i = 1 \mid H_1)$. Hence, $A_i$ is equal to the detection probability $\Pr(S_i = 1 \mid H_1)$.

(ii) All the sensors have the same detection probability: $A_1 = A_2 = A_3 = A$.

(iii) All the sensors have the same transmission power: $P_1 = P_2 = P_3 = P$.

(iv) The noise powers of all the fusion channels are identical: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \sigma^2$.

With these simplifications, we define the signal-to-noise ratio as $\mathrm{SNR} = P/\sigma^2$, and use the notation $\mathbb{E}$ for the mean of random variables.

To further facilitate our computations, we omit the short-term fading in the fusion channels until Section 6, and only take the path loss into account. Hence in (4), the channel fading $h_i = 1$ and the path loss $G_i = d_i^{-\beta}$, where $d_i$ is the distance from the $i$th sensor to the data fusion center and $\beta$ is the path loss exponent. We assume the equal distance from all the sensors to the data fusion center, that is, $d_1 = d_2 = d_3 = d$.

Let $P_t$ be the probability that a transmission on a fusion channel is demodulated correctly at the data fusion center. Throughout this paper, we assume $\pi_0$ and $\pi_1$ are unknown to the receiver. Hence, the optimal demodulation threshold is set to 0. Then, it follows from the BPSK modulation scheme that (cf., e.g., [19])

$$P_t = \Pr(T_i = S_i) = 1 - Q\left(\sqrt{\frac{G_i P}{\sigma^2}}\right) = 1 - Q\left(\sqrt{d^{-\beta}\mathrm{SNR}}\right), \tag{7}$$

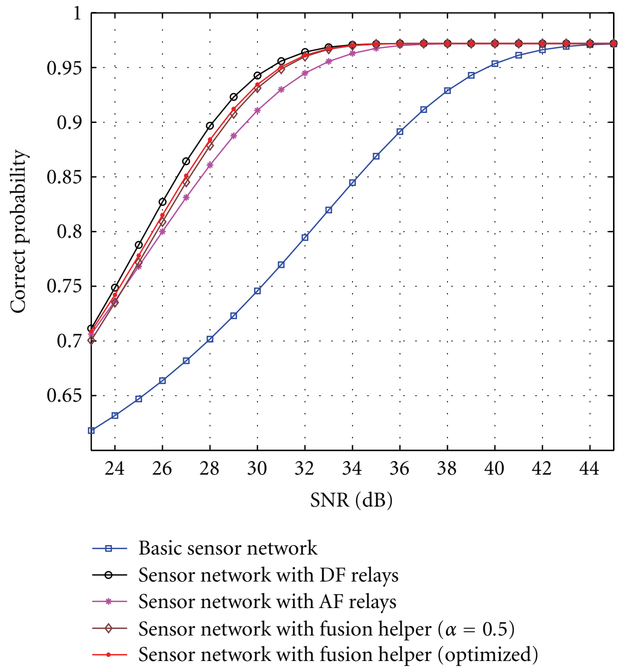


Figure 2: Theoretical correct probability of the overall decision in different sensor networks with Gaussian channels ($A = 0.9, \beta = 3.5, d = 10$).

where $Q(x) = (1/\sqrt{2\pi})\int_x^\infty e^{-t^2/2}dt$ is the Gaussian tail function. It follows from the Markov chain and (7) that

$$\begin{aligned}\Pr(T_i = -1 \mid H_0) &= \Pr(T_i = 1 \mid H_1)\\ &= \Pr(T_i = -1, S_i = -1 \mid H_0)\\ &\quad + \Pr(T_i = -1, S_i = 1 \mid H_0)\\ &= AP_t + (1 - A)(1 - P_t) \triangleq P_B.\end{aligned} \tag{8}$$

By the majority combining rule (5), we have

$$\Pr(U = -1 \mid H_0) = \Pr(U = 1 \mid H_1) = 3P_B^2(1 - P_B) + P_B^3. \tag{9}$$

Hence, it follows from (6) that

$$P_c = 3P_B^2(1 - P_B) + P_B^3. \tag{10}$$

This probability versus SNR is illustrated using the curve with square marker in Figure 2. In plotting this curve, we set $A = 0.9$, $\beta = 3.5$, and $d = 10$. Note that we apply transmit SNR rather than receive SNR in this figure (and subsequent figures) so as to coincide it for the relay and the intermediate fusion helper cases. The difference between transmit SNR and receive SNR in this case is equal to $d^{-\beta} = -35$ dB.

Even though the correct probability of the individual decision is as high as 0.9, we observe from the figure that the correct probability of the overall decision is quite small in the low SNR region. It follows from the majority combining rule that the correct probability of the overall decision is upper bounded by $3A^2(1 - A) + A^3$, which is equal to 0.972 for $A = 0.9$. This upper bound is achieved when the fusion channels are noiseless.

## 3. Sensor Network with Relays

As discussed, the basic sensor network does not perform well at low SNRs. A natural way to increase $P_c$ is via enhancing the sensors' transmission power $P$, and hence the SNR. This approach may be infeasible due to the power limitation of the sensors, as well as the potential interference caused. An approach to improving the transmission reliability without enhancing the sensors' transmission power is by means of relays. The usage of relays for reliable transmissions and throughput increment has been widely studied (e.g., [20–23]), while its application for reliable transmissions on the fusion channels has been adopted in [14, 15].

Consider the sensor network in Figure 1, but with a relay located between every sensor and the data fusion center. It is known that the common relaying schemes include the demodulate-and-forward (DF) scheme and the amplify-and-forward (AF) scheme. We shall characterize the correct probability of the overall decision $P_c$ in the sensor network with relays, using either of these two relaying schemes.

Here, we assume that the distance from a sensor to its serving relay is $\alpha d$, $0 \le \alpha \le 1$, and the distance from a relay to the data fusion center is $(1-\alpha)d$. All the relays have the same transmission power as the sensors.

*3.1. Demodulate-and-Forward Relays.* For the demodulate-and-forward scheme, a relay first demodulates the transmission from a sensor. It then remodulates the binary decision and transmits it to the data fusion center. Note that all the channels to and from the relays are wireless fading channels. The data fusion center demodulates the transmissions from the relays and applies the majority combining rule to make an overall decision.

Denote by $R_i$ the demodulated decision at the $i$th relay. Let $P_{t,1}$ be the probability that a transmission from a sensor is demodulated correctly at the corresponding relay. Let $P_{t,2}$ be the probability that a transmission from a relay is demodulated correctly at the data fusion center. Then, we have

$$P_{t,1} = \Pr(R_i = S_i) = 1 - Q\left(\sqrt{(\alpha d)^{-\beta}\mathrm{SNR}}\right), \quad (11)$$

$$P_{t,2} = \Pr(T_i = R_i) = 1 - Q\left(\sqrt{[(1-\alpha)d]^{-\beta}\mathrm{SNR}}\right). \quad (12)$$

It can be derived that

$$\begin{aligned}\Pr(T_i = -1 \mid H_0) &= \Pr(T_i = 1 \mid H_1)\\ &= AP_{t,1}P_{t,2} + A(1-P_{t,1})(1-P_{t,2})\\ &\quad + (1-A)(1-P_{t,1})P_{t,2}\\ &\quad + (1-A)P_{t,1}(1-P_{t,2})\\ &\triangleq P_R.\end{aligned} \quad (13)$$

Therefore,

$$P_c = 3P_R^2(1-P_R) + P_R^3. \quad (14)$$

*3.2. Amplify-and-Forward Relays.* Let $X_{i,1}$ and $Y_{i,1}$ denote the inputs and the outputs of the channel from the $i$th sensor to its serving relay. Let $X_{i,2}$ and $Y_{i,2}$ denote the inputs and the outputs of the channel from the $i$th relay to the data fusion center. Then, we have

$$Y_{i,1} = \sqrt{G_{i,1}}X_{i,1} + N_{i,1} = \sqrt{G_{i,1}P}S_i + N_{i,1}, \quad (15)$$

$$Y_{i,2} = \sqrt{G_{i,2}}X_{i,2} + N_{i,2}, \quad (16)$$

where $G_{i,1}$ and $G_{i,2}$ represent the path loss on the respective channel and $N_{i,1}$ and $N_{i,2}$ represent the additive white Gaussian noise with distribution $\mathcal{N}(0,\sigma^2)$ on the respective channel. It follows from the path loss model that $G_{i,1} = (\alpha d)^{-\beta}$ and $G_{i,2} = [(1-\alpha)d]^{-\beta}$.

For the amplify-and-forward scheme, a relay amplifies its received signal $Y_{i,1}$ by a factor of $K$ before transmitting it to the data fusion center, that is,

$$X_{i,2} = KY_{i,1} = K\sqrt{G_{i,1}P}S_i + KN_{i,1}. \quad (17)$$

Since the transmission power of a relay is equal to $P$, we obtain that

$$K = \sqrt{\frac{P}{(\alpha d)^{-\beta}P + \sigma^2}} = \sqrt{\frac{\mathrm{SNR}}{(\alpha d)^{-\beta}\mathrm{SNR} + 1}}. \quad (18)$$

Denote by ESNR the equivalent SNR for the transmissions from the sensor to the data fusion center. Then, it follows from (16), (17), and (18) that

$$\begin{aligned}\mathrm{ESNR} &= \frac{\mathbb{E}\left[\left|K\sqrt{G_{i,1}G_{i,2}P}S_i\right|^2\right]}{\mathbb{E}\left[\left|K\sqrt{G_{i,2}}N_{i,1} + N_{i,2}\right|^2\right]}\\ &= \frac{\mathrm{SNR}^2(\alpha d)^{-\beta}[(1-\alpha)d]^{-\beta}}{\mathrm{SNR}(\alpha d)^{-\beta} + \mathrm{SNR}[(1-\alpha)d]^{-\beta} + 1}.\end{aligned} \quad (19)$$

Let $P_{t,A}$ be the probability that a transmission from a sensor is demodulated correctly at the data fusion center. Then, we have $P_{t,A} = 1 - Q(\sqrt{\mathrm{ESNR}})$. By the similar arguments as in Section 2, we derive that

$$P_c = 3P_A^2(1-P_A) + P_A^3, \quad (20)$$

where $P_A = AP_{t,A} + (1-A)(1-P_{t,A})$.

For either DF scheme or AF scheme, the correct probabilities (14) and (20) are ascending functions of the probability of correct reception at the data fusion center. Hence, the optimal relay position to maximize (14) and (20) results from the classic relay positioning problem, which is known [20–23] to be half-way between source and destination. Thereby, the parameter $\alpha$ will be optimized to 0.5 in the following discussions.

The probability (14) is plotted as the curve with circle marker in Figure 2 and the probability (20) is plotted as the curve with star marker in Figure 2. In plotting these curves, we adopt the same parameters as before, that is, $A = 0.9$, $\beta = 3.5$, and $d = 10$. It is seen from the figure that the sensor network with relays (either DF or AF) significantly outperforms the basic sensor network. Furthermore, the sensor network with DF relays performs better than that with AF relays in the operational SNR region (though such conclusion may be contrary at lower SNRs). Hence, we shall focus on the DF relays in the remaining discussions of this paper.

The sensor network with relays achieves the desired correct probability of the overall decision, in the cost of three additional relays. A simplified version [14] of this network is a single relay taking all the relaying functionalities. In other words, the single relay repeats the operations for each of the sensors. Note that the relay makes three transmissions, one for each sensor. This consumes much energy and may be infeasible for low-power relays. Moreover, the multiple transmissions may become a communication bottleneck at the data fusion center if the number of sensors in the network is large.

## 4. Sensor Network with an Intermediate Fusion Helper

Consider the sensor network in Figure 1, but with a single intermediate fusion helper located between all the sensors and the data fusion center. The intermediate fusion helper receives and demodulates the transmissions from all the sensors, and then applies the majority combining rule to make an intermediate decision on whether the signal is present or not. It sends this binary decision to the data fusion center. Subsequently, the data fusion center simply demodulates this message and declares the same decision.

The channels to and from the intermediate fusion helper are wireless fading channels. We assume that the distances from the sensors to the intermediate fusion helper are $\alpha d$ and the distance from the intermediate fusion helper to the data fusion center is $(1-\alpha)d$. The intermediate fusion helper has the same transmission power as the sensors.

Let $P_{t,1}$ be the probability that a transmission from a sensor is demodulated correctly at the intermediate fusion helper. Let $P_{t,2}$ be the probability that a transmission from the intermediate fusion helper is demodulated correctly at the data fusion center. Then, these probabilities follow from (11) and (12).

Denote by $F_i$ the demodulated decision from the $i$th sensor at the intermediate fusion helper, and denote by $U_F$ the intermediate fusion decision made at the intermediate fusion helper. Then, by the majority combining rule,

$$U_F = \begin{cases} -1 & \text{if} \sum_{i=1}^{3} F_i < 0, \\ 1 & \text{if} \sum_{i=1}^{3} F_i \geq 0. \end{cases} \tag{21}$$

It is not difficult to derive

$$\begin{aligned} \Pr(F_i = -1 \mid H_0) &= \Pr(F_i = 1 \mid H_1) \\ &= AP_{t,1} + (1-A)(1-P_{t,1}) \triangleq P_{F,i}, \end{aligned} \tag{22}$$

$$\begin{aligned} \Pr(U_F = -1 \mid H_0) &= \Pr(U_F = 1 \mid H_1) \\ &= 3P_{F,i}^2(1-P_{F,i}) + P_{F,i}^3 \triangleq P_F. \end{aligned} \tag{23}$$

Therefore, the correct probability of the overall decision is obtained as

$$P_c = P_F P_{t,2} + (1-P_F)(1-P_{t,2}). \tag{24}$$

Using the same parameters as before, we plot the probability $P_c$ of the sensor network with an intermediate fusion helper as the curve with diamond marker in Figure 2. It is seen from the figure that the sensor network with an intermediate fusion helper performs almost as good as the sensor network with relays. However, only a single message is transmitted from the intermediate fusion helper to the data fusion center. This reduces the system bandwidth requirement and energy consumption, comparing with the sensor network with relays. Note that intermediate fusion helper performs a bit more processing than DF relay.

*4.1. Optimal Location of the Intermediate Fusion Helper.* In drawing the curve with diamond marker in Figure 2, we locate the intermediate fusion helper in the middle of the sensors and the data fusion center, that is, $\alpha = 0.5$. However, such a location may not be optimal for maximizing $P_c$. In this subsection, we shall examine the optimal location of the intermediate fusion helper.

It is not difficult to derive from (11) and (12) that

$$\frac{\partial P_{t,1}}{\partial \alpha} = -\sqrt{\frac{\beta^2 d^{-\beta}\mathrm{SNR}}{8\pi}}\, e^{-(\alpha d)^{-\beta}\mathrm{SNR}/2}\alpha^{-\beta/2-1}, \tag{25}$$

$$\frac{\partial P_{t,2}}{\partial \alpha} = \sqrt{\frac{\beta^2 d^{-\beta}\mathrm{SNR}}{8\pi}}\, e^{-[(1-\alpha)d]^{-\beta}\mathrm{SNR}/2}(1-\alpha)^{-\beta/2-1}. \tag{26}$$

It follows from (22) and (23) that

$$\begin{aligned} \frac{\partial P_F}{\partial \alpha} &= \frac{\partial P_F}{\partial P_{F,i}} \cdot \frac{\partial P_{F,i}}{\partial \alpha} = 6P_{F,i}(1-P_{F,i})\frac{\partial P_{F,i}}{\partial \alpha} \\ &= 6P_{F,i}(1-P_{F,i})(2A-1)\frac{\partial P_{t,1}}{\partial \alpha}. \end{aligned} \tag{27}$$

Finally, we obtain from (24) that

$$\frac{\partial P_c}{\partial \alpha} = -\frac{\partial P_{t,2}}{\partial \alpha} - \frac{\partial P_F}{\partial \alpha} + 2P_F\frac{\partial P_{t,2}}{\partial \alpha} + 2P_{t,2}\frac{\partial P_F}{\partial \alpha}. \tag{28}$$

By inserting (12), (23), (25), (26), and (27) into (28), and setting it to zero, we obtain the optimal location $\alpha^*$ as a function of $(A, \mathrm{SNR}, \beta, d)$. Note that $P_c$ is not a concave function of $\alpha$ at low SNRs. For simplicity, we focus only on the medium-high SNR range (i.e., SNR $\geq$ 23 dB), which ensures a concave $P_c$ function.

Figure 3 shows the optimal location $\alpha^*$ versus SNR for different values of $A$ and for fixed $\beta = 3.5$ and $d = 10$. It is observed from the figure that $\alpha^*$ is generally larger than 0.5 and tends to 0.5 at high SNR. This indicates that to optimize the performance, the intermediate fusion helper is closer to the data fusion center than to the sensors. A possible reason is that the feature of majority combining rule allows certain level of tolerance for incorrect reception at the intermediate fusion helper, which results in reduced importance of the link from sensors to the intermediate fusion helper. Therefore, the

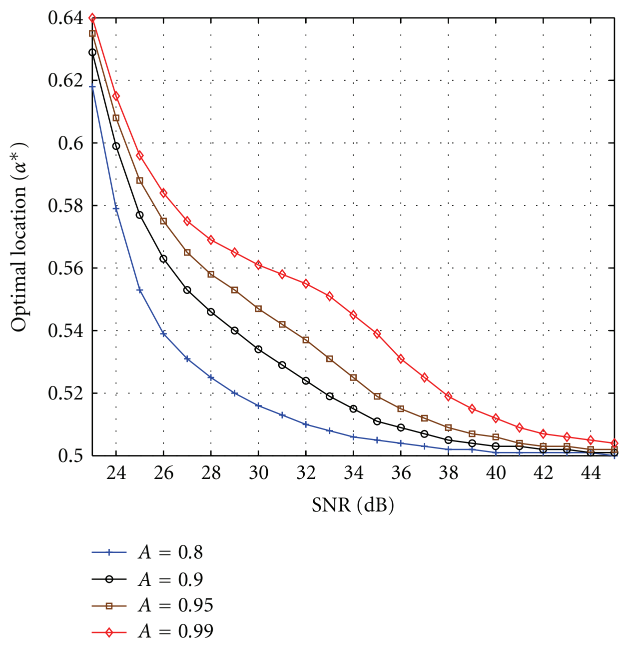


FIGURE 3: Analytical optimal location of the intermediate fusion helper in Gaussian channels ($\beta = 3.5$, $d = 10$).

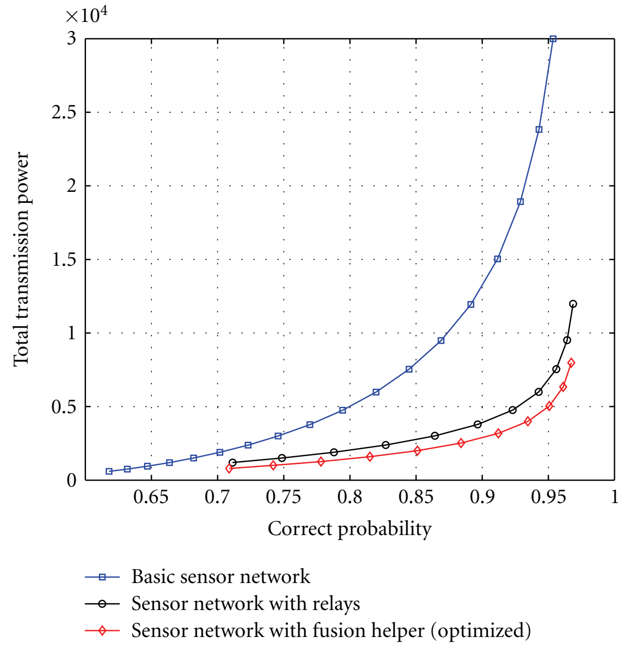


FIGURE 4: The total transmission energy used in different sensor networks (with the unit noise power) with Gaussian channels ($A = 0.9$, $\beta = 3.5$, $d = 10$).

intermediate fusion helper could be located closer to the data fusion center. This argument facilitates our placement of the intermediate fusion helper for any given locations of sensors and data fusion center. For instance, consider a circle with large radius, where sensors are distributed on the arc of the circle and the data fusion center is located at the center of the circle. The distances between sensors are much smaller than the radius of the circle. To achieve the best system performance, the intermediate fusion helper could be placed on the line between the data fusion center and one of the sensors, with the distance to the data fusion center being $\alpha^*$ times the radius of the circle.

The $P_c$ of the sensor network with the optimally located intermediate fusion helper is shown as the curve with dot marker in Figure 2. A few performance gains of the sensor network with optimized intermediate fusion helper location over that with a fixed $\alpha = 0.5$ can be observed from the figure.

*4.2. Transmission Energy of the Sensor Networks.* In the basic sensor network, in order to achieve a desired correct probability of the overall decision, the transmission energy of each sensor needs to be very large. In the sensor network with relays, the multiple relays will consume as much transmission energy as the sensors. The network with intermediate fusion helper requires a single transmission from the intermediate fusion helper. Hence, the total transmission energy consumed in the sensor network with intermediate fusion helper is less than that of the sensor network with relays. Figure 4 shows the total transmission energy used in the three networks, with the assumption of unit noise power. It is seen that to achieve the same correct probability of the overall decision, the network with an intermediate fusion helper consumes the least transmission energy.

The comparison of the basic sensor network, the sensor network with relays and the sensor network with intermediate fusion helper is summarized in Table 1. The basic sensor network is of the lowest complexity and has the lowest bandwidth requirement, but it does not perform well in terms of correct probability of the overall decision. The sensor network with relays achieves high detection performance in the cost of high complexity (in terms of the number of relays) and high bandwidth requirement. The sensor network with an intermediate fusion helper has the similar detection performance as the sensor network with relays, but with much reduced complexity and bandwidth requirement. To achieve the same level of detection performance, the sensor network with intermediate fusion helper consumes the least transmission energy.

## 5. Generalizations

In this section, we evaluate the performance of sensor networks with arbitrary numbers of sensors, and under different information combining rules other than the majority combining rule.

*5.1. Sensor Networks with an Arbitrary Number of Sensors.* Consider a sensor network with $m$ sensors connecting to the data fusion center, for some odd $m$. All the other settings remained intact as in Section 2. The data fusion center

Table 1: Comparison of three sensor networks.

| | Complexity | Detection performance | Bandwidth requirement | Transmission energy for certain detection level |
|---|---|---|---|---|
| Basic sensor network | Low | Bad | Low | High |
| Sensor network with relays | High | Good | High | Medium |
| Sensor network with intermediate fusion helper | Medium | Good | Medium | Low |

applies the majority combining rule to make the overall decision as

$$U = \begin{cases} -1 & \text{if } \sum_{i=1}^{m} T_i < 0, \\ 1 & \text{if } \sum_{i=1}^{m} T_i \geq 0. \end{cases} \tag{29}$$

Hence, we have

$$\begin{aligned} &\Pr(U = -1 \mid H_0) \\ &\quad = \Pr(U = 1 \mid H_1) \\ &\quad = \sum_{j=\lceil m/2 \rceil}^{m} \binom{m}{j} \Pr(T_i = -1 \mid H_0)^j \Pr(T_i = 1 \mid H_0)^{m-j} \\ &\quad = \sum_{j=\lceil m/2 \rceil}^{m} \binom{m}{j} P_B^j (1 - P_B)^{m-j}, \end{aligned} \tag{30}$$

where $P_B$ is defined in (8). The correct probability of the overall decision is given by

$$P_c = \sum_{j=\lceil m/2 \rceil}^{m} \binom{m}{j} P_B^j (1 - P_B)^{m-j}. \tag{31}$$

Following similar arguments, we obtain that for the sensor network with $m$ sensors and $m$ relays, the correct probability of the overall decision at the data fusion center is

$$P_c = \sum_{j=\lceil m/2 \rceil}^{m} \binom{m}{j} P_R^j (1 - P_R)^{m-j}, \tag{32}$$

where $P_R$ is defined in (13).

Consider the sensor network with $m$ sensors and a single intermediate fusion helper. All the other settings remained same as in Section 4. We obtain that

$$\begin{aligned} \Pr(U_F = -1 \mid H_0) &= \Pr(U_F = 1 \mid H_1) \\ &= \sum_{j=\lceil m/2 \rceil}^{m} \binom{m}{j} P_{F,i}^j (1 - P_{F,i})^{m-j} \triangleq P_{F,m}, \end{aligned} \tag{33}$$

where $P_{F,i}$ is defined in (22). Finally,

$$P_c = P_{F,m} P_{t,2} + (1 - P_{F,m})(1 - P_{t,2}). \tag{34}$$

Figure 5 shows the curves of (31), (32), and (34) with $m = 9$. The relays and the intermediate fusion helper are located at the place of $\alpha = 0.5$, while all the other parameters remained same as in Figure 2. In these networks, we know from the majority rule that the correct probability of the overall decision is upper bounded by $\sum_{j=5}^{9} \binom{9}{j} 0.9^j 0.1^{9-j}$. For $A = 0.9$, this upper bound is equal to 0.999, as seen in the figure.

In a sensor network with a single intermediate fusion helper serving a large number of sensors, the single intermediate fusion helper may also occur the communication bottleneck. To alleviate this problem, more than one intermediate fusion helper could be used. Consider a sensor network with $m$ sensors and $n$ intermediate fusion helpers. Each intermediate fusion helper serves $m/n$ sensors. Denote by $U_{F,i}$ the intermediate decision made by the $i$th intermediate fusion helper. It is clear that

$$\begin{aligned} \Pr(U_{F,i} = -1 \mid H_0) &= \Pr(U_{F,i} = 1 \mid H_1) \\ &= \sum_{j=\lceil m/2n \rceil}^{m/n} \binom{m/n}{j} P_{F,i}^j (1 - P_{F,i})^{m/n-j} \\ &\triangleq P_{U,i}, \end{aligned} \tag{35}$$

where $P_{F,i}$ is defined in (22). Denote by $T_i$ the demodulated decision from the $i$th intermediate fusion helper at the data fusion center. Then,

$$\begin{aligned} \Pr(T_i = -1 \mid H_0) &= \Pr(T_i = 1 \mid H_1) \\ &= P_{t,2} P_{U,i} + (1 - P_{t,2})(1 - P_{U,i}) \triangleq P_U, \end{aligned} \tag{36}$$

where $P_{t,2}$ is given by (12). The correct probability of the overall decision is

$$P_c = \sum_{j=\lceil n/2 \rceil}^{n} \binom{n}{j} P_U^j (1 - P_U)^{n-j}. \tag{37}$$

Consider an example of a sensor network with 9 sensors and 3 intermediate fusion helpers. Each intermediate fusion helper serves 3 sensors. The $P_c$ of this network is plotted as a curve with star marker in Figure 5. We observe that the performance of this sensor network is between the sensor network with a single intermediate fusion helper and the sensor network with relays. This implies that with more intermediate fusion helpers, we achieve higher correct probability of the overall decision in the cost of more transmission

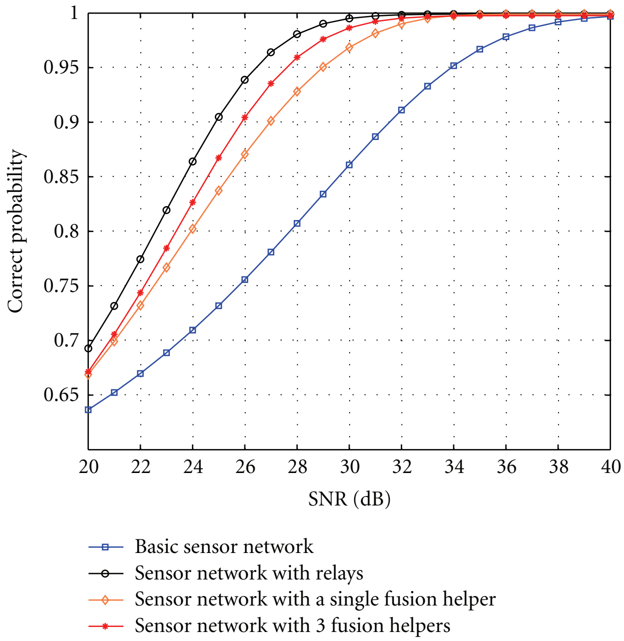


Figure 5: Theoretical correct probability of the overall decision for sensor networks with 9 sensors in Gaussian channels ($A = 0.9$, $\beta = 3.5$, $d = 10$).

energy. In this sense, the sensor network with relays can be regarded as the sensor network with intermediate fusion helpers, where the number of intermediate fusion helpers is same as the number of sensors. In practice, the number of intermediate fusion helpers in a sensor network could be determined as a result of balancing the communication bottleneck, the correct probability of the overall decision, and the total transmission energy.

*5.2. The AND Combining Rule and the OR Combining Rule.* It was assumed in the previous sections that either the data fusion center or the intermediate fusion helper applies the majority combining rule. There are two other usual hard combining rules: the AND rule and the OR rule. We shall compare the performance of different sensor networks under these two information combining rules.

Consider a sensor network with $m$ sensors connecting to the data fusion center. The data fusion center applies the following AND combining rule:

$$U = \begin{cases} -1 & \text{if } \sum_{i=1}^{m} T_i < m, \\ 1 & \text{if } \sum_{i=1}^{m} T_i = m, \end{cases} \tag{38}$$

where $T_i$ is the demodulated decision from the $i$th sensor. We obtain that

$$\begin{aligned} \Pr(U = -1 \mid H_0) &= 1 - \Pr\left(\sum_{i=1}^{m} T_i = m \mid H_0\right) \\ &= 1 - \Pr(T_i = 1 \mid H_0)^m = 1 - (1 - P_B)^m, \\ \Pr(U = 1 \mid H_1) &= \Pr\left(\sum_{i=1}^{m} T_i = m \mid H_1\right) \\ &= \Pr(T_i = 1 \mid H_1)^m = P_B^m, \end{aligned} \tag{39}$$

where $P_B$ is defined in (8). The correct probability of the overall decision is

$$P_c = \pi_0\left[1 - (1 - P_B)^m\right] + \pi_1 P_B^m. \tag{40}$$

Following similar arguments, we obtain that for the sensor network with $m$ sensors and $m$ relays, the correct probability of the overall decision is

$$P_c = \pi_0\left[1 - (1 - P_R)^m\right] + \pi_1 P_R^m, \tag{41}$$

where $P_R$ is defined in (13).

Consider the sensor network with $m$ sensors and a single intermediate fusion helper. The intermediate fusion helper applies the AND combining rule as follows:

$$U_F = \begin{cases} -1 & \text{if } \sum_{i=1}^{m} F_i < m, \\ 1 & \text{if } \sum_{i=1}^{m} F_i = m, \end{cases} \tag{42}$$

where $F_i$ is the demodulated decision at the intermediate fusion helper for the $i$th sensor. We obtain that

$$\begin{aligned} \Pr(U_F = -1 \mid H_0) &= 1 - \Pr\left(\sum_{i=1}^{m} F_i = m \mid H_0\right) \\ &= 1 - (1 - P_{F,i})^m, \\ \Pr(U_F = 1 \mid H_1) &= \Pr\left(\sum_{i=1}^{m} F_1 = m \mid H_1\right) = P_{F,i}^m, \end{aligned} \tag{43}$$

where $P_{F,i}$ is defined in (22). Therefore,

$$\begin{aligned} \Pr(U = -1 \mid H_0) &= P_{t,2}\left[1 - (1 - P_{F,i})^m\right] \\ &\quad + (1 - P_{t,2})(1 - P_{F,i})^m, \\ \Pr(U = 1 \mid H_1) &= P_{t,2}P_{F,i}^m + (1 - P_{t,2})\left(1 - P_{F,i}^m\right). \end{aligned} \tag{44}$$

The correct probability of the overall decision is

$$\begin{aligned} P_c = {} & \pi_0\left[P_{t,2} + (1 - P_{F,i})^m - 2P_{t,2}(1 - P_{F,i})^m\right] \\ & + \pi_1\left[P_{t,2}P_{F,i}^m + (1 - P_{t,2})\left(1 - P_{F,i}^m\right)\right]. \end{aligned} \tag{45}$$

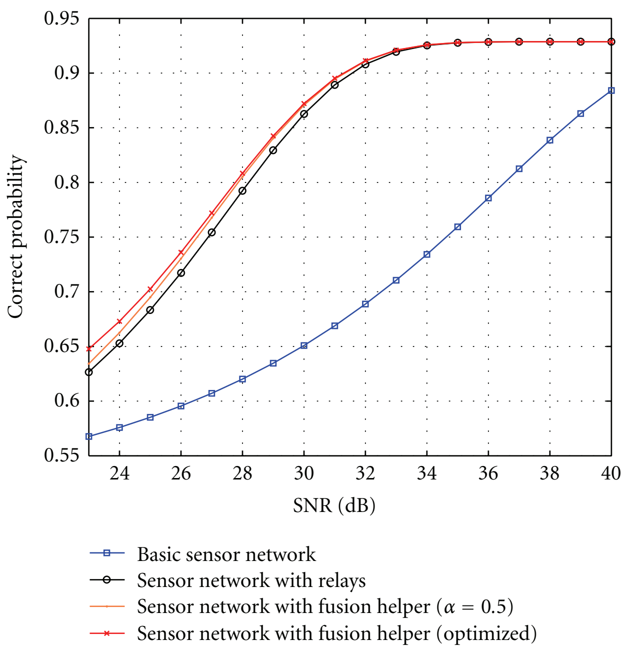


Figure 6: Theoretical correct probability of the overall decision for different sensor networks under the AND combining rule in Gaussian channels ($A = 0.95$, $\beta = 3.5$, $d = 10$, $\pi_0 = \pi_1 = 0.5$).

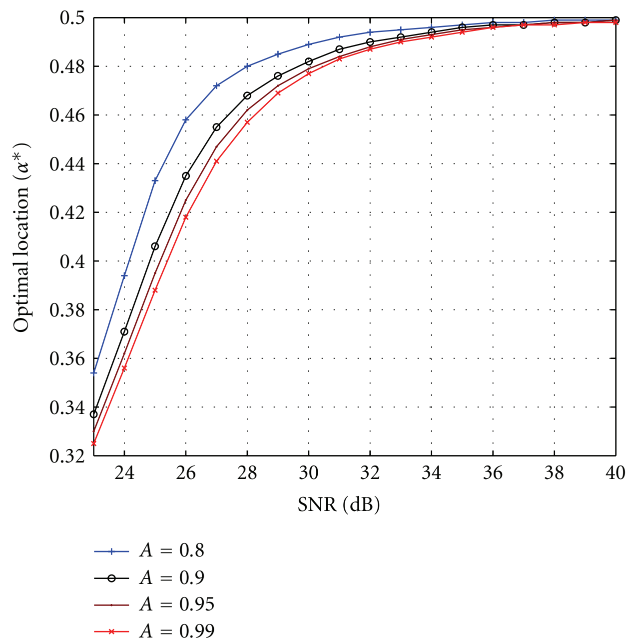


Figure 7: Analytical optimal location of the intermediate fusion helper under the AND combining rule in Gaussian channels ($\beta = 3.5$, $d = 10$, $\pi_0 = \pi_1 = 0.5$).

If the data fusion center or the intermediate fusion helper applies the following OR combining rule:

$$U = \begin{cases} -1 & \text{if } \sum_{i=1}^{m} T_i = -m, \\ 1 & \text{if } \sum_{i=1}^{m} T_i > -m, \end{cases} \tag{46}$$

then by the similar arguments, we obtain that the correct probabilities of the overall decision for the three networks are, respectively, given by (40), (41), and (45), with the exchange of $\pi_0$ and $\pi_1 = 1 - \pi_0$.

Figure 6 shows the correct probability of the overall decision in these networks with 3 sensors under the AND rule. In plotting the curves, we set the parameters $A = 0.95$, $\beta = 3.5$, $d = 10$, and $\pi_0 = \pi_1 = 0.5$. The optimal relay location is still $\alpha^* = 0.5$, while the optimal intermediate fusion helper location is shown in Figure 7. It is seen from Figure 6 that the sensor network with the intermediate fusion helper (either with $\alpha = 0.5$ or with the optimal $\alpha^*$) results in a larger correct probability of the overall decision than the sensor network with relays.

The curves in Figure 7 suggest that with the AND combining rule, the intermediate fusion helper should be located closer to the sensors for the best detection performance. This conclusion is in contrast with that for the majority combining rule. Heuristically, the AND combining rule is more sensitive to correct reception from each single sensor, since any incorrect reception could reverse the decision at the intermediate fusion helper. This implies the link from sensors to intermediate fusion helper is more important, and subsequently, the intermediate fusion helper should be located closer to the sensors.

*5.3. Side Information on Reliability.* In the above discussions, each sensor transmits its binary decision, and the data fusion center or an intermediate fusion helper utilizes the hard combining schemes. In a general case, a sensor could output more than one bit sensing result, and the data fusion center or the intermediate fusion helpers could correspondingly apply different information combining schemes. In this subsection, we make a simple extension to the case of 2-bit sensing results.

Consider a sensor network with 2 sensors connecting to the data fusion center. Besides the binary decision $S$, each sensor generates an additional bit SR to indicate the reliability of its binary decision $S$. Specifically, $\mathrm{SR} = -1$ if $S$ is of low reliability and $\mathrm{SR} = 1$ otherwise.

The reliability information $SR$ may be related with the sensing channel condition. In general, the higher the SNR of a sensing channel, the more reliability of a binary sensing decision. Moreover, the reliability may stem from the sensing metric. For example, in the energy-detector-based sensing, a sensor declares $H_1$ (or $H_0$) when the energy level $E$ is above (or below) a certain threshold Th. To obtain the reliability of the binary decision, the energy level $E$ can be further exploited as follows (cf., [5]). For some $\mathrm{Th}_0$ and $\mathrm{Th}_1$ with $\mathrm{Th}_0 < \mathrm{Th} < \mathrm{Th}_1$, the condition of $\mathrm{Th}_0 < E < \mathrm{Th}_1$ implies that the energy level is near the borderline, and hence, the binary decision is not reliable. Therefore, SR is set as $-1$ under this condition. In contrast, the condition $E \geq \mathrm{Th}_1$ or $E \leq \mathrm{Th}_0$ implies the binary decision is reliable and hence, SR is set as 1.

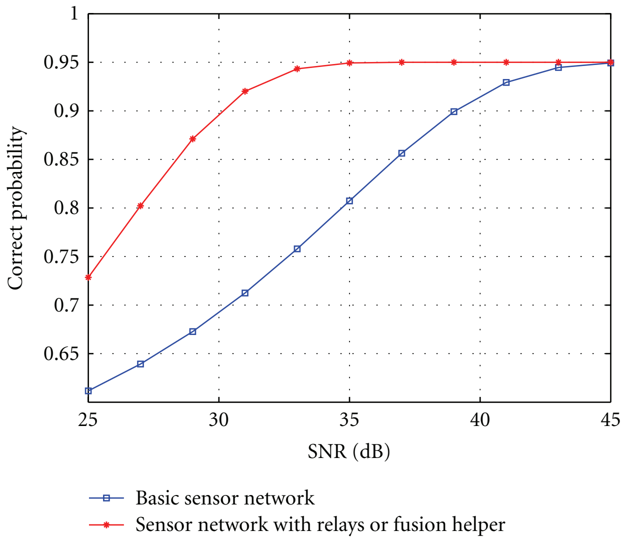


Figure 8: Theoretical correct probability of the overall decision in different sensor networks under the 2-bit combining rule in Gaussian channels ($A = 0.95$, $\beta = 3.5$, $d = 10$).

Under the prerequisite $\Pr(S = -1 \mid H_0) = \Pr(S = 1 \mid H_1) = A$, we assume that

$$\Pr(S = -1, \mathrm{SR} = -1 \mid H_0) = \Pr(S = 1, \mathrm{SR} = -1 \mid H_1) = \frac{A}{2},$$
$$\Pr(S = 1, \mathrm{SR} = -1 \mid H_0) = \Pr(S = -1, \mathrm{SR} = -1 \mid H_1) = \frac{1-A}{2}. \tag{47}$$

This assumption is aimed to maximize the entropy of variable SR, which could be a design criterion of quantizing side information.

Suppose each sensor in the network sends its sensing results $(S_i, \mathrm{SR}_i)$ to the data fusion center. All the other settings are remained intact as in Section 2. The data fusion center demodulates the received signal to decision bit $T_i$ and reliability bit $\mathrm{TR}_i$. Clearly,

$$P_t = \Pr(S_i = T_i) = \Pr(\mathrm{SR}_i = \mathrm{TR}_i) = 1 - Q\left(\sqrt{d^{-\beta}\mathrm{SNR}}\right). \tag{48}$$

The overall decision at the data fusion center is expressed as

$$U = \begin{cases} -1 & \text{if } (T_1, T_2, \mathrm{TR}_1, \mathrm{TR}_2) = (-1,-1,*,*), (-1,1,1,-1), \text{ or } (1,-1,-1,1), \\ 1 & \text{if } (T_1, T_2, \mathrm{TR}_1, \mathrm{TR}_2) = (1,1,*,*), (-1,1,-1,1), \text{ or } (1,-1,1,-1), \\ \Gamma & \text{if } (T_1, T_2, \mathrm{TR}_1, \mathrm{TR}_2) = (1,-1,1,1), (1,-1,-1,-1), (-1,1,-1,-1), \text{ or } (-1,1,1,1), \end{cases} \tag{49}$$

where $*$ means either value in $\{-1, 1\}$, $\Gamma$ is a $\{-1, 1\}$-valued random variable with $\Pr(\Gamma = -1) = \Pr(\Gamma = 1) = 1/2$, and $\Gamma$ is independent of all other variables. The last condition in (49) implies that the data center makes a random guess upon receiving opposite sensing results from the two sensors with the same reliability level.

We could derive from (47) that

$$\Pr(T_i = -1, \mathrm{TR}_i = -1 \mid H_0) = \Pr(T_i = -1, \mathrm{TR}_i = 1 \mid H_0) = \frac{1}{2}P_B, \tag{50}$$

where $P_B$ is given by (8). It follows from (49) that

$$P_c = \Pr(U = -1 \mid H_0) = \Pr(U = 1 \mid H_1) = P_B^2 + P_B(1 - P_B). \tag{51}$$

Following similar arguments, we obtain that for the sensor network with relays, the correct probability of the overall decision is

$$P_c = P_R^2 + P_R(1 - P_R), \tag{52}$$

where $P_R$ is given by (13).

Consider the sensor network with an intermediate fusion helper. All the other settings follow from Section 4, except that the intermediate fusion helper applies the fusion rule as (49). Let $U_F$ be the binary decision made at the intermediate fusion helper. Then,

$$\Pr(U_F = -1 \mid H_0) = \Pr(U_F = 1 \mid H_1) = P_{F,i}^2 + P_{F,i}(1 - P_{F,i}), \tag{53}$$

where $P_{F,i}$ is given by (23). Once the data fusion center demodulates the binary decision $U_F$ transmitted from the intermediate fusion helper, it claims the same decision. Hence,

$$P_c = 1 - P_{t,2} + (2P_{t,2} - 1)\left[P_{F,i}^2 + P_{F,i}(1 - P_{F,i})\right], \tag{54}$$

where $P_{t,2}$ is given by (12). Note that both (52) and (54) can be simplified to $A + (1 - 2A)(P_{t,1} + P_{t,2}) + (4A - 2)P_{t,1}P_{t,2}$, implying the identical correct probability of the overall decision. Figure 8 shows the correct probabilities (51), (52), (54) with $A = 0.95$, $\beta = 3.5$, and $d = 10$.

Finally, it should be mentioned that compared with the sensor network with relays, the sensor network with an intermediate fusion helper uses only 1/4 of the bandwidth at the data fusion center. It is not difficult to extend the above analysis to the networks with more than two sensors.

## 6. Sensor Networks with Rayleigh Fading

In the discussions above, we omitted the short-term fading of the fusion channels for simplicity, that is, by taking $h_i = 1$ in (4). In this section, we shall consider the general case where the fusion channels experience not only path loss, but also independent Rayleigh fading. Specifically, we assume $h_i$ in (4) is a Rayleigh distributed random variable with parameter $\sqrt{1/2}$.

Next we analyze the performance of 3 sensor networks under Rayleigh fading conditions: (1) basic sensor network; (2) sensor network with relays, and (3) sensor network with intermediate fusion helper. In the analysis, we assume the channel condition is perfectly known at the receiver, and hence, the maximum ratio combining is employed.

*6.1. Basic Sensor Network.* After receiving the signals from 3 sensors, the data fusion center applies the maximum ratio combining to obtain

$$\tilde{y} = \begin{bmatrix} h_1^* & h_2^* & h_3^* \end{bmatrix} \begin{bmatrix} h_1 S_1 \\ h_2 S_2 \\ h_3 S_3 \end{bmatrix} \sqrt{GP} + \begin{bmatrix} h_1^* & h_2^* & h_3^* \end{bmatrix} \begin{bmatrix} N_1 \\ N_2 \\ N_3 \end{bmatrix}, \quad (55)$$

where $G = d^{-\beta}$. Then the overall decision at the data fusion center is based on the sign of the real sufficient statistic and can be expressed as

$$U = \begin{cases} -1 & \text{if } r < 0 \\ 1 & \text{if } r \geq 0, \end{cases} \quad (56)$$

where $r$ represents the real part of $\tilde{y}$. Due to the similarity conditions, the correct probability of the overall decision in fading channels is $P_c = \Pr(r \geq 0 \mid H_1)$. Following the Markov Chain, we have

$$\begin{aligned} \Pr(r \geq 0 \mid H_1) = {} & \Pr(r \geq 0 \mid S_1 = S_2 = S_3 = 1)\Pr(S_1 = S_2 = S_3 = 1 \mid H_1) \\ & + \Pr(r \geq 0 \mid S_1 = S_2 = S_3 = -1)\Pr(S_1 = S_2 = S_3 = -1 \mid H_1) \\ & + 3\Pr(r \geq 0 \mid S_1 = 1, S_2 = S_3 = -1)\Pr(S_1 = 1, S_2 = S_3 = -1 \mid H_1) \\ & + 3\Pr(r \geq 0 \mid S_1 = -1, S_2 = S_3 = 1)\Pr(S_1 = -1, S_2 = S_3 = 1 \mid H_1). \end{aligned} \quad (57)$$

It is known that for fixed $S_i, i = 1, 2, 3$, the random variable $r$ conditioned on channel fading, $h_i, i = 1, 2, 3$, is Gaussian distributed with mean $(\sum_{i=1}^{3} |h_i|^2 S_i)\sqrt{GP}$ and variance $(1/2)(\sum_{i=1}^{3} |h_i|^2)\sigma^2$. It is trivial (cf., [19]) that

$$\Pr(r \geq 0 \mid h_i) = Q\left(-\sqrt{\frac{2G \cdot \mathrm{SNR}}{\sum_{i=1}^{3} |h_i|^2}} \sum_{i=1}^{3} |h_i|^2 S_i\right). \quad (58)$$

Let $Q_1 \triangleq \Pr(r \geq 0 \mid S_1 = S_2 = S_3 = -1)$ and $Q_2 \triangleq \Pr(r \geq 0 \mid S_1 = 1, S_2 = S_3 = -1)$. Then by (58),

$$\begin{aligned} Q_1 &= \mathbb{E}_{|h_i|^2}\left[Q\left(\sqrt{2G \cdot \mathrm{SNR} \sum_{i=1}^{3} |h_i|^2}\right)\right], \\ Q_2 &= \mathbb{E}_{|h_i|^2}\left[Q\left(\sqrt{\frac{2G \cdot \mathrm{SNR}}{\sum_{i=1}^{3} |h_i|^2}}\left(|h_2|^2 + |h_3|^2 - |h_1|^2\right)\right)\right]. \end{aligned} \quad (59)$$

Note that both $Q_1$ and $Q_2$ are functions of SNR and the path loss $G$.

Therefore, we obtain

$$\begin{aligned} P_c = {} & A^3(1 - Q_1) + (1 - A)^3 Q_1 + 3A(1 - A)^2 Q_2 \\ & + 3A^2(1 - A)(1 - Q_2). \end{aligned} \quad (60)$$

It is clear that $\lim_{\mathrm{SNR} \to \infty} Q_1 = 0$ and

$$\lim_{\mathrm{SNR} \to \infty} Q_2 = \Pr\left(|h_1|^2 > |h_2|^2 + |h_3|^2\right) = \frac{1}{4}. \quad (61)$$

Therefore,

$$\lim_{\mathrm{SNR} \to \infty} P_c = A^3 + \frac{3}{4}A(1 - A)^2 + \frac{9}{4}A^2(1 - A) \triangleq \eta. \quad (62)$$

It can be shown that $\eta$ is always greater than $A$ for $0.5 < A < 1$. This indicates that the decision made by the data fusion center is more reliable than that at each sensor, despite the fading on the fusion channels. On the other hand, we have $\lim_{\mathrm{SNR} \to 0} Q_1 = \lim_{\mathrm{SNR} \to 0} Q_2 = 1/2$. Hence, $\lim_{\mathrm{SNR} \to 0} P_c = 1/2$.

*6.2. Sensor Network with Relays.* Consider the sensor network with relays, where the distances from a relay to its served sensor and to the data fusion center are $\alpha d$ and $(1 - \alpha)d$, respectively. The data fusion center applies the maximal ratio combining on the signals received from different relays.

Let $R_i$ be the demodulated decision at the $i$th relay, and let $P_{t,1}^f$ be the probability that a transmission from a sensor is demodulated correctly at the the corresponding relay. Then,

$$P_{t,1}^f = 1 - \mathbb{E}_{|h|^2}\left[Q\left(2|h|^2(\alpha d)^{-\beta}\mathrm{SNR}\right)\right], \quad (63)$$

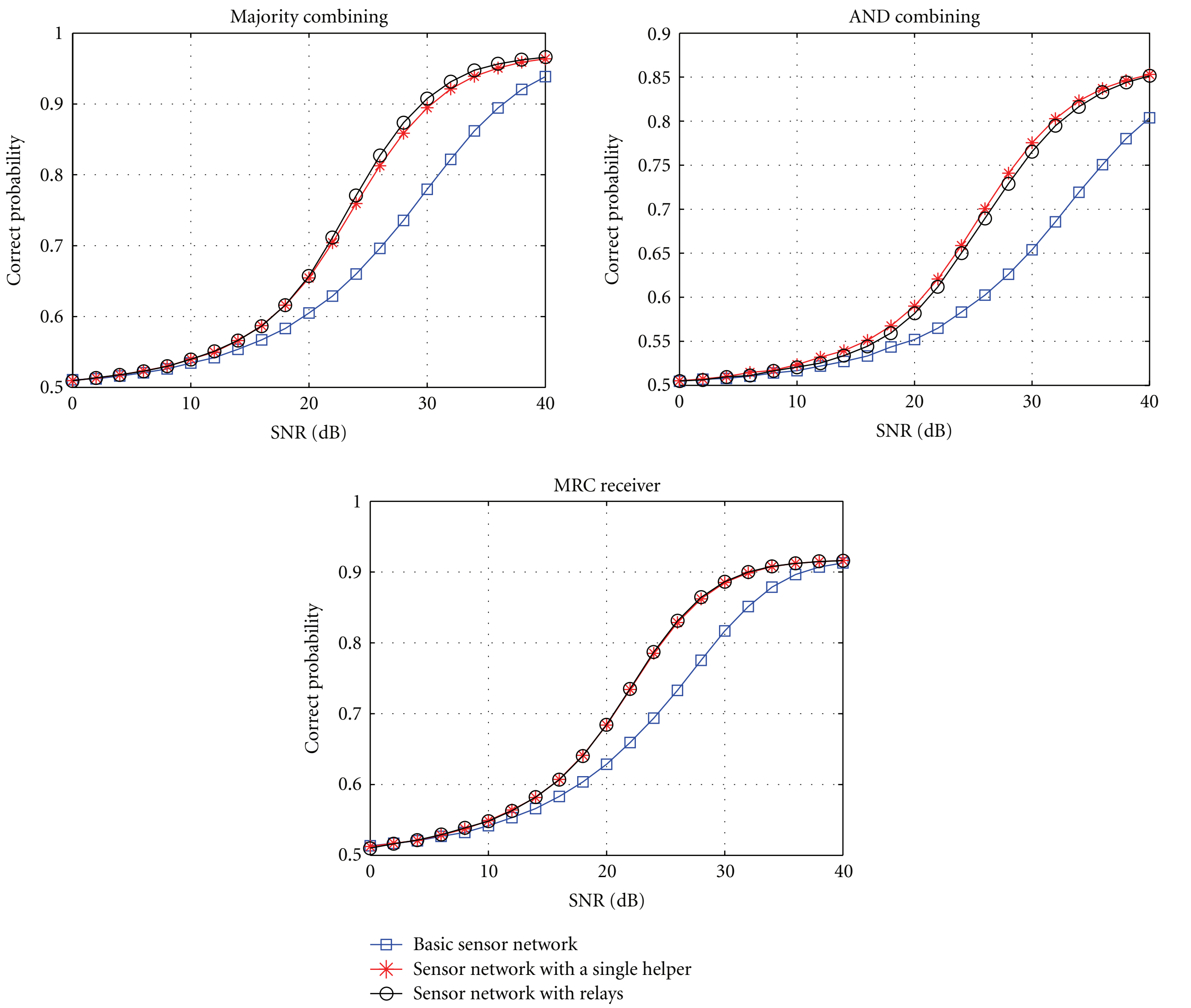


FIGURE 9: Simulated correct probability in three networks with optimized locations of the intermediate fusion helper and relays in Rayleigh fading channels ($A = 0.9$, $\beta = 3.5$, $d = 10$).

where $h$ represents the fading of the channel from sensor to relay. The probability that data fusion center correctly detects the source signal is

$$\begin{aligned} P_c &= Q_1 \Pr(R_1 = R_2 = R_3 = -1 \mid H_1) \\ &\quad + (1 - Q_1)\Pr(R_1 = R_2 = R_3 = 1 \mid H_1) \\ &\quad + 3Q_2 \Pr(R_1 = 1, R_2 = R_3 = -1 \mid H_1) \\ &\quad + 3(1 - Q_2)\Pr(R_1 = -1, R_2 = R_3 = 1 \mid H_1) \\ &= Q_1(1 - P_B)^3 + (1 - Q_1)P_B^3 + 3Q_2(1 - P_B)^2 P_B \\ &\quad + 3(1 - Q_2)P_B^2(1 - P_B), \end{aligned} \tag{64}$$

where $Q_1$ and $Q_2$ are given by (59) with $G$ being $[(1-\alpha)d]^{-\beta}$, and $P_B$ is given by (8) by substituting $P_t$ with $P_{t,1}^f$ in (63). The probability $P_c$ tends to $\eta$ for large SNR and tends to 1/2 for small SNR.

*6.3. Sensor Network with an Intermediate Fusion Helper.* Consider the sensor network with an intermediate fusion helper, where the distances from the intermediate fusion helper to the sensors and to the data fusion center are $\alpha d$ and $(1-\alpha)d$, respectively. The intermediate fusion helper receives the signals from multiple sensors and performs maximum ratio combining.

Similar to Section 6.1, we derive that the correct probability of the decision at the intermediate fusion helper is

$$\begin{aligned} P_{c,\text{in}} &= A^3(1 - Q_1) + (1 - A)^3 Q_1 + 3A(1 - A)^2 Q_2 \\ &\quad + 3A^2(1 - A)(1 - Q_2), \end{aligned} \tag{65}$$

where $Q_1$ and $Q_2$ are given by (59) with $G$ being $(\alpha d)^{-\beta}$.

Hence, the correct probability of the overall decision is

$$P_c = P_{c,\text{in}} P_{t,2}^f + (1 - P_{c,\text{in}})\left(1 - P_{t,2}^f\right), \tag{66}$$

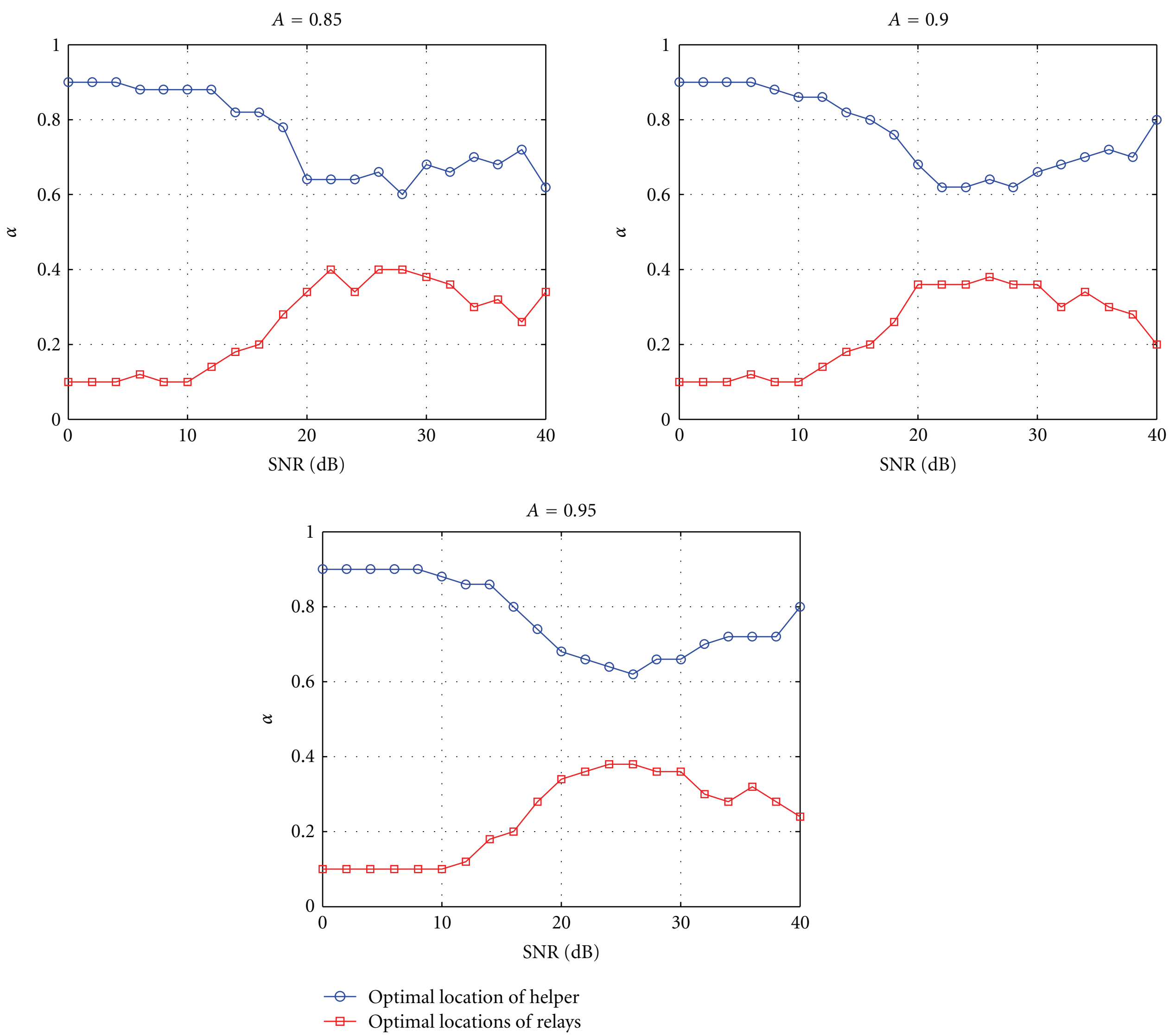


Figure 10: Simulated optimal locations of the intermediate fusion helper and relays using MRC receiver in Rayleigh fading channels ($\beta = 3.5$, $d = 10$).

where $P^f_{t,2}$ is the correct probability of the transmission from the intermediate fusion helper to the data fusion center. It is clear that

$$P^f_{t,2} = 1 - \mathbb{E}_{|h|^2}\left[Q\left(\sqrt{2|h|^2[(1-\alpha)d]^{-\beta}\mathrm{SNR}}\right)\right], \tag{67}$$

where $h$ represents the fading of the channel from the intermediate fusion helper to the data fusion center. Again, the probability $P_c$ tends to $\eta$ for large SNR, and tends to 1/2 for small SNR.

## 7. Simulation Results

In this section, we show our simulation results on the performance of different sensor networks, where the fusion channels experience both path loss and Rayleigh fading.

We consider the sensor networks with 3 sensors, and the data fusion center or the intermediate fusion helper applies 3 different detection algorithms: (i) hard combining using the majority combining rule, (ii) hard combining using the AND combining rule, and (iii) soft combining with maximum ratio combining. The corresponding simulated average correct probabilities of the overall decision are given in Figure 9, with parameters $A = 0.9$, $\beta = 3.5$, $d = 10$, and $\pi_0 = \pi_1 = 0.5$. By comparing the subfigures in Figure 9, we find that the maximum ratio combining receiver performs the best in the low-to-medium SNR region but is outperformed by the majority combining rule at high SNR. This is because with the maximum ratio combining, the network performance is dominated by the worst of three fading channels. In other words, if one fusion channel is in deep fading, the maximum ratio combining receiver is likely to make a wrong decision. Such situation does not apply

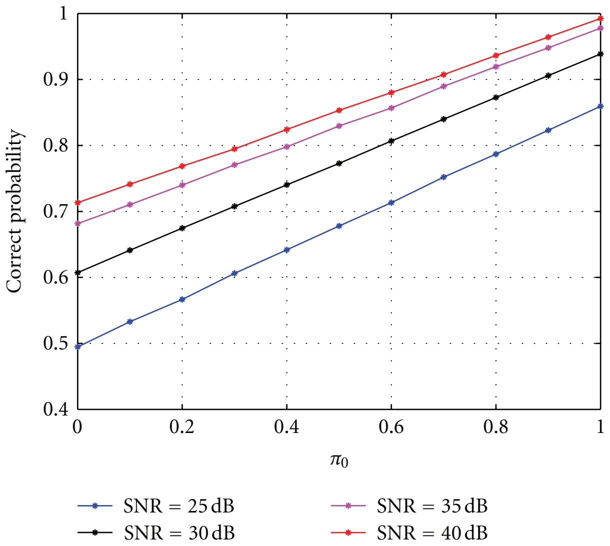


Figure 11: Simulated correct probability of the sensor network with an intermediate fusion helper under the AND combining rule for different $\pi_0$ values in Rayleigh fading channels ($A = 0.95$, $\beta = 3.5$, $d = 10$, $\alpha = 0.5$).

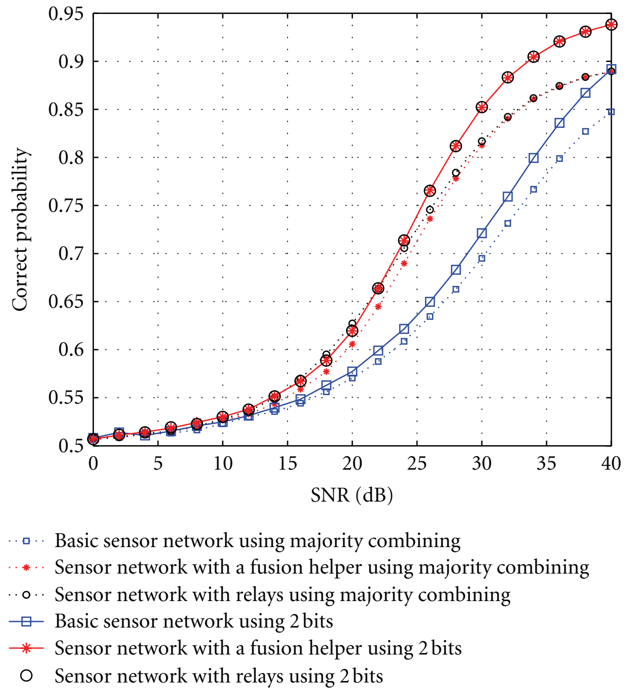


Figure 12: Simulated correct probability in different sensor networks with 2-bit sensing results and majority combining rule in Rayleigh fading channels ($A = 0.9$, $\beta = 3.5$, $d = 10$).

to the majority combining rule. Furthermore, the correct probability of maximum ratio combining receiver is upper bound by $\eta = 0.918$ (for $A = 0.9$), while such bound is equal to 0.972 for the majority combining rule. In consistence with analytical results, the network using the AND combining rule does not perform as well as the other two. For each combining rule, we also observe the similar performance of the networks with relays and with an intermediate fusion helper, which is at least 5 dB better than that of the basic network in the medium-to-high SNR region. Overall, our simulation results manifest the same trend as the analytical conclusions.

Figure 10 shows the optimal locations of the relays and the intermediate fusion helper, at different values of $A$ but with fixed $\beta = 3.5$ and $d = 10$, when the MRC receiver is implemented. It is seen that the optimal intermediate fusion helper location is closer to the data fusion center, while the optimal relay location is closer to the sensors. They both move toward the middle point with the increase of SNR. In general, the (combined) link between sensors and the intermediate fusion helper is more reliable than the link between the intermediate fusion helper and the data fusion center, as there are multiple sensors and the intermediate fusion helper performs information combining. Heuristically, the optimal intermediate fusion helper location is closer to the data fusion center to broaden the bottleneck of the cascaded connection. The similar argument applies to the optimal relay locations. Since the (combined) link between relays and the data fusion center is more reliable than the link between sensor and relay, the optimal relay location is closer to sensors in order to broaden the communication bottleneck.

In the sensor network with a fusion helper using the AND combining rule, the correct probability of the overall decision depends on the *a priori* probabilities $\pi_0$ and $\pi_1$. Figure 11 illustrates such dependence. Due to its property, the AND rule is prone to declare the hypothesis $H_0$. This leads to a higher probability of matching the true hypothesis, with the increase of $\pi_0$, as shown in the figure.

Finally, we consider the network with 2 sensors. The simulated correct probability of 3 different networks using majority combining and 2-bit sensing results are compared in Figure 12. Under the majority combining, we apply the same rule as shown in Section 5.1 except $U = 1$ or $-1$ with equal probability when the sum of the demodulated signals is 0. Given the optimal locations of relays and intermediate fusion helper, the same performance of the network with an intermediate fusion helper and with relays is observed for 2-bit sensing method (i.e., the last two lines in the figure overlap). This can be explained using the similar arguments as in Section 5.3. The simulation results indicate that the 2-bit sensing method generally provides better performance than the majority combining rule. The only exception is for the network with relays in low-to-medium SNR region.

## 8. Conclusions and Future Work

We have considered the problem of using multiple sensors for cooperative spectrum sensing, in which the fusion channels from sensors to data fusion center are wireless fading channels. We examined three different sensor networks: (1) basic sensor network, (2) sensor network with relays, and (3) sensor network with an intermediate fusion helper. Their performance of correct detection probability, energy and spectral efficiency was compared. The comparison was further extended to sensor networks with an arbitrary number of sensors and to sensor networks applying various information combining rules.

Our discussions in this paper were restricted to hard spectrum sensing results. Subsequently, 1-bit sensing result (with or without another bit of reliability information) is sent from sensors to data fusion center. Possible future work could include the performance analysis of different sensor networks with soft spectrum sensing results. Furthermore, though the optimal information combining rules for the basic sensor network and the sensor network with relays have been extensively studied, there is few work on the optimal information combining rules for the sensor network with intermediate fusion helpers, which is worth further investigation. Finally, the performance analysis of a sensor network with mixed usage of relays and intermediate fusion helpers could also be explored.

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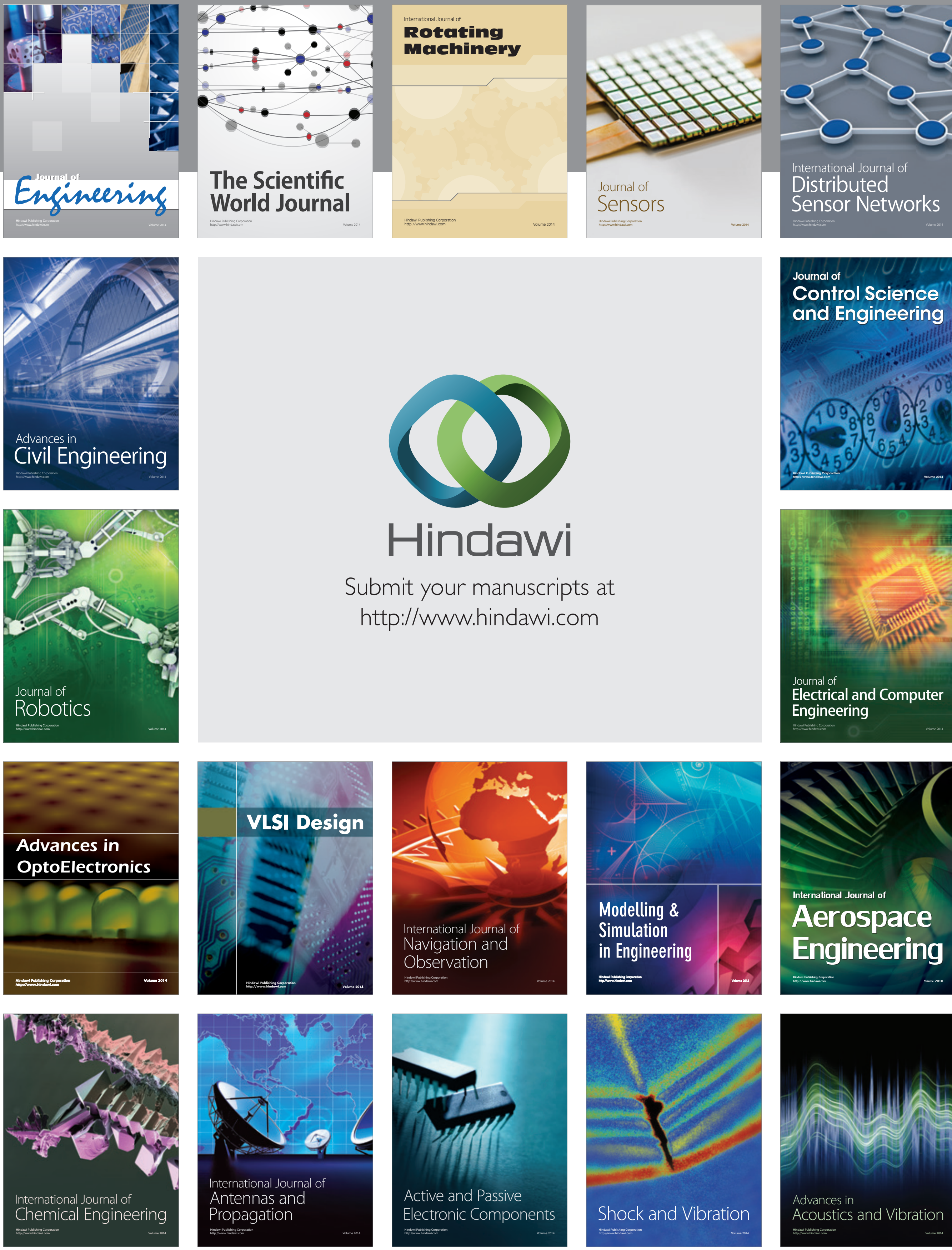
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