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Research Letter

Training Sequence Length Optimization for a Turbo-Detector Using Decision-Directed Channel Estimation

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We consider the problem of optimization of the training sequence length when a turbo-detector composed of a maximum *a posteriori* (MAP) equalizer and a MAP decoder is used. At each iteration of the receiver, the channel is estimated using the hard decisions on the transmitted symbols at the output of the decoder. The optimal length of the training sequence is found by maximizing an effective signal-to-noise ratio (SNR) taking into account the data throughput loss due to the use of pilot symbols.

1. Introduction

To combat the effects of intersymbol interference (ISI) due to the frequency selectivity of mobile radio channels, an equalizer has to be used. In order to efficiently detect the transmitted symbols, the equalizer needs a good estimate of the channel. The channel is classically estimated by using a training sequence (TS) known at the receiver [1]. When the length of the TS increases, the channel estimate becomes more reliable. However, this leads to a loss in terms of data throughput. Thus, instead of using the training sequence only, the information carried by the observations corresponding to the data symbols can also be used to improve iteratively the channel estimation. At each iteration, the channel estimator refines its estimation by using the hard or soft decisions on the data symbols at the output of the data detector or the channel decoder [2–4].

A question that one can ask concerns the length of the TS to choose in order to obtain a satisfactory initial channel estimate without decreasing significantly the data throughput. Several methods have been proposed to answer this question. In [5], a solution based on the maximization of a lower bound of the capacity of the training-based scheme was proposed for a transmission over a frequency selective channel. In [6], we considered the case where a maximum *a posteriori* (MAP) equalizer is used. We proposed to maximize an effective signal-to-noise ratio (SNR) taking into account the loss in terms of data throughput due to the use of the pilot symbols. These studies have been performed for a noniterative receiver. In [2], an iterative data detection and channel estimation scheme was considered for a transmission over a flat fading channel. The TS length optimization was performed by minimizing the ratio of the channel estimation mean square error (MSE) to the data throughput.

In this letter, we consider a coded transmission over a frequency selective channel. At the receiver, a turbo-detector composed of a MAP equalizer and a MAP decoder is used. The channel is iteratively estimated by using hard decisions on the coded bits at the output of the decoder. This estimation strategy is usually referred to as decision-directed or bootstrap estimation. We derive the expression of the equivalent SNR at the output of the MAP equalizer fed with the *a priori* information (from the decoder) and the channel estimate. We define, based on this expression, an effective SNR taking into account the loss in terms of data throughput due to the use of the TS. We propose to find the length of the TS maximizing this expression. We show that when the decisions provided by the decoder are enough reliable, the optimal TS length is equal to its minimum value

$2L-1$, where L is the channel memory. Notice that a similar result was found in [5] by maximizing a lower bound on the training-based channel capacity when the training and data powers are allowed to vary.

Throughout this letter, scalars and matrices are lower and upper case, respectively, and vectors are underlined lower case. The operator $(\cdot)^T$ denotes the transposition.

2. Transmission System Model

As shown in Figure 1, the input information bit sequence is encoded with a convolutional code, interleaved and mapped to the symbol alphabet $\mathcal{A}$. In this letter, we consider the BPSK modulation ($\mathcal{A} = \{-1, 1\}$). The symbols are then transmitted over a multipath channel. We assume that transmissions are organized into bursts of T symbols. The channel is assumed to be invariant during the transmission. The received baseband signal sampled at the symbol rate at time k is

$$y_k = \sum_{l=0}^{L-1} h_l x_{k-l} + n_k, \tag{1}$$

where L is the channel memory and x_k are the transmitted symbols. In this expression, n_k are modeled as independent and identically distributed (iid) samples from a random variable with normal probability density function (pdf) $\mathcal{N}(0, \sigma^2)$, where $\mathcal{N}(\alpha, \sigma^2)$ denotes a Gaussian distribution with mean α and variance σ^2. The term h_l is the lth tap gain of the channel.

The initial channel estimate is provided to the receiver by a least square estimator using T_p training symbols with $2L-1 \leq T_p \leq T$ [1], where T_p is the parameter to be optimized.

3. Decision-Directed Channel Estimation

As shown in Figure 2, we consider a turbo-detector composed of a MAP equalizer and a MAP decoder. At each iteration, the equalizer and the decoder compute *a posteriori* probabilities (APPs) and extrinsic probabilities on the coded bits [7]. They exchange the extrinsic probabilities which will be used as *a priori* probabilities, to improve iteratively their performance. In order to refine the channel estimate, the channel estimator uses the hard decisions on the transmitted coded symbols based on the APPs at the output of the decoder. Indeed, the channel estimator is fed with T_p pilot symbols and δT estimates of the coded symbols coming from the decoder. Let $\underline{x} = (x_{T_p+\delta T-1}, \dots, x_0)^T$ be the sequence containing the T_p training symbols $(x_{T_p-1}, \dots, x_0)^T$ and the δT data symbols $(x_{T_p+\delta T-1}, \dots, x_{T_p})^T$. The output of the channel corresponding to the vector $\underline{x}$ is given by

$$\underline{y} = X\underline{h} + \underline{n}, \tag{2}$$

where $\underline{h} = (h_0, \dots, h_{L-1})^T$ is the vector of channel taps, X is the $(T_p - L + 1 + \delta T) \times L$ Hankel matrix having the first column $(x_{T_p+\delta T-1}, \dots, x_{L-1})^T$ and the last row $(x_{L-1}, \dots, x_0)$; and $\underline{n}$ is the corresponding noise vector.

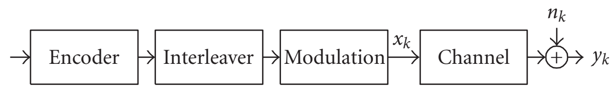

Figure 1: Transmitter structure.

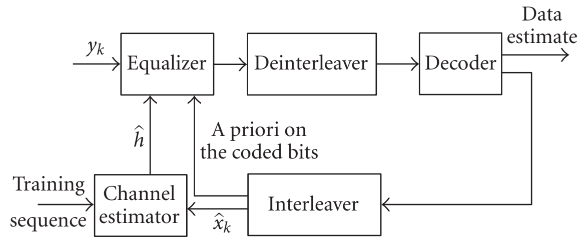

Figure 2: Receiver structure.

In order to estimate the channel, the observation vector $\underline{y}$ is approximated as follows:

$$\underline{y} \approx \hat{X}\underline{h} + \underline{n}, \tag{3}$$

where $\hat{X}$ is the estimated version of the matrix X containing the hard decisions on the coded symbols at the output of the decoder. The iteration process can be repeated several times and here the matrix $\hat{X}$ corresponds to the estimated symbols at the last iteration.

The least square estimate $\underline{\hat{h}} = (\hat{h}_0, \dots, \hat{h}_{L-1})^T$ of $\underline{h}$ is given by [1]

$$\underline{\hat{h}} = (\hat{X}^T \hat{X})^{-1} \hat{X}^T \underline{y}. \tag{4}$$

In general, δT is chosen to give a good complexity/performance trade-off. We suppose in the following that δT is fixed such that $\delta T \gg L$. We also assume that the vector of errors on the coded symbols at the output of the decoder is independent of the noise vector. In average, the errors are assumed to be uniformly distributed over the burst. The channel estimation mean square error (MSE) is given by [3]

$$E(\|\delta\underline{h}\|^2) = \frac{\sigma^2 L}{T_p - L + 1 + \delta T} + 4\frac{E(n^2) + (L-1)E(n)}{(T_p - L + 1 + \delta T)^2}, \tag{5}$$

where $E(\cdot)$ is the mathematical expectation, n is the number of erroneous hard decisions on the coded symbols at the output of the decoder, used by the channel estimator.

Let $\overline{\beta} = E(n/\delta T)$ and $\overline{\beta^2} = E(n^2/\delta T^2)$. Hence, (5) can be rewritten as

$$E(\|\delta\underline{h}\|^2) = \frac{\sigma^2 L}{T_p - L + 1 + \delta T} + 4\frac{\overline{\beta^2}\delta T^2 + (L-1)\overline{\beta}\delta T}{(T_p - L + 1 + \delta T)^2}. \tag{6}$$

4. Performance Analysis of the MAP Equalizer

We want now to study the impact of the *a priori* information and the channel estimation errors on the MAP equalizer performance.

4.1. Equivalent SNR at the Output of the Equalizer

We assume that the *a priori* (extrinsic) log likelihood ratios (LLRs) at the input of the equalizer, fed back from the decoder, are iid samples from a random variable with the conditional pdf $\mathcal{N}(\pm 2/\sigma_a^2, 4/\sigma_a^2)$ [8–10]. The equivalent signal-to-noise ratio at the output of the MAP equalizer fed with the *a priori* LLRs from the decoder and the channel estimate (from the decision-directed channel estimator) can be approximated at high SNR by

$$\text{SNR}_{\text{eq,DD}} = \left(\frac{d'^2 + 4m'\mu^2}{4\sigma^2}\right)\left(1 + \frac{1}{\sigma^2}\frac{E(\|\delta \underline{h}\|^2)}{1 + 4m'\mu^2/d'^2}\right)^{-1}, \tag{7}$$

where $\mu = \sigma/\sigma_a$ and $E(\|\delta \underline{h}\|^2)$ is the channel estimation MSE given in (6). The quantities m' and d' are defined as

$$(m', d') = \arg \min_{m(\underline{e}), \|\underline{d}(\underline{e})\|} \frac{\sqrt{\|\underline{d}(\underline{e})\|^2 + 4m(\underline{e})\mu^2}}{2\sigma} \times \left(1 + \frac{E(\|\delta \underline{h}\|^2)}{1 + 4m(\underline{e})\mu^2/\|\underline{d}(\underline{e})\|^2}\right)^{-1/2}, \tag{8}$$

where $\underline{e} \in E, E$ is the set of all nonzero error events [11], $m(\underline{e})$ is the number of decision errors in $\underline{e}$, and $\underline{d}(\underline{e})$ is the convolution of $\underline{e}$ with the channel.

Remark 1. We prove the result given in (7) similarly to [12, Proposition 1]. However, in [12], we assumed that the channel was estimated by using a perfect training sequence and the covariance matrix of the channel estimation error was then diagonal. When a decision-directed channel estimator is used, as it is the case in this work, this covariance matrix is not diagonal which makes the proof more complicated. We omit here the proof for the sake of space.

4.2. Simulation Results

In our simulations, we consider the channel with impulse response (0.37; 0.6; 0.6; 0.37). The transmissions are organized into bursts of 256 symbols. The modulation used is the BPSK. We do not consider the channel coding and the turbo-detector yet. Figure 3 shows the bit error rate (*BER*) curves with respect to the SNR at the input of the MAP equalizer when the channel is estimated by the decision-directed channel estimator using $T_p = 15$ pilot symbols and $\delta T = 30$ estimates of the data symbols. The δT estimates of the data information symbols at the input of the channel estimator are generated by making hard decisions on artificial LLRs modeled as iid samples from a random variable with pdf $\mathcal{N}(\pm 2/\sigma_x^2, 4/\sigma_x^2)$ [8–10]. We consider two reliability levels of *a posteriori* information: $\sigma_x^2 = 0.5$ and $\sigma_x^2 = 0.2$ and two reliability levels of *a priori* information: $\mu = 0.1$ and $\mu = 0.5$. The solid lines indicate the theoretical MAP equalizer performance. The dotted ones are obtained by simulations. We note that the theoretical curves approximate well the curves obtained by simulations.

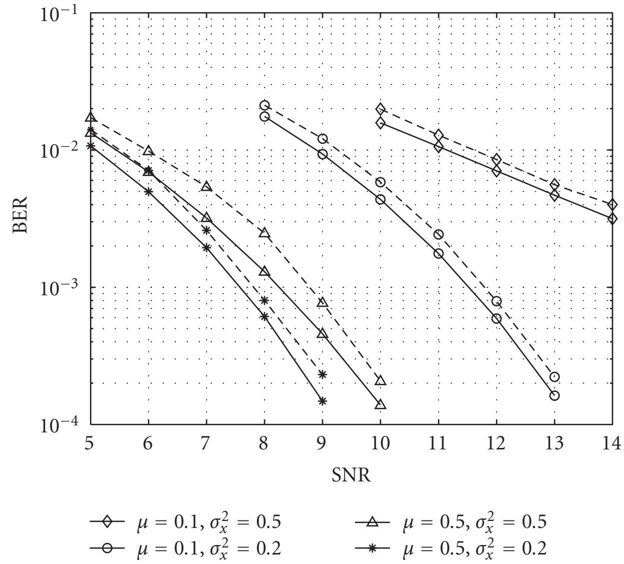

FIGURE 3: Comparison of the equalizer performance (dotted curves) and the theoretical performance (solid curves) when the channel estimate is provided by the decision-directed channel estimator.

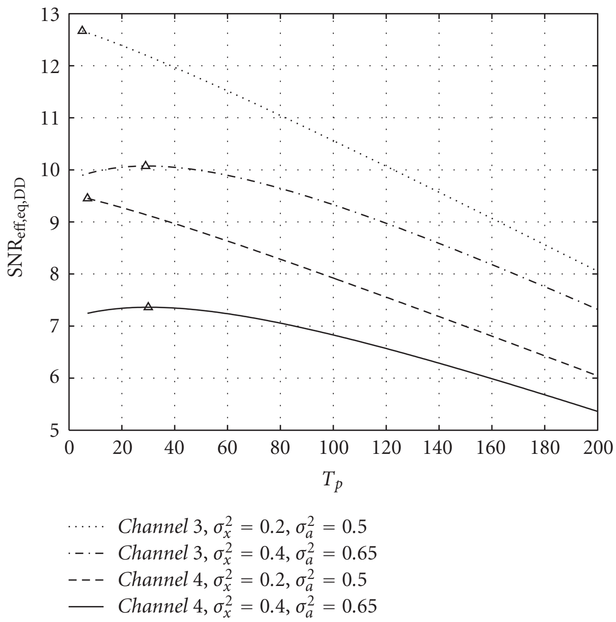

FIGURE 4: $\text{SNR}_{\text{eff,eq,DD}}$ versus T_p for *Channel* 3 and *Channel* 4, SNR = 9 dB, $T = 512$, and $\delta T = 100$.

In the following, we propose to optimize the training sequence length by maximizing an effective signal-to-noise ratio that we will define taking into account the data throughput loss due to the use of the pilot symbols.

5. Optimization of the Training Sequence Length

Increasing the training sequence length leads to an improvement of the channel estimate quality but also to a loss in terms of data throughput. Thus, in order to take into account this loss, we define, based on (7), an effective SNR at the output of the MAP equalizer as

$$\begin{aligned} \mathrm{SNR}_{\mathrm{eff,eq,DD}} &= \frac{T-T_p}{T}\mathrm{SNR}_{\mathrm{eq,DD}} \\ &= \frac{T-Tp}{T}\left(\frac{d'^2+4m'\mu^2}{4\sigma^2}\right) \\ &\quad\times\left(1+\frac{1}{\sigma^2}\frac{E(\|\delta\underline{h}\|^2)}{1+4m'\mu^2/d'^2}\right)^{-1}. \end{aligned} \tag{9}$$

Our aim is to find the TS length maximizing this quantity. Hence, we define the following optimization problem

$$T_p^* = \arg\max_{2L-1\le T_p\le T-\delta T}\mathrm{SNR}_{\mathrm{eff,eq,DD}}. \tag{10}$$

Let $x \in \mathbf{R}_+^*$, $x \ge 2L-1$ and

$$f_1(x) = \frac{T-x}{T}\left(\frac{d'^2+4m'\mu^2}{4\sigma^2}\right)(1+g(x))^{-1}, \tag{11}$$

where $g(x) = (1/\sigma^2(1+4m'\mu^2/d'^2))(\sigma^2 L/(x-L+1+\delta T)+4((\overline{\beta^2}\delta T^2+(L-1)\overline{\beta}\delta T)/(x-L+1+\delta T)^2))$. Thus, $\mathrm{SNR}_{\mathrm{eff,eq,DD}} = f_1(T_p)$.

When $g(x) \ll 1$, $f_1(x)$ can be approximated by

$$f_1(x) \approx \frac{T-x}{T}\left(\frac{d'^2+4m'\mu^2}{4\sigma^2}\right)(1-g(x)) \tag{12}$$

which is a decreasing function.

When the δT decisions on the data symbols added to the training sequence are reliable, $g(T_p) \ll 1$. Thus, the optimal length of the training sequence solution of (10) is

$$T_p^* = 2L-1. \tag{13}$$

When the hard decisions used by the channel estimator are not reliable, the approximation $g(T_p) \ll 1$ becomes inaccurate and the optimization problem cannot be solved analytically.

6. Simulation Results

We first consider a MAP equalizer fed with the channel estimate calculated by the decision-directed channel estimator using T_p pilot symbols and δT estimates of the data symbols. These estimates are obtained by making hard decisions on artificial *a posteriori* LLRs modeled as iid samples from a random variable with pdf $\mathcal{N}(\pm 2/\sigma_x^2, 4/\sigma_x^2)$. The equalizer is also fed with artificial *a priori* LLRs modeled as iid samples from a random variable with pdf $\mathcal{N}(\pm 2/\sigma_a^2, 4/\sigma_a^2)$. Figure 4 shows the effective SNR given in (9) as a function of the training sequence length for *Channel* 3 and *Channel* 4 with respective impulse responses $(0.5; 0.71; 0.5)$ and $(0.37; 0.6; 0.37; 0.6)$, for SNR $= 9$ dB, $T = 512$, and $\delta T = 100$. We consider two values of σ_x^2: $\sigma_x^2 = 0.2$ and $\sigma_x^2 = 0.4$; and two values of σ_a^2: $\sigma_a^2 = 0.5$ and $\sigma_a^2 = 0.65$. We note that the training sequence length maximizing $\mathrm{SNR}_{\mathrm{eff,eq,DD}}$ is equal

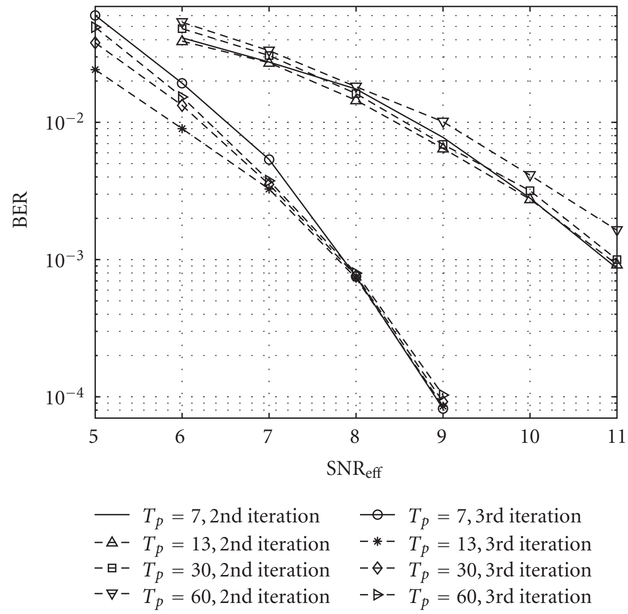

Figure 5: MAP equalizer BER performance versus $\mathrm{SNR}_{\mathrm{eff}}$ for different values of the length of the training sequence after two and three iterations of the iterative receiver for *Channel* 4, $T = 512$ and $\delta T = 100$.

to $2L-1$ when the decisions on the data symbols added to the TS become reliable ($\sigma_x^2 = 0.2$ corresponding to $\overline{\beta} = 0.013$). We also note that the optimal choice of the training sequence length can significantly improve the effective SNR.

Now, we consider the whole system with the channel coding at the transmitter and the iterative receiver composed of a MAP equalizer and a MAP decoder. We use *Channel 4* with impulse response $(0.37; 0.6; 0.6; 0.37)$. The information data are encoded using the rate 1/2 convolutional code with generator polynomials $(7, 5)$ in octal. At the first iteration, the channel is estimated by using the TS [1]. At the next iterations, it is estimated by using the decision-directed technique. Figure 5 shows the BER performance (on the coded bits) at the output of the MAP equalizer after two iterations of the iterative receiver and at the convergence (after three iterations) with respect to $\mathrm{SNR}_{\mathrm{eff}} = ((T - T_p)/T)\mathrm{SNR}$, where SNR is the signal-to-noise ratio at the input of the MAP equalizer, for $T = 512$, $\delta T = 100$ and for different values of the length T_p of the training sequence. The δT estimates of the coded symbols at the input of the channel estimator are obtained by making hard decisions on the *a posteriori* LLRs at the output of the MAP decoder. Simulations confirm that the MAP equalizer best performance is achieved, at high SNR, when $T_p = 2L-1$.

7. Conclusion

In this letter, we considered the problem of optimization of the training sequence length for the iterative receiver composed of a MAP equalizer, a MAP decoder and a decision-directed channel estimator. We proved that the training sequence length maximizing the effective SNR at the output of the MAP equalizer is equal to its minimum value $2L-1$ when the decisions provided by the decoder are reliable. A future work will consider the case where the channel estimator uses soft decisions provided by the decoder on the coded symbols.

References

[1] S. N. Crozier, D. D. Falconer, and S. A. Mahmoud, "Least sum of squared errors (LSSE) channel estimation," *IEE Proceedings F: Radar and Signal Processing*, vol. 138, no. 4, pp. 371–378, 1991.

[2] S. Buzzi, M. Lops, and S. Sardellitti, "Performance of iterative data detection and channel estimation for single-antenna and multiple-antennas wireless communications," *IEEE Transactions on Vehicular Technology*, vol. 53, no. 4, pp. 1085–1104, 2004.

[3] S. Lasaulce and N. Sellami, "On the impact of using unreliable data on the bootstrap channel estimation performance," in *Proceedings of the 4th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC '03)*, pp. 348–352, Rome, Italy, June 2003.

[4] N. Sellami, S. Lasaulce, and I. Fijalkow, "Iterative channel estimation for coded DS-CDMA systems over frequency selective channels," in *Proceedings of the 4th IEEE Workshop on Signal Processing Advances in Wireless Communications (SPAWC '03)*, pp. 80–84, Rome, Italy, June 2003.

[5] H. Vikalo, B. Hassibi, B. Hochwald, and T. Kailath, "On the capacity of frequency-selective channels in training-based transmission schemes," *IEEE Transactions on Signal Processing*, vol. 52, no. 9, pp. 2572–2583, 2004.

[6] I. Hadj Kacem, N. Sellami, A. Roumy, and I. Fijalkow, "Training sequence optimization for frequency selective channels with MAP equalization," in *Proceedings of the 3rd International Symposium on Communications, Control and Signal Processing (ISCCSP '08)*, pp. 532–537, Julians, Malta, March 2008.

[7] A. Picart, P. Didier, and A. Glavieux, "Turbo-detection: a new approach to combat channel frequency selectivity," in *Proceedings of the IEEE International Conference on Communications (ICC '97)*, vol. 3, pp. 1498–1502, Montreal, Canada, June 1997.

[8] M. Tüchler, R. Koetter, and A. C. Singer, "Turbo equalization: principles and new results," *IEEE Transactions on Communications*, vol. 50, no. 5, pp. 754–767, 2002.

[9] S. Ten Brink, "Convergence of iterative decoding," *Electronics Letters*, vol. 35, no. 10, pp. 806–808, 1999.

[10] N. Sellami, A. Roumy, and I. Fijalkow, "A proof of convergence of the MAP turbo-detector to the AWGN case," *IEEE Transactions on Signal Processing*, vol. 56, no. 4, pp. 1548–1561, 2008.

[11] S. Benedetto and E. Biglieri, *Principles of Digital Transmission with Wireless Applications*, Kluwer/Plenum, New York, NY, USA, 1999.

[12] N. Sellami, A. Roumy, and I. Fijalkow, "The impact of both a priori information and channel estimation errors on the MAP equalizer performance," *IEEE Transactions on Signal Processing*, vol. 54, no. 7, pp. 2716–2724, 2006.