

## Research Letter

# Slow Adaptive $M$ -QAM under Third-Party Received Signal Constraints in Shadowing Environments

Yan Li<sup>1,2</sup> and Shaline Kishore<sup>1</sup>

<sup>1</sup>Department of Electrical and Computer Engineering, Lehigh University, Bethlehem, PA 18015, USA

<sup>2</sup>InterDigital Inc., King of Prussia, PA 19406, USA

Correspondence should be addressed to Yan Li, liyan@lehigh.edu

Received 9 March 2008; Accepted 23 May 2008

Recommended by Luca De Nardis

Motivated by recent interest in dynamic spectrum sharing between primary and secondary spectrum users, we investigate slow adaptive quadrature amplitude modulation (QAM) under third-party received power constraints. The optimal rate and power adaptation schemes are derived to maximize the average spectral efficiency (SE) for the secondary users. Additionally, closed-form expressions of the achievable SE are presented for correlated lognormal shadow fading environments. Our analytical and numerical results underscore the importance of exploiting shadowing correlation in the proposed adaptive schemes.

Copyright © 2008 Y. Li and S. Kishore. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

## 1. Introduction

Opportunistic secondary access of licensed spectrum bands permits secondary users to transmit on these spectral bands as long as the primary licensee's interference temperature is below its tolerable level. This spectrum sharing approach has motivated recent studies of the channel capacity with constrained received-signal power at a third-party receiver [1, 2]. Figure 1 shows the associated system model in which a secondary transmitter communicates with a secondary receiver over a licensed-band, resulting in interference to a primary user. The channel gain between the transmitter and the primary receiver is denoted as  $g$ ; the channel gain between this transmitter and the secondary receiver is denoted as  $h$ . In [1], authors assume additive white Gaussian noise (AWGN) channels, where  $g$  and  $h$  are deterministic. In [2], authors assume flat fading channels, where the instantaneous values of  $g$  and  $h$  are known at the transmitter. This letter presents a communication-theoretic study of a similar system model but assuming the transmitter is only aware of slow fading conditions. We assume discrete-time block fading channels with stationary and ergodic time-varying power gains  $g = \bar{g}r_g$  and  $h = \bar{h}r_h$ ;  $r_g$  and  $r_h$  are (unitmean) fast small-scale fading coefficients, and  $\bar{g}$  and  $\bar{h}$  are slow large-scale fading coefficients. Due to (bandwidth, delay) limitations of the feedback channel, we assume only

$\bar{h}$  and its correlation with  $\bar{g}$  are reliably known at the transmitter. The correlation between  $\bar{g}$  and  $\bar{h}$  can be fed back by a band manager [3], which coordinates the two parties. The transmitter then uses this information to apply a slow adaptive quadrature amplitude modulation (QAM) technique [4, 5]. We derive the optimal rate and power adaptation policies that maximize the average spectral efficiency (SE) for secondary users. Under optimal adaptation, we present closed-form expressions for achievable SE in correlated lognormal shadowing environments. The results underscore the importance of knowing/estimating shadow fading correlations in the proposed adaptive scheme.

## 2. Optimal Power and Rate Adaptation

Assuming unit power AWGN at the primary and secondary receivers, the instantaneous BER for  $M$ -QAM can be approximated as [6, 7]

$$\text{BER}_{\text{in}}(h) \approx c_1 \exp\left(-\frac{c_2 h P}{M-1}\right), \quad (1)$$

where  $P$  is the transmit power,  $M$  is the QAM constellation size.  $c_1$  and  $c_2$  are two parameters optimized via curve fitting.

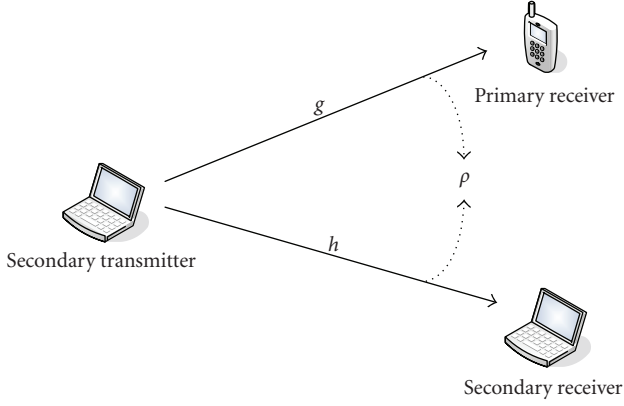


FIGURE 1: System model.

Assuming Nakagami- $m$  fading, the short-term average BER over fast fading can be calculated as

$$\begin{aligned} \text{BER}_{\text{st}}(\bar{h}) &= \int_0^\infty \text{BER}_{\text{in}}(\bar{h}r_h) f_{r_h}(r_h) dr_h \\ &= \int_0^\infty \text{BER}_{\text{in}}(\bar{h}r_h) \frac{m^m r_h^{m-1}}{\Gamma(m)} \exp(-mr_h) dr_h \quad (2) \\ &= c_1 \left( \frac{c_2 \bar{h} P(\bar{h})}{m(M(\bar{h}) - 1)} + 1 \right)^{-m}, \end{aligned}$$

where  $P(\bar{h})$  and  $M(\bar{h})$  are two transmission parameters adaptive to  $\bar{h}$ . We consider here continuous rate adaptation, where the set of signal constellations is unrestricted [7, 8], that is, our results will provide an upper bound if only a discrete finite set of constellations is available. It is worth noting that in [5], exact average BER expression has been derived for discrete rate QAM with diversity reception over flat fading and shadowing channels. We do not apply this accurate expression in our analysis here due to the intractability of noninvertible integrals. Instead, we employ the approximation above as it provides analytical means to derive power and rate adaptation schemes for the system under study.

Given the short-term average BER constraint  $\text{BER}_{\text{st}} = \text{BER}^*$ , the optimal rate adaptation policy must satisfy

$$M(\bar{h}) = 1 + K\bar{h}P(\bar{h}), \quad (3)$$

where

$$K := \frac{c_2}{m[(c_1/\text{BER}^*)^{1/m} - 1]}. \quad (4)$$

Furthermore, the optimal power adaptation policy for maximal long-term average SE is the solution to

$$\max_{P(\bar{h})} \mathbb{E}\{\log_2(1 + K\bar{h}P(\bar{h}))\} \quad (5)$$

$$\text{subject to } \mathbb{E}\{\bar{g}r_g P(\bar{h})\} \leq P_I, \quad P(\bar{h}) \geq 0,$$

where  $P_I$  is the maximal average interference power tolerable at the primary receiver.

For this problem, we can show the following.

*Case 1* (when  $\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}$  is not a constant). The optimal power allocation policy is

$$P^*(\bar{h}) = \frac{1}{K\mathbb{E}\{\bar{g} | \bar{h}\}} \underbrace{\left( \frac{1}{\kappa_0} - \frac{1}{\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}} \right)^+}_{\text{water filling over } \bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}}, \quad (6)$$

where  $(\bullet)^+ = \max(0, \bullet)$ , and  $\kappa_0$  is the solution of  $\mathbb{E}\{\mathbb{E}\{\bar{g} | \bar{h}\} P^*(\bar{h})\} = P_I$ . Note that  $\kappa_0$  is the cutoff ratio of  $\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}$  and

$$P^*(\bar{h}) \begin{cases} \geq 0, & \text{if } \bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\} \geq \kappa_0, \\ = 0, & \text{otherwise.} \end{cases} \quad (7)$$

$P^*(\bar{h})$  is the product of a water-filling solution over  $\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}$  and  $(K\mathbb{E}\{\bar{g} | \bar{h}\})^{-1}$ . Since  $\mathbb{E}\{\bar{g} | \bar{h}\}$  is a function of  $\bar{h}$ ,  $P^*(\bar{h})$  is not a pure water-filling solution in general. However, when  $\bar{g}$  and  $\bar{h}$  are independent such that  $\mathbb{E}\{\bar{g} | \bar{h}\} = \mathbb{E}\{\bar{g}\}$ , (6) reduces to the classic water-filling solution.

Using the optimal power and rate adaptation scheme, the long-term average SE can be expressed as

$$\text{SE}^* = \int_{\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\} \geq \kappa_0} \log_2\left(\frac{\bar{h}}{\kappa_0 \mathbb{E}\{\bar{g} | \bar{h}\}}\right) f_{\bar{h}}(\bar{h}) d\bar{h}. \quad (8)$$

*Case 2* (when  $\bar{h}/\mathbb{E}\{\bar{g} | \bar{h}\}$  is equal to a constant). In this case, the optimal power allocation policy is channel inversion:

$$P^*(\bar{h}) = \frac{RP_I}{\bar{h}}, \quad (9)$$

and the average SE becomes

$$\text{SE}^* = \log_2(1 + KR P_I). \quad (10)$$

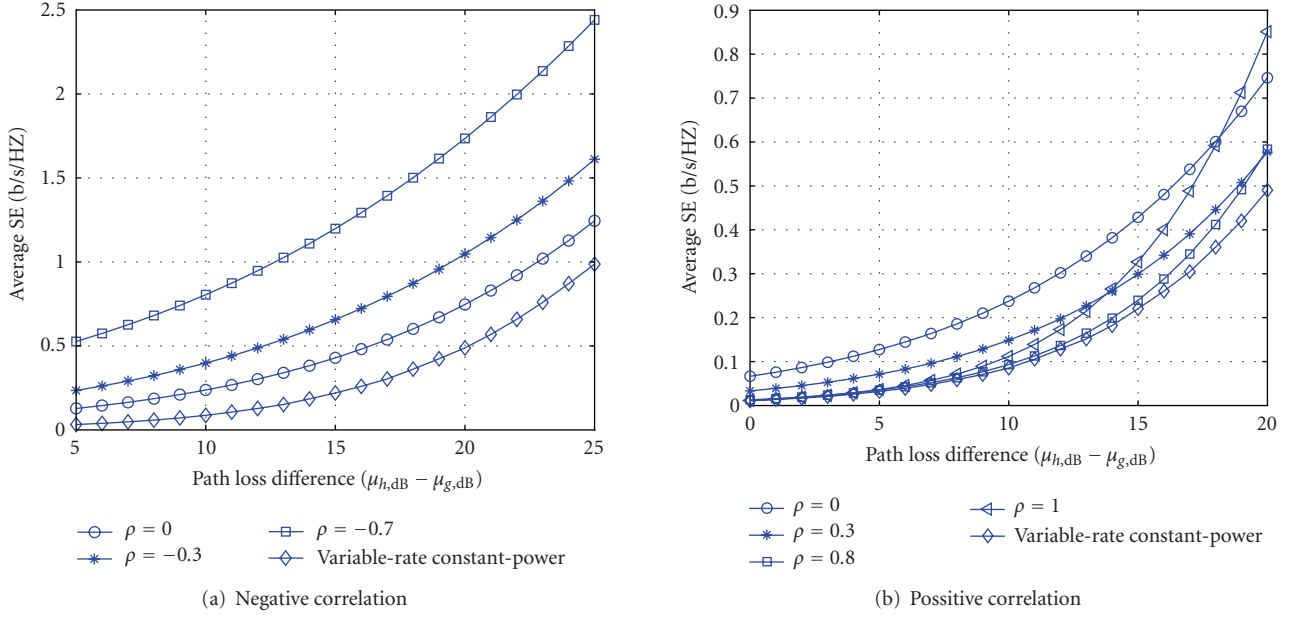
### 3. Lognormal Shadowing

In contrast to the small-scale fading effects, which are typically independent from one terminal to another, shadowing effects tend to be correlated over much larger distances. For example, [9] presents shadowing cross correlation between several base stations and a mobile terminal in a 1900 MHz GSM system; the resulting correlations range from  $-0.34$  to  $0.43$ .

For correlated lognormal shadowing, we can write  $\bar{g} = e^{X_g}$  and  $\bar{h} = e^{X_h}$ , where  $X_g$  and  $X_h$  are jointly Gaussian distributed with means  $\mu_g$  and  $\mu_h$ , variances  $\sigma_g^2$  and  $\sigma_h^2$ , and a correlation coefficient  $\rho$ . Using this model, we can write

$$\mathbb{E}\{\bar{g} | \bar{h}\} = \bar{h}^{(\sigma_g/\sigma_h)\rho} \exp\left[\mu_g - \frac{\sigma_g}{\sigma_h} \rho \mu_h + \frac{(1 - \rho^2)\sigma_g^2}{2}\right]. \quad (11)$$

Based on (11), we observe that the optimal power and rate adaptation scheme depends on the relationship between  $\sigma_h$  and  $\rho\sigma_g$ , which dictates if  $\mathbb{E}\{\bar{g} | \bar{h}\}$  is a linear function of  $\bar{h}$ . We consider all three possible functional relationships between  $\mathbb{E}\{\bar{g} | \bar{h}\}$  and  $\bar{h}$ . Substituting (11) into the analytical expressions presented in Section 2 and resorting to commonly used manipulations, we have the following.


 FIGURE 2: Average SE versus the path loss difference  $\mu_{h,\text{dB}} - \mu_{g,\text{dB}}$ .  $\sigma_{g,\text{dB}} = \sigma_{h,\text{dB}} = 8$  dB.

*Case 1* (when  $\sigma_h > \rho\sigma_g$ ). The probability of service (i.e., the complement of the outage probability) is

$$\Pr\left(\frac{\bar{h}}{\mathbb{E}\{\bar{g} | \bar{h}\}} \geq \kappa_0\right) = \Pr(\bar{h} \geq \exp[\mu_h + \psi(\kappa_0)\sigma_h]) \quad (12)$$

$$= Q[\psi(\kappa_0)],$$

where  $Q(\cdot)$  is the Gaussian right-tail probability function, and

$$\psi(\kappa_0) := \frac{1}{\sigma_h - \rho\sigma_g} \left[ \log \kappa_0 + \mu_g - \mu_h + \frac{(1 - \rho^2)\sigma_g^2}{2} \right]. \quad (13)$$

The achievable average SE is

SE\*

$$= \log_2(e)(\sigma_h - \rho\sigma_g) \left\{ \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\psi^2(\kappa_0)}{2}\right] - \psi(\kappa_0)Q[\psi(\kappa_0)] \right\}, \quad (14)$$

where  $\kappa_0$  is solved from

$$\begin{aligned} KP_I &= K \mathbb{E}\{\bar{g}r_g P^*(\bar{h})\} \\ &= K \mathbb{E}\{\mathbb{E}\{\bar{g} | \bar{h}\} P^*(\bar{h})\} \\ &= \frac{1}{\kappa_0} Q[\psi(\kappa_0)] - \exp\left(\frac{\sigma_g^2 + \sigma_h^2 - 2\rho\sigma_g\sigma_h}{2} + \mu_g - \mu_h\right) \\ &\quad \times Q[\psi(\kappa_0) + \sigma_h - \rho\sigma_g]. \end{aligned} \quad (15)$$

*Case 2* (when  $\sigma_h = \rho\sigma_g$ ). In this case,

$$\frac{\bar{h}}{\mathbb{E}\{\bar{g} | \bar{h}\}} = \exp\left(\mu_h - \mu_g + \frac{\sigma_h^2 - \sigma_g^2}{2}\right) := R. \quad (16)$$

Thus, the optimal adaptation policy is channel inversion, and the achievable SE can be obtained by substituting (16) into (10).

*Case 3* (when  $\sigma_h < \rho\sigma_g$ ). The probability of service is

$$\Pr\left(\frac{\bar{h}}{\mathbb{E}\{\bar{g} | \bar{h}\}} \geq \kappa_0\right) = \Pr(\bar{h} \leq \exp[\mu_h + \psi(\kappa_0)\sigma_h]) \quad (17)$$

$$= 1 - Q[\psi(\kappa_0)].$$

The achievable average SE is

$$\begin{aligned} \text{SE}^* &= \log_2(e)(\rho\sigma_g - \sigma_h) \left\{ \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{\psi^2(\kappa_0)}{2}\right] \right. \\ &\quad \left. + \psi(\kappa_0)\{1 - Q[\psi(\kappa_0)]\} \right\}, \end{aligned} \quad (18)$$

where  $\kappa_0$  is solved from

$$\begin{aligned} KP_I &= \frac{1}{\kappa_0} \{1 - Q[\psi(\kappa_0)]\} - \exp\left(\frac{\sigma_g^2 + \sigma_h^2 - 2\rho\sigma_g\sigma_h}{2} + \mu_g - \mu_h\right) \\ &\quad \times \{1 - Q[\psi(\kappa_0) + \sigma_h - \rho\sigma_g]\}. \end{aligned} \quad (19)$$

## 4. Numerical Results

In our numerical results, we assume  $\text{BER}^* = 10^{-3}$ , the instantaneous BER approximation parameters  $c_1 = 0.2$  and  $c_2 = 1.6$  [7], the tolerable interference-to-noise ratio (INR) at the primary receiver is 0 dB, the fast fading is Rayleigh distributed, and the slow large-scale fading conditions are lognormal. In Figure 2, we plot the average SE versus the

path loss difference  $\mu_{h,\text{dB}} - \mu_{g,\text{dB}}$ , assuming that  $\sigma_{g,\text{dB}} = \sigma_{h,\text{dB}} = 8\text{ dB}$ . Note that  $\mu_{\{g,h\},\text{dB}} = 10 \log_{10}(e)\mu_{\{g,h\}}$  and  $\sigma_{\{g,h\},\text{dB}} = 10 \log_{10}(e)\sigma_{\{g,h\}}$ .  $\bar{g}$  and  $\bar{h}$  are assumed correlated with (a)  $\rho = 0, -0.3, \text{ and } -0.7$ ; and (b)  $\rho = 0, 0.3, 0.8, \text{ and } 1$ . For negative correlations, the average SE increases as  $\rho$  decreases. This is because (1) higher  $\bar{h}$  implies lower  $\bar{g}$ , providing better opportunity for secondary transmissions; and (2) smaller  $\rho$  (or larger  $|\rho|$ ) implies heavier correlation, implying  $\bar{h}$  gives more complete information on  $\bar{g}$  and enables better adaptation over the time-varying channel. As reference, we also plot the average SE achieved using the variable-rate constant-power adaptation scheme. We observe that by adapting the transmit power, we can achieve much higher average SE when the correlation is heavy. When no correlation exists, the increase in SE, achieved via classic water-filling solution, is not significant compared with the constant-power scheme. For positive correlations, we do not see a clear trend between the average SE and  $\rho$  due to two contrary effects. On the one hand,  $\bar{g}$  and  $\bar{h}$  are varying statistically along the same direction; thus, a large  $\rho$  limits good transmission opportunities for the secondary system. On the other hand, heavier correlation provides more complete channel information, aiding in channel adaptation. We can observe that when the path-loss difference is small, the average SE increases as  $\rho$  decreases, which means that the first effect is dominant. However, the second effect becomes dominant as the path-loss difference increases, for example, when  $\mu_{h,\text{dB}} - \mu_{g,\text{dB}}$  is greater than 18 dB, the average SE for  $\rho = 1$  (achieved by the channel inversion) becomes the highest amongst all curves.

## 5. Conclusions

We investigate slow adaptive QAM technique under the third-party received power constraints. Optimal rate and power adaptations are derived to maximize average SE for secondary users. We present closed-form expressions of achievable SE for correlated lognormal shadowing environments. Results underscore the importance of exploiting shadowing correlation in proposed adaptive schemes.

## References

- [1] M. Gastpar, "On capacity under received-signal constraints," in *Proceedings of the 42nd Annual Allerton Conference on Communication, Control and Computing*, pp. 1322–1331, Monticello, Ill, USA, September–October 2004.
- [2] A. Ghasemi and E. S. Sousa, "Fundamental limits of spectrum-sharing in fading environments," *IEEE Transactions on Wireless Communications*, vol. 6, no. 2, pp. 649–658, 2007.
- [3] J. M. Peha, "Approaches to spectrum sharing," *IEEE Communications Magazine*, vol. 43, no. 2, pp. 10–12, 2005.
- [4] S. Vishwanath, S. A. Jafar, and A. Goldsmith, "Adaptive resource allocation in composite fading environments," in *Proceedings of the IEEE Global Telecommunications Conference (GLOBECOM '01)*, vol. 2, pp. 1312–1316, San Antonio, Tex, USA, November 2001.
- [5] A. Conti, M. Z. Win, and M. Chiani, "Slow adaptive M-QAM with diversity in fast fading and shadowing," *IEEE Transactions on Communications*, vol. 55, no. 5, pp. 895–905, 2007.
- [6] X. Qiu and K. Chawla, "On the performance of adaptive modulation in cellular systems," *IEEE Transactions on Communications*, vol. 47, no. 6, pp. 884–895, 1999.
- [7] S. T. Chung and A. J. Goldsmith, "Degrees of freedom in adaptive modulation: a unified view," *IEEE Transactions on Communications*, vol. 49, no. 9, pp. 1561–1571, 2001.
- [8] A. J. Goldsmith and S.-G. Chua, "Variable-rate variable-power MQAM for fading channels," *IEEE Transactions on Communications*, vol. 45, no. 10, pp. 1218–1230, 1997.
- [9] E. Perahia, D. C. Cox, and S. Ho, "Shadow fading cross correlation between basestations," in *Proceedings of the 53rd IEEE Vehicular Technology Conference (VTC '01)*, vol. 1, pp. 313–317, Rhodes, Greece, May 2001.



**Hindawi**

Submit your manuscripts at  
<http://www.hindawi.com>

