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Research Letter

Realization of IIR Decimation Filters Based on Merged Delay Transformation

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A novel realization of IIR decimation filters is proposed which is based on merged delay transformation. The transformation is derived analytically and can be applied directly to first- and second-order IIR filters. Computational efficiency is enhanced because the current output can be directly computed from Mth old output. The output data rate is decreased by M by merging M number of delay elements in the recursive path. The proposed transformation is applied to higher-order IIR filter by decomposing it into parallel first-order and second-order sections. This transformation not only gives better stability for coefficient quantization but also reduces the requirement on processing clock, for sample, rate reduction. Filtering and down sampling are performed in the same stage. Number of multiplications is reduced by 45% as compared to the conventional IIR filters where all output samples are computed.

1. INTRODUCTION

large number of decimation filter realizations exist, that are mainly a combination of identical all-pass subfilters [1], polyphase-FIR filters [2, 3], and comb-FIR-IIR filters [4, 5], and so forth. The conventional realizations consist of cascade of integrators, decimators, and a number of differentiators. The multistage realizations have drawbacks of larger sizes and enhanced complexity of calculations. Although IIR filters are well known for their reduced computational complexity, yet their application in the realization of decimation filters is rarely found in the literature.

This letter proposes a novel technique to realize IIR decimation filter that computes directly the current value of the output without calculating the intermediate outputs. This is possible because with the help of proposed merged delay transformation (MDT), the current output sample becomes dependent only on Mth old output sample and M input samples. Output data rate can be reduced by M by merging the M delay elements in the recursive path. An Nth order IIR filter can be decomposed into N parallel first-order sections with complex coefficients. This is not a problem as two first-order sections having complex conjugate coefficients produce real output for a real input sample. The second-order sections are realized with reduced complexity. The input sampling rate can be M-times higher than the output sampling rate. The computational efficiency is enhanced due to reduction in number of multiplications and parallel realization.

2. MERGED DELAY TRANSFORMATION

A first-order recursive difference equation can be written as follows:

$$y[n] = b_1 y\,[n-1] + a_0 x\,[n]. \tag{1}$$

Here, $x[n]$ and $y[n]$ are the input and output data samples, respectively, where b_1 and a_0 are real constants.

The values of $y[n-1], y[n-2], y[n-3], \dots, y[n-M+1]$ can be replaced successively in (1) and the following general equation can be derived:

$$y[n] = b_1^M\, y[n-M] + \sum_{k=0}^{M-1} b_1^k\, a_0\, x[n-k]. \tag{2}$$

Equation (2) is called as merged delay transformation. Such a direct relationship is not possible for second- and higher-order IIR filters. Equation (2) computes current output $y[n]$

from single Mth previous output and M inputs. The values of intermediate outputs are not required. The input sampling rate can be M-times higher than the output sampling rate. With the help of this transformation, an IIR filter can be transformed into an M-fold decimation filter. The filter structure to realize (2) is shown in Figure 1.

In this figure, output $y[n]$ is fed back after passing through "M" number of unit delay elements. Merging M number of delay elements, output sampling rate can be reduced by M. In this way, we obtain one output sample at every Mth sample of the input data that realizes down sampling of factor M. Intermediate values of output are not computed. This structure performs $M + 1$ multiplications as compared to $2M$ multiplications performed in the IIR decimation filter where all output samples are to be computed.

For higher-order IIR filters, direct application of merged delay transformation is not possible. However, we have proposed a simple technique for the higher-order cases, too. We decompose the Nth order filter into N parallel first-order sections. The coefficients of individual first-order sections may be complex, but they are complex conjugate for a pair of sections. Hence in the implementation, two first-order sections can be combined to form a second-order section to give a real output. The second-order transfer function $H(z)$ can be written in parallel form as follows:

$$\begin{aligned} H(z) &= k + H_1(z) + H_2(z), \text{ where} \\ H_1(z) &= \frac{r_1}{1 - p_1 z^{-1}}, \\ H_2(z) &= \frac{r_2}{1 - p_2 z^{-1}}. \end{aligned} \tag{3}$$

Here, $r_1 = r_{1r} + jr_{1i}$ and $p_1 = p_{1r} + jp_{1i}$ are complex conjugates of r_2 and p_2, respectively. The symbol "j" denotes imaginary operator.

Let $y_1[n]$ and $y_2[n]$ represent the outputs from $H_1(z)$ and $H_2(z)$, respectively. We can apply MDT on each section and obtain the following results for $M = 2$:

$$\begin{aligned} y_1[n] &= (A + jB)y_1[n-2] + (C + jD)x[n] \\ &\quad + (E + jF)x[n-1], \\ y_2[n] &= (A - jB)y_2[n-2] + (C - jD)x[n] \\ &\quad + (E - jF)x[n-1], \end{aligned} \tag{4}$$

where

$$\begin{aligned} A &= p_{1r}^2 - p_{1i}^2, \qquad B = 2p_{1r}p_{1i}, \\ C &= r_{1r}, \qquad D = r_{1i}, \\ E &= r_{1r}p_{1r} - r_{1i}p_{1i}, \qquad F = r_{1i}p_{1r} + r_{1r}p_{1i}, \\ y_{1R}[n] &= A\, y_{1R}[n-2] - B\, y_{1I}[n-2] \\ &\quad + C\, x[n] + E\, x[n-1], \\ y_{1I}[n] &= A\, y_{1I}[n-2] + B\, y_{1R}[n-2] \\ &\quad + D\, x[n] + F\, x[n-1]. \end{aligned} \tag{5}$$

Here, $y_{1R}[n]$ and $y_{1I}[n]$ are the real and imaginary parts of $y_1[n]$, respectively. Similarly expressions for $y_{2R}[n]$ and $y_{2I}[n]$ are obtained. Equation (5), shows that $y[2]$ can be computed from $y[0]$, $y[2]$, $y[4]$, and so on. Thus $y[Mn]$ can be computed without intermediate outputs and this leads to sample rate reduction by M.

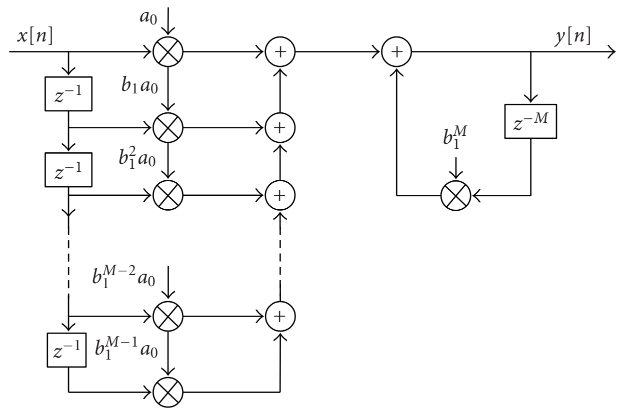

FIGURE 1: Realization of MDT-based M-fold decimation in first-order IIR filter.

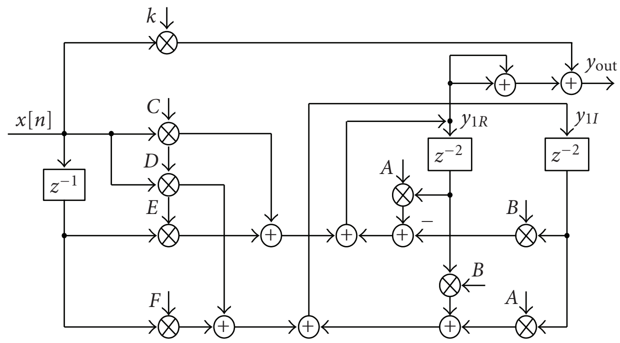

FIGURE 2: Realization of second-order IIR decimation filter with $M = 2$.

It is observed that

$$\begin{aligned} y_{1R}[n] &= y_{2R}[n], \\ y_{1I}[n] &= -y_{2I}[n]. \end{aligned} \tag{6}$$

Since the imaginary parts are equal and opposite, they cancel out at the output. Real parts are equal so computation of only one real part is sufficient to get the output from second-order section. This results in reduction of computational complexity. The output from a second-order section, $y_{\text{out}}[n]$ can be obtained as follows:

$$y_{\text{out}}[n] = k\, x[n] + 2\, y_{1R}[n]. \tag{7}$$

The second-order section of MDT- based IIR decimation filter with $M = 2$ is realized as shown in Figure 2. The filter structure can be drawn for any value of M. The number of multipliers for this structure is equal to $2M + 6$.

With the help of above procedure, an Nth order IIR decimation filter with any integer decimation factor M can be realized.

3. RESULTS AND DISCUSSIONS

The proposed technique is verified using Matlab simulations for a large number of filters. First-order IIR filters of

Table 1: Comparison of number of multiplications for first-order IIR section.

M	No. of multiplications Conventional (polyphase)	Proposed	%age reduction
2	4 (4)	3	25
4	8 (6)	5	37.5
6	12 (8)	7	41.6
8	16 (10)	9	43.7
10	20 (12)	11	45

Butterworth, Chebyshev-I, Chebyshev-II, and Elliptic type are designed for a maximum decimation factor of 10. The output $y[n]$ calculated from original difference equation is compared with the transformed output computed from (2). The peak difference between two outputs for $M = 2$–10, is of the order of 10^{-16}.

Similarly, many second-order filters of different types are designed. These second-order filter are transformed into first-order parallel sections as explained above. Peak difference in the results from direct output and the transformed output for $M = 10$ is again of the order of 10^{-16}.

A fifth-order IIR decimation filter is realized as two second-order sections and one first-order section, all connected in parallel. The transformed output is calculated and compared with the direct output for various decimation factors. The peak error in results for $M = 10$ is of the order of 10^{-13}.

The stability is not a problem in this realization, as we start from a stable system and decompose it into parallel sections. This, in fact, improves stability as the parallel realization is found to be much less sensitive to coefficient quantization [6]. Regarding problem of nonlinear phase in IIR filters, it can be taken care by block filtering technique [7]. It is imperative to point out that the advancement in IIR filter designs must be used to design an effective, stable, linear-phase filter. The technique proposed in this paper enables the use of recursive filter for decimation operation as they are computationally more effective than their FIR equivalent design.

Comparison of multiplication operations with the conventional IIR decimation filter where all output samples are computed is carried out. For comparison with polyphase structures, the number of multiplications is computed as reported in [3] and shown in parenthesis. The results are shown in Table 1.

The reduction as compared to conventional IIR decimation is 45% for $M = 10$. More reduction is possible with higher values of M. The results show that the proposed technique can efficiently realize IIR decimation filters.

4. CONCLUSIONS

A novel technique to efficiently realize IIR decimation filter is presented. First-order recursive equation is transformed by merged delay transformation. Filtering is performed along with down sampling in the same stage. Drawbacks associated with cascade of integrators, down samplers, and differentiators are avoided. For first-order IIR decimation filter, reduction of number of multiplications upto 45% is achieved. Simulation results show a very good agreement with the theory.

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