Hindawi Publishing Corporation
Research Letters in Signal Processing
Volume 2007, Article ID 17206, 5 pages
doi:10.1155/2007/17206

Research Letter

Recursive Estimation and Identification of Time-Varying Long-Term Fading Channels

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Received 19 July 2007; Accepted 11 November 2007

Recommended by Peter Handel

This paper is concerned with modeling of time-varying wireless long-term fading channels, parameter estimation, and identification from received signal strength data. Wireless channels are represented by stochastic differential equations, whose parameters and state variables are estimated using the expectation maximization algorithm and Kalman filtering, respectively. The latter are carried out solely from received signal strength data. These algorithms estimate the channel path loss and identify the channel parameters recursively. Numerical results showing the viability of the proposed channel estimation and identification algorithms are presented.

1. INTRODUCTION

This paper is concerned with the development of time-varying (TV) long-term fading (LTF) wireless channel models based on system identification and estimation algorithms to extract various parameters of the LTF channel using received signal measurements. TV wireless channel models capture both the space and time variations of wireless systems, which are due to the relative mobility of the receiver and/or transmitter and scatterers [1–3]. In the TV models, the statistics of channel are time-varying. This contrasts with the majority of published work that mainly deals with time-invariant (static) random models or simple free space model, where the channel statistics do not depend on time [4–6]. In time-invariant models, channel parameters are random but do not depend on time, and they remain constant throughout the observation and estimation phase. This contrasts with TV models, where the channel dynamics become TV random (stochastic) processes [1–3].

The TV LTF channel model is discussed in [2] and represented by stochastic differential equations (SDEs). We propose to estimate the TV power path loss of the LTF channel and its parameters from received signal strength data, which are usually available or easy to obtain in any wireless network. The expectation maximization (EM) algorithm [7] and Kalman filtering [8] are employed in the identification and estimation of the channel parameters and path loss. The proposed identification and estimation algorithms are recursive and only based on received signal measurements. Numerical results are provided to determine the performance of the proposed estimation algorithm.

The paper is organized as follows. In Section 2, the TV LTF mathematical channel model is introduced. In Section 3, the EM algorithm together with the Kalman filter, to estimate the channel parameters as well as the channel power loss from signal measurements, is developed. In Section 4, numerical results are presented. Finally, Section 5 provides the conclusion.

2. TV LTF WIRELESS CHANNEL MATHEMATICAL MODEL

Wireless channels suffer from short-term fading (STF) due to multipath, and LTF due to shadowing depending on the geographical area. In suburban areas, which are populated with less obstacles like vehicles, buildings, mountains, and so forth, their communication signal undergoes phenomenal LTF (lognormal shadowing) [5]. For such propagation environments, the *random process* power loss (PL) in dB, $\{X(t,\tau)\}_{t\geq 0,\tau\geq\tau_0}$, which is a function of both time t and space represented by the time delay τ, is generated by a mean-reverting version of a general linear time-varying SDE given by [2, 3]

$$\begin{aligned} dX(t,\tau) &= \beta(t,\tau)(\gamma(t,\tau) - X(t,\tau))dt + \delta(t,\tau)dW(t), \\ X(t_0,\tau) &= \mathcal{N}(\overline{PL}(d)[dB];\sigma_{t_0}^2), \end{aligned} \tag{1}$$

where $\{W(t)\}_{t\geq 0}$ is the standard Brownian motion (zero drift, unit variance) which is assumed to be independent of $X(t_0,\tau)$, $\mathcal{N}(\mu;\kappa)$ denotes a Gaussian random variable with mean μ and variance κ, and $\overline{PL}(d)$ $[dB]$ is the average PL in dB. The parameter $\gamma(t,\tau)$ models the average TV PL at distance d from transmitter, which corresponds to $\overline{PL}(d)$ $[dB]$ at d indexed by t. This model tracks and converges to this value as time progresses. The instantaneous drift $\beta(t,\tau)(\gamma(t,\tau) - X(t,\tau))$ represents the local mean while $\beta(t,\tau)$ represents the local standard deviation. Note that $\beta(t,\tau)$ can be selected to control the speed of adjustment towards a specific mean value associated with (1).

In [2, 3], this model is shown to capture the spatiotemporal variations of the propagation environment as the random parameters $\{\beta(t,\tau),\gamma(t,\tau),\delta(t,\tau)\}_{t\geq 0}$ can be used to model the TV characteristics of the LTF channel. The received signal, $y(t)$, at any time t can be expressed as

$$y(t) = s(t)H(t) + v(t), \tag{2}$$

where $s(t)$ is the information signal, $v(t)$ is the channel disturbance at the receiver, and $H(t)$ is the signal attenuation coefficient defined by $H(t) \triangleq e^{kX(t,\tau)}$, where $k = -\ln(10)/20$ [5].

The general spatiotemporal lognormal model in (1) and (2) can be realized by a stochastic state space system given by

$$\begin{aligned} \dot{X}(t,\tau) &= A(t,\tau)X(t,\tau) + B(t,\tau)w(t), \\ y(t) &= s(t)e^{kX(t,\tau)} + D(t)v(t), \end{aligned} \tag{3}$$

where $A(t,\tau) = -\beta(t,\tau)$, $B(t,\tau) = [\delta(t,\tau)\ \ \beta(t,\tau)\gamma(t,\tau)]$, and $w(t) = [dW(t)\ \ 1]^T$.

The above system parameters and state variable values are estimated from received signal measurements. The EM algorithm and Kalman filtering are employed in the system parameters and state estimation, respectively. These algorithms are introduced next.

3. WIRELESS CHANNEL ESTIMATION VIA THE EM ALGORITHM AND KALMAN FILTERING

This section describes the procedure employed to estimate the channel model parameters and states associated with the state space model in (3), based on the EM algorithm [7] together with Kalman filtering [8]. Since the estimation and identification processes are carried out in discrete instants, we consider a sampled version of the state space model (3) in discrete time as

$$\begin{aligned} x_{t+1} &= A_t x_t + B_t w_t, \\ y_t &= s_t e^{kx_t} + D_t v_t, \end{aligned} \tag{4}$$

where $x_t \in \Re^n$ is a state vector, $y_t \in \Re^d$ is a measurement vector, $w_t \in \Re^m$ is a state noise, and $v_t \in \Re^d$ is a measurement noise. Note that the state space model is nonlinear since the output equation in (4) is nonlinear.

The channel parameters $\theta_t = \{A_t, B_t, D_t\}$ are unknown and they are estimated together with the path loss represented by the system states x_t from a finite set of received signal measurement data, $Y_N = \{y_1, y_2, \ldots, y_N\}$. The proposed methodology is recursive and based on the EM algorithm combined with the extended Kalman filter (EKF). The latter is used due to the nonlinear output equation.

3.1. Channel state estimation—The EKF

The EKF approach is based on linearizing the nonlinear system model (4) around the previous estimate. It estimates the channel states x_t for given system parameter $\theta_t = \{A_t, B_t, D_t\}$ and measurements Y_t. It is described by the following equations [8]:

$$\begin{aligned} \hat{x}_{t|t} &= A_t\hat{x}_{t-1|t-1} + P_{t|t}C_t^T D_t^{-2}(y_t - C_t A_t \hat{x}_{t-1|t-1}), \\ \hat{x}_{t|t-1} &= A_t\hat{x}_{t-1|t-1}, \\ C_t &= s_t \frac{d(e^{kx_{t|t}})}{dx_{t|t}}\bigg|_{x_{t|t}=\hat{x}_{t-1|t-1}} = s_t k e^{kx_{t|t}}\big|_{x_{t|t}=\hat{x}_{t-1|t-1}}, \end{aligned} \tag{5}$$

where $t = 0, 1, 2, \ldots, N$, and $P_{t|t}$ is given by

$$\begin{aligned} \overline{P}_{t|t}^{-1} &= P_{t-1|t-1}^{-1} + A_t^T B_t^{-2} A_t, \\ P_{t|t}^{-1} &= C_t^T D_t^{-2} C_t + B_t^{-2} - B_t^{-2}\overline{P}_{t|t} A_t^T B_t^{-2}, \\ P_{t|t-1} &= A_t P_{t-1|t-1} A_t^T + B_t^2, \end{aligned} \tag{6}$$

where $B_t^2 = B_t B_t^T$ and $D_t^2 = D_t D_t^T$. The channel parameters $\theta_t = \{A_t, B_t, D_t\}$ are estimated based on the EM algorithm, which is introduced next.

3.2. Channel parameter estimation—The EM algorithm

The EM algorithm uses a bank of Kalman filters to yield a maximum likelihood (ML) parameter estimate of the state space model. It is an iterative scheme for computing the ML estimate of the system parameters θ_t, given the set of data Y_t. Specifically, each iteration of the EM algorithm consists of two steps: the expectation step and the maximization step [9]. The filtered expectation step only uses filters for the first- and second-order statistics. The algorithm yields parameter estimates with nondecreasing values of the likelihood function and converges under mild assumptions [10]. The expectation step evaluates the conditional expectation of the log-likelihood function given the complete data as

$$\Lambda(\theta_t, \hat{\theta}_t) = E_{\theta_l}\left\{\log\frac{dF_{\theta_t}}{dF_{\hat{\theta}_t}} \mid Y_t\right\}, \tag{7}$$

where $\{F_{\theta_t};\ \theta_t \in \Theta\}$ denotes a family of probability measures induced by the system parameters θ_t, and $\widehat{\theta}_t$ denotes the estimated system parameters at time step t. The maximization step finds that

$$\widehat{\theta}_{t+1} \in \arg\max_{\theta_t \in \Theta} \ \Lambda(\theta_t,\ \widehat{\theta}_t). \tag{8}$$

The expectation and maximization steps are repeated until the sequence of model parameters converges to the real parameters. The EM algorithm is given by [7, 9]

$$\begin{aligned}
\widehat{A}_t &= E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right) \times \left[E\left(\sum_{k=1}^{t} x_k x_k^T \mid Y_t\right)\right]^{-1},\\
\widehat{B}_t^2 &= \frac{1}{t} E\left(\sum_{k=1}^{t} \left((x_k - A_k x_{k-1})(x_k - A_k x_{k-1})^T\right) \mid Y_t\right)\\
&= \frac{1}{t} E\left(\sum_{k=1}^{t} ((x_k x_k^T) - A_k (x_k x_{k-1}^T)^T - (x_k x_{k-1}^T) A_k^T \right.\\
&\qquad \left. + A_k (x_{k-1} x_{k-1}^T) A_k^T) \mid Y_t\right),\\
\widehat{D}_t^2 &= \frac{1}{t} E\left(\sum_{k=1}^{t} \left((y_k - C_k x_k)(y_k - C_k x_k)^T\right) \mid Y_t\right)\\
&= \frac{1}{t} E\left(\sum_{k=1}^{t} \left((y_k y_k^T) - (y_k x_k^T) C_k^T - C_k (y_k x_k^T)^T \right.\right.\\
&\qquad \left.\left. + C_k (x_k x_k^T) C_k^T\right) \mid Y_t\right),
\end{aligned} \tag{9}$$

where $E(\cdot)$ denotes the expectation operator, and $t = 0, 1, 2, \ldots, N$. The system parameters $\{\widehat{A}_t, \widehat{B}_t^2, \widehat{D}_t^2\}$ are computed from the following conditional expectations[7]:

$$\begin{aligned}
L_t^{(1)} &= E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\},\\
L_t^{(2)} &= E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\},\\
L_t^{(3)} &= E\left\{\sum_{k=1}^{t} [x_k^T R x_{k-1} + x_{k-1}^T R^T x_k] \mid Y_t\right\},\\
L_t^{(4)} &= E\left\{\sum_{k=1}^{t} [x_k^T S y_k + y_k^T S^T x_k] \mid Y_t\right\},
\end{aligned} \tag{10}$$

where Q, R, and S are given by

$$\begin{aligned}
&Q = \left\{\frac{e_i e_j^T + e_j e_i^T}{2}\right\}, \qquad R = \left\{\frac{e_i e_j^T}{2}\right\},\\
&S = \left\{\frac{e_i e_k^T}{2}\right\}, \qquad i, j = 1, 2, \ldots, n,\ k = 1, 2, \ldots, d,
\end{aligned} \tag{11}$$

in which e_i is the unit vector in the Euclidean space, that is, $e_i = 1$ in the ith position and 0 elsewhere. For instance, consider the case $n = d = 1$, then $E(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t)$ is

$$E\left(\sum_{k=1}^{t} x_k x_{k-1}^T \mid Y_t\right) = L_t^{(3)}\left[R = \frac{1}{2}\right]. \tag{12}$$

The other terms in (9) can be computed similarly.

The conditional expectations $\{L_t^{(1)}, L_t^{(2)}, L_t^{(3)}, L_t^{(4)}\}$ can be estimated from measurements Y_t as follows.

(1) Filter estimate of $L_t^{(1)}$ is

$$\begin{aligned}
L_t^{(1)} &= E\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\}\\
&= -\frac{1}{2}\mathrm{Tr}(N_t^{(1)} P_{t|t}) - \frac{1}{2}\sum_{k=1}^{t}\mathrm{Tr}(N_{k-1}^{(1)} \overline{P}_{k|k})\\
&\quad - \frac{1}{2}\sum_{k=1}^{t}\left(-2x_{k|k}^T P_{k|k}^{-1} r_k^{(1)} + 2x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(1)} - x_{k|k}^T N_k^{(1)} x_{k|k}\right.\\
&\qquad \left. + x_{k|k-1}^T B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(1)} \overline{P}_{k|k} A_k^T B_k^{-2} x_{k|k-1}\right),
\end{aligned} \tag{13}$$

where $\mathrm{Tr}(\cdot)$ denotes the matrix trace. In (13), $r_k^{(1)}$ and $N_k^{(1)}$ satisfy the following recursions:

$$\begin{aligned}
r_k^{(1)} &= (A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(1)} + 2P_{k|k} Q x_{k|k-1}\\
&\quad - P_{k|k} N_k^{(1)} P_{k|k} C_k^T D_k^{-2} (y_k - C_k x_{k|k-1}),\\
r_{k|k-1}^{(1)} &= A_k r_k^{(1)},\\
r_0^{(1)} &= 0_{m\times 1},\\
N_k^{(1)} &= B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(1)} \overline{P}_{k|k} A_k^T B_k^{-2} - 2Q,\\
N_0^{(1)} &= 0_{m\times m}.
\end{aligned} \tag{14}$$

(2) Filter estimate of $L_t^{(2)}$ is

$$\begin{aligned}
L_t^{(2)} &= E\left\{\sum_{k=1}^{t} x_{k-1}^T Q x_{k-1} \mid Y_t\right\} = E_\theta\{x_0^T Q x_0 \mid Y_t\}\\
&\quad + E_\theta\left\{\sum_{k=1}^{t} x_k^T Q x_k \mid Y_t\right\} - E_\theta\{x_t^T Q x_t \mid Y_t\}.
\end{aligned} \tag{15}$$

Therefore, $L_t^{(2)}$ can be obtained from $L_t^{(1)}$.

(3) Filter estimate of $L_t^{(3)}$ is

$$\begin{aligned}
L_t^{(3)} &= E\left\{\sum_{k=1}^{t} \left(x_k^T R x_{k-1} + x_{k-1}^T R^T x_k\right) \mid Y_t\right\}\\
&= -\frac{1}{2}\mathrm{Tr}(N_t^{(3)} P_{t|t}) - \frac{1}{2}\sum_{k=1}^{t}\mathrm{Tr}(N_{k-1}^{(3)} \overline{P}_{k|k})\\
&\quad - \frac{1}{2}\sum_{k=1}^{t}\left(-2x_{k|k}^T P_{k|k}^{-1} r_k^{(3)} + 2x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(3)} - x_{k|k}^T N_k^{(3)} x_{k|k}\right.\\
&\qquad \left. + x_{k|k-1}^T B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(3)} \overline{P}_{k|k} A_k^T B_k^{-2}\ x_{k|k-1}\right).
\end{aligned} \tag{16}$$

In this case, $r_k^{(3)}$ and $N_k^{(3)}$ satisfies the following recursions:

$$\begin{aligned} r_k^{(3)} &= (A_k - P_{k|k}C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(3)} \\ &\quad - P_{k|k} N_k^{(3)} P_{k|k} C_k^T D_k^{-2} (y_k - C_k x_{k|k-1}) \\ &\quad + (2P_{k|k} R + 2P_{k|k} B_k^{-2} A_k \overline{P}_{k|k} R^T A_k) x_{k-1|k-1}, \\ r_{k|k-1}^{(3)} &= A_k r_k^{(3)}, \\ r_0^{(3)} &= 0_{m\times 1}, \\ N_k^{(3)} &= B_k^{-2} A_k \overline{P}_{k|k} N_{k-1}^{(3)} \overline{P}_{k|k} A_k^T B_k^{-2} \\ &\quad - 2R\overline{P}_{k|k} A_k^T B_k^{-2} - 2B_k^{-2} A_k \overline{P}_{k|k} R^T, \\ N_0^{(3)} &= 0_{m\times m}. \end{aligned} \tag{17}$$

(4) Filter estimate of $L_t^{(4)}$ is

$$\begin{aligned} L_t^{(4)} &= E\left\{\sum_{k=1}^{t} (x_k^T S y_k + y_k^T S^T x_k) \mid Y_t\right\} \\ &= \sum_{k=1}^{t} \left(x_{k|k}^T P_{k|k}^{-1} r_k^{(4)} - x_{k|k-1}^T P_{k|k-1}^{-1} r_{k|k-1}^{(4)} \right), \end{aligned} \tag{18}$$

where $r_k^{(4)}$ satisfies the following recursions:

$$\begin{aligned} r_k^{(4)} &= (A_k - P_{k|k} C_k^T D_k^{-2} C_k A_k) r_{k-1}^{(4)} + 2P_{k|k} S y_k, \\ r_{k|k-1}^{(4)} &= A_k r_k^{(4)}, \\ r_0^{(4)} &= 0_{m\times 1}. \end{aligned} \tag{19}$$

Using the filters for $L_t^{(i)}$ $(i = 1, 2, 3, 4)$ and the extended Kalman filter described in (5) and (6), the system parameters $\theta_t = \{A_t, B_t, D_t\}$ are estimated through the EM algorithm described in (9). Numerical results that show the applicability of the above algorithm are discussed next.

4. NUMERICAL RESULTS

In this section, the accuracy of the EM algorithm together with the extended Kalman filter to estimate channel parameters, as well as channel PL from the received signal measurements, is determined. The measurement data are generated by the system parameters:

$$\begin{aligned} \gamma(t,\tau) &= \gamma_m(\tau)\left(1 + 0.15e^{-2t/T} \sin\left(\frac{10\pi t}{T}\right)\right), \\ \delta(t,\tau) &= 5, \qquad \beta(t,\tau) = 0.2, \end{aligned} \tag{20}$$

where $\gamma_m(\tau)$ is the average PL at a specific location τ and it is chosen to be 25 dB, T is the observation interval, and the variances of the state and measurement noises are 10^{-2} and 10^{-6}, respectively. Figure 1 shows the actual and estimated received signals using the EM algorithm together with the extended Kalman filter for 500 sampled data. From Figure 1, it can be noticed that the received signal has been estimated with very good accuracy. Figure 2 shows the received signal estimates' root mean square error (RMSE) for 100 runs. It can be noticed that it takes just few iterations (less than 15) for the filter to converge, and the steady-state performance of the proposed channel estimation algorithm using the EM together with Kalman filtering is excellent. Since our stochastic model in (3) is first-order, the computational cost of the proposed estimation algorithm is very low, and thus it can be implemented online. Moreover, the filters of the expectation step are recursive and decoupled, and hence easy to implement in parallel on a multiprocessor system [9].

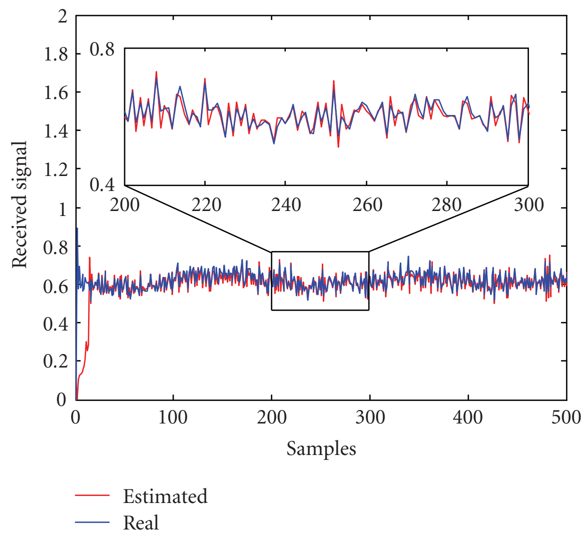

Figure 1: Real and estimated received signals for the channel model.

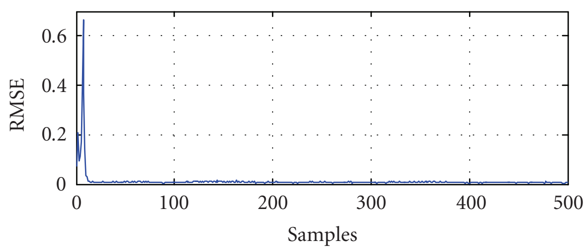

Figure 2: Received signal estimates' RMSE for 100 runs using the EM algorithm together with the extended Kalman filter.

5. CONCLUSION

This paper develops a general scheme for extracting mathematical LTF channel models from noisy received signal measurements. The proposed estimation algorithm is recursive and consists of filtering based on the extended Kalman filter to remove noise from data, and identification based on the EM algorithm to determine the parameters of the model which best describe the measurements. The proposed estimation and parameter identification algorithms estimate the path loss and the channel parameters. Performance of the latter is investigated through a numerical example that shows excellent results. Therefore, the proposed algorithms have good potential for real-time applications. Future work includes combining identification and estimation with other performance requirements, such as power control, admission control, and base station assignment.

ACKNOWLEDGMENTS

This work was supported by Cyprus Research Promotion Foundation under the Grant PLHRO/0506/20 and the University of Cyprus via a Large Size Research Proposal.

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