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Research Letter

Precoded DOSTBC over Rayleigh Channels

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Differential orthogonal space-time block codes (DOSTBC) sent over correlated Rayleigh fading channels are considered in this paper. Approximate expressions for the symbol error rate (SER) are derived for DOSTBC with M-PSK, M-PAM, and M-QAM constellations assuming arbitrary correlation between the transmit and receive antennas. A full memoryless precoder is designed to improve the performance of the DOSTBC over correlated Rayleigh MIMO channels. The proposed precoder design differs from the previous work: (1) our precoder design considers arbitrary correlation in the channels, whereas the previously proposed precoder design considers only transmit correlations in the Kronecker correlation model; (2) the proposed precoder is based on minimizing proposed SER, whereas the previously proposed precoder is based on minimizing the Chernoff bound of approximate SER; (3) we propose precoder design for DOSTBC with M-PSK, M-PAM, and M-QAM constellations, whereas the previously proposed precoder works for DOSTBC with M-PSK only. Additionally, the proposed precoded DOSTBC outperforms the conventional eigenbeamforming-based precoded DOSTBC for the Kronecker model with only transmit correlation.

1. INTRODUCTION

Differential modulation of orthogonal space-time block codes (OSTBC) is a promising technique in the terms of data rate and performance gain for the channels, which remains constant for small number of symbol durations [1–3]. DOSTBC can be decoded without any channel information at the receiver. The performance of DOSTBC suffers due to the channel correlations. In [4], an eigen-beamforming precoder for DOSTBC with M-PSK constellations is proposed. This precoder is based on Chernoff bound of *approximate* SER formulation derived in [4] assuming transmit correlations only in the Kronecker model [5]. However, this model does not always render the multipath structure correctly [6], but it might introduce artificial paths that are not present in the underlying measurement data. Moreover, the Kronecker model underestimates the mutual information of the MIMO channel systematically [6]. In this paper, our main contributions are as follows: (1) we derive approximate SER for DOSTBC for M-PSK, M-QAM, and M-PAM constellations in arbitrarily correlated Rayleigh channels; (2) we also propose a precoder design to improve the ruggedness of DOSTBC against the arbitrary correlations in Rayleigh MIMO channel.

2. DIFFERENTIAL CODING OF OSTBC WITH M-PSK OVER CORRELATED RAYLEIGH CHANNELS

2.1. Differential encoding for OSTBC

Let $\mathbf{S}_k$ be an $n \times n$ OSTBC data matrix obtained at time k from $\mathbf{s}_k = [s_{k,1}, s_{k,2}, \ldots, s_{k,n_s}]^{\mathcal{T}}$, $s_{k,t} \in$ M-PSK constellation and $n_s \leq n$. A differentially encoded data matrix $\mathbf{D}_k \in \mathbb{C}^{b\times n}$ can be obtained from $\mathbf{S}_k$ as

$$\mathbf{D}_k = \mathbf{D}_{k-1}\mathbf{S}_k. \tag{1}$$

It is well known that channel correlation degrades the performance of a wireless MIMO system [5, 7]. For improving the performance of the DOSTBC system, let us multiply DOSTBC $\mathbf{D}_k$ with a full memoryless precoder matrix $\mathbf{F} \in \mathbb{C}^{n_t\times b}$ before it is transmitted at time k. Hence, we finally transmit the precoded DOSTBC matrix $\mathbf{G}_k = \mathbf{F}\mathbf{D}_k$ at any time k.

2.2. Model of correlated channels

We assume flat block-fading correlated MIMO Rayleigh fading channel model [5]. Let the channel $\mathbf{H} \in \mathbb{C}^{n_r\times n_t}$ has zero mean, complex Gaussian circularly distribution with positive semidefinite autocorrelation given by $\mathbf{R} = \mathbb{E}[\text{vec}(\mathbf{H})\text{vec}^{\mathcal{H}}(\mathbf{H})]$ of size $n_t n_r \times n_t n_r$. A channel realization of the correlated channels can be found by $\text{vec}(\mathbf{H}) = \mathbf{R}^{1/2}\text{vec}(\mathbf{H}_w)$, where $\mathbf{R}^{1/2}$ is the unique semipositive definite matrix square root [8] of $\mathbf{R}$ and $\mathbf{H}_w$ has size $n_r \times n_t$ and consists of complex circular Gaussian distributed elements with zero mean and γ^2 variance.

2.3. Decoding of precoded differential OSTBC

The $n_r \times n$ received data matrix at time k is

$$\mathbf{Y}_k = \mathbf{HFD}_k + \mathbf{Q}_k = \mathbf{HG}_k + \mathbf{Q}_k, \tag{2}$$

where $\mathbf{Q}_k$ is an $n_r \times n$ matrix, containing additive white complex-valued Gaussian noise (AWGN), whose elements are i.i.d. Gaussian random variables with zero mean and variance σ^2. $\mathbf{H}$ is assumed stationary over the transmission period of at least two consecutive transmitted matrices ($\mathbf{G}_{k-1}$ and $\mathbf{G}_k$). A differential decoder for precoded DOSTBC with M-PSK constellation can be obtained as follows [9]:

$$\widehat{\mathbf{S}}_k = \arg\min_{\mathbf{S}_k \in \Xi} \|\mathbf{Y}_k - \mathbf{Y}_{k-1}\mathbf{S}_k\|^2, \tag{3}$$

where Ξ is the set of all OSTBC matrices, consisting of symbols belonging to the M-PSK constellation.

3. SER PERFORMANCE ANALYSIS OF PRECODED DOSTBC WITH UNITARY CONSTELLATIONS

As $\mathbf{S}_k$ is OSTBC, therefore, $\mathbf{S}_k\mathbf{S}_k^{\mathcal{H}} = \|\mathbf{s}_k\|^2\mathbf{I}_n$ and it can be expressed in the terms of $\mathbf{s}_k$ as [10]

$$\mathbf{S}_k = [\mathbf{\Phi}_{k,1}\mathbf{s}_k + \mathbf{\Theta}_{k,1}\mathbf{s}_k^*, \dots, \mathbf{\Phi}_{k,n}\mathbf{s}_k + \mathbf{\Theta}_{k,n}\mathbf{s}_k^*], \tag{4}$$

where $\mathbf{\Phi}_{k,m}$ and $\mathbf{\Theta}_{k,m}$ are $n \times n_s$ matrices with real or complex elements, which depend on the OSTBC. If $\mathbf{\Phi}_{k,m} = [\phi_{k,m,1}, \phi_{k,m,2}, \dots, \phi_{k,m,n_s}]$, where $\phi_{k,m,l}$ is $n \times 1$ vector and $\mathbf{\Theta}_{k,m} = [\theta_{k,m,1}, \theta_{k,m,2}, \dots, \theta_{k,m,n_s}]$, where $\theta_{k,m,l}$ is $n \times 1$ vector, the received data matrix and AWGN noise matrix in (2) can be written as $\mathbf{Y}_k = [\mathbf{y}_{k,1}, \mathbf{y}_{k,2}, \dots, \mathbf{y}_{k,n}]$ and $\mathbf{Q}_k = [\mathbf{q}_{k,1}, \mathbf{q}_{k,2}, \dots, \mathbf{q}_{k,n}]$, respectively, where $\mathbf{y}_{k,m}$ and $\mathbf{q}_{k,m}$ are $n_r \times 1$ vectors. Let $\widetilde{\mathbf{s}}_k$ be the transmitted data vector encoded into $\widetilde{\mathbf{S}}_k$ and its estimated value is given by $\widehat{\mathbf{s}}_k$. From (2) and (4) the received data vector corresponding to $\widetilde{\mathbf{s}}_k$ can be given as

$$\mathbf{y}_{k,m} = \mathbf{HFD}_{k-1}(\mathbf{\Phi}_{k,m}\widetilde{\mathbf{s}}_k + \mathbf{\Theta}_{k,m}\widetilde{\mathbf{s}}_k^*) + \mathbf{q}_{km}. \tag{5}$$

We may rewrite the decision metric (3) with the help of (5) as follows:

$$\begin{aligned}\widehat{\mathbf{s}}_k = \arg\min_{\mathbf{s}_k \in \chi^{n_s}} \sum_{m=1}^{n} \Big\| \mathbf{HFD}_{k-1}\Big(\mathbf{\Phi}_{k,m}[\widetilde{\mathbf{s}}_k - \mathbf{s}_k] \\ + \mathbf{\Theta}_{k,m}[\widetilde{\mathbf{s}}_k - \mathbf{s}_k]^*\Big) + \mathbf{e}_{k,m}\Big\|^2,\end{aligned} \tag{6}$$

where χ is M-PSK constellation and $\mathbf{e}_{k,m} = \mathbf{q}_{k,m} - \mathbf{Q}_{k-1}(\mathbf{\Phi}_{k,m}\mathbf{s}_k + \mathbf{\Theta}_{k,m}\mathbf{s}_k^*)$. After several manipulations and neglecting two noise terms which have a diminishing effect at high SNR, a simplified and approximate decision variable corresponding to (6) can be obtained as [7, 11, 12]

$$\mathcal{D}_{\mathbf{s}}(\mathbf{s}_k) = \sum_{t=1}^{n_s} \left|\widetilde{s}_{k,t} - s_{k,t} + \vartheta_{k,t}\right|^2. \tag{7}$$

As $s_{k,t} \in M$-PSK constellation, $\|\mathbf{s}_k\|^2 = n_s w$, where the variance of each symbol $s_{k,t}$ is w. If $w = 1/n_s$, $\|\mathbf{HFD}_{k-1}\|^2 = \|\mathbf{HF}\|^2$ and it can be shown that $\vartheta_{k,t} \sim \mathcal{CN}(0, 2\sigma^2/\|\mathbf{HF}\|^2)$, [12], where σ^2 is the AWGN noise power. Let $\mathbf{\Psi} \triangleq \mathbf{R}^{1/2}[(\mathbf{F}^*\mathbf{F}^{\mathcal{T}}) \otimes \mathbf{I}_{n_r}]\mathbf{R}^{1/2}$ be a positive semidefinite matrix of size $n_t n_r \times n_t n_r$ with the eigen-decomposition $\mathbf{\Psi} = \mathcal{V}\mathbf{\Omega}\mathcal{V}^{\mathcal{H}}$. Define the nonnegative scalar $\beta \triangleq \|\mathbf{HF}\|^2 = \text{vec}^{\mathcal{H}}(\mathbf{H}_w)\mathbf{\Psi}\,\text{vec}(\mathbf{H}_w)$. It can be seen from (7) that the MIMO system is collapsed into n_s single-input single-output (SISO) systems having the following output-input relationship:

$$y_{kt} = \sqrt{\beta}s_{k,t} + v_{k,t}, \quad \forall t \in \{1, 2, \dots, n_s\}, \tag{8}$$

where $v_{k,t} = \vartheta_{k,t}\|\mathbf{HF}\|^2$ is distributed as $\mathcal{CN}(0, 2\sigma^2)$. For the SISO system of (8) the probability of error given $\|\mathbf{HF}\|^2$ can be written as [13, equation (8.23)]

$$\mathbb{P}_e\{\beta\} = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \exp\left\{-\frac{w\sin^2(\pi/M)}{2\sigma^2\sin^2\psi}\beta\right\}d\psi. \tag{9}$$

Next, the SER can be obtained by averaging (9) over the channel as follows:

$$\mathbb{E}_{\mathbf{H}}[\mathbb{P}_e\{\beta\}] = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \mathcal{M}_\beta\left(\frac{\alpha_{\text{PSK}}}{\sin^2\psi}\right)d\psi, \tag{10}$$

where $\alpha_{\text{PSK}} = w\sin^2(\pi/M)/2\sigma^2$ and $\mathcal{M}_\beta(\cdot)$ is moment generating function (MGF) of β. From the definition of β, it is seen that $\beta = \text{vec}^{\mathcal{H}}(\mathbf{H}'_w)\mathbf{\Omega}\text{vec}(\mathbf{H}'_w)$ [8], where $\text{vec}(\mathbf{H}'_w)$ is distributed as $\mathcal{CN}(\mathbf{0}_{n_t n_r \times 1}, \mathbf{I}_{n_t n_r})$. It is clear from the latest expression of β that it can be expressed as a weighted sum of squares of the absolute value of independent complex Gaussian variables. Hence, the MGF of β can be written as [13] $\mathcal{M}_\beta(s) = 1/\prod_{i=0}^{n_t n_r - 1}(1 - \omega_i s)$, where ω_i are nonnegative eigenvalues of $\mathbf{\Psi}$ and $s = \alpha_{\text{PSK}}/\sin^2\psi$. From (10) and using the property of eigen-decomposition [8] we can write the approximate SER of DOSTBC with M-PSK constellation as

$$\text{SER} = \frac{1}{\pi}\int_0^{(M-1)\pi/M} \frac{d\psi}{\det\left(\mathbf{I}_{n_t n_r} + (\alpha_{\text{PSK}}/\sin^2\psi)\mathbf{\Psi}\right)}. \tag{11}$$

4. PRECODED DOSTBC FOR M-PAM AND M-QAM

Let $s_{k,t} \in M$-QAM or M-PAM constellation, then we modify the differential encoding of (1) to avoid the power fluctuation from one DOSTBC block to other as follows:

$$\mathbf{D}_k = \mathbf{D}_{k-1}\frac{\mathbf{S}_k}{\|\mathbf{s}_{k-1}\|} = \mathbf{D}_{k-1}\breve{\mathbf{S}}_k. \tag{12}$$

For the simplicity of analysis we use the following differential decoder for our analysis and simulation of DOSTBC with nonunitary (M-PAM and M-QAM) constellations:

$$\widehat{\mathbf{S}}_k = \arg\min_{\mathbf{S}_k \in \Xi} \left\| \mathbf{Y}_k - \mathbf{Y}_{k-1}\breve{\mathbf{S}}_k \right\|^2, \tag{13}$$

where Ξ is the set of all OSTBC matrices, consisting of symbols belonging to the finite M-QAM or M-PAM constellation. It is shown in [3, Section 5] that for DOSTBC with nonunitary constellations, (13) can be used as the suboptimal decoder, which performs almost as same as the maximum likelihood (ML) optimal decoder. Following the same procedure as in Section 3, we can obtain a simplified and approximate decision variable for the case of DOSTBC with nonunitary constellations also as [7, 11, 12]

$$\mathcal{D}_{\mathbf{s}}(\mathbf{s}_k) = \sum_{t=1}^{n_s} \left| \widetilde{s}_{k,t} - s_{k,t} + \vartheta'_{k,t} \right|^2, \tag{14}$$

where $\vartheta'_{k,t} \sim \mathcal{CN}(0, \sigma^2[1+f]/\|\mathbf{HF}\|^2)$ with $f = \mathbb{E}_{\mathbf{s}}\{\|\mathbf{s}_k\|^2/\|\mathbf{s}_{k-1}\|^2\}$, where $\mathbb{E}_{\mathbf{s}}\{\cdot\}$ is the expectation over the original information symbols. f depends on the type and size of nonunitary signal constellation, and can be calculated numerically. For 16-QAM its value is approximately 1.308. Following the same procedure as in Section 3 and using [13, equations (8.5) and (8.12)], the SER of the precoded DOSTBC with nonunitary constellations can be written as

$$\begin{aligned}
\text{SER} &= \frac{2}{\pi}\frac{M-1}{M}\int_0^{\pi/2} \frac{d\psi}{\det\left(\mathbf{I}_{n_t n_r} + (\alpha_{\text{PAM}}/\sin^2\psi)\boldsymbol{\Psi}\right)}, \\
\text{SER} &= \frac{4}{\pi}\frac{\sqrt{M}-1}{\sqrt{M}}\left[\frac{1}{\sqrt{M}}\int_0^{\pi/4} \frac{d\psi}{\det\left(\mathbf{I}_{n_t n_r} + (\alpha_{\text{QAM}}/\sin^2\psi)\boldsymbol{\Psi}\right)}\right. \\
&\qquad \left. + \int_{\pi/4}^{\pi/2} \frac{d\psi}{\det(\mathbf{I}_{n_t n_r} + (\alpha_{\text{QAM}}/\sin^2\psi)\boldsymbol{\Psi})}\right],
\end{aligned} \tag{15}$$

where $\alpha_{\text{PAM}} = 3w/\sigma^2(M^2-1)[1+f]$ and $\alpha_{\text{QAM}} = 3w/2\sigma^2(M-1)[1+f]$.

5. PRECODER DESIGN FOR DOSTBC

Let us assume that the receiver can estimate the channel correlation matrix $\mathbf{R}$ and noise variance σ^2, and feed these back to the transmitter. Based on these assumptions, a precoder can be designed for the DOSTBC based on the SER formulations of (11) and (15).

5.1. Problem formulation

By using the properties of orthogonal STBC, it can be shown from (1) and (12) that the average power of the kth DOSTBC block for M-PSK, M-PAM, and M-QAM constellation is $\mathbb{E}\{\mathbf{D}_k\mathbf{D}_k^{\mathcal{H}}\} = n_s w \mathbf{I}_n$. If we set $w = 1/n_s$, the average power constraint on the transmitted block $\mathbf{G}_k = \mathbf{FD}_k$ can be expressed as $\text{Tr}\{\mathbf{FF}^{\mathcal{H}}\} = P$, where P is the average power used by the transmitted block $\mathbf{G}_k$. The goal is to design a precoder $\mathbf{F}$ such that the approximate SER is minimized subject to the average transmitted power constraint:

$$\min_{\{\mathbf{F}\in\mathbb{C}^{n_t\times b}\}} \text{SER}, \quad \text{subject to } \text{Tr}\{\mathbf{FF}^{\mathcal{H}}\} = P. \tag{16}$$

5.2. Precoder design

The constrained maximization problem (16) can be converted into an unconstrained optimization problem by introducing a positive Lagrange multiplier μ [14],

$$\mathcal{L}(\mathbf{F}, \mathbf{F}^*) = \text{SER} + \mu\text{Tr}\{\mathbf{FF}^{\mathcal{H}}\}. \tag{17}$$

To find an optimal solution of $\mathbf{F}$, we need to find the matrix derivation of the objective funcion in (17) with respect to $\mathbf{F}^*$, that is, $\mathcal{D}_{\mathbf{F}^*}\mathcal{L}(\mathbf{F}, \mathbf{F}^*)$ [15] and equate the result to zero. It can be shown with the help of [15, Table IV] that $\mathcal{D}_{\mathbf{F}^*}\mathcal{L}(\mathbf{F}, \mathbf{F}^*) = \mathbf{0}_{1\times n_t b}$ is equivalent to

$$\text{vec}(\mathbf{F}) = \mathcal{D}_{\mathbf{F}^*}\text{SER}(\mathbf{F}, \mathbf{F}^*), \tag{18}$$

where $\text{SER}(\mathbf{F}, \mathbf{F}^*)$ is the SER as a function of $\mathbf{F}$ and $\mathbf{F}^*$. Define the $n_t^2 \times n_t^2 n_r^2$ matrix as $\mathcal{K} = [\mathbf{I}_{n_t^2} \otimes \text{vec}^{\mathcal{T}}(\mathbf{I}_{n_r})][\mathbf{I}_{n_t} \otimes \mathbf{K}_{n_t,n_r} \otimes \mathbf{I}_{n_r}]$ and an intermediate vector variable $\mathbf{g}(\mathbf{F}, \psi, \alpha, \lambda)$ of size $bn_t \times 1$ as

$$\begin{aligned}
\mathbf{g}(\mathbf{F}, \psi, \alpha, \lambda) &= \lambda[\mathbf{F}^{\mathcal{T}} \otimes \mathbf{I}_{n_t}]\mathcal{K}\left[\mathbf{R}^{1/2} \otimes (\mathbf{R}^{1/2})^*\right] \\
&\quad \times \frac{\text{vec}\left(\left[\mathbf{I}_{n_t n_r} + (\alpha/\sin^2(\psi))\boldsymbol{\Psi}^*\right]^{-1}\right)}{\sin^2(\psi)\det(\mathbf{I}_{n_t n_r} + (\alpha/\sin^2(\psi))\boldsymbol{\Psi})},
\end{aligned} \tag{19}$$

where $\mathbf{K}_{p,q}$ is a commutation matrix [16] of size $pq \times pq$ and λ is a scalar which is chosen to satisfy the power constraint. After many manipulations and using the results given in [15, 17], we can write the first-order derivative for DOSTBC using PSK, PAM, or QAM constellation as

$$\begin{aligned}
\mathcal{D}_{\mathbf{F}^*}\text{SER}(\mathbf{F}, \mathbf{F}^*) &= \int_0^{(M-1)\pi/M} \mathbf{g}(\mathbf{F}, \psi, \alpha_{\text{PSK}}, \lambda)\,d\psi, \\
\mathcal{D}_{\mathbf{F}^*}\text{SER}(\mathbf{F}, \mathbf{F}^*) &= \int_0^{\pi/2} \mathbf{g}(\mathbf{F}, \psi, \alpha_{\text{PAM}}, \lambda)\,d\psi, \\
\mathcal{D}_{\mathbf{F}^*}\text{SER}(\mathbf{F}, \mathbf{F}^*) &= \frac{1}{\sqrt{M}}\int_0^{\pi/4} \mathbf{g}(\mathbf{F}, \psi, \alpha_{\text{QAM}}, \lambda)\,\text{d}\psi \\
&\quad + \int_{\pi/4}^{\pi/2} \mathbf{g}(\mathbf{F}, \psi, \alpha_{\text{QAM}}, \lambda)\,d\psi.
\end{aligned} \tag{20}$$

We can use a fixed-point iteration method to find the precoder matrix from (18), (19), and (20). An iterative method for fast convergence is explained in a pseudocode provided in Algorithm 1. The initial value for the precoder matrix should be chosen appropriately. The trivial precoder matrix $\mathbf{F} = \sqrt{P}[\mathbf{I}_{\max\{n_t,b\}}]_{n_t\times b}$ can be used as the initialization matrix. Here, $[\mathbf{I}_{\max\{n_t,b\}}]_{n_t\times b}$ is a matrix of the size $n_t \times b$ taken from an identity matrix $\mathbf{I}_{\max\{n_t,b\}}$, where $\max\{n_t, b\}$ returns the maximum value of n_t and b. When the precoder matrix has been found for a certain channel conditions, it can be used as the initial precoder matrix for other channel conditions that are close to the one already optimized. The parameter ε decides the termination of the iterative algorithm.

Step 1: Initialization
Choose OSTBC, P, w, n_t, n_r, ϵ, and signal constellation
Estimate $\mathbf{R}$ and σ^2
Initialize $\mathbf{F}$ to an already optimized precoder or to $\mathbf{F}_0 = \sqrt{P}[\mathbf{I}_{\max\{n_t,b\}}]_{n_t \times b}$
Step 2: Precoder Optimization
$i := 0$
repeat
  $i := i + 1$
  Calculate the right-hand side of (20).
  Normalize precoder matrix $\mathbf{F}_i$ such that:
   $\|\mathbf{F}_i\| = \sqrt{P}$,
until $\|\mathbf{F}_i - \mathbf{F}_{i-1}\| < \epsilon$
The optimized precoder is given by $\mathbf{F} = \mathbf{F}_i$

Algorithm 1: Pseudocode of the numerical precoder optimization algorithm.

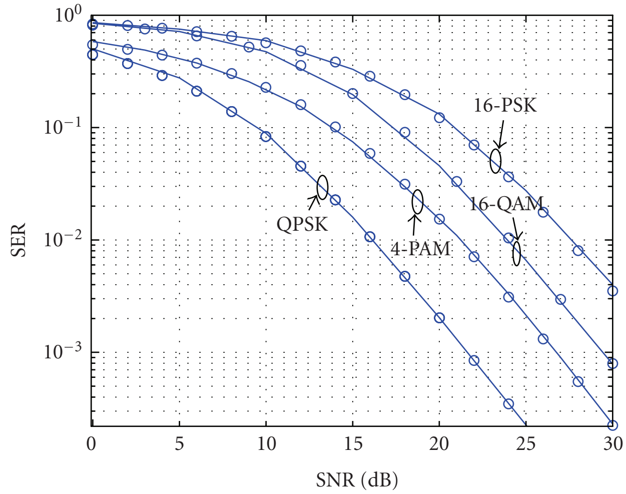

Figure 1: Analytical ∘ and simulated − SER of DOSTBC in uncorrelated channels.

In the numerical experiments, we observed that the proposed fixed-point algorithm always converged. However, the optimized $\mathbf{F}$ is not unique. We have experimented with many different initialization matrices and never experienced any divergence of the proposed fixed-point algorithm. Let $\mathbf{F}_1$ and $\mathbf{F}_2$ be two precoder matrix solutions obtained after applying the fixed-point algorithm with two different initialization matrices. It can be seen from [17, Lemma I] that $\mathbf{F}_1\mathbf{V}_1$ and $\mathbf{F}_2\mathbf{V}_2$ are also the optimal precoder. Let $\mathbf{F}_1 = \mathbf{U}_1\mathbf{\Lambda}_1\mathbf{V}_1^{\mathcal{H}}$ and $\mathbf{F}_2 = \mathbf{U}_2\mathbf{\Lambda}_2\mathbf{V}_2^{\mathcal{H}}$ be the singular value decompositions of $\mathbf{F}_1$ and $\mathbf{F}_2$, respectively. Then we have observed from the numerical experiments that $\mathbf{U}_1 = \mathbf{U}_2$ and $\mathbf{\Lambda}_1 = \mathbf{\Lambda}_2$ subject to the reordering of the eigenvalues, multiplication of a point on the unit circle, and normalization.

6. PERFORMANCE RESULTS AND COMPARISONS

In all the simulations we assumed that the channel is constant over the transmission periods of two DOSTBC blocks. Figure 1 shows the comparison of analytical SER (11) and (15), and simulated SER of differential orthogonal STBC with 4, 16-PSK, 4-PAM, and 16-QAM constellations for Alamouti code, $n_t = 2$, $n_r = 1$, $\mathbf{R} = \mathbf{I}_{n_t n_r}$, $\mathbf{F} = \sqrt{P}\mathbf{I}_{n_t}$. It can be seen from Figure 1 that the experimental results closely follow our analytical formulation of SER from moderate to high SNR values. Figure 2 shows comparisons of the proposed precoded DOSTBC and eigen-beamforming precoder-based DOSTBC of [4] for Alamouti code with $n_t = 2$ and $n_r = 1$, QPSK constellation, $[\mathbf{R}_t]_{i,j} = (0.7)^{i-j}$, $1 \le \{i, j\} \le n_t$, and $\mathbf{R}_r = \mathbf{I}_{n_r}$. Apparently, the proposed precoded DOSTBC outperforms the existing precoded DOSTBC. In Figure 3, we have shown the performance of differential system based on Alamouti code with $n_t = 2$ and $n_r = 2$, and 16-QAM constellation. In this case, the channel is assumed to be correlated with $[\mathbf{R}]_{i,j} = (\rho)^{i-j}$, $1 \le \{i, j\} \le n_t n_r$, with $\rho \in \{0.9, 0.99, 0.99999\}$. It can be seen from Figure 3 that the proposed precoded DOSTBC exhibits performance gain as compared to the unprecoded DOSTBC in correlated channels.

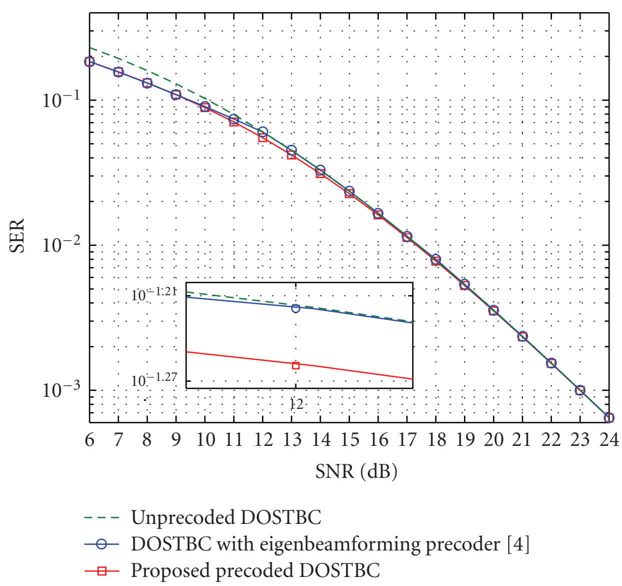

Figure 2: Comparison of the proposed precoded DOSTBC with the eigenbeamforming-based precoded differential system of [4] over channels with only transmit correlation.

7. CONCLUSIONS

We have derived approximate SER expressions for the precoded DOSTBC with unitary and nonunitary constellations for *arbitrary* joint correlations in the transmitter and the receiver. We have also proposed a precoder design for the DOSTBC to improve the performance over arbitrarily correlated Rayleigh fading MIMO channels. The proposed precoded differential system not only works well in arbitrarily

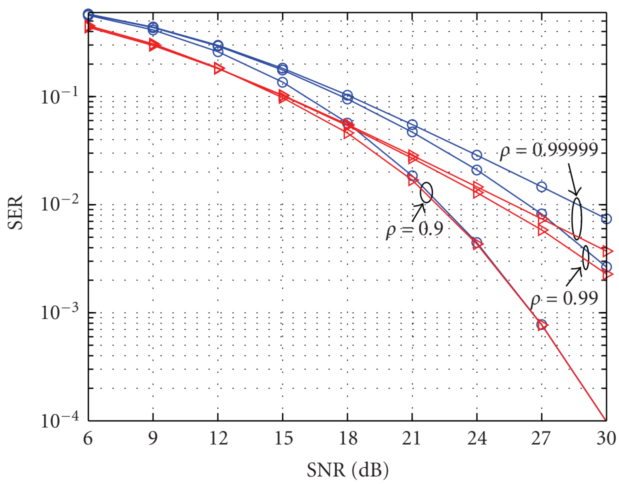

FIGURE 3: Analytical performance of the proposed precoded DOSTBC with 16-QAM in generally correlated channels; $-\circ-$ unprecoded differential system and $-\Delta-$ precoded differential system.

correlated channels but it also performs better than the previously proposed eigenbeamforming-based precoded differential codes with the Kronecker correlation channel model with only transmit correlation.

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