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Research Article

Symbol Error Probability of DF Relay Selection over Arbitrary Nakagami-*m* Fading Channels

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We present a new analytical expression for the moment generating function (MGF) of the end-to-end signal-to-noise ratio of dual-hop decode-and-forward (DF) relaying systems with relay selection when operating over Nakagami-m fading channels. The derived MGF expression, which is valid for arbitrary values of the fading parameters of both hops, is subsequently utilized to evaluate the average symbol error probability (ASEP) of M-ary phase shift keying modulation for the considered DF relaying scheme under various asymmetric fading conditions. It is shown that the MGF-based ASEP performance evaluation results are in excellent agreement with equivalent ones obtained by means of computer simulations, thus validating the correctness of the presented MGF expression.

1. Introduction

Cooperative communication through relay nodes has been shown to be capable of extending the radio coverage and improving the reliability of emerging wireless systems [1–3]. One of the bandwidth efficient dual-hop cooperative techniques combines the decode-and-forward (DF) relaying protocol with relay selection (RS) [4]. The performance of this technique has been studied in [4] over Rayleigh fading channels and in [5–7] for the more general Nakagami-m fading channel model. However, in the latter works analytical expressions for the moment generating function (MGF) of the end-to-end signal-to-noise ratio (SNR) have been presented, which are valid only for the special case where the Nakagami m-parameter of both hops takes integer values. In [5], the authors based their analysis on the tight approximation for the end-to-end SNR presented in [8], whereas [6, 7] utilized the end-to-end SNR characterization of [9]. Nevertheless, in realistic wireless communication scenarios, estimators for m from field measurement data typically result in arbitrary m values [10]. Moreover, restricting m to only integer values severely limits the advantageous property of the Nakagami-m fading distribution to adequately approximate the Rice and Hoyt ones [11]. Very recently, based on [9], the authors in [12] investigated the error probability of opportunistic DF relaying over Nakagami-m fading channels with arbitrary m.

In this paper, capitalizing on the approach of [8], we present a new closed-form representation for the MGF of the end-to-end SNR of dual-hop DF RS-based systems which is valid for arbitrary-valued parameters for both Nakagami-m faded hops. In addition, the derived expression is utilized to evaluate the average symbol error probability (ASEP) of M-ary phase shift keying (PSK) modulation for the considered relaying scheme. Numerically evaluated ASEP results match perfectly with equivalent results obtained from computer simulations and clearly demonstrate that ASEP is rather sensitive to even slight variations of any of the hops' fading parameter.

The remainder of this paper is organized as follows. Section 2 presents in brief the corresponding signal and system model. A new closed-form representation for the

MGF of DF with RS over arbitrary Nakagami-m fading channels is derived in Section 3. An application of the derived expression in evaluating the ASEP performance of M-PSK is demonstrated in Section 4, while closing remarks are provided in Section 5.

2. Signal and System Model

Consider a wireless dual-hop DF cooperative system comprising a source node S, a destination node D, and L half-duplex relays, each of which is denoted by R_k, $k = 1, 2, \ldots, L$. During the first transmission time slot, S broadcasts a symbol to the relays. In the second time slot, the relay with the most favorable channel to D is selected to transmit, while S remains silent. Node D is assumed to possess perfect channel state information so that maximum-likelihood combining of the signals from S and the selected relay can be employed.

The wireless channels between any pair of system nodes are assumed to be subject to the Nakagami-m fading distribution [13]. Furthermore, transmission is corrupted by additive white Gaussian noise (AWGN) with single-sided power spectral density N_0. The fading parameters for the $S \rightarrow D$, $S \rightarrow R_k$, and $R_k \rightarrow D$ channels are denoted by m_0, m_{1k}, and m_{2k}, respectively, whereas the corresponding instantaneous received SNRs as γ_0, γ_{1k}, and γ_{2k}, respectively. It is noted that unlike many other studies [4–7], no restrictions are applied on the Nakagami-m fading parameters of both hops. In other words, m_0 and each $m_{\ell k}$, with $\ell = 1, 2$ and $k = 1, 2, \ldots, L$, can take any arbitrary positive real value as long as it obeys the necessary condition for the Nakagami-m distribution, that is, $m_0, m_{\ell k} \geq 0.5$. The average received SNRs at the first and second hop are defined as $\overline{\gamma}_0 = \Omega_{SD} E_s / N_0$, $\overline{\gamma}_{1k} = \Omega_{SR_k} E_s / N_0$, and $\overline{\gamma}_{2k} = \Omega_{R_k D} E_s / N_0$, respectively, where Ω_{AB} denotes the average fading power of the $A \rightarrow B$ channel and E_s is the energy of the symbol transmitted. Clearly, γ_0 and $\gamma_{\ell k}$'s are Gamma distributed random variables which are assumed to be statistically independent.

As shown in [8], a dual-hop transmission $S \rightarrow R_k \rightarrow D$ employing the DF protocol can be modeled as a single-hop transmission $S \rightarrow D$ with the equivalent instantaneous received SNR at D bounded as

$$\gamma_{\min} - \frac{3.24}{M} < \gamma_k \leq \gamma_{\min}. \tag{1}$$

In (1), $\gamma_{\min} = \min\{\gamma_{1k}, \gamma_{2k}\}$ and M denotes the transmitted signal's modulation order. For relatively large SNR and M values, setting $\gamma_k = \gamma_{\min}$ yields a tight approximation for γ_k [8]. To this effect, by using RS, the corresponding received SNR at D can be accurately approximated as $\gamma_{\mathrm{RS}} = \max_{k=1,2,\ldots,L} \gamma_k$ [5]. Utilizing next a time-diversity version of maximal-ratio combining [11], the instantaneous received SNR at D's output can be expressed as

$$\gamma_D = \gamma_0 + \gamma_{\mathrm{RS}}. \tag{2}$$

3. MGF of DF RS in Nakagami-m Fading

By differentiating [5, equation (3)], a closed-form expression for the probability density function (PDF) of γ_k for any arbitrary value of $m_{1k}, m_{2k} \geq 0.5$ can be obtained as follows:

$$f_{\gamma_k}(x) = \sum_{p=1}^{2} \frac{a_{pk}^{m_{pk}}}{\Gamma(m_{pk})} x^{m_{pk}-1} \exp(-a_{pk} x) \times G\left[m_{(3-p)k}, a_{(3-p)k} x\right], \tag{3}$$

where $a_{1k} = m_{1k}/\overline{\gamma}_{1k}$, $a_{2k} = m_{2k}/\overline{\gamma}_{2k}$, and $G(c, dx) = \Gamma(c, dx)/\Gamma(c)$, with $c, d \in \mathbb{R}$, while $\Gamma(\cdot)$ and $\Gamma(\cdot,\cdot)$ denote the incomplete [14, equation (8.310)] and the upper incomplete Gamma function [14, equation (8.350.2)], respectively. Substituting [5, equation (3)] and (3) into [5, equation (6)] and after some algebraic manipulations, the following expression for the PDF of γ_{RS} is deduced:

$$\begin{aligned} f_{\gamma_{\mathrm{RS}}}(x) = {} & \sum_{p=1}^{2} \sum_{k=1}^{L} \sum_{l=0}^{L} \widetilde{\sum} \prod_{t=1}^{l} \frac{(-1)^l a_{pk}^{m_{pk}}}{l!\,\Gamma(m_{pk})} x^{m_{pk}-1} \\ & \times \exp(-a_{pk} x)\, G\left[m_{pn_t}, a_{pn_t} x\right] \\ & \times G\left[m_{(3-p)k}, a_{(3-p)k} x\right] G\left[m_{(3-p)n_t}, a_{(3-p)n_t} x\right], \end{aligned} \tag{4}$$

where symbol $\widetilde{\sum}$ is used for short-hand representation of multiple summations of the form $\sum_{n_1=1}^{L} \sum_{n_2=1}^{L} \cdots \sum_{n_l=1}^{L}$ with $n_1 \neq n_2 \neq \cdots \neq n_l$.

In order to derive an explicit expression for the MGF of γ_{RS}, which is defined as $M_{\gamma_{\mathrm{RS}}}(s) = \int_0^{\infty} f_{\gamma_{\mathrm{RS}}}(x) \exp(sx)\, dx$ [11, equation (1.2)], for arbitrary valid values of both m_{1k} and m_{2k}, one is required to analytically evaluate integrals that involve combinations of arbitrary powers, exponentials, and $G(\cdot,\cdot)$ functions. To this end, by expressing all $G(\cdot,\cdot)$'s in (4) according to [14, equation (8.354.2)], utilizing [14, equation (8.310)], and performing some rather long but basic algebraic manipulations, one obtains the following explicit expression for $M_{\gamma_{\mathrm{RS}}}(s)$:

$$\begin{aligned} M_{\gamma_{\mathrm{RS}}}(s) = {} & \sum_{k=1}^{L} \sum_{l=0}^{L} \widetilde{\sum} \prod_{t=1}^{l} \frac{(-1)^l}{l!} \prod_{\ell=1}^{2} \frac{1}{\Gamma(m_{\ell k}) \Gamma(m_{\ell n_t})} \\ & \times \left\{ a_{1k}^{m_{1k}} \sum_{p=1}^{8} \mathscr{A}_p + a_{2k}^{m_{2k}} \sum_{p=1}^{8} \mathscr{B}_p \right\} \end{aligned} \tag{5}$$

which is valid for $a_{1k}, a_{2k} \neq s$ for all $k = 1, 2, \ldots, L$. In (5), parameters $\mathscr{A}_p$ and $\mathscr{B}_p$, with $p = 1, 2, \ldots, 8$, are given by

$$\begin{Bmatrix} \mathscr{A}_1 \\ \mathscr{B}_1 \end{Bmatrix} = \prod_{\ell=1}^{2} \Gamma(m_{\ell n_t}) \Gamma(\Xi) H_3(1, 1, 1), \tag{6}$$

$$\left\{\begin{matrix}\mathscr{A}_2\\ \mathscr{B}_2\end{matrix}\right\} = -\Gamma\left(m_{2n_t}\right)\Gamma\left(\Xi\right)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}a_{1n_t}^{m_{1n_t}+i}H_2\left(m_{1n_t}+i,1,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_3\\ \mathscr{B}_3\end{matrix}\right\} = -\prod_{\ell=1}^{2}\Gamma\left(m_{\ell n_t}\right)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}^{\Xi+i}H_2\left(\Xi+i,1,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_4\\ \mathscr{B}_4\end{matrix}\right\} = \Gamma\left(m_{2n_t}\right)\sum_{i,j=0}^{\infty}\frac{(-1)^{i+j}}{i!j!}a_{1n_t}^{m_{1n_t}+j}\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}^{\Xi+i} \times H_1\left(\Xi+i,m_{1n_t}+j,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_5\\ \mathscr{B}_5\end{matrix}\right\} = -\Gamma\left(m_{1n_t}\right)\Gamma\left(\Xi\right)\sum_{i=0}^{\infty}\frac{(-1)^i}{i!}a_{2n_t}^{m_{2n_t}+i}H_2\left(m_{2n_t}+i,1,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_6\\ \mathscr{B}_6\end{matrix}\right\} = \Gamma\left(\Xi\right)\sum_{i,j=0}^{\infty}\frac{(-1)^{i+j}}{i!j!}a_{1n_t}^{m_{1n_t}+i}a_{2n_t}^{m_{2n_t}+j} \times H_1\left(m_{1n_t}+i,m_{2n_t}+j,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_7\\ \mathscr{B}_7\end{matrix}\right\} = \Gamma\left(m_{1n_t}\right)\sum_{i,j=0}^{\infty}\frac{(-1)^{i+j}}{i!j!}\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}^{\Xi+i}a_{2n_t}^{m_{2n_t}+j} \times H_1\left(\Xi+i,m_{2n_t}+j,1\right),$$

$$\left\{\begin{matrix}\mathscr{A}_8\\ \mathscr{B}_8\end{matrix}\right\} = -\sum_{i,j,z=0}^{\infty}\frac{(-1)^{i+j+z}}{i!j!z!}\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}^{\Xi+i}a_{1n_t}^{m_{1n_t}+j}a_{2n_t}^{m_{2n_t}+z} \times H_0\left(\Xi+i,m_{1n_t}+j,m_{2n_t}+z\right). \tag{7}$$

In (6)-(7) the notation $\left\{\begin{smallmatrix}x\\ y\end{smallmatrix}\right\} = \left\{\begin{smallmatrix}c\\ d\end{smallmatrix}\right\}$ is used for short-hand representation of $x = c$ and $y = d$, and symbols Ξ and Υ represent $\left\{\begin{smallmatrix}m_{2k}\\ m_{1k}\end{smallmatrix}\right\}$ and $\left\{\begin{smallmatrix}a_{1k}\\ a_{2k}\end{smallmatrix}\right\}$, respectively, whereas the function of s denoted by $H_n(\kappa,\lambda,\mu)$, with $\kappa,\lambda,\mu \in \mathbb{R}$ and $n = 0, 1, 2$ and 3, is defined as

$$H_n\left(\kappa,\lambda,\mu\right) = \frac{\Gamma\left(\left\{\begin{smallmatrix}m_{1k}\\ m_{2k}\end{smallmatrix}\right\}+\kappa+\lambda+\mu-n\right)}{\kappa\lambda\mu(\Upsilon-s)^{\left\{\begin{smallmatrix}m_{1k}\\ m_{2k}\end{smallmatrix}\right\}+\kappa+\lambda+\mu-n}}. \tag{8}$$

Importantly, after careful inspection of $\mathscr{A}_p$'s and $\mathscr{B}_p$'s for $p = 2,3,\ldots,8$ in (5), it follows that they can all be expressed in terms of well-known generalized hypergeometric functions [14, equation (9.14)]. To this end, for the $\mathscr{A}_p$'s and $\mathscr{B}_p$'s with $p = 2, 3$, and 5, by expressing every term of the form $(x + y)$ as $(x + y)!/(x + y - 1)!$ and using the identity $\Gamma(x + y) = (x)_y\Gamma(y)$, where $(x)_y$ is the Pochhammer symbol [14, page xliii], yields

$$\left\{\begin{matrix}\mathscr{A}_2\\ \mathscr{B}_2\end{matrix}\right\} = \Gamma\left(m_{2n_t}\right)\Gamma\left(\Xi\right)I\left(m_{1n_t},a_{1n_t}\right),$$

$$\left\{\begin{matrix}\mathscr{A}_3\\ \mathscr{B}_3\end{matrix}\right\} = \prod_{\ell=1}^{2}\Gamma\left(m_{\ell n_t}\right)I\left(\Xi,\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}\right), \tag{9}$$

$$\left\{\begin{matrix}\mathscr{A}_5\\ \mathscr{B}_5\end{matrix}\right\} = \Gamma\left(m_{1n_t}\right)\Gamma\left(\Xi\right)I\left(m_{2n_t},a_{2n_t}\right),$$

where $I(\kappa,\lambda)$ is given by

$$I\left(\kappa,\lambda\right) = -\lambda^{\kappa}H_2(\kappa,1,1)\,{}_2F_1\left(\left\{\begin{matrix}m_{1k}\\ m_{2k}\end{matrix}\right\}+\kappa,\kappa;\kappa+1;\frac{\lambda}{s-\Upsilon}\right). \tag{10}$$

In (10), ${}_2F_1(\cdot)$ represents the Gauss hypergeometric function [14, equation (9.14.2)]. Likewise, for the $\mathscr{A}_p$'s and $\mathscr{B}_p$'s with $p = 4, 6$, and 7, each infinite series can be ultimately expressed in terms of the Appell hypergeometric function $F_2(\cdot)$ [14, equation (9.180.2)] as

$$\left\{\begin{matrix}\mathscr{A}_4\\ \mathscr{B}_4\end{matrix}\right\} = \Gamma\left(m_{2n_t}\right)J\left(\Xi,m_{1n_t},\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\},a_{1n_t}\right),$$

$$\left\{\begin{matrix}\mathscr{A}_6\\ \mathscr{B}_6\end{matrix}\right\} = \Gamma\left(\Xi\right)J\left(m_{1n_t},m_{2n_t},a_{1n_t},a_{2n_t}\right), \tag{11}$$

$$\left\{\begin{matrix}\mathscr{A}_7\\ \mathscr{B}_7\end{matrix}\right\} = \Gamma\left(m_{1n_t}\right)J\left(\Xi,m_{2n_t},\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\},a_{2n_t}\right),$$

where $J(\kappa,\lambda,\mu,\nu)$, with $\nu \in \mathbb{R}$, is given by

$$J\left(\kappa,\lambda,\mu,\nu\right) = \mu^{\kappa}\nu^{\lambda}H_1\left(\kappa,\lambda,1\right) \times F_2\left(\left\{\begin{matrix}m_{1k}\\ m_{2k}\end{matrix}\right\}+\kappa+\lambda;\kappa,\lambda;\kappa+1,\lambda+1;\frac{\mu}{s-\Upsilon},\frac{\nu}{s-\Upsilon}\right). \tag{12}$$

Finally, by using once more all aforementioned identities, it follows that the $\mathscr{A}_8$ and $\mathscr{B}_8$ coefficients can be expressed in terms of the generalized Lauricella function $F_A^{(3)}(\cdot)$ [15, equation (1.1)], yielding

$$\left\{\begin{matrix}\mathscr{A}_8\\ \mathscr{B}_8\end{matrix}\right\} = -a_{1n_t}^{\lambda}a_{2n_t}^{\mu}\left\{\begin{matrix}a_{2k}\\ a_{1k}\end{matrix}\right\}^{\Xi}H_0\left(\Xi,\lambda,\mu\right) \times F_A^{(3)}\left(\begin{matrix}\left\{\begin{smallmatrix}m_{1k}\\ m_{2k}\end{smallmatrix}\right\}+\Xi+\lambda+\mu;\Xi,\lambda,\mu;\\ \Xi+1,\lambda+1,\mu+1;\frac{\left\{\begin{smallmatrix}a_{2k}\\ a_{1k}\end{smallmatrix}\right\}}{s-\Upsilon},\frac{a_{1n_t}}{s-\Upsilon},\frac{a_{2n_t}}{s-\Upsilon}\end{matrix}\right). \tag{13}$$

To this effect, substituting (6), (9), (11), and (13) to (5), a novel expression for $M_{\gamma_{\mathrm{RS}}}(s)$ is deduced which is valid for arbitrary $m_{1k}, m_{2k} \geq 0.5$. It is noted that in Appendices A and B we present MATLAB routines for computational efficient implementations of functions $F_2(\cdot)$ and $F_A^3(\cdot)$, respectively.

Using the previously derived $M_{\gamma_{\mathrm{RS}}}(s)$ expression, the MGF of γ_0 and by recalling that γ_{RS} and γ_0 are statistically independent, the MGF of γ_D is straightforwardly deduced, namely,

$$M_{\gamma_D}\left(s\right) = \left(\frac{m_0}{\overline{\gamma}_0}\right)^{m_0}\left(s+\frac{m_0}{\overline{\gamma}_0}\right)^{-m_0}M_{\gamma_{\mathrm{RS}}}\left(s\right). \tag{14}$$

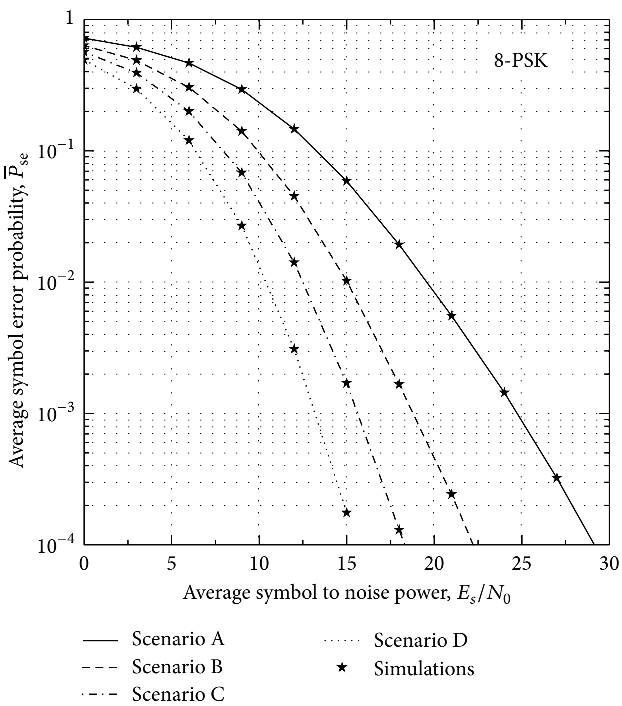

Figure 1: Average symbol error probability, $\overline{P}_{se}$, of 8-PSK of dual-hop DF relaying with RS as a function of the average symbol to noise power, E_s/N_0, for $L = 3$ relays and various Nakagami-m fading conditions.

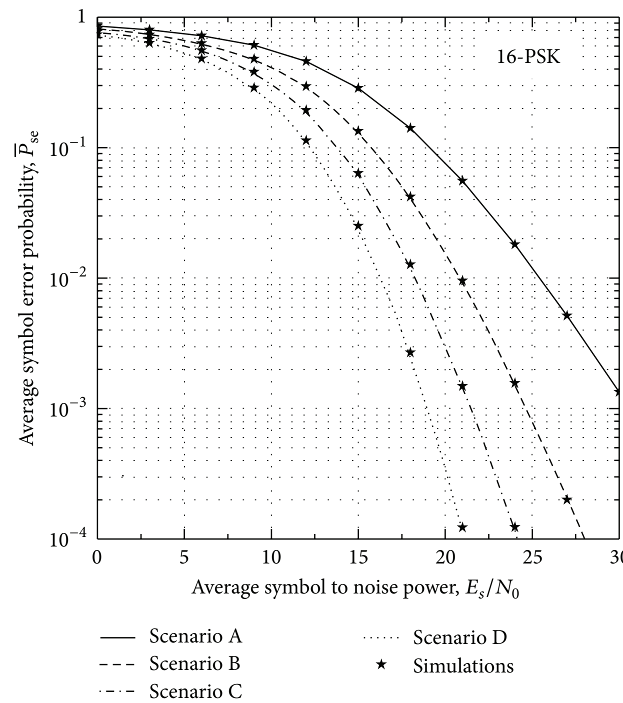

Figure 2: Average symbol error probability, $\overline{P}_{se}$, of 16-PSK of dual-hop DF relaying with RS as a function of the average symbol to noise power, E_s/N_0, for $L = 3$ relays and various Nakagami-m fading conditions.

4. ASEP Performance Evaluation

The ASEP performance of various modulation schemes for the considered dual-hop DF relaying system with RS over Nakagami-m fading channels can be directly evaluated using the $M_{\gamma_D}(s)$ expression and the MGF-based approach presented in [11, Chapter 1]. For example, the ASEP of M-PSK modulation is easily obtained as

$$\overline{P}_{se} = \frac{1}{\pi} \int_0^{\pi-\pi/M} M_{\gamma_D} \left[\frac{\sin^2(\pi/M)}{\sin^2(\theta)} \right] d\theta. \quad (15)$$

Let us assume a relaying system with $L = 3$ relays, no direct link $S \rightarrow D$, and the following four asymmetric fading scenarios: (i) Scenario A: $m_{11} = \Omega_{11} = 0.6$, $m_{12} = \Omega_{12} = 1.1$, $m_{13} = \Omega_{13} = 1.7$, and $m_{2k} = \Omega_{2k} = 0.7$ for $k = 1, 2$, and 3; (ii) Scenario B: $m_{11} = \Omega_{11} = 1.1$, $m_{12} = \Omega_{12} = 2.1$, $m_{13} = \Omega_{13} = 3.1$, and $m_{2k} = \Omega_{2k} = 1.1$ for $k = 1, 2$, and 3; (iii) Scenario C: $m_{11} = \Omega_{11} = 1.1$, $m_{12} = \Omega_{12} = 2.3$, $m_{13} = \Omega_{13} = 3.8$, and $m_{2k} = \Omega_{2k} = 1.8$ for $k = 1, 2$, and 3; and (iv) Scenario D: $m_{11} = \Omega_{11} = 1.6$, $m_{12} = \Omega_{12} = 3.3$, $m_{13} = \Omega_{13} = 4.2$, and $m_{2k} = \Omega_{2k} = 2.6$ for $k = 1, 2$ and 3. The ASEP performance, $\overline{P}_{se}$, of 8-, 16- and 32-PSK modulation is depicted in Figures 1, 2, and 3, respectively, as a function of the average symbol to noise power, E_s/N_0, over various Nakagami-m fading conditions. As expected, $\overline{P}_{se}$ improves with increasing E_s/N_0 and/or decreasing M and/or increasing any of the fading parameters. Furthermore, all figures clearly depict the excellent agreement between the numerically evaluated ASEP results and the equivalent ones obtained from Monte Carlo simulations. In addition, it is observed that the higher is the E_s/N_0, the more sensitive is $\overline{P}_{se}$ to slight variations of the fading conditions. More specifically, for the case of 8-PSK modulation, it can be observed that the differences in the $\overline{P}_{se}$ curves among the four considered fading scenarios for the low E_s/N_0 regime, for example, at 5 dB, are 51% between Scenarios A and B; 52% between Scenarios B and C; and 54% between Scenarios C and D. For the same modulation order, the differences in the high E_s/N_0 regime, for example, at 20 dB, are: 98% between Scenario A and B; 100% between Scenario B and C; and 103% between Scenario C and D. Likewise, for the case of 16-PSK and for $E_s/N_0 = 5$ dB, the differences in the $\overline{P}_{se}$ curves for the different scenarios are 23% between Scenarios A and B; 29% between Scenarios B and C; and 36% between Scenarios C and D. For 16-PSK modulation and for $E_s/N_0 = 20$ dB, the differences among the various $\overline{P}_{se}$ curves are 84% between Scenarios A and B; 90% between Scenarios B and C; and 98% between Scenarios C and D. Finally, for 32-PSK modulation, it is evident that the $\overline{P}_{se}$ curves differ at $E_s/N_0 = 5$ dB: 11% between Scenarios A and B; 14% between Scenarios B and C; 19% between Scenarios C and D whereas, for $E_s/N_0 = 20$ dB, the $\overline{P}_{se}$ differences among scenarios are: 46% between Scenarios A and B; 62% between Scenarios B and C; and 79% between Scenario C and D. It is evident from the above quantitative results that the sensitivity of the ASEP on the fading severity parameter m is high in all

```
Function F2 = Appell(a,b1,b2,c1,c2,x,y);
f1 = gamma(c1).*gamma(c2);
f2 = gamma(b1).*gamma(b2);
f3 = gamma(c1-b1).*gamma(c2-b2);
f = f1./(f2.*f3);
Q = @(u,v)f.*u.^(b1-1).*(v.^(b2-1)).*...
   ((1-u).^(c1-b1-1)).*...
   ((1-v).^(c2-b2-1))./...
   ((1-u.*x-v.*y).^a);
F2 = dblquad(Q,0,1,0,1);
```

Algorithm 1: MATLAB Program for evaluating $F_2(\cdot)$.

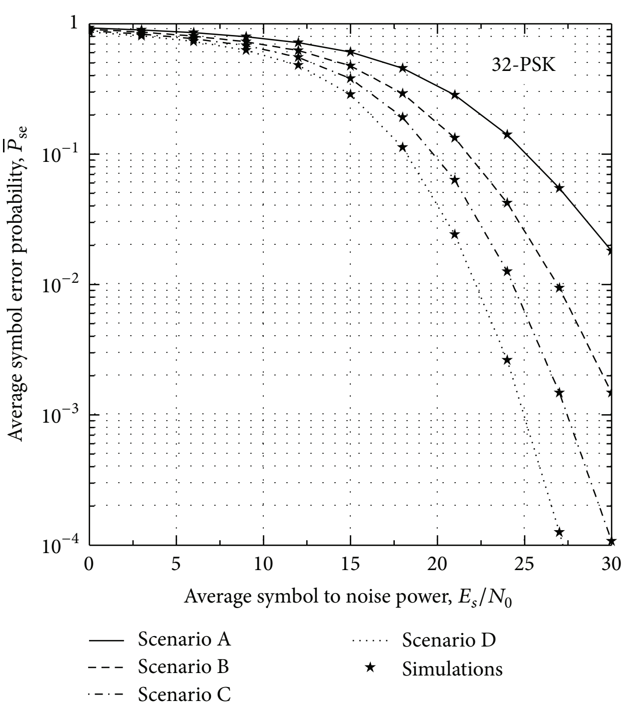

Figure 3: Average symbol error probability, $\overline{P}_{se}$, of 32-PSK of dual-hop DF relaying with RS as a function of the average symbol to noise power, E_s/N_0, for $L = 3$ relays and various Nakagami-m fading conditions.

considered communication scenarios and particularly in the case of modulation schemes with low M and/or high E_s/N_0 values.

5. Conclusion

This work was devoted to the derivation of a new analytical expression for the MGF of the end-to-end SNR of dual-hop DF relaying communication systems with RS over Nakagami-m fading conditions. The presented expression involves well-known generalized hypergeometric functions and is valid for any arbitrary value of the fading parameters of both hops. Using the MGF-based approach, the ASEP of M-PSK modulation for the considered system was evaluated and a perfect match with equivalent computer simulated performance results was shown. More importantly, it was evident that the ASEP is sensitive to even slight variations of any of the hops' fading parameters and particularly for low order modulation schemes and high SNR values.

Appendices

A. MATLAB Program for the Evaluation of $F_2(\cdot)$

The Appell hypergeometric function $F_2(\cdot)$ is defined by the double infinite series given by [14, equation (9.180.2)]. An equivalent expression for $F_2(\cdot)$ in a double integral form with finite limits is given by [14, equation (9.184.2)]. The latter expression can be easily evaluated using the MATLAB function shown in Algorithm 1.

B. MATLAB Program for the Evaluation of $F_A^{(3)}(\cdot)$

The generalized Lauricella function $F_A^{(3)}(\cdot)$ is expressed according to the triple infinite series given by [15, equation (1.1)]. With the aid of the integral representation for $F_2(\cdot)$ in [14, equation (9.184.2)], the following triple integral form for $F_A^{(3)}(\cdot)$ with finite limits is deduced:

$$F_A^{(3)}(a; b_1, b_2, b_3; c_1, c_2, c_3; x, y, z) = \int_0^1 \int_0^1 \int_0^1 f(u, v, w)\, du\, dv\, dw, \tag{B.1}$$

where $a, b_1, b_2, b_3, c_1, c_2, c_3, x, y, z \in \mathbb{R}$ and

$$f(u, v, w) = \frac{u^{b_1-1} v^{b_2-1} w^{b_3-1} (1-u)^{c_1 b_1 - 1}}{\Gamma(b_1)\Gamma(b_2)\Gamma(b_3)(c_1)_{-b_1}(c_2)_{-b_2}} \times \frac{(1-v)^{c_2-b_2-1}(1-w)^{c_3-b_3-1}}{(c_3)_{-b_3}(1-ux-vy-wz)^a}. \tag{B.2}$$

To this effect, the MATLAB function shown in Algorithm 2 evaluates $F_A^{(3)}(\cdot)$ using (B.1).

```
Function FA = Lauricella (a,b1,b2,b3,c1,c2,c3,x,y,z);
f1 = gamma(c1).*gamma(c2).*gamma(c3);
f2 = gamma(b1).*gamma(b2).*gamma(b3);
f3 = gamma(c1-b1).*gamma(c2-b2).*gamma(c3-b3);
f = f1./(f2.*f3);
Q = @(u,v,w)f.*(u.^(b1-1)).*...
    (v.^(b2-1)).*(w.^(b3-1)).*...
    ((1-u).^(c1-b1-1)).*...
    ((1-v).^(c2-b2-1)).*...
    ((1-w).^(c3-b3-1))./...
    ((1-u.*x-v.*y-w.*z).^a);
FA = triplequad(Q,0,1,0,1,0,1);
```

Algorithm 2: MATLAB Program for evaluating $F_A^{(3)}(\cdot)$.

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