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Research Article

Throughput Analysis and Optimization of Relay Selection Techniques for Millimeter Wave Communications

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In this paper, we optimize the throughput of millimeter wave communications using relay selection techniques. We study opportunistic amplify and forward (OAF), opportunistic decode and forward (ODF), and partial and reactive relay selection (PRS and RRS). Our analysis is valid for interference-limited millimeter wave communications. We suggest a new optimal power allocation (OPA) strategy that offers significant performance enhancement with respect to uniform power allocation (UPA). The proposed OPA offers up to 2 dB gain with respect to UPA. Our analysis is confirmed with extensive simulation results for Nakagami fading channels.

1. Introduction

Millimeter wave (mmWave) communications offer high data rates of several Gb/s [1–3]. Millimeter wave communication operates on an important bandwidth from 3 to 300 GHz [1–6]. Cooperation is mandatory in mmWave communications because the mmWave signals cannot penetrate through walls [1–7]. Dual relaying can be used where the signal goes from a source S to relay R_k and then to the destination D. Multihop multibranch relaying allows to extend the coverage of mmWave communications [7–12]. Amplify and forward relays use an adaptive or fixed gain. When the gain is adaptive, these are called nonblind relays. Otherwise, relays with fixed gain are less complex and are called blind relays. AF can be implemented with relay selection techniques such as OAF, PRS, and RRS.

In decode and forward (DF) relaying, each relay decodes the signal and is allowed to transmit only if it has correctly decoded the packet [13–15]. In opportunistic DF (ODF), the chosen relay offers the best SINR of the second hop between relays and destination D [15–18].

Optimal power allocation has not been yet proposed for mmWave communication with an analysis at the packet level. In previous studies, power allocation has been optimized to minimize the symbol or bit error probabilities [19]. The innovations of the paper are as follows:

(i) We derive the throughput and packet error probability (PEP) of millimeter wave communications for OAF, ODF, PRS, and RRS. Previous papers [1–10] studied only the symbol and bit error probabilities. To the best of our knowledge, the PEP has not been yet derived in closed form for mmWave communication with OAF, ODF, PRS, and RRS.

(ii) We propose a new optimal power allocation (OPA) strategy that offers up to 2 dB gains with respect to uniform power allocation (UPA). Our optimization is performed at the packet level.

(iii) Our analysis is valid for quadrature amplitude modulation (QAM), amplitude-shift keying (ASK), and phase-shift keying (PSK) modulations with any number of relays having arbitrary positions.

The paper contains seven sections. The signal to interference plus noise ratio (SINR) is analyzed in Section 2. Section 3 derives the cumulative distribution function (CDF) of SINR of different relay selection techniques. In Section 4, we derive the throughput of mmWave communications. Section 5 suggests an optimal power allocation strategy, while Section 6 gives some simulation and theoretical results. The last section summarizes the obtained results.

2. SINR Statistics

The system model illustrated in Figure 1 contains a source S, M relays R_k, and a destination D. It is assumed that the received signal $k-$th relay R_k is affected by P_{R_k} interferers as shown in Figure 1. In interference-limited millimeter wave communications, the SINR at k-th relay is equal to [9]

$$\Gamma_{SR_k} = \frac{E_S\left|h_{SR_k}\right|^2}{N_0 + I_{R_k}}, \quad (1)$$

where E_S is the transmitted energy per symbol of the source S, h_{SR_k} is the channel coefficient between S and R_k, N_0 is the power spectral density (PSD) of additive white Gaussian noise (AWGN), and I_{R_k} is the interference term at R_k written as follows [9]:

$$I_{R_k} = \sum_{p=1}^{P_{R_k}} E_p\left|g_{p,R_k}\right|^2, \quad (2)$$

where P_{R_k} is the number of interferers at R_k, E_p is the transmitted energy per symbol of p-th interferer, and g_{p,R_k} is the channel coefficient between p-th interferer and relay R_k.

We derive the statistics of an upper bound of SINR:

$$\Gamma_{SR_k} = \frac{E_S\left|h_{SR_k}\right|^2}{N_0 + I_{R_k}} < \Gamma_{SR_k}^{\text{up}} = \frac{E_S\left|h_{SR_k}\right|^2}{I_{R_k}}. \quad (3)$$

For Nakagami fading channels, $U_k = E_S|h_{SR_k}|^2$ follows a gamma distribution $G(\alpha_{1,k}, \beta_{1,k})$ defined as

$$f_{U_k}(x) = \frac{x^{\alpha_{1,k}} e^{-(x/\beta_{1,k})}}{\Gamma(\alpha_{1,k})\beta_{1,k}^{\alpha_{1,k}}}, \quad (4)$$

where $\alpha_{1,k} > 0.5$ is the m-fading figure of $S - R_k$ link ($\alpha_{1,k} = 1$ corresponds to Rayleigh channels),

$$\beta_{1,k} = \frac{E_S E\left(\left|h_{SR_k}\right|^2\right)}{\alpha_{1,k}} = \frac{\overline{Y_k}}{\alpha_{1,k}}. \quad (5)$$

The interference term is expressed as

$$I_{R_k} = \sum_{p=1}^{P_{R_k}} E_p\left|g_{p,R_k}\right|^2 = \sum_{p=1}^{P_{R_k}} I_{p,k}, \quad (6)$$

where $I_{p,k} = E_p|g_{p,R_k}|^2$.

It is assumed that I_{R_k} is the sum of independent and identically distributed (i.i.d) gamma random variables (r.v) $I_{p,k}$ that follows a $G(a_k, b_k)$ where $a_k > 0.5$ and

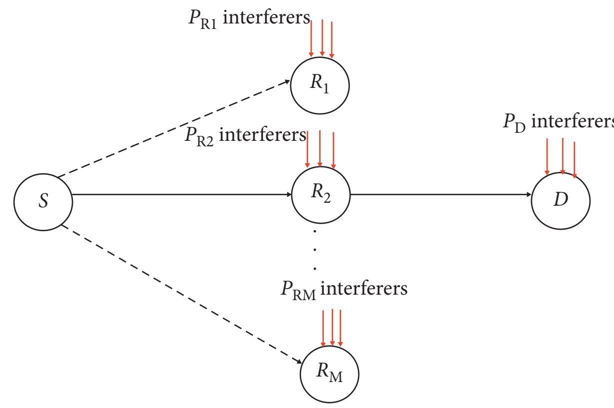

Figure 1: Network model.

$$b_k = \frac{E\left(I_{p,k}\right)}{a_k}. \quad (7)$$

The sum of P_{R_k} i.i.d gamma r.v. $I_{p,k}$ is a gamma r.v. $G(P_{R_k}a_k, b_k)$. Therefore, $\Gamma_{SR_k}^{\text{up}}$ in (3) is the quotient of two gamma r.v. that follows a general prime distribution with PDF [20]:

$$f_{\Gamma_{SR_k}^{\text{up}}}(x) = \frac{x^{\alpha_{1,k}-1}}{w_k^{\alpha_{1,k}} B\left(\alpha_{1,k}, A_k\right)}\left(1 + \frac{x}{w_k}\right)^{-B_k}, \quad (8)$$

where

$$B\left(\alpha_{1,k}, A_k\right) = \frac{\Gamma\left(\alpha_{1,k}\right)\Gamma\left(A_k\right)}{\Gamma\left(\alpha_{1,k} + A_k\right)}, \quad (9)$$

$$A_k = P_{R_k}a_k, \quad (10)$$

$$B_k = A_k + \alpha_{1,k}, \quad (11)$$

$$w_k = \frac{\beta_{1,k}}{b_k}. \quad (12)$$

The CDF of SINR Γ_{SR_k} is obtained by a simple primitive of PDF. We use the following result [21]:

$${}_2F_1(a, b; c; z) = \frac{1}{B(b, c-b)}\int_0^1 x^{b-1}(1-x)^{c-b-1}(1-zx)^{-a}dx, \quad (13)$$

where $b > 0$, $c > 0$, and ${}_2F_1(a, b; c; z)$ is the hypergeometric function.

We use (13) to express the CDF of SINR as

$$F_{\Gamma_{SR_k}^{up}}(x) = \frac{x^{\alpha_{1,k}}}{B\left(\alpha_{1,k}, A_k\right)w_k^{\alpha_{1,k}}}\,{}_2F_1\left(B_k, \alpha_{1,k}; \alpha_{1,k}+1, \frac{-x}{w_k}\right). \quad (14)$$

Using (8), the asymptotic PDF is expressed as

$$f_{\Gamma_{SR_k}^{\text{up}}}(x) \approx \frac{x^{\alpha_{1,k}-1}}{B\left(\alpha_{1,k}, A_k\right)w_k^{\alpha_{1,k}}}. \quad (15)$$

By a primitive, we deduce the asymptotic CDF expressed as

$$F_{\Gamma^{\text{up}}_{SR_k}}(x) \approx \frac{x^{\alpha_{1,k}}}{\alpha_{1,k} B\left(\alpha_{1,k}, A_k\right) w_k^{\alpha_{1,k}}}. \quad (16)$$

These asymptotic expressions will be used to derive the optimal power allocation (OPA) strategy for cooperative mmWave communications.

It is assumed that the interference term at D is the sum of P_D i.i.d gamma r.v. with distribution $G(c, d)$. The CDF of SINR of the second hop between R_k and D is written similarly to (8)

$$f_{\Gamma^{\text{up}}_{R_kD}}(x) = \frac{x^{\alpha_{2,k}-1}}{u_k^{\alpha_{2,k}} B\left(\alpha_{2,k}, E\right)}\left(1 + \frac{x}{u_k}\right)^{-D_k}, \quad (17)$$

where $\alpha_{2,k} > 0.5$ is the m-fading figure of $R_k - D$ link,

$$E = P_D c, \quad (18)$$

$$D_k = E + \alpha_{2,k}, \quad (19)$$

$$u_k = \frac{\beta_{2,k}}{d}, \quad (20)$$

$$\beta_{2,k} = \frac{E_{R_k} E\left(\left|h_{R_kD}\right|^2\right)}{\alpha_{2,k}} = \frac{\overline{Z}_k}{\alpha_{2,k}}. \quad (21)$$

3. Performance Analysis of Relay Selection Techniques

In this section, we assume that a single relay is selected. There is no interference between the signals transmitted by different relays as a single relay is selected to amplify or decode the source packet. Therefore, our analysis is valid for any number of relays.

3.1. Opportunistic AF Relaying. For AF relaying, the SINR between the source S, relay R_k, and destination D is expressed as [22]

$$\Gamma_{SR_kD} = \frac{\Gamma_{SR_k}\Gamma_{R_kD}}{\Gamma_{SR_k} + \Gamma_{R_kD} + 1}. \quad (22)$$

The SINR can be tightly upper bounded by [22]

$$\Gamma_{SR_kD} < \Gamma^{\text{up}}_{SR_kD} = \min\left(\Gamma_{SR_k}, \Gamma_{R_kD}\right). \quad (23)$$

The CDF of SINR is expressed as

$$F_{\Gamma_{SR_kD}}(x) > F_{\Gamma^{\text{up}}_{SR_kD}}(x), \quad (24)$$

where

$$F_{\Gamma^{\text{up}}_{SR_kD}}(x) = 1 - P\left(\min\left(\Gamma_{SR_k}, \Gamma_{R_kD}\right) > x\right). \quad (25)$$

Assuming that Γ_{SR_k} and Γ_{R_kD} are independent, we have

$$F_{\Gamma^{\text{up}}_{SR_kD}}(x) = 1 - \left[1 - F_{\Gamma_{SR_k}}(x)\right]\left[1 - F_{\Gamma_{R_kD}}(x)\right]. \quad (26)$$

In OAF, the chosen relay R_{sel} offers the best end-to-end SINR

$$\Gamma_{SR_{\text{sel}}D} = \max_{k\in\{1,\ldots,M\}} \Gamma_{SR_kD} < \Gamma^{\text{up}}_{SR_{\text{sel}}D} = \max_{k\in\{1,\ldots,M\}} \Gamma^{\text{up}}_{SR_kD}. \quad (27)$$

Using (27) and assuming that the SINRs for different relays are independent, the CDF of SINR is expressed as

$$F_{\Gamma_{SR_{\text{sel}}D}}(x) > F_{\Gamma^{\text{up}}_{SR_{\text{sel}}D}}(x) = \prod_{n=1}^{M} F_{\Gamma^{\text{up}}_{SR_nD}}(x) = \prod_{n=1}^{M} \cdot \left[1 - \left(1 - F_{\Gamma_{SR_n}}(x)\right)\left(1 - F_{\Gamma_{R_nD}}(x)\right)\right]. \quad (28)$$

3.2. Partial Relay Selection. In PRS, the selected relay offers the best SINR of the first hop. Let p_k be the probability that relay R_k is selected and Γ_k the corresponding SINR is expressed as

$$\Gamma_k = \frac{\Gamma^{\max}\Gamma_{R_kD}}{\Gamma^{\max} + \Gamma_{R_kD} + 1}, \quad (29)$$

where $\Gamma^{\max}$ is the SINR at relay R_k which is the highest SINR between S and relays,

$$\Gamma^{\max} = \max_{j\in\{1,\ldots,M\}} \Gamma_{SR_j}. \quad (30)$$

The SINR Γ_k can be tightly upper bounded by [22]

$$\Gamma_k < \Gamma^{\text{up}}_k = \min\left(\Gamma^{\max}, \Gamma_{R_kD}\right). \quad (31)$$

If $\Gamma^{\max}$ and Γ_{R_kD} are independent, we have

$$F_{\Gamma_k}(x) > F_{\Gamma^{up}_k}(x) = 1 - P\left(\Gamma_{R_kD} > x\right) P\left(\Gamma^{\max} > x\right). \quad (32)$$

We have

$$P\left(\Gamma^{\max} > x\right) = 1 - P\left(\Gamma^{\max} \le x\right) = 1 - \prod_{j=1}^{M} F_{\Gamma_{SR_j}}(x). \quad (33)$$

Using (32) and (33), we have

$$F_{\Gamma_k}(x) > F_{\Gamma^{\text{up}}_k}(x) = 1 - \left[1 - \prod_{j=1}^{M} F_{\Gamma_{SR_j}}(x)\right]\left[1 - F_{\Gamma_{R_kD}}(x)\right]. \quad (34)$$

The CDF of SINR is expressed as

$$F_{\Gamma_{SR_{\text{sel}}D}}(x) = \sum_{k=1}^{M} p_k F_{\Gamma_k}(x). \quad (35)$$

The probability to select relay R_k is expressed as follows:

$$p_k = P\left(\Gamma_{SR_k} > \max_{j\neq k} \Gamma_{SR_j}\right). \quad (36)$$

Let $X = \max_{j\neq k} \Gamma_{SR_j}$, we can write

$$p_k = \int_0^{+\infty} f_X(x) P\left(\Gamma_{SR_k} > x\right) dx = \int_0^{+\infty} f_X(x)\left[1 - F_{\Gamma_{SR_k}}(x)\right] dx, \quad (37)$$

where $f_X(x)$ is the PDF of X.

Assuming that the SINRs of the first hop are independent, the CDF of X is written as

$$F_X(x) = \prod_{q=1, q\neq k}^{M} F_{\Gamma_{SR_q}}(x). \quad (38)$$

We have

$$f_X(x) = \sum_{q=1, q\neq k}^{M} f_{\Gamma_{SR_q}}(x) \prod_{m=1, m\neq k, m\neq q}^{M} F_{\Gamma_{SR_m}}(x). \quad (39)$$

For PRS, the CDF of SINR at D is written as (35) with p_k given in (37).

3.3. Reactive Relay Selection. In RRS, the chosen relay offers the best SINR between relays and destination. Let r_k be the probability that relay R_k is selected and Γ_k the corresponding SINR is expressed as

$$\Gamma_k = \frac{\Gamma_{SR_k}\Gamma^{\max}}{\Gamma^{\max} + \Gamma_{SR_k} + 1}, \quad (40)$$

where $\Gamma^{\max}$ is the highest SINR of the second hop

$$\Gamma^{\max} = \max_{j\in\{1,\dots,M\}} \Gamma_{R_j D}. \quad (41)$$

The SINR Γ_k can be tightly upper bounded by [22]

$$\Gamma_k < \Gamma_k^{\text{up}} = \min\left(\Gamma^{\max}, \Gamma_{SR_k}\right). \quad (42)$$

If $\Gamma^{\max}$ and Γ_{SR_k} are independent, we have

$$F_{\Gamma_k}(x) > F_{\Gamma_k^{\text{up}}}(x) = 1 - P\left(\Gamma_{SR_k} > x\right) P\left(\Gamma^{\max} > x\right). \quad (43)$$

We have

$$P\left(\Gamma^{\max} > x\right) = 1 - \prod_{j=1}^{M} F_{\Gamma_{R_j D}}(x). \quad (44)$$

Using (43) and (44), we have

$$F_{\Gamma_k}(x) > F_{\Gamma_k^{\text{up}}}(x) = 1 - \left[1 - \prod_{j=1}^{M} F_{\Gamma_{R_j D}}(x)\right]\left[1 - F_{\Gamma_{SR_k}}(x)\right]. \quad (45)$$

The CDF of SINR is expressed as

$$F_{\Gamma_{SR_{\text{sel}}D}}(x) = \sum_{k=1}^{M} r_k F_{\Gamma_k}(x). \quad (46)$$

The probability to select relay R_k is expressed as follows:

$$r_k = P\left(\Gamma_{R_k D} > \max_{j\neq k} \Gamma_{R_j D}\right). \quad (47)$$

Let $Y = \max_{j\neq k} \Gamma_{R_j D}$, we can write

$$r_k = \int_0^{+\infty} f_Y(y) P\left(\Gamma_{R_k D} > y\right) dy = \int_0^{+\infty} f_Y(y)\left[1 - F_{\Gamma_{R_k D}}(y)\right] dy, \quad (48)$$

where $f_Y(y)$ is the PDF of Y.

Assuming that the SINRs of the second hop are independent, the CDF of Y is written as

$$F_Y(y) = \prod_{q=1, q\neq k}^{M} F_{\Gamma_{R_q D}}(y). \quad (49)$$

We have

$$f_Y(y) = \sum_{q=1, q\neq k}^{M} f_{\Gamma_{R_q D}}(y) \prod_{m=1, m\neq k, m\neq q}^{M} F_{\Gamma_{R_m D}}(y). \quad (50)$$

For RRS, the CDF of SINR at D is written as (46) with r_k given in (48).

3.4. Performance Analysis of Opportunistic DF. In ODF, we activate the relay with largest SNR of the second hop. This relay should correctly decode S packet. The PEP is written as

$$\text{PEP} = \sum_{v\in\{1,\dots,M\}} P(v)\,\text{PEP}(v), \quad (51)$$

where

$$P(v) = \prod_{n\in v} \left(1 - \text{PEP}_n\right) \prod_{q\notin v} \text{PEP}_q, \quad (52)$$

where PEP_q is the PEP at relay R_q.

$\text{PEP}(v)$ is the PEP when v is the set of relays having correctly decoded expressed as [23]

$$\text{PEP}(v) < \prod_{k\in v} P_{\Gamma_{R_k D}}(T_0), \quad (53)$$

where T_0 is a waterfall threshold

$$T_0 = \int_0^{+\infty} t(x) dx, \quad (54)$$

where $t(x)$ is the PEP for SINR x. It is detailed in the next section for different modulations.

4. Throughput of Cooperative mmWave Communications

The PEP is expressed as

$$P_{\text{bloc}} = \int_0^{+\infty} f_\Gamma(x) t(x) dx, \quad (55)$$

where $f_\Gamma(x)$ is the PDF of SINR Γ and $t(x)$ is the PEP for SINR x.

For K-QAM, we have

$$t(x) = 1 - \left[1 - 2\left(1 - \frac{1}{\sqrt{K}}\right)\text{erfc}\left(\sqrt{x\frac{3\log_2(K)}{2(K-1)}}\right)\right]^N, \quad (56)$$

where N is packet length in symbols and K is the constellation size.

For K-ASK, we have

$$t(x) = 1 - \left[1 - \left(1 - \frac{1}{K}\right)\operatorname{erfc}\left(\sqrt{x\frac{3\log_2(K)}{(K^2-1)}}\right)\right]^N. \quad (57)$$

For K-PSK modulation, we have

$$t(x) = 1 - \left[1 - \operatorname{erfc}\left(\sqrt{x\log_2(K)\sin^2\left(\frac{\pi}{K}\right)}\right)\right]^N. \quad (58)$$

The PEP can be upper bounded by [23]

$$\text{PEP} \le \int_0^{T_0} f_\Gamma(x)dx = F_\Gamma(T_0), \quad (59)$$

where $F_\Gamma(x)$ is the CDF of SINR and T_0 is a waterfall threshold [23]:

$$T_0 = \int_0^{+\infty} t(x)dx. \quad (60)$$

In order to compute the PEP, we have to only derive the CDF of SINR and use (51). The throughput is written as

$$\text{Thr} = \frac{\log_2(K)}{2}(1 - \text{PEP}), \quad (61)$$

where coefficient 0.5 is due to the fact that half the frame duration is used for transmission by the source and the other half by the selected relay.

The throughput of OAF, PRS, and RRS are given in (61) where the PEP is deduced from CDF of SINR as (59). The CDF of SINR of OAF, PRS, and RRS are respectively given in (28), (35), and (46). The throughput of ODF is given in (61) with PEP expressed as (51).

5. Optimal Power Allocation

For each candidate relay, we suggest an optimal power allocation that minimizes the asymptotic packet error probability expressed as

$$F_{\Gamma^{\text{up}}_{SR_kD}}(T_0) \simeq F_{\Gamma_{SR_k}}(T_0) + F_{\Gamma_{R_kD}}(T_0) \approx \frac{T_0^{\alpha_{1,k}}}{\alpha_{1,k}B(\alpha_{1,k}, A_k)w_k^{\alpha_{1,k}}} + \frac{T_0^{\alpha_{2,k}}}{\alpha_{2,k}B(\alpha_{2,k}, E)u_k^{\alpha_{2,k}}}. \quad (62)$$

Using the definitions of u_k (20) and w_k (12), we have

$$F_{\Gamma^{\text{up}}_{SR_kD}}(x) \approx \frac{T_0^{\alpha_{1,k}} b_k^{\alpha_{1,k}} \alpha_{1,k}^{\alpha_{1,k}}}{\alpha_{1,k}B(\alpha_{1,k}, A_k)\overline{Y}_k^{\alpha_{1,k}}} + \frac{T_0^{\alpha_{2,k}} d^{\alpha_{2,k}} \alpha_{2,k}^{\alpha_{2,k}}}{\alpha_{2,k}B(\alpha_{2,k}, E)\overline{Z}_k^{\alpha_{2,k}}}. \quad (63)$$

Let $a_S = (E_S/E_s)$ be the fraction of power allocated to the source. E_S is the transmitted energy per symbol of S and $E_s = E_S + E_{R_k}$ is the transmitted energy per symbol. E_{R_k} is the transmitted energy per symbol of R_k. Similarly, $a_{R_k} = (E_{R_k}/E_s)$ is the fraction of power allocated to R_k. We deduce

$$\overline{Y}_k = a_S E_s E\left(\left|h_{SR_k}\right|^2\right), \quad \overline{Z}_k = a_{R_k} E_s E\left(\left|h_{R_kD}\right|^2\right). \quad (64)$$

Therefore, the outage probability (63) to be minimized is expressed as

$$F_{\Gamma^{\text{up}}_{SR_kD}}(x) \approx \frac{\lambda_k}{a_S^{\alpha_{1,k}}} + \frac{\mu_k}{a_{R_k}^{\alpha_{2,k}}}, \quad (65)$$

under the constraint that $a_S + a_{R_k} = 1$ and where

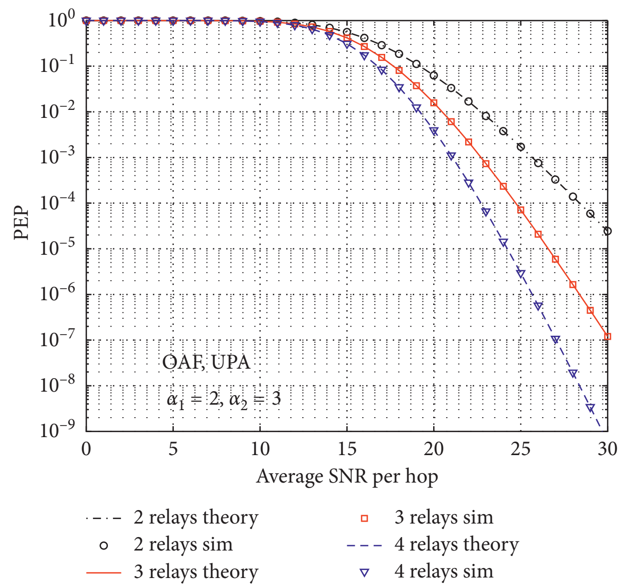

Figure 2: PEP of OAF for Nakagami fading channels.

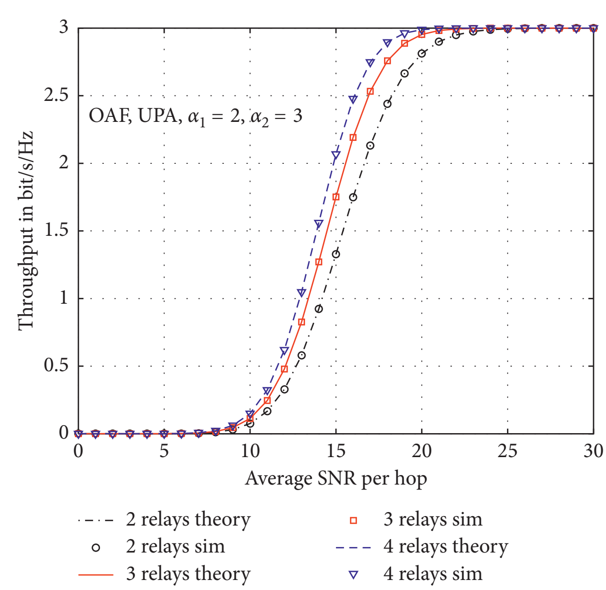

Figure 3: Throughput of OAF for Nakagami fading channels.

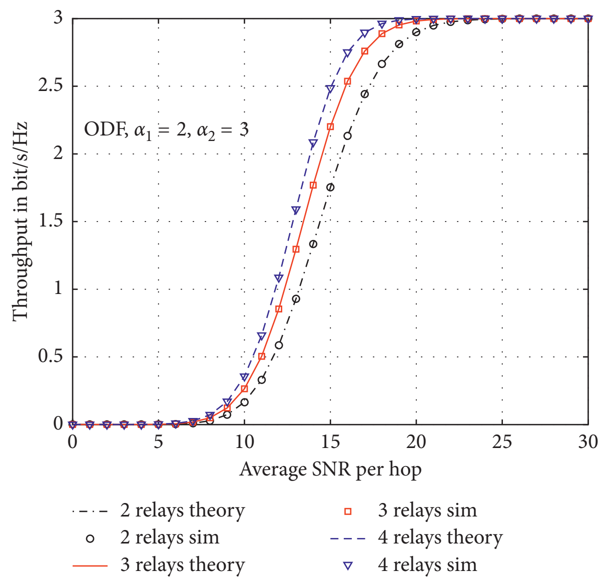

Figure 4: Throughput of ODF for Nakagami fading channels.

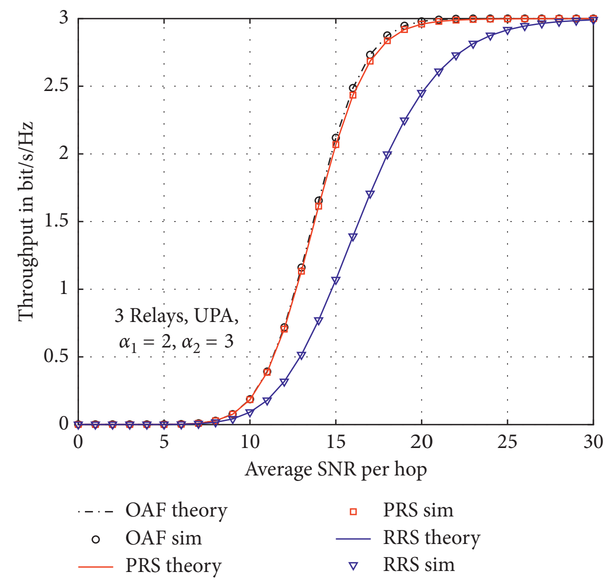

Figure 6: Throughput of OAF, PRS, and RRS for Nakagami fading channels: relays close to the destination.

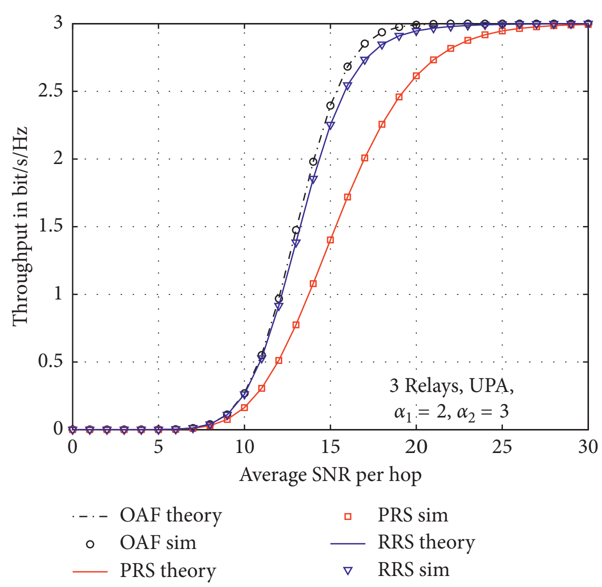

Figure 5: Throughput of OAF, PRS, and RRS for Nakagami fading channels: relays close to the source.

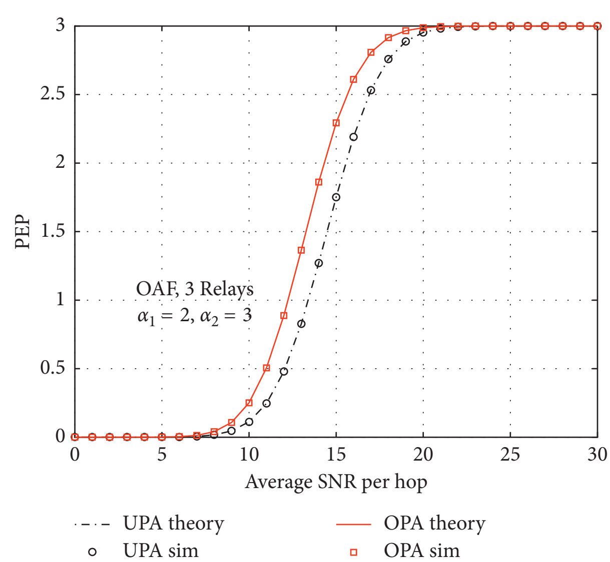

Figure 7: Throughput of OAF with UPA and OPA for Nakagami fading channels.

$$\lambda_k = \frac{T_0^{\alpha_{1,k}} b_k^{\alpha_{1,k}} \alpha_{1,k}^{\alpha_{1,k}-1}}{B\left(\alpha_{1,k}, A_k\right) E_s^{\alpha_{1,k}} \left[E\left(\left|h_{SR_k}\right|\right)\right]^{\alpha_{1,k}}},$$
$$\mu_k = \frac{T_0^{\alpha_{2,k}} d^{\alpha_{2,k}} \alpha_{2,k}^{\alpha_{2,k}-1}}{B\left(\alpha_{2,k}, E\right) E_s^{\alpha_{2,k}} \left[E\left(\left|h_{R_kD}\right|\right)\right]^{\alpha_{2,k}}}. \quad (66)$$

The Lagrangian of the problem is expressed as

$$J = \frac{\lambda_k}{a_S^{\alpha_{1,k}}} + \frac{\mu_k}{a_{R_k}^{\alpha_{2,k}}} + \rho\left(a_S + a_{R_k} - 1\right). \quad (67)$$

The derivative of J is expressed as

$$\frac{\partial J}{\partial a_S} = \frac{-\lambda_k}{a_S^{\alpha_{1,k}+1}} + \rho = 0, \quad (68)$$

$$\frac{\partial J}{\partial a_{R_k}} = \frac{-\mu_k}{a_{R_k}^{\alpha_{2,k}+1}} + \rho = 0. \quad (69)$$

When the fading figure of $S - R_k$ link is the same as that of $R_k - D$, i.e., $\alpha_{1,k} = \alpha_{2,k} = \alpha_k$, we obtain the OPA strategy

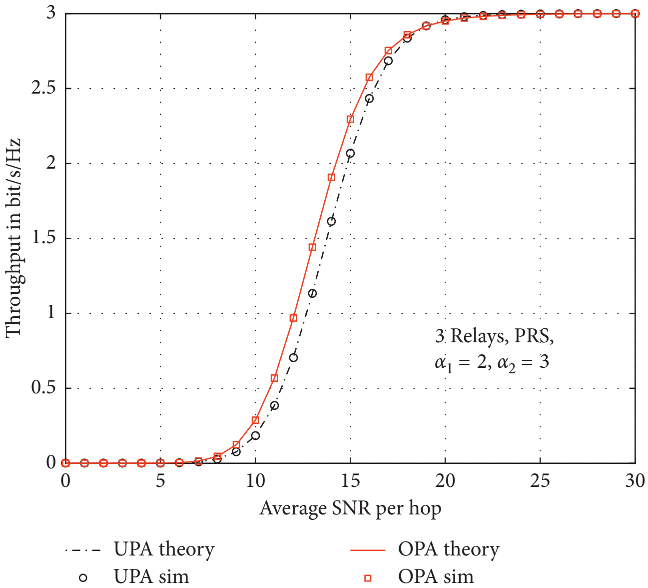

Figure 8: Throughput of PRS with UPA and OPA for Nakagami fading channels.

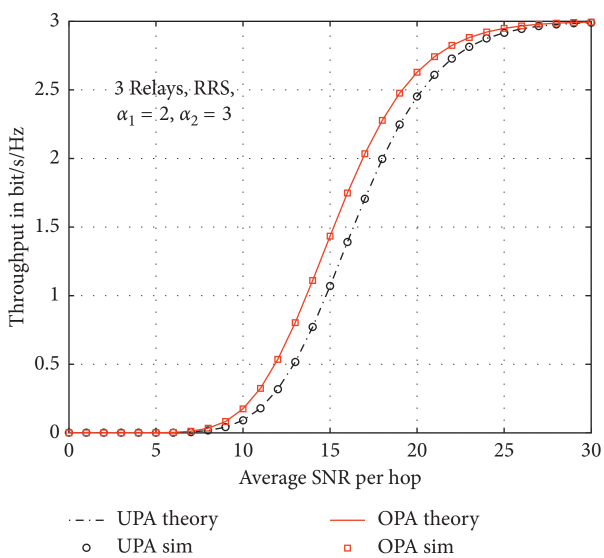

Figure 9: Throughput of RRS with UPA and OPA for Nakagami fading channels.

$$a_S = \frac{\lambda_k^{(1/\alpha_k+1)}}{\mu_k^{(1/\alpha_k+1)} + \lambda_k^{(1/\alpha_k+1)}},$$
$$a_{R_k} = \frac{\mu_k^{(1/\alpha_k+1)}}{\mu_k^{(1/\alpha_k+1)} + \lambda_k^{(1/\alpha_k+1)}}. \quad (70)$$

If $\alpha_{1,k} \neq \alpha_{2,k}$, we deduce from (68) and (69) that we have to solve with the Newton algorithm the following equation:

$$a_S + a_S^{((\alpha_{1,k}+1)/(\alpha_{2,k}+1))} \left(\frac{\mu_k \alpha_{2,k}}{\lambda_k \alpha_{1,k}}\right)^{(1/(\alpha_{2,k}+1))} = 1. \quad (71)$$

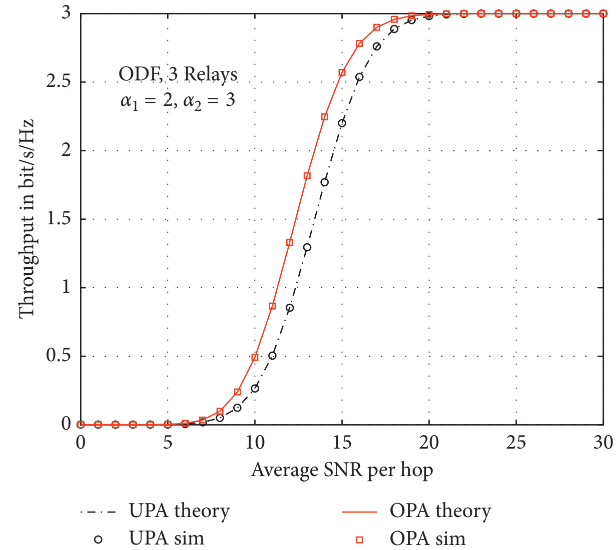

Figure 10: UPA and OPA for ODF: Nakagami fading channels.

We deduce the fraction of power allocated to relay $a_{R_k} = 1 - a_S$.

6. Theoretical and Simulation Results

We plot theoretical curves with MATLAB and made some simulations for 64-QAM modulation. We have varied the number of relays $M = 2, 3, 4$. The path loss exponent is equal to 3. The results are valid for Nakagami channels with $\alpha_{1,k} = 2$ and $\alpha_{2,k} = 2$. The fading figure of interference terms is $a_k = 2$.

Figures 2 and 3 show the PEP and throughput of OAF for different number of relays M. There are two interferers. The distance between S and R_k is $d_{SR_k} = 0.3$. The distance between R_k and D is $d_{R_kD} = 1 - d_{SR_k} = 0.7$. We notice that the throughput improves as M increases due to cooperative diversity. A good accordance between theoretical and simulation results is observed.

Figure 4 shows the throughput of ODF in the same context as Figures 2 and 3. We notice a good accordance between theoretical and simulation results. Also, the throughput improves as the number of relays is increased.

Figure 5 compares the throughput of OAF, PRS, and RRS for $d_{R_kD} = 1 - d_{SR_k} = 0.7$. There are three relays and two interferers. We observe that OAF offers the highest throughput. RRS offers better performance than PRS because the relays are close to S. Figure 6 shows that PRS offers a higher throughput than RRS when relays are close to D, $d_{R_kD} = 1 - d_{SR_k} = 0.2$. OAF offers the highest throughput since it selects the relay with the largest end-end SINR.

Figures 7–10 show the throughput of OAF, PRS, RRS, and ODF for optimal or uniform power allocation (OPA or UPA) with two interferers and three relays. Figures 7–10 show that the proposed OPA allows up to 2 dB gain with respect to UPA.

7. Conclusion

In this paper, we have optimized the throughput of millimeter wave communications using OAF, ODF, PRS, and RRS. Our analysis is valid for interference-limited millimeter wave communications with any number of relays with arbitrary positions. The proposed optimal power allocation (OPA) strategy offers up to 2 dB gain with respect to uniform power allocation (UPA). Our analysis was confirmed with extensive simulation results in the presence of Rayleigh fading channels. The main contribution of our paper is to suggest an optimal power allocation to enhance the throughput at the packet level, while previous studies perform the optimization to minimize bit or symbol error probabilities [19]. Besides, our results showed that OAF offers higher throughput than PRS and RRS as it selects the relay with highest SINR at destination, while PRS and RRS use only the SINR of the first and second hops.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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