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Research Article

A Simplified Network Model for Travel Time Reliability Analysis in a Road Network

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This paper proposes a simplified network model which analyzes travel time reliability in a road network. A risk-averse driver is assumed in the simplified model. The risk-averse driver chooses a path by taking into account both a path travel time variance and a mean path travel time. The uncertainty addressed in this model is that of traffic flows (i.e., stochastic demand flows). In the simplified network model, the path travel time variance is not calculated by considering all travel time covariance between two links in the network. The path travel time variance is calculated by considering all travel time covariance between two adjacent links in the network. Numerical experiments are carried out to illustrate the applicability and validity of the proposed model. The experiments introduce the path choice behavior of a risk-neutral driver and several types of risk-averse drivers. It is shown that the mean link flows calculated by introducing the risk-neutral driver differ as a whole from those calculated by introducing several types of risk-averse drivers. It is also shown that the mean link flows calculated by the simplified network model are almost the same as the flows calculated by using the exact path travel time variance.

1. Introduction

Conventional frameworks for analyzing and modeling transportation systems have been confined to average representations of the network state (e.g., average link flow or average travel flow). For instance, in the traditional traffic assignment model, one can obtain a deterministic prediction of a future flow on a certain link in the network based on average origin-destination (O-D) flows, link capacities, and a form of proportional path choice model (either deterministic user equilibrium (DUE) or stochastic user equilibrium (SUE)). This represents a deterministic view of the environment and the modeler's postulation that the variability or uncertainty in the system is not influential in system design and evaluation.

There has been a growing concern over the uncertainty of travel time in transport systems and its effect on the reliability of transport services [1]. Research on network reliability has begun to address this problem [2–5]. From the traveler's perspective, the issue of travel time reliability has been a major concern. Travelers may experience excessive variability of travel time from day to day on the same trips [6–8].

In transport modeling, some advances have been made toward incorporating traffic flow uncertainties into the network modeling framework (i.e., developing a stochastic network model). Watling [9] proposed a second-order network equilibrium model that explicitly considers random path choice behavior. His model, in contrast to the conventional SUE model, uses path choice probability, as predicted by a SUE model, to define stochastic path flows that follow a multinomial distribution. The path flows derived using the traditional SUE model are, in fact, the expected flows of this multinomial distribution. In this model, the drivers choose their paths so as to minimize their perceived long-run expected travel costs. With a nonlinear travel cost function, this long-run expected travel cost will differ from the equilibrium cost computed by the conventional SUE model [10].

Clark and Watling [6] extended this stochastic network model to the case with a Poisson distribution of O-D flows. Similarly, Nakayama and Takayama [11] proposed a stochastic network model with random path choice behavior but using a binomial distribution of O-D flows. These models fully represent stochastic/uncertain path choice behavior with uncertain flows. Lo et al. [12] extended their original model to consider the concept of travel time budget in path choice decision-making. Shao et al. [13] adopted a similar postulation of the central limit theorem to derive the normal distribution of the path travel time but with O-D flow distribution (normal distribution). Szeto and Solayappan [14] proposed a nonlinear complementarity problem formulation for the risk-aversive stochastic transit assignment problem. Sumalee et al. [15] address stochastic flow and stochastic capacity in a multimodal network. Uchida et al. [16] address network design problem in a stochastic multimodal network model. Uchida [17] proposed a model which simultaneously estimates the value of travel time and of travel time reliability based on the risk-averse driver's path choice behavior. Uchida [18] proposed a network equilibrium model which estimates travel time reliability from the observed link flows in the network. Kato and Uchida [19] proposed a benefit estimation method that considers travel time reliability.

In the context of the advancements in theoretical studies on travel time variability in road networks, transportation benefit-cost analysis (BCA) considering travel time variability (https://sites.google.com/site/benefitcostanalysis/benefits/travel-time-reliability; [20]) is now becoming a big concern. The traditional DUE traffic assignment model has been widely used to estimate the value or benefits of a policy, program, or project considering no travel time variability. If we consider the value of travel time variability in estimating the benefit of a policy, the DUE traffic assignment model cannot be applied, since it does not address the stochastic nature of a travel time variability. Therefore, a plausible network equilibrium model which is well established in terms of theory and practice is needed. For BCA in which travel time variability is considered, a measure of travel time variability, which is discussed in the next section, needs to be determined. Once the travel time variability measure is determined, then a network equilibrium model which combines risk-averse driver's path choice behavior with the generalized travel time, which is defined as a mean travel time plus a travel time variability measure multiplied by a calibration parameter, is developed. If we put more weight on the accuracy than the practicality of a network equilibrium model, then the validity of BCA may increase; however, the costs required for calculating BCA may increase, and vice versa. Therefore, the modeler has to consider the trade-off between accuracy and practicality. The objective of this study is to propose a simplified and plausible network equilibrium model which takes into account both the risk-averse driver's path choice behavior and the travel time variability. The difference between this study and the other studies that the authors have presented is that we propose a model that can be applied to a large network problem. The stochastic network models that the authors have developed require path enumeration in the network. However, the enumeration of all possible paths is difficult in the case of a large network. Therefore, a stochastic network model for travel time reliability analysis that solves a large network problem is demanded.

This paper starts by examining several measures of travel time variability in the next section. Based on the discussion provided in the next section, we will employ a travel time variance as a measure of travel time variability in this study. Then, link and path travel time under stochastic demand flows are formulated in Section 3. In Section 4, two network equilibrium models under stochastic demand flows are formulated considering a risk-averse driver's path choice behavior in a road network. The first model introduces path travel time variance which is calculated considering all travel time covariances between two links in the network. The generalized travel time in this model is not additive since the generalized path travel time is not equal to the sum of the generalized link travel times related to that path. The second model, which we propose in this study, is a simplified version of the first model. In this model, the path travel time variance is not calculated by considering all travel time covariance between two links in the network. The path travel time variance is calculated by considering all travel time covariance between two adjacent links in the network. The generalized path travel time in this model is additive. It is shown that a unique solution is provided by the simplified network model. Numerical experiments are carried out to illustrate the applicability and validity of the proposed model. Finally, concluding remarks are provided in Section 6.

2. Measures of Travel Time Variability

We will briefly review how to obtain a measure of travel time variability based on the expected utility maximization principle. Vickrey [21] considered a separable or additive utility function which is a sum of utilities obtained from time spent at an origin and time spent at a destination of a trip. Using such formulation of utility, it is possible to consider a driver who chooses a departure time optimally in order to maximize expected utility when facing uncertain travel time. Noland and Small [22], Bates et al. [23], Fosgerau and Karlström [24], Fosgerau and Engelson [25], and Engelson [26] have shown how measures of travel time variability can be derived from the drivers' scheduling preferences.

A popular formulation of scheduling preferences is the α-β-γ preference in which the marginal utility of time (MUT) at the origin is constant and that at the destination is a step function [27, 28]. By assuming an exponential travel time distribution or a uniform travel time distribution, Noland and Small [22] derived the scheduling utility which is linear in (μ, σ), where μ and σ are the mean and standard deviation (SD) of the stochastic travel time. Fosgerau and Karlström [24] generalized this result to any travel time distributions. Fosgerau and Engelson [25] considered the value of travel time reliability under scheduling preferences that were defined in terms of linear MUTs being at the origin and at the destination. They found that the scheduling utility was linear in (μ, μ^2, σ^2) and that this result was independent of the shape of a travel time distribution.

Engelson [26] derived the scheduling utility for the two cases when the MUTs at both the origin and the destination are either quadratic or exponential in form, and demonstrated special cases when the scheduling utility is additive. The necessary condition when the scheduling utility is additive is that the MUT at the origin is a positive constant.

Engelson and Fosgerau [29] derived a measure of travel time variability for travelers equipped with scheduling preferences defined in terms of MUT and who chose departure time optimality. In the case of $h(t) = h_0$ and $w(t) = h_0 + (\gamma/\beta) \cdot (e^{\beta\cdot(t-t_0)} - 1)$ which are, respectively, MUT at the origin and that at the destination evaluated at clock time t (the parameters in the MUTs satisfy the following conditions: $h_0 > 0$, $\beta \geq 0$, $\gamma > 0$, and $h(t_0) = w(t_0)$), the scheduling utility is

$$\begin{aligned} u(t,T) &= \int_t^{t_0} h_0 d\tau \\ &\quad + E\left[\int_{t_0}^{t+T}\left(h_0 + \frac{\gamma}{\beta}\cdot e^{\beta\cdot(\tau-t_0)} - \frac{\gamma}{\beta}\right)d\tau\right] \\ &= h_0\cdot\mu - \frac{\gamma}{\beta}\cdot(t - t_0 + \mu) + \frac{\gamma}{\beta^2} \\ &\quad \cdot\left(e^{\beta\cdot(t-t_0)}\cdot E\left[e^{\beta\cdot T}\right] - 1\right), \end{aligned} \tag{1}$$

where T is an independently distributed random travel time with a mean value of μ and an SD of σ and t_0 is the time such that the individual prefers being at the origin before this time and at the destination after this time. From the first-order condition, the scheduling utility is derived as

$$u(T) = h_0\cdot\mu + \frac{\gamma}{\beta^2}\cdot\ln E\left[e^{\beta\cdot(T-\mu)}\right], \tag{2}$$

where $\ln E[e^{\beta\cdot(T-\mu)}]$ is the cumulant-generating function (CGF) of the centralized travel time distribution. A limiting case of $\beta \to 0$ yields

$$\lim_{\beta\to 0} u(T) = h_0\cdot\mu + \frac{\gamma}{2}\cdot\sigma^2. \tag{3}$$

If the travel time distribution has compact support, then the scheduling utility is finite for any β and can be presented as the convergent Taylor series

$$u(T) = h_0\cdot\mu + \frac{\gamma}{2}\cdot\sigma^2 + \gamma\cdot\sum_{n=3}^{\infty}\frac{\beta^{n-2}\cdot k_n}{n!}, \tag{4}$$

where k_n is the cumulant of order n of the travel time distribution. If the travel time follows a normal distribution, however, the normal distribution does not have compact support, and for any β the scheduling utility is

$$u(T) = h_0\cdot\mu + \frac{\gamma}{2}\cdot\sigma^2. \tag{5}$$

A special case of this problem with constant $h(t) \equiv \alpha$ and the two-valued function $w(t) = \alpha - \beta$ for $t < t_0$ and $w(t) = \alpha + \beta$ for $t \geq t_0$ leads to the α-β-γ preferences model. It was reported that (2) and (3) have advantages over the α-β-γ preferences model. First, they do not depend on the shape of the travel time distribution. The second is additivity with respect to parts of trip with independent travel time, which is an important property to analyze travel time reliability in a road network. Travel time variance as a measure of travel time variability may be criticized for not taking into account the skewness of the travel time distribution [30]. The CGF depends on the skewness ($k_3/\sigma^3 = E[(T-\mu)^3]/\sigma^3$) of the travel time distribution for nonzero β. However, the travel time covariance between two links is not taken into account in a CGF in which independent link travel time is assumed. The effect of link travel time covariance terms on the path travel time variance becomes larger than that of the skewness as the number of links in a path increases. For example, if a path is comprised of n links, the number of link travel time covariance terms taken into account in calculating the path travel time variance is ${}_nC_2$. We recognize that the travel time covariance between two links is a more important factor in analyzing travel time reliability in a road network than the skewness of the travel time distribution.

Hjorth et al. [31] analyzed the stated preference data by applying the scheduling preferences model that assumes MUTs at the origin and at the destination. They have shown that the value of travel time variability can be proportional to the variance of travel time. This result can partially support the use of travel time variance as a measure of travel time variability.

The additivity of the scheduling utility is a convenient property for a network equilibrium model from the practical viewpoint (calculation cost efficiency, ease of handling the network equilibrium model, etc.). If we employ an SD related measure of the travel time variability (SD, percentile value, etc.), unrealistic drivers' path choice behavior as shown next may be generated.

The following example is cited from Cominetti and Torrico [32]. The generalized travel time of a random travel time T is given by

$$c(T) = \mu + \widehat{\omega}\cdot\sigma, \tag{6}$$

where we assume $\widehat{\omega} = 1$ without loss of generality. We consider then the traffic situation shown in Figure 1 in which a road network consists of three nodes and three links, and the stochastic link travel time is shown. In the network, link travel time is denoted by T_i ($i = 1,\ldots,3$), where i is link number, which follows the normal distribution $N(\mu,\sigma^2)$ with a mean of μ and a variance of σ^2. From the link travel times shown in the figure, we obtain $c(T_1) = 12$, $c(T_2) = 10 + \sqrt{5}$, $c(T_1 + T_3) = 21 + \sqrt{3}$, and $c(T_2 + T_3) = 20 + \sqrt{7}$. From these four generalized travel times, the following results can be obtained. The minimum generalized travel time between nodes 1 and 2 is 12, and the minimum path consists of link 1. The minimum generalized travel time between nodes 1 and 3 is $\min(c(T_1+T_3)\quad c(T_2+T_3)) = 20 + \sqrt{7}$, and the minimum path consists of links 2 and 3. This example shows that a driver in a network with an origin node 1 and a destination node 2 will choose the path comprising link 1 if he/she prefers smaller generalized travel time. However, the same driver in the network whose origin and destination nodes are, respectively, 1 and 3 will choose the path comprising two links 2 and 3. However, the path choice criterion is clear, and the path choice behavior of the driver is somehow unrealistic. Even though the mean deviation is used instead of SD, such an unrealistic case can occur since the generalized path travel

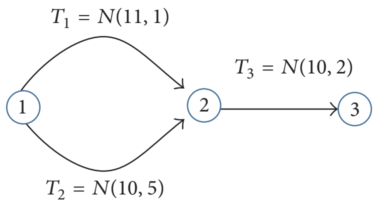

FIGURE 1: A paradoxical path choice.

time is not equal to the sum of the generalized travel times of the links that comprise that path in the case of mean deviation.

We examined the convenience and importance of the additivity of the generalized path travel time when addressing it in a network problem. In the following, we formulate some network equilibrium models in which travel time variance is employed as a measure of the travel time variability in reference to (5). According to (5), the generalized travel time is given by

$$c = \frac{u}{h_0} = \mu + \omega \cdot \sigma^2, \tag{7}$$

where

$$\omega = \frac{\gamma}{2 \cdot h_0}. \tag{8}$$

3. Link and Path Travel Times under Stochastic Flows

3.1. Notation. The notations below are used in this paper.

A: Set of links in the network

I: Set of O-D pairs in the network

J_i: Set of paths between O-D pair i

δ_{aj}: Variable that equals 1 if link a is part of path j and equals 0 otherwise

V_a: Stochastic flow of link a

v_a: Mean flow of link a

v_{ab}: Mean flow that passes through both links a and b

c_a: Capacity of link a

F_{ij}: Stochastic flow of path j between O-D pair i

f_{ij}: Mean flow of path j between O-D pair i

Q_i: Stochastic flow for O-D pair i

q_i: Mean flow for O-D pair i

Q: Stochastic total O-D flow

q: Total mean O-D flow

p_{ij}: Path choice probability for O-D pair i choosing path j

p_i: Proportion of mean flow for O-D pair i, q_i, to total mean flow, q

cv_i: Coefficient of variation of random flow Q_i

Ξ_{ij}: Stochastic travel time of path j which serves O-D pair i

c_{ij}: Generalized travel time of path j which serves O-D pair i.

3.2. Stochastic Traffic Flows. An O-D flow, Q_i, is assumed to be a random variable with a mean of $E[Q_i] = q_i$ and a variance of $\text{var}[Q_i] = (cv_i \cdot q_i)^2$, where cv_i is the coefficient of variation of the random flow Q_i. Following Lam et al. [33], the stochastic flow on path $j \in J_i$, F_{ij}, is then given by

$$F_{ij} = p_{ij} \cdot Q_i \quad \forall i \in I,\ \forall j \in J_i. \tag{9}$$

F_{ij} is a random variable with a mean of $f_{ij} = p_{ij} \cdot q_i \geq 0$ and a covariance of $\text{cov}[F_{ij}, F_{ik}] = p_{ij} \cdot p_{ik} \cdot \text{var}[Q_i]$, where $p_{ij}(j \in J_i)$ is path choice probability which can be determined by a path choice model (DUE, SUE, etc.). The following flow conservation law holds for each O-D pair:

$$\sum_{j \in J_i} f_{ij} = q_i \quad \forall i \in I. \tag{10}$$

The variance of F_{ij} is given by

$$\begin{aligned} \text{var}\left[F_{ij}\right] &= \text{var}\left[p_{ij} \cdot Q_i\right] = \left(p_{ij}\right)^2 \cdot \text{var}\left[Q_i\right] \\ &= \left(cv_i \cdot f_{ij}\right)^2 \quad \forall i \in I,\ \forall j \in J_i. \end{aligned} \tag{11}$$

The conservation of the path flow variance in relation to the O-D flow variance holds (Appendix A). The stochastic flow of link a, V_a, is given by

$$V_a = \sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot F_{ij} \quad \forall a \in A. \tag{12}$$

The mean and covariance of the stochastic link flow are then

$$v_a = \sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot f_{ij} = \sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot p_{ij} \cdot q_i \quad \forall a \in A, \tag{13}$$

$$\text{cov}\left[V_a, V_b\right] = \text{var}\left[\sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot \delta_{bj} \cdot F_{ij}\right] \quad \forall a, b \in A, \tag{14}$$

where $\sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot \delta_{bj} \cdot F_{ij}$ is the sum of all stochastic path flows that pass through both links a and b.

3.3. Stochastic Link Travel Time and Stochastic Path Travel Time. In this study, link travel time is represented by the following BPR function [34]:

$$t_a\left(v_a\right) = t_a^0 \cdot \left(1 + \kappa \cdot \left(\frac{v_a}{c_a}\right)^\lambda\right) \quad \forall a \in A, \tag{15}$$

where t_a^0 is free flow travel time of link a and $\kappa(\geq 0)$ and $\lambda(\geq 1)$ are calibration parameters. By substituting v_a in (15) with V_a, we obtain

$$t_a(V_a) = t_a^0 + \widehat{\kappa}_a \cdot (V_a)^\lambda \quad \forall a \in A, \tag{16}$$

where $\widehat{\kappa}_a = t_a^0 \cdot \kappa/(c_a)^\lambda$.

Next, we will show how to calculate both a mean value and variance of the stochastic link travel time shown by (16). By performing an mth-order ($m \geq 1$) Taylor expansion to (16) at $V_a = v_a$, we obtain

$$t_a(V_a) = \sum_{k=0}^{m} b_{ka} \cdot (V_a - v_a)^k \quad \forall a \in A, \tag{17}$$

where b_{ka} is the coefficient of the kth term of the Taylor expansion given by

$$\begin{aligned} b_{ka} &= \frac{1}{k!} \cdot \left.\frac{\partial^{(k)} t_a(V_a)}{\partial V_a^{(k)}}\right|_{V_a = v_a} \\ &= \begin{cases} t_a(v_a) & \text{if } k = 0 \\ \dfrac{\widehat{\kappa}_a \cdot \prod_{l=1}^{k}(\lambda - l + 1)}{k!} \cdot v_a^{\lambda - k} & \text{otherwise.} \end{cases} \end{aligned} \tag{18}$$

The mean link travel time is then calculated as

$$E[t_a(V_a)] = \sum_{k=0}^{m} b_{ka} \cdot E\left[(V_a - v_a)^k\right] \quad \forall a \in A. \tag{19}$$

The travel time covariance between two links is

$$\begin{aligned} \operatorname{cov}[t_a(V_a), t_b(V_b)] = {} & \sum_{k=0}^{m}\sum_{l=0}^{m} b_{ka} \cdot b_{lb} \\ & \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right] \\ & - E[t_a(V_a)] \cdot E[t_b(V_b)] \\ & \forall a, b \in A. \end{aligned} \tag{20}$$

As shown in Clark and Watling [6], (19) and (20) can be calculated by applying a method proposed by Isserlis [35] given the moments of V_a by assuming that the link flow follows normal distribution [17, 18, 33]. This assumption was supported by Rakha et al. [36], which demonstrated that the normality assumption may be sufficient from a practical standpoint given its computational simplicity. Equation (19) can be calculated as follows:

$$E[t_a(V_a)] = \begin{cases} b_{0a} + \displaystyle\sum_{k=1}^{m/2} \prod_{l=1}^{k} (2l-1) \cdot b_{2k,a} \cdot (\operatorname{var}[V_a])^k & m\text{: even number} \\ b_{0a} + \displaystyle\sum_{k=1}^{(m-1)/2} \prod_{l=1}^{k} (2l-1) \cdot b_{2k,a} \cdot (\operatorname{var}[V_a])^k & m\text{: odd number} \end{cases} \quad \forall a \in A. \tag{21}$$

The results of $m = 4$ are provided in Appendix B.

We now assume that the coefficient of each O-D flow takes a specific value. By applying this assumption to (14) (i.e., $cv_i = cv \ \forall i \in I$), we obtain

$$\operatorname{cov}[V_a, V_b] = \operatorname{var}\left[\sum_{i \in I}\sum_{j \in J_i} \delta_{aj} \cdot \delta_{bj} \cdot F_{ij}\right] = (cv \cdot v_{ab})^2 \quad \forall a, b \in A, \tag{22}$$

where

$$v_{ab} = \sum_{i \in I}\sum_{j \in J_i} \delta_{aj} \cdot \delta_{bj} \cdot f_{ij} \quad \forall a, b \in A. \tag{23}$$

v_{ab} in (23) is the mean flow that passes through both links a and b. If $a = b$ in (22), then we obtain $\operatorname{var}[V_a] = (cv \cdot v_a)^2 \ \forall a \in$ A.

In fact, this assumption can be justified if we regard total O-D flow in the network, $Q = \sum_{i \in I} Q_i$, as a random variable with a mean of $E[Q] = q(= \sum_{i \in I} q_i)$ and a variance of $\operatorname{var}[Q] = (cv \cdot q)^2$, where $Q_i = p_i \cdot Q$. $p_i = q_i/q$ is the proportion of the O-D flow, q_i, to total O-D flow, q. In this case, all O-D flows are statistically dependent on each other. The conservation of the O-D flow variance in relation to the total O-D flow variance holds (Appendix C).

By substituting (22) into (19) and (20), we obtain

$$E[t_a(V_a)] = t_a(v_a) + \sum_{k=1} \prod_{l=1}^{k} (2l-1) \cdot \widehat{b}_{2k,a} \cdot (v_a)^\lambda \quad \forall a \in A, \tag{24}$$

$$\operatorname{cov}[t_a(V_a), t_b(V_b)] = \sum_{k=1}^{m} c_k^m \cdot (v_a)^{\lambda - k} \cdot (v_b)^{\lambda - k} \cdot (v_{ab})^{2k} \quad \forall a, b \in A, \tag{25}$$

where

$$\widehat{b}_{k,a} = \frac{\widehat{\kappa}_a \cdot (cv)^k \cdot \prod_{l=1}^{k}(\lambda - l + 1)}{k!}. \tag{26}$$

c_k^m in (25) is the coefficient for the kth term. The results of $m = 4$ are provided in Appendix D.

Note that it is shown from (24) and (25) that the mean, variance, or covariance of link travel time is expressed by using only mean link flow(s) with some given parameters, and it will be shown that both $E[t_a(V_a)]$ and $\text{cov}[t_a(V_a), t_b(V_b)]$ are increasing functions with respect to v_a and v_{ab}, respectively. It will be shown that these two mathematical properties are convenient for developing a network equilibrium model. The most dominant reason for these two properties is that the coefficients of variation of all O-D flows are assumed to be the same. Thanks to this assumption, any moments for the stochastic link flow can be calculated by using its mean value. Also, as far as the Taylor series expansion provides good approximation, the method presented in this study can be applied to any functional forms. Since the Taylor series expansion can approximate well the function of $f(x) = 1/(1 - x)$ for $0 < x \leq 1$, the proposed method can be applied to the Davidson type link cost function. From (25), $\text{cov}[t_a(V_a), t_b(V_b)] > 0$ if and only if $v_{ab} > 0$. From (23), if $v_{ab} > 0$, then $v_a > 0$ and $v_b > 0$; however, the inverse relationship does not always hold for any two links in the network (i.e., even though $v_a > 0$ and $v_b > 0$, v_{ab} can be zero). These two mathematical properties show that the travel time covariance of two links, $\text{cov}[t_a(V_a), t_b(V_b)]$, is greater than zero if and only if v_{ab} is greater than zero and that v_a and v_a can influence the travel time covariance of two links, $\text{cov}[t_a(V_a), t_b(V_b)]$, if and only if v_{ab} is greater than zero. Therefore, a calculation of v_{ab} is important to calculate $\text{cov}[t_a(V_a), t_b(V_b)]$ in the network. As discussed in the next section, the calculation of v_{ab} for two adjacent links in the network is easily implemented since there is no need to enumerate a path set in the network. In contrast, the calculation of v_{ab} for two unconnected links in the network becomes more difficult than that for two adjacent links since that may need to enumerate a path set in the network.

For notational simplicity, in the rest of the paper, $E[t_a(V_a)]$, $\text{var}[t_a(V_a)]$, and $\text{cov}[t_a(V_a), t_b(V_b)]$ are denoted by $\hat{t}_a(v_a)$, $\sigma_a^2(v_a)$, and $\sigma_{ab}(v_{ab}; v_a, v_b)$, respectively. The travel time of path j which serves O-D pair i (Ξ_{ij}) is given by

$$\Xi_{ij} = \sum_{a \in A} t_a(V_a) \cdot \delta_{aj} \quad \forall i \in I, \ \forall j \in J_i. \tag{27}$$

The mean path travel time and path travel time variance are, respectively, given by

$$\begin{aligned} E[\Xi_{ij}] &= E\left[\sum_{a \in A} t_a(V_a) \cdot \delta_{aj}\right] = \sum_{a \in A} \hat{t}_a(v_a) \cdot \delta_{aj} \\ &\qquad \forall i \in I, \ \forall j \in J_i, \\ \text{var}[\Xi_{ij}] &= \text{var}\left[\sum_{a \in A} t_a(V_a) \cdot \delta_{aj}\right] \\ &= \sum_{a \in A} \sum_{b \in A} \delta_{aj} \cdot \delta_{bj} \cdot \sigma_{ab}(v_{ab}, v_a, v_b) \\ &= \sum_{a \in A} \sigma_a^2(v_a) \cdot \delta_{aj} + 2 \sum_{a \in A} \sum_{b(\neq a) \in A} \delta_{aj} \cdot \delta_{bj} \\ &\quad \cdot \sigma_{ab}(v_{ab}, v_a, v_b) \quad \forall i \in I, \ \forall j \in J_i. \end{aligned} \tag{28}$$

The path travel time covariance is

$$\begin{aligned} \text{cov}[\Xi_{ij}, \Xi_{ik}] &= \text{cov}\left[\sum_{a \in A} \delta_{aj} \cdot t_a(V_a), \sum_{b \in A} \delta_{bj} \cdot t_b(V_b)\right] \\ &= \sum_{a \in A} \sum_{b \in A} \delta_{aj} \cdot \delta_{bk} \cdot \sigma_{ab}(v_{ab}, v_a, v_b) \\ &\qquad \forall i \in I, \ \forall j \in J_i, \ \forall k \in J_i. \end{aligned} \tag{29}$$

For calculation methods of the mean travel time and travel time variance when each link flow in the network follows a lognormal distribution, the reader is referred to Tani and Uchida [37] in which each link capacity in the network is also assumed to follow a lognormal distribution.

4. Network Equilibrium Model under Stochastic Flows

4.1. DUE Principle. A risk-averse driver may take into account both a mean travel time and travel time variability in his/her path choice decision. Travel time variance is employed as a measure of the travel time variability in this study. The generalized travel time of path j which serves O-D pair i (Ξ_{ij}) is defined as

$$c_{ij} = E[\Xi_{ij}] + \omega \cdot \text{var}[\Xi_{ij}], \tag{30}$$

where $\omega \geq 0$ is a relative weight assigned to $\text{var}[\Xi_{ij}]$. The risk-averse driver assumed in this study chooses the path with lower path travel time variance if the mean path travel times of all the alternative paths are the same. Such risk-averse driver's path choice problem based on the DUE principle can be formulated as follows:

$$\begin{aligned} c_{ij}^* &= \pi_i \quad \text{if } f_{ij}^* > 0, \\ c_{ij}^* &\geq \pi_i \quad \text{if } f_{ij}^* = 0 \\ &\forall i \in I, \ \forall j \in J_i, \end{aligned} \tag{31}$$

subject to (10), (13), and (23), where π_i is the minimum generalized travel time of O-D pair i. The superscript $*$ is used to denote the variables that are obtained at equilibrium. It is known that this problem is equivalent to the following nonlinear complementary problem (NCP):

$$\begin{aligned} f_{ij}^* \cdot (c_{ij}^* - \pi_i) &= 0, \\ c_{ij}^* - \pi_i &\geq 0, \\ f_{ij}^* &\geq 0 \\ &\forall i \in I, \ \forall j \in J_i, \end{aligned} \tag{32}$$

subject to (10), (13), and (23). The equilibrium path flows can be obtained by solving the following variational inequality (VI) problem [38].

Find $\mathbf{f}^* \in \Omega_f$ such that

$$\sum_{i \in I} \sum_{j \in J_i} \left(f_{ij} - f_{ij}^* \right) \cdot c_{ij}^* \geq 0, \quad \forall \mathbf{f} \in \Omega_f, \tag{33}$$

where $\Omega_f = \{\mathbf{f} \mid \sum_{j \in J_i} f_{ij} = q_i \; \forall i \in I, \; f_{ij} \geq 0 \; \forall i \in I, \; \forall j \in J_i\}$ and $\mathbf{f} = (f_{ij})_{i \in I, j \in J_i}$.

There are efficient solution algorithms for solving the DUE traffic assignment problem. However, most of such algorithms cannot be applied to solve the VI problem shown above in which the path travel time variance is nonadditive due to the link travel time covariance between two links in the network. Therefore, path-based solution algorithms [39] which, in general, require enumeration of a path set need to be applied in order to solve the VI problem. However, the enumeration of all possible paths is almost impossible for the case of a large network. Therefore, in the next section, we will propose a simplified network model in which only the covariance terms between two adjacent links in the network are taken into account in calculating the path travel variance by considering practicality. Thus, an efficient link-based algorithm for DUE traffic assignment problem can be applied to the simplified network model.

4.2. Simplification. Paths enumeration may be required for solving the VI problem presented in the previous section. Enumeration of all noncyclic paths in a large road network is impossible. From a practical standpoint, it may be reasonable to enumerate several paths for each O-D pair (e.g., two or three paths for each O-D pair). However, different solutions can be obtained depending on paths enumerated, and that may be a troublesome issue in estimating the benefit of a project.

If we assume that the link travel time follows an independent distribution, then path travel time variance is the sum of the link travel time variance related to that path. In this case, the path choice problem can be formulated as the following convex programming problem:

$$\min \quad z = \sum_{a \in A} \int_0^{v_a} g_a(w) \, dw, \tag{34}$$

subject to (10), (13), and (23), where $g_a(v_a) = \hat{t}_a(v_a) + \omega \cdot \sigma_a^2(v_a)$. This problem has the same mathematical structure as the standard DUE traffic assignment model and thus can be solved easily by applying standard link-based algorithms (MSA (Method of Successive Averages), the Frank-Wolfe algorithm, etc.) [40]. However, ignoring all travel time covariance in (28) may bring about unrealistic solutions.

Rakha et al. [36] presented extensive evidence of a significant correlation between travel times of two adjacent links in the network. Although they analyzed travel time variability over vehicles, this evidence may support travel time variability over days. By utilizing this evidence, we now take into account all travel time covariance between two adjacent links in the network when calculating the path travel time variance. The corresponding path choice problem can be then formulated as the following link-based VI problem: simplified network model (SNM).

Find $\mathbf{v}^* \in \Omega_v$ and $\hat{\mathbf{v}}^* \in \Omega_{\hat{v}}$ such that

$$\sum_{a \in A} \left(\left(v_a - v_a^* \right) \cdot g_a\left(v_a^* \right) + \sum_{b \in \theta(a)} \left(v_{ab} - v_{ab}^* \right) \cdot g_{ab}\left(v_{ab}^*, v_a^*, v_b^* \right) \right) \geq 0, \quad \forall \mathbf{v} \in \Omega_v, \; \forall \hat{\mathbf{v}} \in \Omega_{\hat{v}}, \tag{35}$$

where

$$\begin{aligned} g_{ab}\left(v_{ab}, v_a, v_b\right) &= 2 \cdot \omega \cdot \sigma_{ab}\left(v_{ab}, v_a, v_b\right) \\ \Omega_v &= \left\{ \mathbf{v} \mid v_a = \sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot f_{ij} \quad \forall \mathbf{f} \in \Omega_f, \; \forall a \in A \right\}, \\ \Omega_{\hat{v}} &= \left\{ \hat{\mathbf{v}} \mid v_{ab} = \sum_{i \in I} \sum_{j \in J_i} \delta_{aj} \cdot \delta_{aj} \cdot f_{ij} \quad \forall \mathbf{f} \in \Omega_f, \; \forall a \in A, \; \forall b \in \theta(a) \right\}, \\ \mathbf{v} &= \left(v_a\right)_{a \in A}, \\ \hat{\mathbf{v}} &= \left(v_{ab}\right)_{a \in A, b \in \theta(a)}. \end{aligned} \tag{36}$$

$\theta(a)$ is the set of links which are adjacent to link a in front of it. SNM includes no stochastic variable, although SNM analyzes travel time reliability in the network.

To solve SNM, one can apply a network representation which may be used when addressing intersection delays, in which dummy links connecting between all adjacent links in the network are added to the original network. Consider an original network that consists of a set of links and a set of nodes. It is assumed that each node in the network also has an identical number. Consider then a directed link in the original network. We can find each link of which origin node number is the same as the destination node of the link. By using this relationship, we can construct the augmented network (e.g., right-hand side of Figure 2), that corresponds to the original network (e.g., left-hand side of Figure 2), by inserting a directed dummy link between these two links. In the augmented network, the generalized travel time of link a $\forall a \in A$ is $g_a(v_a)$ and that of dummy link ab $\forall a \in A$, $\forall b \in \theta(a)$ is $g_{ab}(v_{ab}, v_a, v_b)$. Once the augmented network is constructed, v_{ab} is easily calculated as the flow of link ab. By applying this network representation, SNM can be solved by a diagonalization method in which v_a and v_b in $g_{ab}(v_{ab}, v_a, v_b)$ are regarded as constant terms [41–45]. Also, MSA can be applied for solving SNM. Due to the mathematical structure of $g_{ab}(v_{ab}, v_a, v_b)$, however, SNM has the same mathematical property as an asymmetric DUE traffic assignment problem which can have multiple solutions. Therefore, SNM may have multiple solutions. Next, we will examine the uniqueness of the solution of SNM.

If both generalized link travel time functions, $g_a(v_a)$ $\forall a \in A$ and $g_{ab}(v_{ab}, v_a, v_b)$ $\forall a \in A$, $\forall b \in \theta(a)$, in the augmented network are monotone functions, SNM has a unique

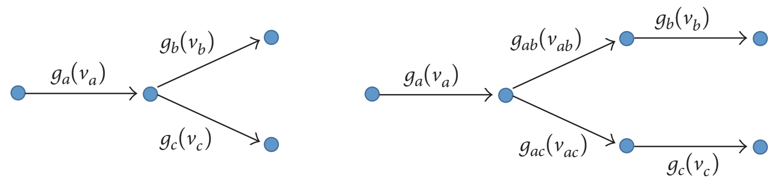

Figure 2: Network representation for addressing intersection delays.

solution. Or equivalently, if the Jacobian matrix $\nabla \mathbf{g}(\mathbf{v}, \hat{\mathbf{v}})$, where $\mathbf{g} = ((g_a)_{a \in A}, (g_{ab})_{a \in A, b \in \theta(a)})$, is positive-definite, SNM has a unique solution. The Jacobian matrix $\nabla \mathbf{g}(\mathbf{v}, \hat{\mathbf{v}})$ is given by the following lower triangle matrix:

$$\nabla \mathbf{g}(\mathbf{v}, \hat{\mathbf{v}}) = \begin{pmatrix} \frac{dg_1}{dv_1} & & & & \\ \mathbf{0} & \ddots & & \mathbf{0} & \\ 0 & 0 & \frac{dg_{|A|}}{dv_{|A|}} & & \\ \frac{\partial g_{1,\theta(1)}}{\partial v_1} & \cdots \frac{\partial g_{1,\theta(1)}}{\partial v_{\theta(1)}} \cdots & 0 & \frac{\partial g_{1,\theta(1)}}{\partial v_{1,\theta(1)}} & \\ \vdots & \vdots & \vdots & \vdots & \ddots \\ 0 & \cdots \frac{\partial g_{|A|,\theta(|A|)}}{\partial v_{\theta(|A|)}} \cdots & \frac{\partial g_{|A|,\theta(|A|)}}{\partial v_{|A|}} & 0 & \cdots \frac{\partial g_{|A|,\theta(|A|)}}{\partial v_{|A|,\theta(|A|)}} \end{pmatrix}. \quad (37)$$

If every eigenvalue of a Jacobian matrix is positive, the Jacobian matrix is positive-definite. The eigenvalues of a lower triangle matrix are equal to the values of diagonal elements of the matrix. In SNM, the following two conditions hold for each link in the augmented network (see Appendix E for the proofs):

$$\frac{\partial g_{ab}(v_{ab}, v_a, v_b)}{\partial v_{ab}} > 0 \quad \forall a, \ b \in A, \qquad \frac{\partial g_a(v_a)}{\partial v_a} > 0 \quad \forall a \in A. \quad (38)$$

Therefore, $\nabla \mathbf{g}(\mathbf{v}, \hat{\mathbf{v}})$ is positive-definite and thus SNM has a unique solution. If the link travel time covariance for any two links in the network is taken into consideration in calculating path travel time variance, the uniqueness of the solution is not obtained.

Since SNM calculates the stochastic variables at the equilibrium, the stability as well as the uniqueness of the solution is guaranteed in SNM. Even in a problem in a static context, the stability and uniqueness of the solution can be analyzed by applying theory in a dynamic context. This is true for our static problem.

5. Numerical Experiments

5.1. Settings. In this section, we carry out numerical experiments for illustrating the application and validity of SNM. We adopt the network of Nguyen and Dupuis [46] with 4 O-D pairs, 25 paths, and 19 directed links (Figure 3). The link sequences of the paths are shown in Table 1.

We employed the mean link travel time and travel time covariance shown in (24) and (25) that are calculated by assuming $m = 4$, respectively. Parameters for the BPR function, κ and λ, are 2.62 and 5, respectively. The coefficient of variation of total O-D flow, cv, is 0.1. The other parameters used in (24) and (25) are shown in Table 2.

In this study, we prepared four experimental cases by changing the effect of the path travel time variance on the driver's path choice behavior in the network. In the first case, we assume a risk-neutral driver who chooses a path based only on mean path travel time in the network. Therefore, the relative weight assigned to path travel time variance, ω, in (30) is 0 in this case. The other three cases assume three types of risk-averse drivers. The relative weight assigned to var$[\Xi_{ij}]$ is assumed as 0.3 in these three cases. In the second case, we assume that there is no travel time correlation between two different links in the network. In the third case, SNM is employed (i.e., all travel time covariance between two adjacent links in the network is introduced to calculate the path travel time variance). In the fourth case, all travel time covariance between two different links in the network is introduced to calculate the path travel time variance. In the latter three cases, the path travel time variance is calculated as

$$\operatorname{var}[\Xi_{ij}] = \sum_{a \in A} \sum_{b \in B} \operatorname{cov}[t_a(V_a), t_b(V_b)] \cdot \delta_{aj} \cdot \delta_{bj} \quad \forall i \in I, \ \forall j \in J_i \quad (39)$$

by using both the relative weight assigned to path travel time variance, ω, and the set of link(s), B, shown in Table 3.

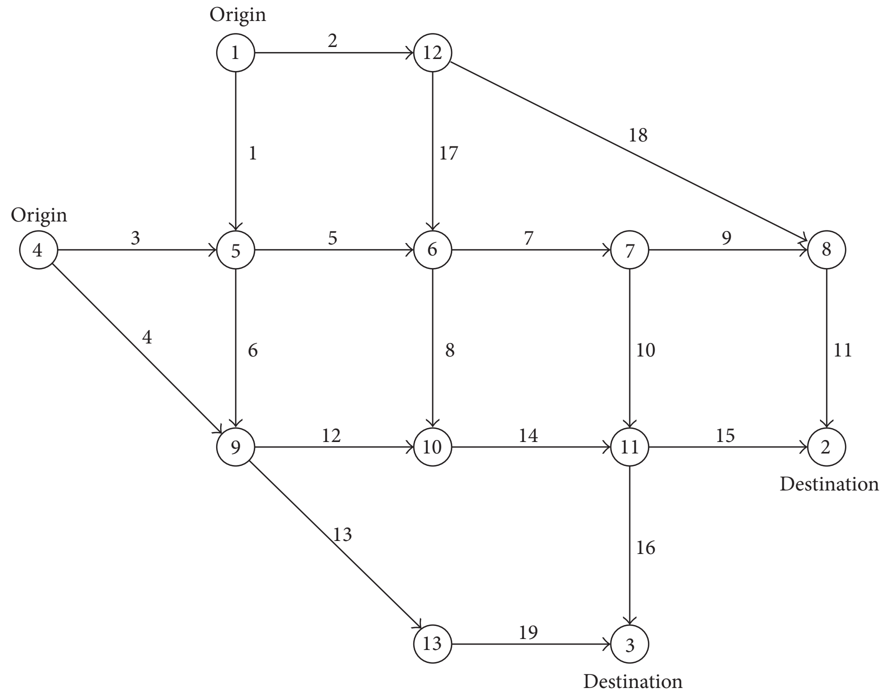

Figure 3: The Nguyen and Dupuis network.

Table 1: Paths and link sequences.

O-D	O-D pair	Path	Link seq.
(1)	1-2	(1)	2-18-11
		(2)	1-5-7-9-11
		(3)	1-5-7-10-15
		(4)	1-5-8-14-15
		(5)	1-6-12-14-15
		(6)	2-17-7-9-11
		(7)	2-17-7-10-15
		(8)	2-17-8-14-15
(2)	4-2	(9)	4-12-14-15
		(10)	3-5-7-9-11
		(11)	3-5-7-10-15
		(12)	3-5-8-14-15
		(13)	3-6-12-14-15
(3)	1-3	(14)	1-6-13-19
		(15)	1-5-7-10-16
		(16)	1-5-8-14-16
		(17)	1-6-12-14-16
		(18)	2-17-7-10-16
		(19)	2-17-8-14-16
(4)	4-3	(20)	4-13-19
		(21)	4-12-14-16
		(22)	3-6-13-19
		(23)	3-5-7-10-16
		(24)	3-5-8-14-16
		(25)	3-6-12-14-16

Table 2: Link travel time parameters.

Link	Free-flow travel time t_a^0	Capacity c_a
(1)	10	1500
(2)	10	1500
(3)	10	1500
(4)	20	1500
(5)	10	1500
(6)	10	1500
(7)	10	1500
(8)	10	1500
(9)	10	1500
(10)	10	1500
(11)	10	1500
(12)	10	1500
(13)	20	1500
(14)	10	1500
(15)	10	1500
(16)	10	1500
(17)	10	1500
(18)	40	1500
(19)	10	1500

For solving the first three cases, we employed MSA as a link-based solution algorithm. The final case can be solved by minimizing the gap function for NCP [17, 47, 48]. In fact, the paths set shown in Table 1 is prepared only for the final case.

Table 3: Assumptions of each case.

Case	Relative weight (ω)	Set of link(s) (B)
2	0.3	a
3	0.3	$\theta(a)$
4	0.3	A

Table 4: Mean path flows.

O-D	Path	Case 1	Case 2	Case 3	Case 4
(1)	(1)	499	658	643	641
	(2)	81	58	**0**	75
	(3)	88	64	165	117
	(4)	45	47	**0**	**0**
	(5)	112	59	61	**0**
	(6)	22	76	88	**0**
	(7)	127	38	42	168
	(8)	25	**0**	**0**	**0**
(2)	(9)	265	276	491	321
	(10)	411	229	294	313
	(11)	157	236	173	88
	(12)	16	126	**0**	127
	(13)	151	133	41	152
(3)	(14)	317	413	304	268
	(15)	79	107	45	100
	(16)	36	58	296	210
	(17)	145	108	24	122
	(18)	248	255	330	296
	(19)	174	59	**0**	5
(4)	(20)	591	491	441	481
	(21)	121	197	28	155
	(22)	149	120	273	268
	(23)	**0**	87	18	**0**
	(24)	96	4	30	**0**
	(25)	43	100	210	97

Table 5: Generalized path travel time.

O-D	Path	Case 1	Case 2	Case 3	Case 4
(1)	(1)	70.5	75.9	77.5	80.0
	(2)	70.5	75.9	**77.5**	80.0
	(3)	70.5	75.9	77.5	80.0
	(4)	70.5	75.9	**77.5**	**80.1**
	(5)	70.5	75.9	77.5	**80.2**
	(6)	70.5	75.9	77.5	**81.1**
	(7)	70.5	75.9	77.5	80.0
	(8)	70.5	**75.9**	**78.5**	**80.1**
(2)	(9)	72.5	79.1	81.3	85.1
	(10)	72.5	79.1	81.3	85.1
	(11)	72.5	79.1	81.3	85.1
	(12)	72.5	79.1	**81.3**	85.1
	(13)	72.5	79.1	81.3	85.1
(3)	(14)	69.8	75.8	77.7	80.6
	(15)	69.8	75.8	77.7	80.6
	(16)	69.8	75.8	77.7	80.6
	(17)	69.8	75.8	77.7	80.6
	(18)	69.8	75.8	77.7	80.6
	(19)	69.8	75.8	**78.7**	80.6
(4)	(20)	71.8	79.0	81.5	85.1
	(21)	71.8	79.0	81.5	85.1
	(22)	71.8	79.0	81.5	85.1
	(23)	**71.8**	79.0	81.5	**85.2**
	(24)	71.8	79.0	81.5	**85.1**
	(25)	71.8	79.0	81.5	85.1

Only case 4 cannot be solved by MSA which does not require paths enumeration and thus is applicable to a large network. On the other hand, it is difficult to solve the problem of a large network using the method based on the gap function that requires paths enumeration.

5.2. Results. Table 4 shows the mean path flows for each case. For the first three cases, since we applied MSA to solve corresponding path choice problems, the path flows for each case were not uniquely determined. Even so, presenting path flows for the first three cases may be useful to understand roughly how path flows change according to different expressions of path travel time variance. Also, by presenting the path flows for cases 1–3 to which MSA was applied, it is easy to understand that the generalized travel times for the paths between each O-D pair that are used by the drivers are the same although both the mean travel time and travel time variance of a path between the O-D pair can be different from the others. In contrast, the path flows for case 4 were uniquely determined, since we applied a path-based algorithm when solving its path choice problem. In all cases, if a path flow is zero, such path flow is denoted by bold figures in Table 4 so that we can know that the corresponding path is not chosen by the drivers in the network. Table 5 shows generalized path travel times for all cases which are calculated by using both the mean path travel times shown in Table 6 and the path travel time variance shown in Table 7. It is observed that the paths chosen by the drivers have the minimum generalized travel time and that the paths which are not chosen by the drivers have generalized travel times which are equal to or greater than the minimum generalized travel time. The generalized path travel times for unused paths are denoted by bold figures in Table 5. It is shown in Tables 4, 6, and 8 that although the mean path flows of each case are different from the other cases, mean link flows are similar among all cases. Since mean travel time of a path is calculated by using mean link flows, therefore similar mean path travel times are obtained among the four cases.

For O-D pair 1, the flows of paths 4, 5, and 8 in cases 2–4 are smaller than those in case 1 whereas the flows of path 1 in cases 2–4 are larger than that in case 1. These differences can be explained as follows. The travel time variances of path 1 in cases 2–4, which are denoted by bold figures in Table 7, are much smaller than those of paths 2–8, although the mean

Table 6: Mean path travel time.

O-D	Path	Case 1	Case 2	Case 3	Case 4
(1)	(1)	70.5	**71.8**	**72.2**	**72.4**
	(2)	70.5	68.5	68.6	68.7
	(3)	70.5	68.8	68.7	68.6
	(4)	70.5	68.2	68.5	68.8
	(5)	70.5	68.7	68.4	68.4
	(6)	70.5	67.7	68.5	68.7
	(7)	70.5	68.0	68.6	68.6
	(8)	70.5	67.5	68.4	68.8
(2)	(9)	72.5	69.9	70.0	70.0
	(10)	72.5	70.6	71.1	71.3
	(11)	72.5	70.9	71.1	71.2
	(12)	72.5	70.3	70.9	71.4
	(13)	72.5	70.8	70.8	71.0
(3)	(14)	69.8	68.0	67.2	66.8
	(15)	69.8	68.7	68.8	68.8
	(16)	69.8	68.2	68.6	69.0
	(17)	69.8	68.6	68.5	68.6
	(18)	69.8	68.0	68.7	68.8
	(19)	69.8	67.4	68.5	69.0
(4)	(20)	71.8	69.2	68.8	68.4
	(21)	71.8	69.8	70.1	70.2
	(22)	71.8	70.1	69.6	69.4
	(23)	71.8	70.9	71.2	71.4
	(24)	71.8	70.3	71.0	71.6
	(25)	71.8	70.7	71.0	71.2

travel time of path 1 is longer than those of paths 2–8, which are denoted by bold figures in Table 6. However, path 1 has longer mean travel time than paths 2–8, and that path is more reliable in terms of travel time variance than paths 2–8 in cases 2–4. In total, the path choice probabilities of path 1 in cases 2–4 are larger than that in case 1. In contrast, since the drivers in case 1 choose their paths based only on mean travel times, the flow of path 1 is larger than those in cases 2–4. This path choice switch can be observed in the other O-D pairs (e.g., from path 10 to path 9 for O-D pair 2, from paths 17 and 19 to paths 16 and 18 for O-D pair 3, and from paths 20 and 24 to path 25 for O-D pair 4.

We can find from Table 6 a tendency of the mean path travel time in case 1 to be greater than those in cases 2–4. Surprisingly, this tendency holds for all paths in the network except for path 1. Obviously, this tendency was derived from the introduction of travel time variance to the drivers' path choice behavior. We will then check how total mean travel time in the network is shortened by the introduction of travel time variance. An index for the total mean travel time for cases $n \in \{1, \ldots, 4\}$, TTT_n, can be given by

$$\mathrm{TTT}_n = \sum_{a \in A} \hat{t}_a (v_a) \cdot v_a. \tag{40}$$

By using both the mean link flows shown in Table 8 and the mean link travel time shown in Table 9, TTT_n $\forall n \in \{1, \ldots, 4\}$ are calculated as 2.847×10^5, 2.789×10^5, 2.797×10^5, and 2.794×10^5, respectively. An introduction of travel time variance to the path choice behavior model may bring about

Table 7: Path travel time variance.

O-D	Path	Case 1	Case 2	Case 3	Case 4
(1)	(1)	0.0	**13.5**	**17.6**	**25.5**
	(2)	0.0	24.8	29.6	37.8
	(3)	0.0	23.6	29.5	38.0
	(4)	0.0	25.6	30.1	37.8
	(5)	0.0	24.0	30.4	39.3
	(6)	0.0	27.3	30.0	41.3
	(7)	0.0	26.2	29.8	38.1
	(8)	0.0	28.1	33.6	37.8
(2)	(9)	0.0	30.7	37.8	50.4
	(10)	0.0	28.5	34.3	46.0
	(11)	0.0	27.3	34.2	46.3
	(12)	0.0	29.2	34.8	45.8
	(13)	0.0	27.7	35.0	47.2
(3)	(14)	0.0	26.1	35.1	46.2
	(15)	0.0	23.5	29.7	39.3
	(16)	0.0	25.5	30.3	38.8
	(17)	0.0	23.9	30.6	40.2
	(18)	0.0	26.1	30.0	39.4
	(19)	0.0	28.0	33.9	38.9
(4)	(20)	0.0	32.8	42.6	55.7
	(21)	0.0	30.6	38.0	49.7
	(22)	0.0	29.8	39.8	52.5
	(23)	0.0	27.2	34.4	46.0
	(24)	0.0	29.1	35.0	45.2
	(25)	0.0	27.6	35.2	46.5

Table 8: Mean link flows.

Link	Case 1	Case 2	Case 3	Case 4
(1)	904	914	896	890
(2)	1,096	1,086	1,104	1,110
(3)	1,024	1,036	1,040	1,044
(4)	976	964	960	956
(5)	1,010	1,017	1,021	1,028
(6)	918	933	914	906
(7)	1,215	1,151	1,157	1,155
(8)	392	295	325	342
(9)	514	363	383	387
(10)	701	788	774	768
(11)	1,013	1,021	1,026	1,028
(12)	837	873	855	846
(13)	1,057	1,024	1,019	1,016
(14)	1,229	1,167	1,181	1,188
(15)	987	979	974	972
(16)	943	976	981	984
(17)	597	428	461	469
(18)	499	658	643	641
(19)	1,057	1,024	1,019	1,016

Table 9: Mean link travel time.

Link	Case 1	Case 2	Case 3	Case 4
(1)	12.3	12.4	12.2	12.1
(2)	16.0	15.7	16.2	16.4
(3)	14.3	14.5	14.6	14.7
(4)	26.7	26.3	26.2	26.1
(5)	14.0	14.1	14.2	14.4
(6)	12.5	12.7	12.4	12.3
(7)	20.0	17.7	17.9	17.8
(8)	10.0	10.0	10.0	10.0
(9)	10.1	10.0	10.0	10.0
(10)	10.6	11.2	11.1	11.0
(11)	14.1	14.2	14.3	14.4
(12)	11.6	11.9	11.7	11.6
(13)	30.0	28.6	28.4	28.2
(14)	20.7	18.2	18.7	19.0
(15)	13.6	13.4	13.3	13.3
(16)	12.8	13.4	13.4	13.5
(17)	10.3	10.1	10.1	10.1
(18)	40.5	41.9	41.7	41.6
(19)	15.0	14.3	14.2	14.1

an efficient use of the road network in terms of the mean travel time. This effect is similar to the one obtained by applying the system optimal principle (i.e., Wardrop's second principle). From the mean link flows shown in Table 8, it is observed that the mean flows of links 5, 9, 11, 12, 16, and 18 in cases 2–4 are larger than those in case 1, whereas those of links 4, 7, 8, 13, 14, 15, and 17 in cases 2–4 are smaller than those in case 1. These changes happened by the path flow changes in cases 1–4 which we have discussed.

As discussed in the previous section, if we do not consider the effect of travel time reliability on the driver's path choice behavior in the network, unrealistic results may be obtained. However, the introduction of all travel time covariance between two different links in the network to the calculation of the path travel time variance is difficult to implement. Therefore, we proposed SNM, in which all travel time covariance between two adjacent links in the network is considered in calculating the path travel time variance. Our concern is now how far the results obtained in cases 1–3 are from the results obtained in case 4. The results obtained in case 4 can be regarded as the exact solution, since it is obtained by adopting the path travel time variance denoted by (28). Table 10 shows the coefficients of correlation of mean link flows among four cases. The bold figures in the table show the coefficients of correlation between case 4 and the other three cases. One can see that SNM (case 3) reproduces almost the same mean link flows as case 4, since the coefficient of correlation between case 4 and case 3 is 1 whereas the other coefficients of correlation are less than 1. Note that the coefficient of correlation between case 4 and case 1 is the smallest. Almost the same results were obtained by calculating the coefficients of correlation of mean link travel times. These results may bring about two practical implications that all travel time covariance between two different links in the network may not be required in calculating mean link flows. Instead, all travel time covariance between two adjacent links in the network is required in calculating mean link flows, and that calculation of all travel time covariance between two adjacent links in the network is easy even in a large network.

Table 10: Coefficients of correlation of mean link flows.

	Case 1	Case 2	Case 3	Case 4
Case 1	1.000			
Case 2	0.957	1.000		
Case 3	0.973	0.995	1.000	
Case 4	**0.969**	**0.997**	**1.000**	1.000

6. Conclusions

In this study, we proposed a simplified network model, that is, SNM, for travel time reliability analysis. The uncertainty addressed in this model is that of O-D flows. In this model, the generalized path travel time is a linear combination of mean path travel time and path travel time variance. In calculating the path travel time variance, we consider all travel time covariance between two adjacent links in the network in SNM. A risk-averse driver in the network is assumed. By applying a network representation used for addressing intersection delays, SNM can be solved by applying a standard link-based algorithm. The other property of SNM which needs to be emphasized here is that its formulation requires only mean network flows. This property may be important for practitioners, since once the coefficient of variation of total O-D flow is determined, one can apply SNM to real road network analysis for which network data sets for a conventional network model (e.g., a deterministic/stochastic user equilibrium traffic assignment model) are already prepared.

Numerical experiments are carried out for illustrating the applications and validity of SNM. The experiments assumed four types of drivers in the network. The first type of driver is a risk-neutral driver who chooses a path based only on mean path travel time. The other three types of drivers are risk-averse drivers who choose their paths based on both mean path travel time and path travel time variance. The second type of driver's path travel time variance is calculated by assuming statistically independent link travel time. The third type of driver's path travel time variance is calculated by considering all travel time covariance between two adjacent links in the network. The fourth type of driver's path travel time variances is calculated by considering all travel time covariance between two different links in the network. The path travel time variance of the fourth type of driver is the exact one in the network. It is shown that mean network flows obtained by assuming the risk-neutral driver differ as a whole from those obtained by assuming the risk-averse drivers. These differences in mean network flows are generated by the effects of travel time reliability on path choice behavior by the driver. It is also shown that mean link flows obtained by assuming the third type of driver, that is, mean link flows calculated by SNM, are almost the same as the mean link flows calculated by assuming the fourth type of driver. In a practical sense, it may be difficult to calculate network flows in a large road network by assuming the fourth type of driver. In contrast, SNM can be easily applied to a large road network in calculating network flows.

In this study, we recognize that the introduction of path travel time variance considering all travel time covariance between two links in a road network to the generalized path travel time is more important in expressing the driver's route choice behavior than the introduction of the skewness of link travel time to the generalized path travel time. Therefore, SNM introduces the path travel time variance considering all travel time covariance between two adjacent links in the network to the generalized path travel time. It is interesting to see how the path choice probabilities which are calculated by assuming both the statistically independent link travel time and the skewness of link travel time differ from the probabilities calculated by SNM. In light of this, there is a need for a network equilibrium model that introduces a driver's path choice preference based on (2). A deterministic path choice model based on Wardrop's first principle is employed in this study in order to express the driver's path choice behavior in the network. The introduction of stochastic models, for example, logit-based models or probit-based models, to SNM is needed, in order to express the driver's perception error on the generalized travel time. These two challenges are our future tasks.

Appendix

A. The Conservation of the Path Flow Variance in Relation to the O-D Flow Variance

O-D flow variance is calculated as the sum of corresponding path flow variance as follows:

$$\begin{aligned}
\operatorname{var}\left[\sum_{j\in J_i} F_{ij}\right] &= \sum_{j\in J_i} \operatorname{var}\left[F_{ij}\right] \\
&\quad + 2\sum_{j_1\in J_i}\sum_{j_2(\neq j_1)\in J_i} \operatorname{cov}\left[F_{ij_1}, F_{ij_2}\right] \\
&= \left(p_{ij}\right)^2 \cdot \operatorname{var}\left[Q_i\right] + 2\sum_{j_1\in J_i}\sum_{j_2(\neq j_1)\in J_i} p_{ij_1} \\
&\quad \cdot p_{ij_2} \cdot \operatorname{var}\left[Q_i\right] \\
&= \left(cv_i \cdot q_i\right)^2 \cdot \left(\sum_{j\in J_i} p_{ij}\right)^2 = \left(cv_i \cdot q_i\right)^2 \\
&= \operatorname{var}\left[Q_i\right] \quad \forall i \in I.
\end{aligned} \tag{A.1}$$

B. The Results of (19) and (20) for $m = 4$

Mean and variance/covariance of link travel time obtained by performing the fourth-order Taylor expansion to (16) are, respectively, given by

$$\begin{aligned}
&E\left[t_a\left(V_a\right)\right] = b_{0a} + b_{2a} \cdot E\left[\left(V_a - v_a\right)^2\right] + b_{4a} \\
&\quad \cdot E\left[\left(V_a - v_a\right)^4\right] = b_{0a} + b_{2a} \cdot \operatorname{var}\left[V_a\right] + 3 \cdot b_{4a} \\
&\quad \cdot \left(\operatorname{var}\left[V_a\right]\right)^2, \\
&\operatorname{cov}\left[t_a\left(V_a\right), t_b\left(V_b\right)\right] = b_{0a} \cdot b_{0b} + b_{0a} \cdot b_{2b} \\
&\quad \cdot E\left[\left(V_b - v_b\right)^2\right] + b_{0a} \cdot b_{4b} \cdot E\left[\left(V_b - v_b\right)^4\right] + b_{1a} \\
&\quad \cdot b_{1b} \cdot E\left[\left(V_a - v_a\right) \cdot \left(V_b - v_b\right)\right] + b_{1a} \cdot b_{3b} \\
&\quad \cdot E\left[\left(V_a - v_a\right) \cdot \left(V_b - v_b\right)^3\right] + b_{2a} \cdot b_{0b} \\
&\quad \cdot E\left[\left(V_a - v_a\right)^2\right] + b_{2a} \cdot b_{2b} \cdot E\left[\left(V_a - v_a\right)^2\right. \\
&\quad \left. \cdot \left(V_b - v_b\right)^2\right] + b_{2a} \cdot b_{4b} \cdot E\left[\left(V_a - v_a\right)^2\right. \\
&\quad \left. \cdot \left(V_b - v_b\right)^4\right] + b_{3a} \cdot b_{1b} \cdot E\left[\left(V_a - v_a\right)^3 \cdot \left(V_b - v_b\right)\right] \\
&\quad + b_{3a} \cdot b_{3b} \cdot E\left[\left(V_a - v_a\right)^3 \cdot \left(V_b - v_b\right)^3\right] + b_{4a} \cdot b_{0b} \\
&\quad \cdot E\left[\left(V_a - v_a\right)^4\right] + b_{4a} \cdot b_{2b} \cdot E\left[\left(V_a - v_a\right)^4\right. \\
&\quad \left. \cdot \left(V_b - v_b\right)^2\right] + b_{4a} \cdot b_{4b} \cdot E\left[\left(V_a - v_a\right)^4\right. \\
&\quad \left. \cdot \left(V_b - v_b\right)^4\right] - \left(b_{0a} + b_{2a} \cdot E\left[\left(V_a - v_a\right)^2\right] + b_{4a}\right. \\
&\quad \left. \cdot E\left[\left(V_a - v_a\right)^4\right]\right) \cdot \left(b_{0b} + b_{2b} \cdot E\left[\left(V_b - v_b\right)^2\right] + b_{4b}\right. \\
&\quad \left. \cdot E\left[\left(V_b - v_b\right)^4\right]\right) = b_{1a} \cdot b_{1b} \cdot \operatorname{cov}\left[V_a, V_b\right] + 3 \cdot b_{1a} \\
&\quad \cdot b_{3b} \cdot \operatorname{var}\left[V_b\right] \cdot \operatorname{cov}\left[V_a, V_b\right] + 2 \cdot b_{2a} \cdot b_{2b} \\
&\quad \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2 + 12 \cdot b_{2a} \cdot b_{4b} \cdot \operatorname{var}\left[V_b\right] \\
&\quad \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2 + 3 \cdot b_{3a} \cdot b_{1b} \cdot \operatorname{var}\left[V_a\right] \\
&\quad \cdot \operatorname{cov}\left[V_a, V_b\right] + b_{3a} \cdot b_{3b} \cdot \left(9 \cdot \operatorname{var}\left[V_a\right] \cdot \operatorname{var}\left[V_b\right]\right. \\
&\quad \left. \cdot \operatorname{cov}\left[V_a, V_b\right] + 6 \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^3\right) + 12 \cdot b_{4a} \cdot b_{2b} \\
&\quad \cdot \operatorname{var}\left[V_a\right] \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2 + b_{4a} \cdot b_{4b} \cdot \left(3 \cdot 8\right. \\
&\quad \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^4 + 3 \cdot 24 \cdot \operatorname{var}\left[V_a\right] \cdot \operatorname{var}\left[V_b\right] \\
&\quad \left. \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2\right).
\end{aligned} \tag{B.1}$$

In the above calculations, we applied the following moment calculations [35]:

$$\begin{aligned}
&E\left[\left(V_a - v_a\right) \cdot \left(V_b - v_b\right)^3\right] = 3 \cdot \operatorname{var}\left[V_b\right] \cdot \operatorname{cov}\left[V_a, V_b\right], \\
&E\left[\left(V_a - v_a\right)^2 \cdot \left(V_b - v_b\right)^2\right] = \operatorname{var}\left[V_a\right] \cdot \operatorname{var}\left[V_b\right] + 2 \\
&\quad \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2, \\
&E\left[\left(V_a - v_a\right)^2 \cdot \left(V_b - v_b\right)^4\right] = 3 \cdot \left(\operatorname{var}\left[V_b\right]\right)^2 \\
&\quad \cdot \operatorname{var}\left[V_a\right] + 12 \cdot \operatorname{var}\left[V_b\right] \cdot \left(\operatorname{cov}\left[V_a, V_b\right]\right)^2,
\end{aligned}$$

$$E\left[(V_a-v_a)^3\cdot(V_b-v_b)^3\right]=9\cdot\operatorname{var}[V_a]\cdot\operatorname{var}[V_b]\cdot\operatorname{cov}[V_a,V_b]+6\cdot(\operatorname{cov}[V_a,V_b])^3,$$
$$E\left[(V_a-v_a)^4\right]=3\cdot(\operatorname{var}[V_a])^2,$$
$$E\left[(V_a-v_a)^4\cdot(V_b-v_b)^4\right]=3\cdot\left(8\cdot(\operatorname{cov}[V_a,V_b])^4+24\cdot\operatorname{var}[V_a]\cdot\operatorname{var}[V_b]\cdot(\operatorname{cov}[V_a,V_b])^2+3\cdot(\operatorname{var}[V_a])^2\cdot(\operatorname{var}[V_b])^2\right). \tag{B.2}$$

A case of $a=b$ in $\operatorname{cov}[t_a(V_a),t_b(V_b)]$ yields

$$\operatorname{var}[t_a(V_a)]=(b_{1a})^2\cdot\operatorname{var}[V_a]+\left(2\cdot3\cdot b_{1a}\cdot b_{3a}+2\cdot(b_{2a})^2\right)\cdot(\operatorname{var}[V_a])^2+\left(2\cdot12\cdot b_{2a}\cdot b_{4a}+9\cdot(b_{3a})^2+6\cdot(b_{3a})^2\right)\cdot(\operatorname{var}[V_a])^3+(3\cdot8+3\cdot24)\cdot(b_{4a})^2\cdot(\operatorname{var}[V_a])^4. \tag{B.3}$$

C. The Conservation of the O-D Flow Variance in Relation to the Total O-D Flow Variance under the Stochastic Total O-D Flow

Total O-D flow variance is calculated as the sum of O-D flow variance as follows:

$$\operatorname{var}\left[\sum_{i\in I}Q_i\right]=\sum_{i\in I}\operatorname{var}[Q_i]+2\sum_{i_1\in I}\sum_{i_2(\neq i_1)\in I}\operatorname{cov}[Q_{i_1},Q_{i_2}]=\sum_{i\in I}\operatorname{var}\left[\sum_{j\in J_i}F_{ij}\right]+2\sum_{i_1\in I}\sum_{i_2(\neq i_1)\in I}\operatorname{cov}\left[\sum_{j_1\in J_{i_1}}F_{i_1j_1},\sum_{j_2\in J_{i_2}}F_{i_2j_2}\right]=\sum_{i\in I}(cv\cdot p_i\cdot q)^2\cdot\left(\sum_{j\in J_i}p_{ij}\right)^2+2\sum_{i_1\in I}\sum_{i_2(\neq i_1)\in I}\left(p_{i_1}\sum_{j_1\in J_{i_1}}p_{i_1j_1}\right)\cdot\left(p_{i_2}\sum_{j_2\in J_{i_2}}p_{i_2j_2}\right)\operatorname{var}[Q]=\sum_{i\in I}(cv\cdot p_i\cdot q)^2+2\sum_{i_1\in I}\sum_{i_2(\neq i_1)\in I}p_{i_1}\cdot p_{i_2}\cdot(cv\cdot q)^2=(cv\cdot q)^2\cdot\left(\sum_{i\in I}(p_i)^2+2\sum_{i_1\in I}\sum_{i_2(\neq i_1)\in I}p_{i_1}\cdot p_{i_2}\right)=(cv\cdot q)^2=\operatorname{var}[Q]. \tag{C.1}$$

D. The Results of (24) and (25) for $m=4$

By using (23), the mean link travel time and link travel time covariance can be, respectively, calculated as

$$E[t_a(V_a)]=b_{0a}+b_{2a}\cdot\operatorname{var}[V_a]+3\cdot b_{4a}\cdot(\operatorname{var}[V_a])^2=b_{0a}+b_{2a}\cdot(cv\cdot v_a)^2+3\cdot b_{4a}\cdot(cv\cdot v_a)^4=b_{0a}+\widehat{b}_{2a}\cdot(v_a)^\lambda+3\cdot\widehat{b}_{4a}\cdot(v_a)^\lambda,$$

$$\operatorname{cov}\left[t_{a(V_a)},t_{b(V_b)}\right]=b_{1a}\cdot b_{1b}\cdot\operatorname{cov}[V_a,V_b]+3\cdot b_{1a}\cdot b_{3b}\cdot\operatorname{var}[V_b]\cdot\operatorname{cov}[V_a,V_b]+2\cdot b_{2a}\cdot b_{2b}\cdot(\operatorname{cov}[V_a,V_b])^2+12\cdot b_{2a}\cdot b_{4b}\cdot\operatorname{var}[V_b]\cdot(\operatorname{cov}[V_a,V_b])^2+3\cdot b_{3a}\cdot b_{1b}\cdot\operatorname{var}[V_a]\cdot\operatorname{cov}[V_a,V_b]+b_{3a}\cdot b_{3b}\cdot\left(9\cdot\operatorname{var}[V_a]\cdot\operatorname{var}[V_b]\cdot\operatorname{cov}[V_a,V_b]+6\cdot(\operatorname{cov}[V_a,V_b])^3\right)+12\cdot b_{4a}\cdot b_{2b}\cdot\operatorname{var}[V_b]\cdot(\operatorname{cov}[V_a,V_b])^2+b_{4a}\cdot b_{4b}\cdot\left(3\cdot8\cdot(\operatorname{cov}[V_a,V_b])^4+3\cdot24\cdot\operatorname{var}[V_a]\cdot\operatorname{var}[V_b]\cdot(\operatorname{cov}[V_a,V_b])^2\right)=\widehat{b}_{1a}\cdot\widehat{b}_{1b}\cdot(cv\cdot v_{ab})^2+3\cdot\widehat{b}_{1a}\cdot\widehat{b}_{3b}\cdot(cv\cdot v_b)^2\cdot(cv\cdot v_{ab})^2+2\cdot\widehat{b}_{2a}\cdot\widehat{b}_{2b}\cdot(cv\cdot v_{ab})^4+12\cdot\widehat{b}_{2a}\cdot\widehat{b}_{4b}\cdot(cv\cdot v_b)^2\cdot(cv\cdot v_{ab})^4+3\cdot\widehat{b}_{3a}\cdot\widehat{b}_{1b}\cdot(cv\cdot v_a)^2\cdot(cv\cdot v_{ab})^2+\widehat{b}_{3a}\cdot\widehat{b}_{3b}\cdot\left(9\cdot(cv\cdot v_a)^2\cdot(cv\cdot v_b)^2\cdot(cv\cdot v_{ab})^2+6\cdot(cv\cdot v_{ab})^6\right)+12\cdot\widehat{b}_{4a}\cdot\widehat{b}_{2b}\cdot(cv\cdot v_a)^2\cdot(cv\cdot v_{ab})^4+\widehat{b}_{4a}\cdot\widehat{b}_{4b}\cdot\left(3\cdot8\cdot(cv\cdot v_{ab})^8+3\cdot24\cdot(cv\cdot v_a)^2\cdot(cv\cdot v_b)^2\cdot(cv\cdot v_{ab})^4\right)=\widehat{b}_{1a}\cdot\widehat{b}_{1b}\cdot(v_a)^{\lambda-1}\cdot(v_a)^{\lambda-1}\cdot(v_{ab})^2+3\cdot\widehat{b}_{1a}\cdot\widehat{b}_{3b}\cdot(v_a)^{\lambda-1}\cdot(v_a)^{\lambda-1}\cdot(v_{ab})^2+2\cdot\widehat{b}_{2a}\cdot\widehat{b}_{2b}\cdot(v_a)^{\lambda-2}\cdot(v_a)^{\lambda-2}\cdot(v_{ab})^4+12\cdot\widehat{b}_{2a}\cdot\widehat{b}_{4b}\cdot(v_a)^{\lambda-2}\cdot(v_a)^{\lambda-2}\cdot(v_{ab})^4+3\cdot\widehat{b}_{3a}\cdot\widehat{b}_{1b}\cdot(v_a)^{\lambda-1}\cdot(v_a)^{\lambda-1}\cdot(v_{ab})^2+9\cdot\widehat{b}_{3a}\cdot\widehat{b}_{3b}\cdot(v_a)^{\lambda-1}\cdot(v_a)^{\lambda-1}\cdot(v_{ab})^2+6\cdot\widehat{b}_{3a}\cdot\widehat{b}_{3b}\cdot(v_a)^{\lambda-3}\cdot(v_a)^{\lambda-3}\cdot(v_{ab})^6$$

$$+ 12 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{2b} \cdot (v_a)^{\lambda-2} \cdot (v_a)^{\lambda-2} \cdot (v_{ab})^4 + 3 \cdot 8 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{4b} \cdot (v_a)^{\lambda-4} \cdot (v_a)^{\lambda-4} \cdot (v_{ab})^8 + 3 \cdot 24 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{4b} \cdot (v_a)^{\lambda-2} \cdot (v_a)^{\lambda-2} \cdot (v_{ab})^4 = c_1^4 \cdot (v_a)^{\lambda-1} \cdot (v_b)^{\lambda-1} \cdot (v_{ab})^2 + c_2^4 \cdot (v_a)^{\lambda-2} \cdot (v_b)^{\lambda-2} \cdot (v_{ab})^4 + c_3^4 \cdot (v_a)^{\lambda-3} \cdot (v_b)^{\lambda-3} \cdot (v_{ab})^6 + c_4^4 \cdot (v_a)^{\lambda-4} \cdot (v_b)^{\lambda-4} \cdot (v_{ab})^8, \tag{D.1}$$

where

$$\begin{aligned} c_1^4 &= \widehat{b}_{1a} \cdot \widehat{b}_{1b} + 3 \cdot \widehat{b}_{1a} \cdot \widehat{b}_{3b} + 3 \cdot \widehat{b}_{3a} \cdot \widehat{b}_{1b} + 9 \cdot \widehat{b}_{3a} \cdot \widehat{b}_{3b}, \\ c_2^4 &= 2 \cdot \widehat{b}_{2a} \cdot \widehat{b}_{2b} + 12 \cdot \widehat{b}_{2a} \cdot \widehat{b}_{4b} + 12 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{2b} + 3 \cdot 24 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{4b}, \\ c_3^4 &= 6 \cdot \widehat{b}_{3a} \cdot \widehat{b}_{3b}, \\ c_4^4 &= 3 \cdot 8 \cdot \widehat{b}_{4a} \cdot \widehat{b}_{4b}. \end{aligned} \tag{D.2}$$

A condition of $a = b$ in $\text{cov}[t_a(V_a), t_b(V_b)]$ yields

$$\text{var}[t_a(V_a)] = \left(\widehat{c}_1^4 + \widehat{c}_2^4 + \widehat{c}_3^4 + \widehat{c}_4^4\right) \cdot (v_a)^{2\lambda} \quad \forall a \in A, \tag{D.3}$$

where

$$\begin{aligned} \widehat{c}_1^4 &= \left(\widehat{b}_{1a} + 3\widehat{b}_{3a}\right)^2, \\ \widehat{c}_2^4 &= 2 \cdot \left(\widehat{b}_{2a} + 6 \cdot \widehat{b}_{4a}\right)^2, \\ \widehat{c}_3^4 &= 6 \cdot \left(\widehat{b}_{3a}\right)^2, \\ \widehat{c}_4^4 &= 3 \cdot 8 \cdot \left(\widehat{b}_{4a}\right)^2. \end{aligned} \tag{D.4}$$

E. The Proofs for (38)

Firstly, we will prove $\partial \widehat{t}_a(v_a)/\partial v_a > 0$. By differentiating both sides of (19) with respect to v_a, we obtain

$$\begin{aligned} \frac{\partial E[t_a(V_a)]}{\partial v_a} &= \sum_{k=0}^{m} \frac{\partial b_{ka}}{\partial v_a} \cdot E\left[(V_a - v_a)^k\right]\Big|_{V_a=v_a} + \sum_{k=0}^{m} b_{ka} \cdot \left.\frac{\partial E\left[(V_a - v_a)^k\right]}{\partial V_a}\right|_{V_a=v_a} \\ &= \frac{\partial b_{0a}}{\partial v_a} + \sum_{k=1}^{m} b_{ka} \cdot k \cdot E\left[(V_a - v_a)^{k-1}\right]\Big|_{V_a=v_a} \\ &= \frac{\partial b_{0a}}{\partial v_a} + b_{1a} > 0 \quad \forall a \in A, \end{aligned} \tag{E.1}$$

where

$$\begin{aligned} \frac{\partial b_{0a}}{\partial v_a} &= \frac{\partial t_a(v_a)}{\partial v_a}, \\ b_{1a} &= \widehat{\kappa}_a. \end{aligned} \tag{E.2}$$

Therefore,

$$\frac{\partial \widehat{t}_a(v_a)}{\partial v_a} = \frac{\partial E[t_a(V_a)]}{\partial v_a} > 0. \tag{E.3}$$

Next, we will prove $\partial \sigma_a^2(v_a)/\partial v_a > 0$. By assuming $V_a = V_b$ in (20) and by differentiating both sides of (20) with respect to v_a, we obtain

$$\begin{aligned} \frac{\partial\, \text{var}[t_a(V_a)]}{\partial v_a} &= \frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{la} \cdot E\left[(V_a - v_a)^{k+l}\right] - (E[t_a(V_a)])^2}{\partial v_a} \\ &= \sum_{k=0}^{m} \sum_{l=0}^{m} \frac{\partial b_{ka}}{\partial v_a} \cdot b_{la} \cdot E\left[(V_a - v_a)^{k+l}\right]\Big|_{V_a=v_a} + \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot \frac{b_{la}}{\partial v_a} \cdot E\left[(V_a - v_a)^{k+l}\right]\Big|_{V_a=v_a} + \sum_{k=0}^{m} \sum_{l=0}^{m} (k+l) \cdot b_{ka} \cdot b_{la} \\ &\cdot E\left[(V_a - v_a)^{k+l-1}\right]\Big|_{V_a=v_a} - 2 \cdot E[t_a(V_a)] \cdot \left.\frac{\partial E[t_a(V_a)]}{\partial V_a}\right|_{V_a=v_a} = \frac{\partial b_{0a}}{\partial v_a} \cdot b_{0a} + b_{0a} \cdot \frac{\partial b_{0a}}{\partial v_a} + 2 \cdot b_{0a} \\ &\cdot b_{1a} - 2b_{0a} \cdot b_{1a} = 2 \cdot \frac{\partial b_{0a}}{\partial v_a} \cdot b_{0a} > 0 \quad \forall a \in A. \end{aligned} \tag{E.4}$$

Therefore,

$$\frac{\partial \sigma_a^2(v_a)}{\partial v_a} = \frac{\partial\, \text{var}[t_a(V_a)]}{\partial v_a} > 0. \tag{E.5}$$

Finally, we will prove $\partial \sigma_{ab}(v_{ab}, v_a, v_b)/\partial v_{ab} > 0$. By differentiating both sides of (20) with respect to v_{ab}, we obtain

$$\begin{aligned} \frac{\partial\, \text{cov}[t_a(V_a), t_b(V_b)]}{\partial v_{ab}} &= \frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right] - E[t_a(V_a)] \cdot E[t_b(V_b)]}{\partial v_{ab}} \\ &= \frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]}{\partial v_{ab}} \end{aligned}$$

$$= \frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]}{\partial v_a} \cdot \frac{\partial v_a}{\partial v_{ab}} + \frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]}{\partial v_b} \cdot \frac{\partial v_b}{\partial v_{ab}}. \quad (E.6)$$

It is shown that $\mathrm{cov}[t_a(V_a), t_b(V_b)] > 0$ if $v_{ab} > 0$ and $\mathrm{cov}[t_a(V_a), t_b(V_b)] = 0$ otherwise. Thus, we will consider only the condition of $v_{ab} > 0$ that is equivalent to $\partial v_a/\partial v_{ab} = \partial v_b/\partial v_{ab} = 1$ in the following discussion. The first term of the right-hand side of the equation above can be calculated as follows:

$$\begin{aligned} &\frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]}{\partial v_a} \\ &= \sum_{k=0}^{m} \sum_{l=0}^{m} \frac{\partial b_{ka}}{\partial v_a} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]\Big|_{V_a=v_a, V_b=v_b} \\ &\quad + \sum_{k=1}^{m} \sum_{l=0}^{m} k \cdot b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^{k-1} \cdot (V_b - v_b)^l\right]\Big|_{V_a=v_a, V_b=v_b} \\ &= \frac{\partial b_{0a}}{\partial v_a} \cdot b_{0b} + b_{1a} \cdot b_{0b} > 0. \end{aligned} \quad (E.7)$$

In the same manner, the following relationship is also obtained:

$$\frac{\partial \sum_{k=0}^{m} \sum_{l=0}^{m} b_{ka} \cdot b_{lb} \cdot E\left[(V_a - v_a)^k \cdot (V_b - v_b)^l\right]}{\partial v_b} > 0. \quad (E.8)$$

Therefore,

$$\frac{\partial \sigma_{ab}(v_{ab}, v_a, v_b)}{\partial v_{ab}} = \frac{\partial \mathrm{cov}\left[t_a(V_a), t_b(V_b)\right]}{\partial v_{ab}} > 0. \quad (E.9)$$

By using the results shown above, the following two conditions are obtained:

$$\begin{aligned} &\frac{\partial g_{ab}(v_{ab}, v_a, v_b)}{\partial v_{ab}} = 2 \cdot \omega \cdot \frac{\partial \sigma_{ab}(v_{ab}, v_a, v_b)}{\partial v_{ab}} > 0 \quad \forall a, b \in A, \\ &\frac{\partial g_a(v_a)}{\partial v_a} = \frac{\partial \hat{t}_a(v_a)}{\partial v_a} + \omega \cdot \frac{\partial \sigma_a^2(v_a)}{\partial v_a} > 0 \quad \forall a \in A. \end{aligned} \quad (E.10)$$

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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