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Thermal Buoyancy Effects in Rotating Non-Isothermal Flows

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The present paper is concerned with the non-isothermal flow mechanisms in rotating systems with emphasis on the rotation-induced thermal buoyancy effects stemming from the coexistence of rotational body forces and the non-uniformity of the fluid temperature field. Non-isothermal flow in rotating ducts of radial and parallel modes and rotating cylindrical configurations, including rotating cylinders and disk systems, are considered. Previous investigations closely related to the rotational buoyancy are surveyed. The mechanisms of the rotation-induced buoyancy are manifested by the author's recent theoretical results and scaling analyses pertaining to the rotation-induced buoyancy in rotating ducts and two-disk systems. Finally, the open issues for future researches in this area are proposed.

Keywords Non-isothermal flow, Rotation-induced buoyancy, Scaling analysis, Rotating ducts, Rotating cylinders, Rotating disks

INTRODUCTION

Rotating flow is an important branch of fluid dynamics and is full of complex physics. In practical applications, rotating thermal flows occur frequently in a variety of rotating machinery. For a long time this intriguing branch of thermal flows has attracted much attention of researchers. In past decades, there have appeared several books and review articles on hydrodynamic and heat transfer characteristics of rotating flows. For example, Greenspan (1968) published a monograph on the theory of rotating fluids. Zandbergen and Dijkstra (1987) presented a review of swirling flow; Morris (1981) and Owen and Rogers (1989) provided up-to-date (through the year of publication) information of flow and heat transfer in rotating ducts and rotor-stator systems, respectively. Hwang and Soong (1992) surveyed literature of convective heat transfer in radial and parallel rotating ducts and dealt with the experimental techniques for heat transfer measurement. A more recent review article by Yang et al. (1994) comprehensively discussed fluid flow and heat transfer characteristics inside rotating channels.

Different from the previous reviewed works mentioned above, the present work confines itself to the thermal buoyancy effects in rotating non-isothermal flows, in which the coexistence of the rotational forces and fluid temperature gradients leads to the emergence of rotation-induced buoyancy effects. Since the rotational forces are spatially varying with the local flow information, the buoyancy effects induced are obviously more sophisticated than the gravitational buoyancy resulted from constant gravity in a conventional natural/mixed convection flow. The coupling nature of the thermal flow phenomena with the rotational buoyancy is very influential to transport phenomena in rotating systems.

The present paper deals with non-isothermal flow mechanisms in rotating systems with emphasis on the rotation-induced thermal buoyancy effects. Non-isothermal flow in rotating ducts of radial and parallel modes and rotating cylindrical configurations, including rotating cylinders and disk systems, are considered. Previous investigations closely related to rotational buoyancy are surveyed. The mechanisms of the rotation-induced buoyancy are manifested by reorganized results of the author's previous theoretical works and scaling analyses pertaining to the influences of the rotation-induced buoyancy in rotating ducts and two-disk systems.

ROTATION-INDUCED BUOYANCY

In a non-isothermal flow field, fluid density varies spatially with local temperature. As the density or temperature gradient is imposed normal to the body force in the field, a buoyancy-driven fluid motion would emerge and play a significant role in the transport phenomena of the flow field. As an illustrative example, Fig. 1 shows non-isothermal fluid flows between two boundaries

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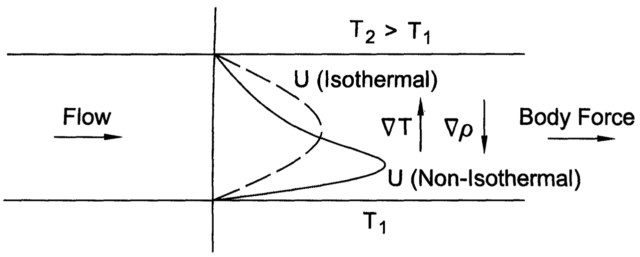

FIGURE 1
Thermal buoyancy effect with the presence of body force and temperature/density gradient.

at different wall temperatures and $T_2 > T_1$. The fluid near the cold wall is denser than that adjacent to the hot wall. Therefore, the buoyancy due to body force produces an assisting effect on the cooler fluid but an opposing effect on the hotter fluid. To explore the thermal buoyancy in the flow field, the baseline case of incompressible fluid flow is employed. Governing equations formulated with respect to a reference frame rotating at the same rate as the system, say $\mathbf{\Omega_1}$, are written as

$$\nabla \bullet \mathbf{V} = 0, \quad [1]$$

$$\rho(\mathbf{V} \bullet \nabla)\mathbf{V} + \rho\mathbf{\Omega_1} \times (\mathbf{\Omega_1} \times \mathbf{R}) + 2\rho\Omega_1 \times \mathbf{V} = -\nabla p + \rho\,\mathbf{g} + \mu\nabla^2\mathbf{V}, \quad [2]$$

$$(\mathbf{V} \bullet \nabla)T = \alpha\nabla^2 T, \quad [3]$$

where $\rho\,\mathbf{g}$ is the gravitational force, and $\rho\mathbf{\Omega_1} \times (\mathbf{\Omega_1} \times \mathbf{R})$ and $2\rho\mathbf{\Omega_1} \times \mathbf{V}$, respectively, are the centrifugal and Coriolis forces owing to *system rotation*. In a cylindrical problem, centrifugal and Coriolis forces due to curvilinear motion of the fluid, $\rho V^2/R$ and $\rho UV/R$ included implicitly in the inertial term $\rho(\mathbf{V} \bullet \nabla)\mathbf{V}$, are also regarded as the body forces for buoyancy (Soong and Chyuan, 1998a).

To explore thermal buoyancy effects, there are two approaches to constructing theoretical models. One is to use a full compressible version of Navier-Stokes equations, in which the density variation with temperature and pressure is automatically taken into account. The thermal buoyancy effect in a compressible flow analysis is usually measured by $\Delta\rho/\rho_r$ or $\Delta T/T_r$. Another approach is to invoke a Boussinesq approximation in the incompressible or constant-property flow model, in which the buoyancy effect is usually measured by Grashof, Rayleigh, or thermal Rossby number. In some early literature, e.g. Trefethen (1955), it was intended to express the effects of rotation by an analogy to conventional non-rotating natural convection or to curved duct flow. It is not simple, however. By inspecting the rotational force terms mentioned above, it is obvious that, besides the density variation, the rotational body forces vary with the local position $\mathbf{R}$ and the local fluid velocity $\mathbf{V}$. The situation is definitely more complicated than the buoyancy produced in a constant gravitational force field.

To facilitate an analysis of rotation-induced buoyancy, the assumption of Boussinesq fluids is useful and usually implemented in analyses. By invoking a Boussinesq approximation, density variation is considered only for the body force terms. A linear density-temperature relation, $\rho = \rho_r[1 - \beta(T - T_r)]$, is employed, where the subscript r denotes a reference state and $\beta = -1/\rho_r(\partial\rho/\partial T)_P$ is the thermal expansion coefficient. The reference state is of conditions $\mathbf{V} \equiv \mathbf{0}$, $T = T_r$, $\rho = \rho_r$ and $P = P_r$. At the reference state, the original momentum equation can be reduced to $\nabla P_r/\rho_r = -\mathbf{\Omega_1} \times (\mathbf{\Omega_1} \times \mathbf{R}) + \mathbf{g}$, which is the conservative part of the centrifugal and gravitational forces in flow field. Finally, a vector form of the Navier-Stokes equation with explicit buoyancy terms can be formulated, viz.,

$$(\mathbf{V} \bullet \nabla)\mathbf{V} = -\nabla p_d/\rho_r + \nu\nabla^2\mathbf{V} + \beta(T - T_r)\mathbf{\Omega_1} \times (\mathbf{\Omega_1} \times \mathbf{R}) - 2[1 - \beta(T - T_r)]\mathbf{\Omega_1} \times \mathbf{V} + \beta(T - T_r)\mathbf{g}, \quad [4]$$

where $\mathbf{R}$ is the position vector, and $P_d = P - P_r$ denotes the pressure departure from the reference state. For rapidly rotating systems, the gravity term can be neglected for the relatively small effect. Taking curl of equation (4) leads to the vorticity transport equation,

$$(\mathbf{V} \bullet \nabla)\boldsymbol{\xi} = (\boldsymbol{\xi} \bullet \nabla)\mathbf{V} + \nu\nabla^2\boldsymbol{\xi} + \nabla \times [\beta(T - T_r)\mathbf{\Omega_1} \times (\mathbf{\Omega_1} \times \mathbf{R}) - 2\nabla \times \{[1 - \beta(T - T_r)]\mathbf{\Omega_1} \times \mathbf{V}\} \quad [5]$$

The last two terms are vorticity generation by centrifugal buoyancy and Coriolis effect, respectively. It is noted that for isothermal flow the centrifugal force becomes a conservative force and has no contribution to the vorticity generation; whereas the Coriolis force works no matter if the flow is isothermal or not.

NON-ISOTHERMAL FLOWS IN ROTATING DUCTS

Literature Survey

Rotating Ducts of Radial Mode

Considering the orientation, the rotation of the duct can be of the radial, parallel, slant, and axial modes. The duct could be straight, curved, spiral, and of various cross-section shapes. In the monograph (Morris, 1981) and the review articles (Hwang and Soong, 1992; Yang et al., 1994), a comprehensive discussion of the effects of rotational forces can be found. There is no intent to perform an exhaustive survey of the subject but only to mention the previous studies closely related to the rotation-induced buoyancy effects.

Schmidt's work (1951) on heat transfer in a rapidly rotating turbine blade might be the earliest study dealing with centrifugal buoyancy. Morris and Ayhan (1979, 1982) first proposed experimental evidence of the significance of centrifugal buoyancy in radially rotating ducts. Later, with a model of a radially rotating pipe, Siegel (1985) performed a perturbation analysis to investigate the centrifugal buoyancy effect on fully developed flow and heat transfer in the ducts. Soong and Hwang (1990, 1993) developed a similarity analysis of mixed convection in a rotating flat-channel with and without wall transpiration. For triangular ducts, Clifford et al. (1985) measured the heat transfer performance at various rotation rates. The buoyancy effects on both radially outward and inward main flows were considered in some investigations (Iskakov and Trusin, 1985; Harasgama and Morris, 1988; Guidez, 1989; Wagner et al., 1991a). By changing the wall-to-fluid temperature difference but with other conditions fixed, Soong et al. (1991) explored the influences of centrifugal buoyancy on heat transfer performance. All of the studies demonstrated that, in laminar and transitional flows, centrifugal buoyancy degrades heat transfer performance in a radially outward flow and enhances it in a radially inward flow. As the Reynolds number increases, however, the adverse effect of the centrifugal buoyancy on heat transfer in radially outward flow is alleviated, and the situation may even be reversed in turbulent flows.

Centrifugal buoyancy effects on heat transfer in more complicated situations have been studied in a number of investigations, e.g. uneven wall heating conditions (Han and Zhang, 1992), a multi-pass duct with and without rib turbulators (Wagner et al., 1991b; Johnson et al., 1994; Fann et al., 1994; Ekkad and Han, 1997), a serpentine duct with compressed air flow (Hwang et al., 2001), etc. Numerical computations were carried out for fully developed flow and heat transfer (Hwang and Jen, 1990), a developing thermal flow in entrance region (Jen and Lavine, 1992; Jen et al., 1992; Yan and Soong, 1995), a flow with wall transpiration effects (Yan, 1994, 1995) and radiation effects (Yan et al., 1999). Dutta et al. (1996) used previous experimental and numerical data to explore buoyancy induced flow separation in radially rotating ducts with radially outward and inward flows. Most recently, by using a large eddy simulation technique, Murata and Mochizuki (2001) studied aiding and opposing effects of the centrifugal buoyancy on turbulent heat transfer in a radially rotating square duct with transverse or angled rib turbulators.

Rotating Ducts of Parallel Mode

For rotating ducts of parallel mode, Morris' analysis (1965) for low rates of heating was the first to consider centrifugal buoyancy. Mori and Nakayama (1967) and Nakayama (1968), respectively, employed a boundary layer approach to analyze laminar and turbulent convection with centrifugal buoyancy in a circular pipe. An experiment with parameter ranges covering laminar and turbulent flow regimes in a parallel rotating pipe was conducted (Humphreys et al., 1967) and their data corroborated the heat transfer enhancement due to centrifugal buoyancy. Skiadarresis and Spalding (1976) and Majumdar et al. (1976) computed laminar and turbulent heat transfer in circular and square ducts by finite difference solutions of a model with rotation-induced buoyancy. Based on their numerical predictions, they claimed that the rotation-induced buoyancy effects are more remarkable in laminar cases and the Coriolis effect is negligibly small in the parameter range they studied. With consideration of compressibility, Neti et al. (1985) investigated convective heat transfer in a parallel rotating rectangular duct of aspect ratio 2 by numerical computations. Although the density variation was included, only limited data on the centrifugal buoyancy effects were offered. Developing heat transfer rates and pressure drop with centrifugal buoyancy in a rotating duct of parallel mode were measured by Levy et al. (1986). Mahadevappa et al. (1996) performed computations for fully developed flows in rectangular and elliptic ducts and found that the friction factor and heat transfer in an elliptic duct are both higher than that in a rectangular duct. By imposing various thermal boundary conditions, Soong and Yan (1999) investigated the axial evolution of secondary motion and its effects on heat transfer performance during the consideration of centrifugal buoyancy.

Rotating Curved Ducts

Recently, mixed convection and flow transitions in a rotating curved circular tube were analyzed by a regular perturbation method (Wang and Cheng, 1996). For the rotating curved square duct, buoyancy-force-driven flow mode transition (Wang, 1997) and rotational instability (Wang, 1999) were both investigated by finite volume solutions. In this flow configuration, centrifugal buoyancy is also exerted in a cross flow plane similar to that in a parallel mode.

Scaling Analysis of Rotational Buoyancy in Duct Flows

The non-isothermal flow considered is steady and laminar in heated rotating ducts at a constant angular speed $\mathbf{\Omega}_1$ about an axis normal or parallel to the duct axis. In a most recent work, Soong and Yan (2001) performed a scaling analysis by using the unified governing equations for the rotating ducts of radial and parallel modes with the coordinate systems defined in Figs. 2 and 3, respectively. The Cartesian coordinate system attaches at the center of the inlet plane which is a distance Z_o from the axis of rotation. The eccentricity E of the duct is define as $E = Z_o + L/2$, where L is the duct length. The inlet mean velocity V_m is $V_m = W_o$ for orthogonal or radial mode and $V_m = V_o$ for parallel mode, and the inlet fluid lies at a uniform temperature T_o. Assume the gravitational effect is negligibly small in a rapidly rotating case. By using the definitions of the rate vector $\mathbf{\Omega}_1 = \Omega_1\mathbf{j}$, the velocity vector $\mathbf{V} = U\mathbf{i} + V\mathbf{j} + W\mathbf{k}$, and the position vector $\mathbf{R} = X\mathbf{i} + Y\mathbf{j} + (Z + Z_o)\mathbf{k}$ with respect to the axis of rotation for calculating the centrifugal force, unified Navier-Stokes/Boussinesq equations for thermal flow in the rotating duct of radial and parallel modes can be depicted in the following scalar form:

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} + \frac{\partial W}{\partial Z} = 0, \qquad [6]$$

$$U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} + W\frac{\partial U}{\partial Z} = \nu\nabla^2 U - \frac{1}{\rho_r}\frac{\partial P_d}{\partial X} - \beta(T - T_r)\Omega_1^2 X - 2[1 - \beta(T - T_r)]\Omega_1 W, \qquad [7]$$

$$U\frac{\partial V}{\partial X} + V\frac{\partial V}{\partial Y} + W\frac{\partial V}{\partial Z} = \nu\nabla^2 V - \frac{1}{\rho_r}\frac{\partial P_d}{\partial Y}, \qquad [8]$$

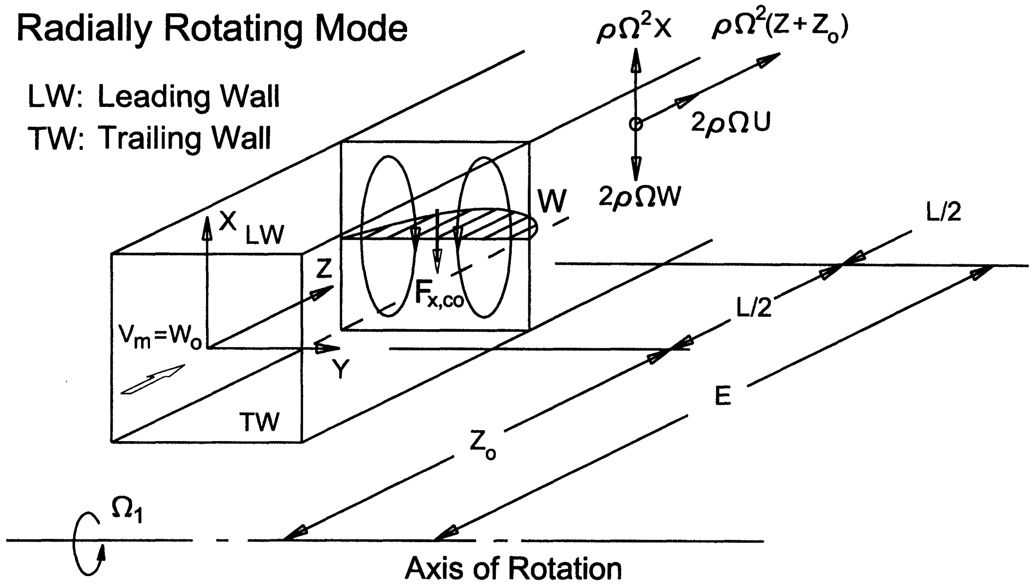

FIGURE 2
Rotating duct of radial mode.

$$U\frac{\partial W}{\partial X}+V\frac{\partial W}{\partial Y}+W\frac{\partial W}{\partial Z}=\nu\nabla^2W-\frac{1}{\rho_r}\frac{\partial P_d}{\partial Z}$$
$$-\beta(T-T_r)\Omega_1^2(Z+Z_o)+2[1-\beta(T-T_r)]\Omega_1U, \quad [9]$$

$$U\frac{\partial T}{\partial X}+V\frac{\partial T}{\partial Y}+W\frac{\partial T}{\partial Z}=\alpha\nabla^2T. \quad [10]$$

Let the duct have a large enough eccentricity E, i.e. $E \sim Z_o \gg H \sim D$. The Z-component of the centrifugal buoyancy is dominant since

$$F_{z,c}/F_{x,c} = \beta(T-T_r)\Omega^2(Z+Z_o)/\beta(T-T_r)\Omega^2X$$
$$\sim (Z+Z_o)/X \sim \varepsilon \gg 1, \quad [11]$$

where $\varepsilon \equiv E/D$ is the dimensionless eccentricity of the duct.

Rotating Ducts of Radial Mode

The mean velocity V_m of the main flow is the inlet velocity W_o, i.e. $V_m = W_o$. The characteristic x-component of velocity on a cross-flow plane is $U_c < W_o$, then

$$F_{z,co}/F_{x,co} \sim U/W = U_c/W_m. \quad [12]$$

In general, main flow is stronger than the cross flow, i.e. $U_c/V_m < 1$, and thus the Coriolis effect in the X-direction is more important. Since the secondary flow is characterized by the fluid motion in the central region ($Y \approx 0$) of the cross flow plane, influences of pressure gradient and viscous forces in the cross flow plane are neglected. Therefore, the inertial force is balanced with the summation of the Coriolis force and the centrifugal buoyancy, i.e.

$$U_c^2/X_c \sim \left|-2(1-\beta\Delta T_c)\Omega_1W_c - \beta\Delta T_c\Omega_1{}^2X_c\right|. \quad [13]$$

In a radial duct, the characteristic velocity W_c is selected as the main flow velocity, $W_c = V_m$. With the present definition of a thermal Rossby number, $B \equiv \beta\Delta T_c$, and Rotation number, $Ro \equiv \Omega_1D/V_m$, one has the velocity of the secondary flow in cross-flow plane is

$$U_c/V_m \sim [2(1-B)Ro + BRo^2]^{1/2}. \quad [14]$$

Considering the validity of the Boussinesq approximation, B has to be small enough, say $B \leq O(10^{-1})$. Using a typical case of

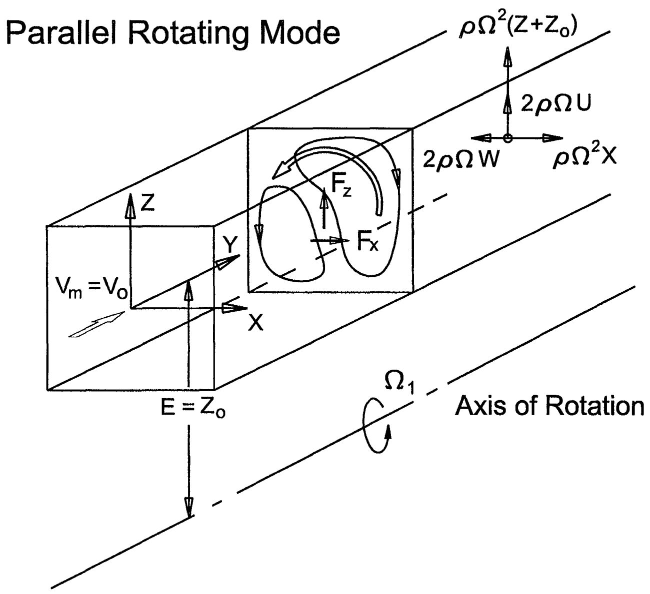

FIGURE 3
Rotating duct of parallel mode.

$Ro \sim O(10^{-1})$ and $B \sim O(10^{-1})$ as an illustrative example, one has

$$U_c/V_m \sim 0.4 \sim O(10^{-1}), \quad [15]$$

in which the Coriolis effect is the major contribution to the generation of the secondary motion in cross section of the duct and, for the parameter ranges considered above, the secondary flow velocity can be estimated by

$$U_c/V_m \sim \sqrt{2(1-B)Ro}. \quad [16]$$

The influence of the centrifugal buoyancy $\beta(T - T_r)\Omega_1^2(Z + Z_o) \sim BE\Omega_1^2$ in a radially rotating duct is mainly on the main flow direction and modifies the longitudinal flow and heat transfer directly.

Rotating Duct of Parallel Mode

For the parallel mode, the main-flow mean velocity V_m is the entrance velocity V_o. In the entrance region, the cross flow is not yet built up, i.e. $U, W \ll V_m$, the Coriolis effects are small. The centrifugal buoyancy is the major driving force in the cross flow plane. The order of the inertial term $W\partial W/\partial Z \sim W_c^2/D$ and the centrifugal buoyancy term $\beta(T - T_r)\Omega_1^2(Z + Z_o) \sim \beta\Delta T_c\Omega_1^2 E$ are comparable. Therefore, we have

$$W_c/V_m \sim Ro\sqrt{B\varepsilon}. \quad [17]$$

For the case of $B \sim O(10^{-1})$, $Ro \sim O(10^{-1})$ and $\varepsilon \sim O(10)$, the cross flow velocity is of the order $W_c/V_m \sim O(10^{-1})$. Once the secondary motion emerges, with the Coriolis force in the cross flow plane, the secondary flow velocity in the Z-direction can be expressed as

$$W_c/V_m \sim BRo^2\varepsilon(U_c/V_m)^{-1} + 2(1-B)Ro. \quad [18]$$

From the X-momentum equation, the scale of U_c/V_m can be derived as

$$U_c/V_m \sim |BRo^2 + 2(1-B)Ro(W_c/V_m)|^{1/2}. \quad [19]$$

Then Eq. (18) becomes

$$W_c/V_m \sim |BRo^2\varepsilon[BRo^2 + 2(1-B)Ro(W_c/V_m)]^{-1/2} + 2(1-B)Ro|. \quad [20]$$

For a typical case with $B = 0.1$ and $Ro \sim 0.1$, the estimate of the characteristic velocity of the secondary flow in cross-flow plane is

$$W_c/V_m \sim 0.23 \sim O(10^{-1}). \quad [21]$$

In the above scaling analysis, it is demonstrated that, in a rotating duct of parallel mode, the centrifugal buoyancy effect plays a major role in the emergence of the secondary flow in the entrance region. Once the secondary flow is built up, the Z-component of the Coriolis force, $2\rho_r\Omega_1 U$, becomes significant to the contribution of the secondary motion. Additionally, secondary flow in the parallel mode is more complicated than that in radial mode, due to resultant forces F_x and F_z in Fig. 3, by which the secondary vortices may become asymmetric and irregular.

Vorticity Generation in Rotating Ducts

Rotational forces usually generate vorticity in flow fields. With consideration of Coriolis buoyancy, the dimensionless form of centrifugal and Coriolis generation of vorticity in a recent analysis (Soong and Yan, 2001) can be expressed as

$$B\varepsilon\,\nabla \times [\boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})\theta] = B\{[-(z + z_o)\partial\theta/\partial y]\mathbf{i} + [(z + z_o)\partial\theta/\partial x - x\partial\theta/\partial z]\mathbf{j} + x(\partial\theta/\partial y)\mathbf{k}\}, \quad [22]$$

and

$$-2Ro\nabla \times [(1 - B\theta)\boldsymbol{\omega} \times \mathbf{v}] = -2Ro[(1 - B\theta)\partial\mathbf{v}/\partial y - B(\partial\theta/\partial y)\mathbf{v}],$$

where $\boldsymbol{\omega} \equiv \boldsymbol{\Omega}_1/|\boldsymbol{\Omega}_1| = \mathbf{j}$ is the unit vector of the rotating speed and the lower case letters denote dimensionless variables defined in the nomenclature. It is observed that the vorticity generation due to centrifugal buoyancy and Coriolis effects depend strongly on the development of the velocity and temperature fields. In a duct of uniform wall temperature, temperature gradients tend to vanish at the fully-developed condition, while all vorticity generation due to temperature nonuniformity diminishes. For a duct rotating in a parallel mode, the contribution of Coriolis force vanishes, since velocity gradients in the main flow (y-) direction all approach zero in a fully developed flow region. More detailed discussion on secondary vortices in rotating ducts can be found in the author's recent works (Soong and Yan, 1999, 2001).

NON-ISOTHERMAL FLOWS IN ROTATING CYLINDRICAL CONFIGURATIONS

The so-called rotating cylindrical configurations include rotating cylindrical containers and rotating disk systems. Figure 4 shows a rotating cylinder of radius R_o and height S. The rotating disk systems can be as simple as a single disk or as complex as two co-axial disks rotating at different rates shown in Fig. 5. Geometrically, for a large radius-to-height aspect ratio, $R_o/S \gg 1$, the side-wall effect could be insignificant and the problem is likely to be a special case of a two-disk system in Fig. 5 with $\Omega_1 = \Omega_2$. Actually, that is not always true because of the presence of the side wall. Therefore, in the following literature survey, rotational convection in cylinders and disk systems are given as two separate sections.

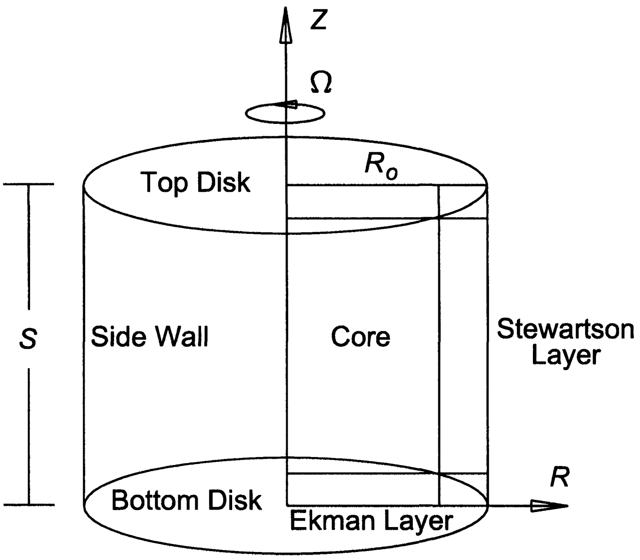

FIGURE 4
Rotating cylinder.

Literature Survey

Rotating Cylinders

Ostrach and Braum (1958) first studied centrifugal-driven convection in a rotating container with fluid heated differentially in the vertical. Barcilon and Pedlosky (1967a, 1967b) employed linear analysis to study the stratified fluid motion in a rotating cylinder. In a successive work (1967c), they performed a nonlinear analysis for stratified fluid heated from above in a rapidly rotating cylinder with an insulated side-wall. They concluded that the buoyancy becomes important as $PrBFr^{-1} \gg Ek^{1/2}$, where $Fr \equiv \Omega^2 S/g$ and $Ek \equiv \nu/\Omega S^2$ are Froude number and Ekman number, respectively, and S denotes the height of the cylinder. In, this flow configuration with fluid heated from above, centrifugal force pumping the colder (denser) fluid radially outward along the Ekman layer on the bottom disk, then through the vertical side-wall layers, momentum and heat are transported to the Ekman layer on the upper disk. Barcilon and Pedlosky advocated that this flow configuration is essentially distinct from the infinite disk flow without a vertical boundary.

Homsy and Hudson's study (1969) considered buoyancy arose from centrifugal force and revealed that the side-wall thermal boundary condition (insulated, adiabatic, or perfectly conducting) is very important. The critical parameters governing the centrifugal driven convection are the radius- or diameter-to-height aspect ratio, R_o/S or D/S, and the dimensionless group $PrBEk$. Their successive work (Homsy and Hudson, 1971a) further explored the effects of thermal boundary conditions of side-wall and the bottom and top disks on the heat transfer performance. In a further analysis, Homsy and Hudson (1971b) demonstrated that the Nusselt numbers of the top and bottom disks of a cylinder with a perfectly conducting side wall approach that for the radially unbounded system as the cylinder radius increased. However, it is not the case for the cylinders with an insulated side-wall. Hudson et al. (1978) conducted an experiment for investigation of this centrifugal convection in a rotating cylinder. Guo and Zhang (1992) numerically studied fluid flow and heat transfer in a vertical rotating cylinder with and without gravity. To examine the validity of the similarity solution in finite geometry, Brummell et al. (2000) numerically investigated the effects of various side-wall conditions. Their comparisons showed that the similarity solutions do describe the motions over three-quarters of the inner part of the cylinder provided that the aspect ratio $R_o/S > 10$ or so.

The above-mentioned works are all employed a Boussinesq approximation in the analysis. Non-Boussineq compressibility effects were examined in a series of studies by Sakurai and Matsuda (1974), Matsuda et al. (1976) and Matsuda and Hashimoto (1976). They found that the critical parameter for this case is $(\gamma - 1)PrG_o\, Ek^{-1/2}/4\gamma$, where γ is the ratio of specific heats and G_o is the square of a rotational Mach number. As the above mentioned parameter is larger than unity, the coupled effect of the fluid compressibility and the thermal condition on the bottom and top disks suppresses the flow in the cylinder.

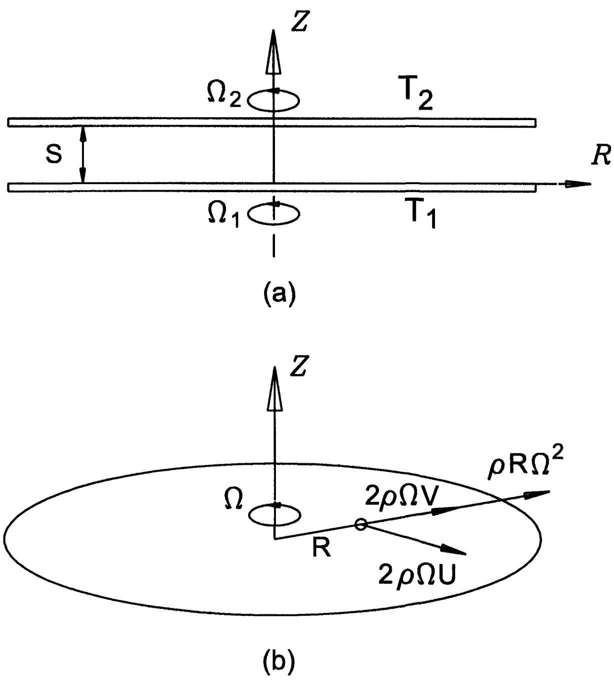

FIGURE 5
Rotating disk systems. (a) Two co-axial disk rotating at different rates; (b) Rotational forces on fluid particles at a rate Ω.

Rotating Disk Systems

Rotating free-disk flow was first analyzed by von Karman (1921), and two theoretical arguments on flow structure between two co-axial disks were proposed (Batchelor, 1951; Sewartson, 1953). After these classic works, a large number of investigations on rotating-disk flows have appeared in the past decades. Most of the studies were concerned with hydrodynamic natures only. Non-isothermal flow effect between two rotating disks can be traced back to the work of Ostrach and Braum (1958). For co-rotation of two disks, they claimed that the Coriolis force would inhibit the radial flow and heat transfer. In the case of co-rotating disks with a large ratio of disk radius to spacing between disks, i.e. $R_o/S \gg 1$, the isothermal fluid rotates at the same rate with the disks and the whole system behaves as a solid body. Once an axial temperature gradient is imposed, the centrifugal buoyancy effect leads to a *natural convection* flow between two disks and the state of solid body rotation cannot be retained anymore. This class of thermally induced flow related to a single disk rotating with its environment was also studied by using compressible boundary layer flow equations (Riley, 1967) as well as by invoking a Bousinnesq approximation (Hudson, 1968a).

Hudson (1968b) performed an asymptotic analysis for two-disk problem with consideration of the rotational buoyancy effects. He disclosed that significant convection occurs as the condition of $PrB^*Re_\Omega{}^{1/2} \gg 1$ and the second order expression of Nusselt number is $\mathrm{Nu} = (PrB^*Re_\Omega{}^{1/2}/2)(1 - PrB^*/2)$, in which $B^* \equiv \beta(T_2 - T_1)/2$ and $Re_\Omega \equiv \Omega_1 S^2/\nu$. Chew (1981) studied non-isothermal flow between two co-rotating infinite disks with a radial temperature distribution $T_w = T_r + CR^n$. It was claimed that, even with the presence of the radial temperature/density gradient in the flow field, there is no rotational buoyancy effect as long as the radial temperature distributions of the two disks are the same. If there is an axial temperature gradient between the disks, however, the rotational buoyancy emerges and alters the flow structure. Chew addressed that the forced, natural, and mixed convection are all the possible flow structures between two heated disks. Soong (1996a), using a similarity model of centrifugal buoyancy (i.e. only buoyancy induced by centrifugal force was considered) to disclose the buoyancy influences on the flow structure and heat transfer characteristics between two disks at different thermal and rotational boundary conditions. Effects of unsteadiness (Soong and Ma, 1995) and Prandtl number (Soong, 1996b) also were studied. To include more complete buoyancy effects, a similarity model with consideration of buoyancy effects stemming from all rotational body forces was developed recently (Soong and Chyuan, 1998a) and the scales of each body force component were analyzed. By using this new model, thermal and solutal buoyancy effects were explored (Soong and Chyuan, 1998b; 1998c).

As to mixed convection between two co-rotating finite disks with effects of radial through-flow and centrifugal buoyancy, numerical computations were conducted for cases of symmetric heating (Soong and Yan, 1993), asymmetric heating and flow reversal phenomena (Soong and Yan, 1994), and a porous disks with wall transpiration (Yan and Soong, 1997). For co-rotating disk systems with fixed and rotating enclosures. Herrero et al. (1994, 1999) explored by numerical computations the thermal flow structure and heat transfer in the a rotating cavity heated differentially in the axial direction. They found that centrifugal buoyancy has a significant influence on the flow structure, especially in symmetry-breaking bifurcation of the flow pattern.

Scaling Analysis of Rotational Buoyancy Effects in Rotating Disk Systems

Figure 5 (a) shows schematically a system of two disks rotating independently. A cylindrical rotating frame (R, Z) is fixed on the disk 1 and rotating with it. Let the disk 1 be a reference and $\Omega_1 > 0$ in the sense of the rotational speed vector. The rate of disk 2, Ω_2, can be positive, zero, and negative for the conditions of co-rotation, rotor-stator, and counter-rotation of the system. Figure 5 (b) draws the rotational forces exerted on a moving fluid particle at a radial distance R and local rotating rate Ω. Under the assumptions of laminar, steady, axisymmetric, and constant-property flow, the governing equations can be depicted as (Soong and Chyuan, 1998a):

$$\frac{\partial}{\partial R}(RU) + \frac{\partial}{\partial Z}(RW) = 0, \qquad [23]$$

$$\rho_r U\frac{\partial U}{\partial R} + \rho_r W\frac{\partial U}{\partial Z} = \rho_r[1 - \beta(T - T_r)]\frac{V^2}{R} - \frac{\partial P_d}{\partial R} - \rho_r\beta(T - T_r)R\Omega_1^2 + \rho_r[1 - \beta(T - T_r)]\,2\Omega_1 V + \mu\left\{\frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial(RU)}{\partial R}\right] + \frac{\partial^2 U}{\partial Z^2}\right\}, \qquad [24]$$

$$\rho_r U\frac{\partial V}{\partial R} + \rho_r W\frac{\partial V}{\partial Z} = -\rho_r[1 - \beta(T - T_r)]\frac{UV}{R} - \rho_r[1 - \beta(T - T_r)]\,2\Omega_1 U + \mu\left\{\frac{\partial}{\partial R}\left[\frac{1}{R}\frac{\partial(RV)}{\partial R}\right] + \frac{\partial^2 V}{\partial Z^2}\right\}, \qquad [25]$$

$$\rho_r U\frac{\partial W}{\partial R} + \rho_r W\frac{\partial W}{\partial Z} = -\frac{\partial P_d}{\partial Z} + \rho_r\beta(T - T_r)g + \mu\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial W}{\partial R}\right) + \frac{\partial^2 W}{\partial Z^2}\right], \qquad [26]$$

$$\rho_r c_p U\frac{\partial T}{\partial R} + \rho_r c_p W\frac{\partial T}{\partial Z} = k\left[\frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial T}{\partial R}\right) + \frac{\partial^2 T}{\partial Z^2}\right]. \qquad [27]$$

By using the variables and parameters: $\eta = Z/S$, $F = U/R\Omega_1$, $G = V/R\Omega_1$, $H = W/(\nu\Omega_1)^{1/2}$, $\theta = (T - T_1)/\Delta T_c$, $Re_\Omega = S^2\Omega/\nu$, and $B = \beta\Delta T_c = \beta(T_2 - T_1)$, this set of equations can be transformed to a similarity form:

$$H'''' = Re_\Omega^{1/2}HH''' + 4Re_\Omega^{3/2}[(1 + G)G' - B(G'\theta + G\theta')] - 2BRe_\Omega^{3/2}\theta' - 2BRe_\Omega^{3/2}G(2G'\theta + G\theta'), \qquad [28]$$

$$G'' = Re_\Omega^{1/2}[HG' - (1 + G)H' + BH'\theta + BGH'\theta], \qquad [29]$$

$$\theta'' = PrRe_\Omega^{1/2}H\theta'. \qquad [30]$$

In the previous work (Soong and Chyuan, 1998a), the buoyancy effects rising from body forces in radial, tangential, and axial directions were examined. The radial components of body forces consist of the centrifugal and Coriolis forces due to disk rotation, $F_{rb,ce}$ and $F_{rb,co}$, and the centrifugal force due to curvilinear motion of the fluids, $F_{rb,cu}$,

$$F_{rb,\mathrm{ce}} = -\rho_r\beta(T - T_r)R\Omega_1^2 \sim -\rho_r\beta\Delta T_c R\Omega_1^2, \quad [31]$$

$$F_{rb,\mathrm{cu}} = -\rho_r\beta(T - T_r)V^2/R \sim -\rho_r\beta\Delta T_c V_c^2/R, \quad [32]$$

$$F_{rb,\mathrm{co}} = -\rho_r\beta(T - T_r)2\Omega_1 V \sim -\rho_r\beta\Delta T_c 2\Omega_1 V_c. \quad [33]$$

Tangential components of buoyancy forces induced by the curvilinear motion of the fluids, $F_{tb,\mathrm{cu}}$ and the Coriolis force, $F_{tb,\mathrm{co}}$ are

$$F_{tb,\mathrm{cu}} = \rho_r\beta(T - T_r)UV/R \sim \rho_r\beta\Delta T_c U_c V_c/R, \quad [34]$$

$$F_{tb,\mathrm{co}} = \rho_r\beta(T - T_r)\,2\Omega_1 U \sim \rho_r\beta\Delta T_c 2\Omega_1 U_c. \quad [35]$$

The only buoyancy force in the axial direction is the gravitational buoyancy. In the following text, assume the gravity effect can be neglected with little effect. Denote the starred quantities as the force components normalized by the radial centrifugal buoyancy $\rho_r\beta\Delta T_c R{\Omega_1}^2$, i.e. $F^* = F/\rho_r\beta\Delta T_c R{\Omega_1}^2$ and consider $V = R(\Omega - \Omega_1)$ with respect to the rotating frame. In terms of the similarity variable for the local tangential velocity, $G = V/R\Omega_1 = (R\Omega - R\Omega_1)/R\Omega_1 = (\Omega - \Omega_1)/\Omega_1$, the scales of the buoyancy components can be rewritten in the following form:

$$\begin{aligned} &F^*_{rb,\,\mathrm{ce}} \sim -1, \\ &F^*_{rb,\,\mathrm{cu}} \sim -[(\Omega - \Omega_1)/\Omega_1]^2 = -G^2, \\ &F^*_{rb,\,\mathrm{co}} \sim -2(\Omega - \Omega_1)/\Omega_1 = -2G, \\ &F^*_{tb,\,\mathrm{co}} \sim 2/Ro, \\ &F^*_{tb,\,\mathrm{cu}} \sim 2[(\Omega - \Omega_1)/\Omega_1]/Ro, = 2G/Ro. \end{aligned} \quad [36]$$

Co-rotating Disks at the Same Rate

For two infinite disks rotating at the same rate and the same sense, the isothermal flow is relatively stationary with respect to the rotating frame and lies at a conductive state, i.e. $\theta = \eta$ and $U = V - R\Omega_1 = W = 0$ or $F = G = H = 0$. As an axial temperature gradient is imposed, the rotation-induced buoyancy emerges and drives the fluid to move radially outward in an Ekman layer on the cold disk and radially inward along the hot disk. The Ekman layer thickness δ_E has the order of $\delta_E/S \sim Re^{-1/2}$, e.g. a correlation $\delta_E/S = 0.782Re^{-1/2}$ for co-rotating disks was proposed by Soong (1996). For weak convection induced by rotational buoyancy, the temperature is approximately at the conductive state, i.e. $\theta \sim \eta$.

As the rigid-body rotation ($U = V = W = 0$) is modified in the presence of the rotational buoyancy, from radial and tangential momentum equations with the inetial force balanced by the buoyancy forces, we have

$$\begin{aligned} \rho_r U\partial U/\partial R &\sim \rho_r\beta(T - T_r)V^2/R + \rho_r\beta(T - T_r)R\Omega_1^2 \\ &\quad + \rho_r\beta(T - T_r)\,2\Omega_1 V \\ &\sim \rho_r B\theta(V^2/R + R\Omega_1^2 + 2\Omega_1 V); \end{aligned} \quad [37]$$

$$\rho_r U\partial V/\partial R \sim \rho_r\beta(T - T_r)UV/R + \rho_r\beta(T - T_r)2\Omega_1 U, \quad [38]$$

or in terms of the thermal Rossby number B and the similarity variables,

$$F \sim \pm\sqrt{B\theta}(1 + G), \quad [39]$$

$$G \sim 2B\theta/(1 - B\theta). \quad [40]$$

Using the values of F and G, the axial velocity gradient can be estimated by $H' = 2Re_\Omega^{1/2}F \sim \pm 2Re_\Omega^{1/2}\sqrt{B\theta}(1 + G)$. Soong and Chyuan (1998a) used a case of $Re_\Omega = 300$, $Pr = 0.7$, and $B = 0.1$ to show that the above estimates of δ_E, F, G, and H' are all of reasonable agreement with the numerical solutions of similarity equations. Furthermore, with $G \sim O(10^{-2})$ in this typical example, it is obvious that, compared to the radial centrifugal buoyancy $\rho_r\beta\Delta T_c R\Omega_1^2$, the order-of-magnitudes of the other radial components $F^*_{rb,\mathrm{cu}} = -G^2 \sim O(10^{-4})$ and $F^*_{rb,\mathrm{co}} = -2G \sim O(10^{-2})$ are negligibly small. It implies that in the co-rotating disk systems the major contribution to the rotational convection is the centrifugal buoyancy induced by system rotation.

Disks Rotating at Different Rates

For the two disks rotating independently, the rotational buoyancy between two disks is more complicated. In the case of two disks rotating at different rates, even if the fluid is isothermal, a flow motion can be driven. Therefore, between two independently rotating disks, the rotation-induced flow can be driven by two mechanisms. One is the unbalanced pumping effect in the Ekman layers of the two disks; the other is the rotational buoyancy effect. The tangential velocity of the fluid and the local buoyancy effect could be spatially different in the flow field. The buoyancy forces for various rotation rates of the local fluids can be estimated by Eqn. (36). For example, as local rotation rate of the fluid is $\Omega_1/2$, G is -0.5 by definition. Therefore, with $F^*_{rb,\mathrm{ce}} \sim -1$, we have $F^*_{rb,\mathrm{cu}} \sim -G^2 = -0.25$ and $F^*_{rb,\mathrm{co}} \sim -2G = 1$. The centrifugal buoyancy due to system rotation is no longer dominant and the rotational force effects become more complicated. It should be noted that, in the cases of two disks rotating at different rates, all the rotational forces should be regarded as the sources of the thermal buoyancy effect.

CONCLUSIONS

The present work has presented a literature survey of previous works related to the study of rotation-induced buoyancy effects in rotating systems. With scaling analysis of rotational buoyancy forces in rotating ducts and rotating disk systems, the author has tried to offer physical insights into the qualitative nature of the rotating non-isothermal flow behaviors. Although numerous studies on the subject have been conducted on this complicated flow with interaction of rotational forces and thermal effects, much is still incomplete and poorly understood. There are still a number of questions open to future investigation. Especially, rotational buoyancy-induced instabilities, rotational buoyancy effects in turbulent flows, and turbulence modeling in numerical simulation of rotating flows are the topics most worthwhile to be investigated.

Recently, thermal flow instability in a rotating configuration attracted lots of attention. Hart (2000) investigated the interaction of centrifugal convection and thermal instability in a rotating cylinder. Turkyilmazoglu et al. (1999) performed a linear stability analysis for the absolute and convective instabilities in the compressible boundary layer of a rotating disk. It is concluded that the overall effect of compressibility is to reduce the extend of absolute instability at higher Mach numbers. The wall heating effect enhances the absolute instability regime for the parameter range studied. A series of linear stability analyses (Soong and Tuliszka-Sznitko, 2000; Tuliszka-Sznitko and Soong, 2000; Tuliska-Sznitko et al., 2001) of mixed convection between rotor-stator disks have been performed for the basic flow states from Soong and Chyuan (1998a). It is found that rotational buoyancy indeed has significant influences on the instability of the non-isothermal flow between coaxial rotating disks. To this class of complicated flow, however, much more effort must be made for better understanding of the underlying physics.

The influences of rotational buoyancy on the transport phenomena in turbulent flow are another important issue worthy to be investigated. The assisting and opposing effects on the thermal flow adjacent to the wall could interact with the wall turbulence and the resultant effects must be very distinct from the buoyancy effects in laminar and transitional flows. The details of the nature of the coupling are not completely understood. Numerical simulation of the rotating non-isothermal flow needs a reliable turbulence model with the inclusion of the effects of rotational buoyancy forces. To develop a turbulence model and numerical techniques would be significant in improving numerical prediction of rotating non-isothermal flows. Experimental work for high confidence measurements of flow and heat transfer characteristics in rapidly rotating systems is most needed for the usefulness in either understanding of the flow physics or development of a turbulence model.

NOMENCLATURE

B	thermal Rossby number, $\beta \Delta T_c$
B^*	alternative thermal Rossby number, $\beta(T_2 - T_1)/2$
D	hydraulic diameter of duct
E	eccentricity defined as the distance of duct center to axis of rotation
F, G, H	dimensionless velocity functions in radial, azimuthal and axial direction
F_{rb}, F_{rb}^*	dimensional and normalized radial buoyancy force, $F_{rb}^* = F_{rb}/\rho_r \beta \Delta T_c R \Omega_1^2$
F_{tb}, F_{tb}^*	dimensional and normalized tangential buoyancy force, $F_{tb}^* = F_{tb}/\rho_r \beta \Delta T_c R \Omega_1^2$
F_{zb}, F_{zb}^*	dimensional and normalized axial buoyancy force, $F_{zb}^* = F_{zb}/\rho_r \beta \Delta T_c R \Omega_1^2$
Fr	Froude number, $R\Omega_1^2/g$
g	gravitational acceleration
h	heat transfer coefficient
k	thermal conductivity
Nu	Nusselt number, hD/k
P	pressure
P_d	pressure departure from the reference state, $P - P_r$
Pr	Prandtl number, ν/α
Re	Reynolds number based on mean velocity of main flow, $V_m D/\nu$
Re_Ω	rotational Reynolds number, $D^2\Omega_1/\nu$ for ducts and $S^2\Omega_1/\nu$ for disks
Ro	rotation number, $D\Omega_1/V_m$
R, Z	cylindrical radial and axial coordinates
S	cylinder height or spacing between two disks
T	temperature
ΔT_c	characteristic temperature difference, $T_w - T_o$ or $T_2 - T_1$
U, V, W	velocity components
V_m	mean velocity of main flow in ducts
u, v, w	dimensionless velocity components
X, Y, Z	Cartesian coordinates
x, y, z	dimensionless coordinates
Z_o	distance from center of inlet plane to axis of rotation

Greek Symbols

α	thermal diffusivity, $k/\rho c_p$
β	thermal-expansion coefficient, $-(1/\rho_r)(\partial_\rho/\partial T)_P$
γ	ratio of specific heats
δ	boundary layer thickness
ε	dimensionless eccentricity of the duct, E/D
η	dimensionless axial coordinate, Z/S
μ	dynamic viscosity
ν	kinematic viscosity
θ	dimensionless temperature function, $(T - T_r)/\Delta T_c$
Ω	rate of rotation

Subscripts

1	disk 1
2	disk 2
c	characteristic quantity
ce	centrifugal buoyancy component due to disk rotation
co	buoyancy component due to Coriolis force
cu	buoyancy component due to curvilinear motion of the fluids
E	Ekman layer

o	inlet condition
r	reference state
w	wall condition
Ω	rotation

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