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Research Article

Variational Iteration Method and Differential Transformation Method for Solving the SEIR Epidemic Model

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The aim of the present study is to analyze and find a solution for the model of nonlinear ordinary differential equations (ODEs) describing the so-called coronavirus (COVID-19), a deadly and most parlous virus. The mathematical model based on four nonlinear ODEs is presented, and the corresponding numerical results are studied by applying the variational iteration method (VIM) and differential transformation method (DTM).

1. Introduction

The whole world is experiencing hardship due to coronavirus (COVID-19), which was first identified in Wuhan, China, in the month of December 2019. It has been considered that COVID-19 originated from wild animals (bats [1]) and transmitted to humans as numerous infected patients claimed that they had been to a local wet market in Wuhan during the end of November [2]. Later, some investigators confirmed that the virus transmission occurs from person to person [3].

Mathematical models can simulate the effects of a disease at many levels, ranging from how the disease influences the interaction between cells in a single patient (within-host models) to how it spreads across several geographically separated populations (metapopulation models). Models simulating the disease spread within and among populations, such as those used to forecast the COVID-19 outbreak [4], are typically based on the SEIR model.

The SEIR model is based on the division of the population under study into four compartments: an individual can either be susceptible (S), exposed to the disease but not yet infectious (E), infectious (I), or recovered (R). The SEIR model can represent many human infectious diseases [5–9]. In this paper, we focus, analyze, and find a solution for the model of nonlinear ordinary differential equations (ODEs) describing the deadly and most parlous coronavirus (COVID-19). A mathematical model based on the four nonlinear ODEs is presented, and the corresponding numerical results are studied by applying the variational iteration method (VIM) and differential transformation method (DTM).

The VIM was developed by He [10, 11]. In recent years, a great deal of attention has been devoted to the study of this method. The reliability of the method and the reduction in the size of the computational domain make this method applicable to a wide range of model predictions. This method is based on the use of restricted variations and a correction functional, and it was found to have wide applications in finding a solution for the nonlinear ordinary and partial differential equations [12–14]. This method does not depend on small parameters in the differential equation and provides a solution (or an approximation to it) as a sequence of iterations. The method does not require that the nonlinearities be differentiable with respect to the dependent variable and its derivatives [15, 16].

The DTM is a numerical method for solving differential equations. The concept of the differential transformation was first proposed by Zhou [17], and its main application therein is solving both linear and nonlinear initial value problems in electric circuit analysis. The DTM provides in a fast manner exact values of the n^{th} derivative of an analytical function at a point in terms of known and unknown boundary conditions. This method constructs, for differential equations, an analytical solution in the form of a polynomial.

2. The SEIR Model

The SEIR model in epidemiology for the spread of an infectious disease is described by the following system of differential equations:

$$\begin{cases} \dfrac{\mathrm{d}S}{\mathrm{d}t}(t) = -\beta S(t) I(t), \\[2ex] \dfrac{\mathrm{d}E}{\mathrm{d}t}(t) = \beta S(t) I(t) - \alpha E(t), \\[2ex] \dfrac{\mathrm{d}I}{\mathrm{d}t}(t) = \alpha E(t) - \gamma I(t), \\[2ex] \dfrac{\mathrm{d}R}{\mathrm{d}t}(t) = \gamma I(t). \end{cases} \tag{1}$$

Here, β, α, and γ are positive parameters and S, E, I, and R denote the fractions of the population that are susceptible, exposed, infectious, and recovered, respectively.

A schematic diagram of the disease transmission among the individuals is shown in Figure 1 using the SEIR model.

For more information about the model refer to [18].

The SEIR model of the novel coronavirus (COVID-19) can be represented as follows:

(1) The rate of change in the number of susceptible people = the susceptible portion of the population × the average number of people infected by an infectious person over the average duration of infection × the number of people infected by infectious people − the susceptible portion of the population × the rate of infectious animal source + travelers entering − percentage of population traveling out × the number of susceptible people + natural birth rate × the total number of population − the death rate of susceptible people × the number of susceptible people:

$$\begin{aligned} \frac{\mathrm{dS}}{\mathrm{d}t}(t) &= -\beta \frac{\mathrm{S}(t)}{N} \mathrm{I}(t) - \frac{Z}{N}\mathrm{S}(t) + \left(\rho_{\mathrm{I}} + \rho_{\mathrm{E}}\right) \\ &\quad - \left(\frac{\varrho_{\mathrm{I}}}{N} + \frac{\varrho_{\mathrm{E}}}{N}\right)\mathrm{S}(t) + \nu N(t) - \mu \mathrm{S}(t). \end{aligned} \tag{2}$$

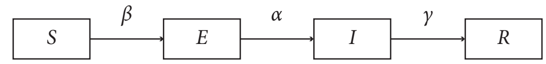

Figure 1: SEIR compartmental model.

(2) The rate of change in the number of exposed people = the susceptible portion of the population × the average number of people infected by an infectious person over the average duration of infection × the number of people infected by infectious people + the susceptible portion of the population × the rate of infectious animal source − the number of exposed people over the average latency period − percentage of population traveling out × the number of exposed people − the death rate of the exposed people × the number of exposed people − testing and therapy rate × the number of exposed people:

$$\begin{aligned} \frac{\mathrm{d}E}{\mathrm{d}t}(t) &= \beta \frac{S(t)}{N} I(t) + \frac{Z}{N} S(t) - \alpha E(t) \\ &\quad - \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right) E(t) - \mu E(t) - \sigma E(t). \end{aligned} \tag{3}$$

(3) The rate of change in the number of infected people = the number of exposed people over the average latency period − the number of infected people over the average duration of infection − percentage of population traveling out × the number of infected people − the death rate of the infected people × the number of infected people:

$$\frac{\mathrm{d}I}{\mathrm{d}t}(t) = \alpha E(t) - \gamma I(t) - \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right) I(t) - \mu I(t). \tag{4}$$

(4) The rate of change in the number of recovered people = the number of infected people over the average duration of infection − the death rate of the recovered people × the number of recovered people + testing and therapy rate × the number of exposed people Figure 2:

$$\frac{\mathrm{d}R}{\mathrm{d}t}(t) = \gamma I(t) - \mu R(t) + \sigma E(t). \tag{5}$$

The transitions between model classes can now be expressed by the following system of first-order differential equations (Table 1):

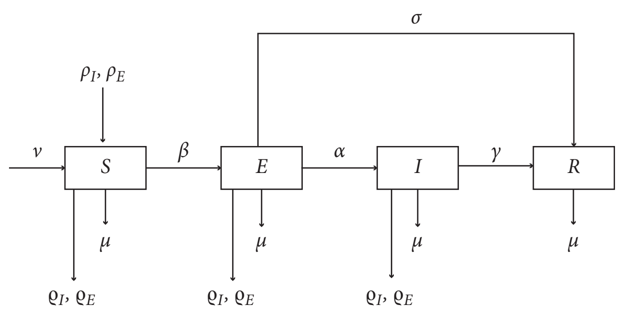

FIGURE 2: The SEIR compartmental model of the novel coronavirus (COVID-19).

TABLE 1: Parameters and initial conditions of the SEIR model.

Parameter	Definition
β	Average number of people infected by an infectious person over the average duration of infection
α	Number of exposed people over the average latency period
γ	Number of infected people over the average duration of infection
μ	The death rate of people
ν	The birth rate parameter of people
σ	The cure rate of people
Z	Force of infection in the baseline scenario
ρ_I	Average daily number of international inbound air passengers
ρ_E	Average daily number of domestic inbound air passengers
ϱ_I	Average daily number of international outbound air passengers
ϱ_E	Average daily number of domestic outbound travellers
S_0	Number of susceptible people at time $t = 0$
E_0	Number of asymptomatic and noninfectious people at time $t = 0$
I_0	Number of asymptomatic but infectious people at time $t = 0$
R_0	Number of recovered people at time $t = 0$
N	Total number of population, $N(t) = S(t) + E(t) + I(t) + R(t)$

$$\begin{cases} \dfrac{dS}{dt}(t) = -\beta \dfrac{S(t)}{N} I(t) - \dfrac{Z}{N} S(t) + (\rho_I + \rho_E) - \left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) S(t) + \nu N(t) - \mu S(t), \\ \dfrac{dE}{dt}(t) = \beta \dfrac{S(t)}{N} I(t) + \dfrac{Z}{N} S(t) - \alpha E(t) - \left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) E(t) - \mu E(t) - \sigma E(t), \\ \dfrac{dI}{dt}(t) = \alpha E(t) - \gamma I(t) - \left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) I(t) - \mu I(t), \\ \dfrac{dR}{dt}(t) = \gamma I(t) - \mu R(t) + \sigma E(t), \end{cases} \tag{6}$$

with the initial conditions

$$\begin{aligned} S(0) &= S_0, \\ E(0) &= E_0, \\ I(0) &= I_0, \\ R(0) &= R_0. \end{aligned} \tag{7}$$

3. The Variational Iteration Method

To illustrate the basic concepts of the VIM, we consider the following general nonlinear differential equation:

$$LU(t) + NU(t) = F(t), \tag{8}$$

where L is a linear operator, N is a nonlinear operator, and $F(t)$ is a known analytical function. We can construct a correction functional according to the variational method as follows:

$$U_{n+1}(t) = U_n(t) + \int_0^t \lambda\{LU_n(s) + N\tilde{U}_n(s) - F(s)\} ds, \tag{9}$$

where λ is the general Lagrange multiplier [19], which can be identified optimally via the variational theory, U_n is the n^{th} approximate solution, and $\tilde{U}_n$ denotes a restricted variation, which means $\delta N\tilde{U}_n = 0$. Successive approximations, U_{n+1}, will be obtained by applying the obtained Lagrange multiplier and a properly chosen initial approximation U_0. Consequently, the solution is given by $U = \lim_{n\longrightarrow\infty} U_n$. For solving equation (6) by means of the VIM, we construct the correctional functional as follows:

$$\begin{cases} S_{n+1}(t) = S_n(t) + \int_0^t \lambda_1(\tau)\left\{\frac{dS_n(\tau)}{d\tau} + \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) + \frac{Z}{N}\widetilde{S}_n(\tau) - (\rho_I + \rho_E) + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right)\widetilde{S}_n(\tau) - \nu\widetilde{N}_n(\tau) + \mu\widetilde{S}_n(\tau)\right\}d\tau, \\ E_{n+1}(t) = E_n(t) + \int_0^t \lambda_2(\tau)\left\{\frac{dE_n(\tau)}{d\tau} - \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) - \frac{Z}{N}\widetilde{S}_n(\tau) + \left(\alpha + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right) + \mu + \sigma\right)\widetilde{E}_n(\tau)\right\}d\tau, \\ I_{n+1}(t) = I_n(t) + \int_0^t \lambda_3(\tau)\left\{\frac{dI_n(\tau)}{d\tau} - \alpha\widetilde{E}_n(\tau) + \left(\gamma + \frac{\varrho_I}{N} + \frac{\varrho_E}{N} + \mu\right)\widetilde{I}_n(\tau)\right\}d\tau, \\ R_{n+1}(t) = R_n(t) + \int_0^t \lambda_4(\tau)\left\{\frac{dR_n(\tau)}{d\tau} - \gamma\widetilde{I}_n(\tau) + \mu\widetilde{R}_n(\tau) - \sigma\widetilde{E}_n(\tau)\right\}d\tau. \end{cases} \tag{10}$$

Here, λ_1, λ_2, λ_3, and λ_4 are general Lagrange multipliers. Making the above correction functional stationary with respect to $S_n(t)$, $E_n(t)$, $I_n(t)$, and $R_n(t)$, noticing that $\delta\widetilde{S}_n(\tau) = \delta\widetilde{E}_n(\tau) = \delta\widetilde{I}_n(\tau) = \delta\widetilde{R}_n(\tau) = 0$, yields

$$\begin{cases} \delta S_{n+1}(t) = \delta S_n(t) + \delta\int_0^t \lambda_1(\tau)\left\{\frac{dS_n(\tau)}{d\tau} + \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) + \frac{Z}{N}\widetilde{S}_n(\tau) - (\rho_I + \rho_E) + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right)\widetilde{S}_n(\tau) - \nu\widetilde{N}_n(\tau) + \mu\widetilde{S}_n(\tau)\right\}d\tau = 0, \\ \delta E_{n+1}(t) = \delta E_n(t) + \delta\int_0^t \lambda_2(\tau)\left\{\frac{dE_n(\tau)}{d\tau} - \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) - \frac{Z}{N}\widetilde{S}_n(\tau) + \left(\alpha + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right) + \mu + \sigma\right)\widetilde{E}_n(\tau)\right\}d\tau = 0, \\ \delta I_{n+1}(t) = \delta I_n(t) + \delta\int_0^t \lambda_3(\tau)\left\{\frac{dI_n(\tau)}{d\tau} - \alpha\widetilde{E}_n(\tau) + \left(\gamma + \frac{\varrho_I}{N} + \frac{\varrho_E}{N} + \mu\right)\widetilde{I}_n(\tau)\right\}d\tau = 0, \\ \delta R_{n+1}(t) = \delta R_n(t) + \delta\int_0^t \lambda_4(\tau)\left\{\frac{dR_n(\tau)}{d\tau} - \gamma\widetilde{I}_n(\tau) + \mu\widetilde{R}_n(\tau) - \sigma\widetilde{E}_n(\tau)\right\}d\tau = 0. \end{cases} \tag{11}$$

Therefore, the Lagrange multiplier can readily be identified: $\lambda_1(\tau) = -1$, $\lambda_2(\tau) = -1$, $\lambda_3(\tau) = -1$, and $\lambda_4(\tau) = -1$. Consequently, the iteration formula can be obtained as follows:

$$\begin{cases} S_{n+1}(t) = S_n(t) - \int_0^t \left\{\frac{dS_n(\tau)}{d\tau} + \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) + \frac{Z}{N}\widetilde{S}_n(\tau) - (\rho_I + \rho_E) + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right)\widetilde{S}_n(\tau) - \nu\widetilde{N}_n(\tau) + \mu\widetilde{S}_n(\tau)\right\}d\tau, \\ E_{n+1}(t) = E_n(t) - \int_0^t \left\{\frac{dE_n(\tau)}{d\tau} - \beta\frac{\widetilde{S}_n(\tau)}{N}\widetilde{I}_n(\tau) - \frac{Z}{N}\widetilde{S}_n(\tau) + \left(\alpha + \left(\frac{\varrho_I}{N} + \frac{\varrho_E}{N}\right) + \mu + \sigma\right)\widetilde{E}_n(\tau)\right\}d\tau, \\ I_{n+1}(t) = I_n(t) - \int_0^t \left\{\frac{dI_n(\tau)}{d\tau} - \alpha\widetilde{E}_n(\tau) + \left(\gamma + \frac{\varrho_I}{N} + \frac{\varrho_E}{N} + \mu\right)\widetilde{I}_n(\tau)\right\}d\tau, \\ R_{n+1}(t) = R_n(t) - \int_0^t \left\{\frac{dR_n(\tau)}{d\tau} - \gamma\widetilde{I}_n(\tau) + \mu\widetilde{R}_n(\tau) - \sigma\widetilde{E}_n(\tau)\right\}d\tau. \end{cases} \tag{12}$$

With initial approximations $S(0) = 2500$, $E(0) = 1$, $I(0) = 1$, $R(0) = 0$, and $N = 2502$, which in turn gives successive approximations, and considering the following values for parameters (see [20]) $\beta = 0.8$, $\alpha = 0.75$,

$\sigma = 0.1$, $\gamma = 0.05$, $\nu = 0.009/N$, $\mu = 0.01$, $Z = 0.001$, $\rho_I = 0.15$, $\rho_E = 0.15$, $\varrho_I = 0.01$, and $\varrho_I = 0.03$, we obtain

$$\begin{cases} S_1(t) = 2500 - 25.53132774t, \\ E_1(t) = 1 - 0.059656274t, \\ I_1(t) = 1 + 0.689984012t, \\ R_1(t) = 0.15t. \end{cases} \tag{13}$$

Similarly, we get the following system after two terms:

$$\begin{cases} S_2(t) = 2500 - 25.53132774t - 0.143869926t^2 + 1.87756013310^{-3}t^3, \\ E_2(t) = 1 - 0.059656274t + 0.29733881t^2 - 1.87756013310^{-3}t^3, \\ I_2(t) = 1 + 0.689984012t - 0.043076138t^2, \\ R_2(t) = 0.15t + 0.013516786t^2. \end{cases} \tag{14}$$

For the solution after three terms, we can write

$$\begin{cases} S_3(t) = 2500 - 25.53132774t - 0.143869926t^2 + 0.013851166t^3 - 8.48550280810^{-5}t^4 \\ \qquad -4.79158897410^{-7}t^5 + 4.31004739710^{-9}t^6, \\ E_3(t) = 1 - 0.059656274t + 0.29733881t^2 - 0.09860941t^3 + 4.8381106310^{-4}t^4 \\ \qquad -4.79158897410^{-7}t^5 + 4.31004739710^{-9}t^6, \\ I_3(t) = 1 + 0.689984012t - 0.043076138t^2 + 0.074410709t^3 - 3.52042524910^{-4}t^4, \\ R_3(t) = 0.15t + 0.013516786t^2 + 9.1483020810^{-3}t^3 - 4.69390033310^{-5}t^4. \end{cases} \tag{15}$$

4. The Differential Transformation Method

The basic definition and the fundamental theorems of the DTM and its applicability to various kinds of differential equations are given in [17, 21]. According to the operations of differential transformation given in Table 1 in [21], we have the following recurrence relation:

$$\begin{cases} S(k+1) = \dfrac{1}{k+1}\left[-\dfrac{\beta}{N}\displaystyle\sum_{m=0}^{k} S(m)I(k-m) - \dfrac{Z}{N}S(k) + (\rho_I + \rho_E)\delta(k) + \nu N(k) - \left[\left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) + \mu\right]S(k)\right], \\ E(k+1) = \dfrac{1}{k+1}\left[\dfrac{\beta}{N}\displaystyle\sum_{m=0}^{k} S(m)I(k-m) + \dfrac{Z}{N}S(k) - \left[\alpha + \left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) + \mu + \sigma\right]E(k)\right], \\ I(k+1) = \dfrac{1}{k+1}\left[\alpha E(k) - \left[\gamma + \left(\dfrac{\varrho_I}{N} + \dfrac{\varrho_E}{N}\right) + \mu\right]I(k)\right], \\ R(k+1) = \dfrac{1}{k+1}\left[\gamma I(k) - \mu R(k) + \sigma E(k)\right]. \end{cases} \tag{16}$$

The inverse differential transformation of $S(k)$ is defined as follows: when t_0 is taken as zero, the given function $S(t)$ is declared by a finite series, and the above equation can be written in the form

$$S(t) = \sum_{k=0}^{\infty} S(k)t^k. \tag{17}$$

By solving the above equations for $S(k+1)$, $E(k+1)$, $I(k+1)$, and $R(k+1)$ up to order 3, we obtain the functions of $S(k)$, $E(k)$, $I(k)$, and $R(k)$, respectively:

$$\begin{aligned} S(t) &= \sum_{k=0}^{3} S(k)t^k, \\ E(t) &= \sum_{k=0}^{3} E(k)t^k, \\ I(t) &= \sum_{k=0}^{3} I(k)t^k, \\ R(t) &= \sum_{k=0}^{3} R(k)t^k. \end{aligned} \tag{18}$$

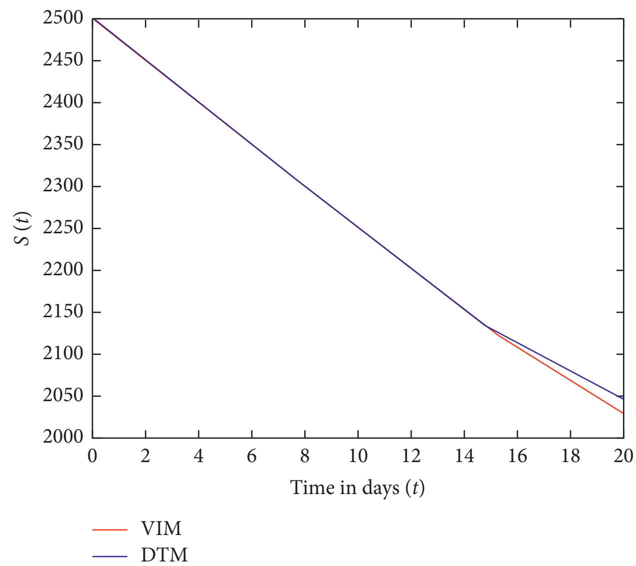

FIGURE 3: Comparison of solutions of the proposed model (6) for compartment S using VIM and DTM.

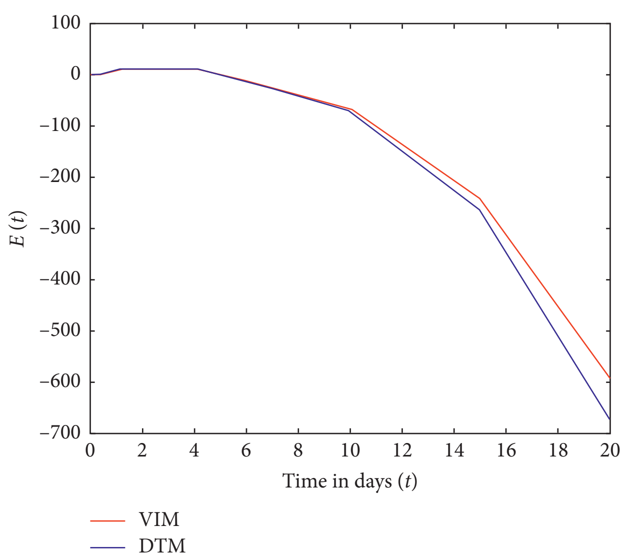

FIGURE 4: Comparison of solutions of the proposed model (6) for compartment E using VIM and DTM.

With initial approximations, $S(0) = 2500$, $E(0) = 1$, $I(0) = 1$, $R(0) = 0$, and $N = 2502$ and parameters $\beta = 0.8$, $\alpha = 0.75$, $\sigma = 0.1$, $\gamma = 0.05$, $\nu = 0.009/N$, $\mu = 0.01$, $Z = 0.001$, $\rho_I = 0.15$, $\rho_E = 0.15$, $\varrho_I = 0.01$, and $\varrho_I = 0.03$, and applying the conditions in equations (16) and (18), we obtain the approximate solution after three terms as follows:

$$\begin{cases} S(t) = 2500 - 25.53132774t - 0.143869688t^2 + 0.013851335t^3, \\ E(t) = 1 - 0.059656274t + 0.297292652t^2 - 0.099439782t^3, \\ I(t) = 1 + 0.689984012t - 0.043076138t^2 + 0.075184915t^3, \\ R(t) = 0.15t + 0.013516786t^2 + 9.1467634810^{-3}t^3. \end{cases} \quad (19)$$

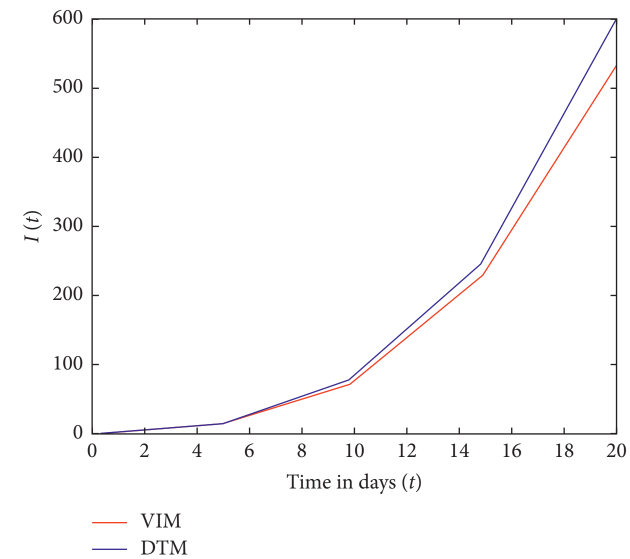

FIGURE 5: Comparison of solutions of the proposed model (6) for compartment I using VIM and DTM.

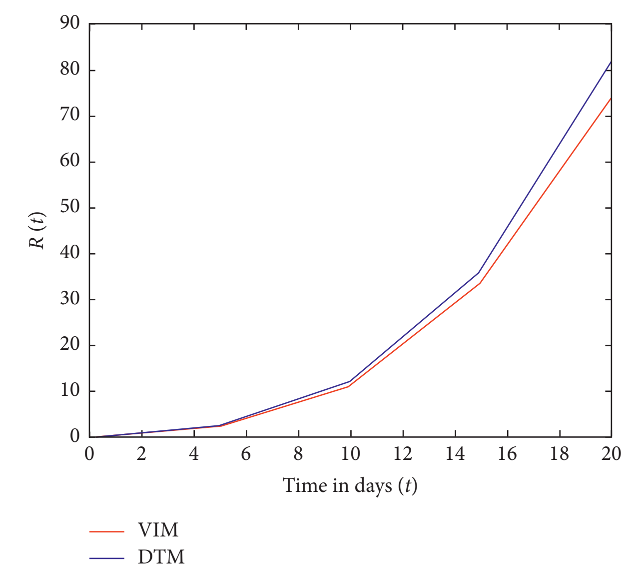

FIGURE 6: Comparison of solutions of the proposed model (6) for compartment R using VIM and DTM.

5. Conclusions

In this paper, we have developed the SEIR model of the COVID-19 epidemic in China that incorporates key features of this pandemic. For solving this model, we used the variational iteration method (VIM) and differential transformation method (DTM). It is found that these methods are effective in providing analytic form solutions for such problems. The comparison of the results obtained by these two methods is in excellent agreement.

For further research, we propose the study of the fractional-order model using the Caputo–Fabrizio derivative [22, 23]. In addition, we propose to extend the results of the present paper and combine them with the results in [6] (Figures 3–6).

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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