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Research Article

On the Control of Coefficient Function in a Hyperbolic Problem with Dirichlet Conditions

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This paper presents theoretical results about control of the coefficient function in a hyperbolic problem with Dirichlet conditions. The existence and uniqueness of the optimal solution for optimal control problem are proved and adjoint problem is used to obtain gradient of the functional. However, a second adjoint problem is given to calculate the gradient on the space $W_2^1(0,l)$. After calculating gradient of the cost functional and proving the Lipschitz continuity of the gradient, necessary condition for optimal solution is constructed.

1. Introduction

Hyperbolic boundary value problems have appeared as mathematical modelling of physical phenomena like small vibration of a string, in the fields of science and engineering. There has been much attention to studies related to optimal control problems involving hyperbolic problems [1]. There have been many studies about optimal control for hyperbolic systems which are considered [2–4].

Some of these important studies can be summarized as follows.

Hasanov [5] has considered problem of controlling the function $w := \{F(x,t); f(t)\}$ for the following problem:

$$\begin{aligned} &u_{tt} = \left(k(x)\,u_x\right)_x + F(x,t), \quad (x,t) \in (0,l)\times(0,T)\\ &u(x,0) = u_0(x),\\ &u_t(x,0) = u_1(x), \quad x \in (0,l)\\ &-k(0)\,u_x(0,t) = f(t),\\ &k(l)\,u_x(l,t) = 0 \quad t \in (0,T) \end{aligned} \tag{1}$$

with the conditions

$$\begin{aligned} u(x,T) &= \mu(x),\\ u_t(x,T) &= \nu(x), \end{aligned} \tag{2}$$

using the functionals

$$\begin{aligned} J_1(w) &= \int_0^l \left[u(x,T;w) - \mu(x)\right]^2 dx,\\ J_2(w) &= \int_0^l \left[u_t(x,T;w) - \nu(x)\right]^2 dx,\\ J_3(w) &= J_1(w) + J_2(w). \end{aligned} \tag{3}$$

Majewski [6] has controlled the function $u(x, y) \in L_2(P, \mathbb{R}^M)$ for hyperbolic equation:

$$\frac{\partial^2 z}{\partial x \partial y}(x, y) = \tilde{f}\left(x, y, z(x, y), \frac{\partial z}{\partial x}(x, y), \frac{\partial z}{\partial y}(x, y), u(x, y)\right) \tag{4}$$

$$z(x, 0) = z(0, y) = 0, \quad \forall x, y \in [0, 1]$$

using the functional

$$J^k(z(\cdot), u(\cdot)) = \int_P F^k(x, y, z(x, y), u(x, y))\, dx\, dy. \quad k = 0, 1, 2, \ldots \tag{5}$$

Yeloğlu and Subaşı [7] have dealt with determination pair $w := \{f(x, t), h(x)\}$ in the following problem:

$$\begin{aligned} &p(x) u_{tt} = (k(x) u_x)_x + f(x, t), \quad (x, t) \in \Omega_T \\ &u(x, 0) = g(x), \\ &u_t(x, 0) = h(x), \quad x \in (0, l) \\ &u(0, t) = 0, \\ &u(l, t) = 0, \quad t \in (0, T] \end{aligned} \tag{6}$$

for the functional

$$J_\alpha(w) = \int_0^l [u(x, T; w) - y(x)]^2 dx + \alpha \|w\|_W^2. \tag{7}$$

Kröner [8] has specified the function $u(t) \in L_2(0, T)$ for nonlinear hyperbolic equation:

$$\begin{aligned} &y_{tt} - A(u, y) = f \\ &y(0) = y_0(u), \\ &y_t(0) = y_1(u) \end{aligned} \tag{8}$$

using the functional

$$J(u, y^1) = \int_0^T J_1(y^1(t))\, dt + J_2(y^1(T)) + \frac{\alpha}{2}\|u\|_U^2. \tag{9}$$

Tagiyev [9] has studied the problem of controlling the coefficients $v = (k(x), q(x, t)) \in L_\infty(\Omega) \times L_\infty(Q_T)$ for linear hyperbolic equation:

$$\begin{aligned} &\frac{\partial^2 u}{\partial t^2} - \sum_{i=1}^n \frac{\partial}{\partial x_i}\left(k_i(x)\frac{\partial u}{\partial x_i}\right) + q(x, t) u = f(x, t), \quad (x, t) \in Q_T \\ &u|_{t=0} = \varphi_0(x), \\ &\left.\frac{\partial u}{\partial t}\right|_{t=0} = \varphi_1(x), \\ &u|_{S_T} = 0 \end{aligned} \tag{10}$$

using the functional

$$J(v) = \alpha_0 \int_{Q_T} |u(x, t, v) - z_0(x, t)|^2 dx\, dt + \alpha_1 \int_\Omega |u(x, T, v) - z_1(x)|^2 dx. \tag{11}$$

2. Statement of the Problem

In this study, we deal with the process of vibration in finite homogeneous string, occupying the interval $(0, l)$. As the control function, we take the transverse elastic force which is in the coefficient of the vibration problem. Also, we propose the usage of a more regular space than the space of square integrable functions in the cost functional. In general, this process exposes some difficulties in the stage of acquiring the gradient. This study offers a second adjoint problem to overcome this case.

In the domain $\Omega := (x, t) \in (0, l) \times (0, T)$, we consider the functional

$$J_\alpha(q) = \int_0^l [u(x, T; q) - y(x)]^2 dx + \alpha \|q - r\|_{W_2^1(0,l)}^2 \tag{12}$$

on the set

$$Q = \left\{q(x) : q(x) \in W_2^1(0, l),\ 0 < q_1 \le q(x) \le q_2,\ \|q(x)\|_{W_2^1(0,l)} \le q_3\right\} \tag{13}$$

subject to the hyperbolic problem

$$u_{tt} - u_{xx} + q(x) u = 0, \quad (x, t) \in \Omega \tag{14}$$

$$\begin{aligned} &u(x, 0) = \varphi_1(x), \\ &u_t(x, 0) = \varphi_2(x), \quad x \in (0, l) \end{aligned} \tag{15}$$

$$\begin{aligned} &u(0, t) = 0, \\ &u(l, t) = 0, \quad t \in (0, T). \end{aligned} \tag{16}$$

Here $y(x) \in L_2(0, l)$ is the desired target function to which $u(x, T)$ must be close enough. The function $r(x) \in W_2^1(0, l)$ is an initial guess for optimal solution. $\alpha > 0$ is regularization parameter. $q_1, q_2, q_3 > 0$ are given positive numbers.

The initial status functions are in the following spaces:

$$\begin{aligned} &\varphi_1(x) \in W_2^1(0, l), \\ &\varphi_2(x) \in L_2(0, l). \end{aligned} \tag{17}$$

The aim of this study is to deal with the problem of

$$J_{\alpha^*} = \inf_{q \in Q} J_\alpha(q) = J_\alpha(q^*) \tag{18}$$

under conditions (12)-(17).

Namely, we want to control the transverse elastic force on the space $W_2^1(0,l)$ and the solution $u(x,T)$ corresponding to this control function must be close enough to $y(x)$ in $L_2(0,l)$. In order to get a stable solution, we choose the space $W_2^1(0,l)$ which is more regular than $L_2(0,l)$.

The inner product and norm in $W_2^1(0,l)$ are defined, respectively, as

$$(f,g)_{W_2^1(0,l)} = \int_0^l \left[f(x) . g(x) + \frac{df(x)}{dx} . \frac{dg(x)}{dx} \right] dx,$$
$$\|f\|^2_{W_2^1(0,l)} = \int_0^l \left([f(x)]^2 + \left[\frac{df(x)}{dx} \right]^2 \right) dx. \quad (19)$$

The paper is organized as follows: in Section 3, we obtain the generalized solution for hyperbolic problem. In Section 4, we prove the existence and uniqueness of the optimal solution. In Section 5, we obtain the adjoint problem for the optimal control problem and find the gradient of the functional. The main contribution of this paper is executed in this section. Because the controls are chosen in the space $W_2^1(0,l)$, getting the gradient of the functional necessitates finding a second adjoint problem. In the last section, we demonstrate the Lipschitz continuity of the gradient and state the necessary condition for optimal solution.

3. Solvability of the Problem

In this section, we first give the definition of the generalized solution for hyperbolic problem.

The generalized solution of problem (14)-(15) is the function $u \in \overset{o}{W}{}_2^{1,1}(\Omega)$ satisfying the following integral equality:

$$\int_\Omega [-u_t\eta_t + u_x\eta_x + q(x)u\eta]\, dx\, dt = \int_0^l \varphi_2(x)\eta(x,0)\, dx \quad (20)$$

for $\forall\eta \in \overset{o}{W}{}_2^{1,1}(\Omega)$, $\eta(x,T) = 0$.

It can be seen in [10] that solution in the sense of (20) exists, is unique, and satisfies the following inequality:

$$\max_{0\le t\le T} \left(\|u(\cdot,t)\|^2_{L_2(0,l)} + \|u_t(\cdot,t)\|^2_{L_2(0,l)} + \|u_x(\cdot,t)\|^2_{L_2(0,l)} \right) \le c_1 \left(\|\varphi_1\|^2_{W_2^1(0,l)} + \|\varphi_2\|^2_{L_2(0,l)} \right) \quad (21)$$

where $c_1 = \max\{3c_0, 3c_0/q_1\}$ and $c_0 = \max\{1, q_2\}$ or

$$\|u\|^2_{W_2^{1,1}(\Omega)} \le c_2 \left(\|\varphi_1\|^2_{W_2^1(0,l)} + \|\varphi_2\|^2_{L_2(0,l)} \right). \quad (c_2 = c_1T) \quad (22)$$

Since φ_1 and φ_2 are given functions, it can be written as follows:

$$\|u\|^2_{W_2^{1,1}(\Omega)} \le c_3. \quad (23)$$

Now, we give an increment $\delta q(x) \in W_2^1(0,l)$ to the control function $q(x)$ such as $q+\delta q \in Q$. Then the difference function $\delta u = \delta u(x,t) = u(x,t;q+\delta q) - u(x,t;q)$ is the solution of the following difference initial-boundary problem:

$$\delta u_{tt} - \delta u_{xx} + [q(x) + \delta q(x)]\delta u + \delta q(x)u = 0 \quad (24)$$

$$\delta u(x,0) = 0, \quad \delta u_t(x,0) = 0 \quad (25)$$

$$\delta u(0,t) = 0, \quad \delta u(l,t) = 0. \quad (26)$$

By considering (23), we obtain that the solution of above difference initial-boundary problem holds the following inequality:

$$\max_{0\le t\le T} \left(\|\delta u(.,t)\|^2_{L_2(0,l)} \right) \le c_4 \|\delta q\|^2_{W_2^1(0,l)}. \quad (27)$$

Here $c_4 = (t^3 2l/3)c_3$ is independent from δq.

4. Existence and Uniqueness of the Optimal Solution

To demonstrate the existence and the uniqueness of optimal solution for problem (12)-(17), it is enough to show that conditions of the following theorem given by Goebel [11] hold.

Theorem 1. *Let H be a uniformly convex Banach space and the set Q be a closed, bounded, and convex subset of H. If $\alpha > 0$ and $\beta \ge 1$ are given numbers and the functional $J(q)$ is lower semicontinuous and bounded from below on the set Q, then there is a dense set G of H that the functional*

$$J_\alpha(q) = J(q) + \alpha \|q - r\|^\beta_H \quad (28)$$

takes its minimum on the set Q for $\forall r \in G$. If $\beta > 1$ then minimum is unique.

Before showing that these conditions have been satisfied, we prove that the functional

$$J(q) = \int_0^l [u(x,T;q) - y(x)]^2\, dx \quad (29)$$

is continuous. For this, we write the following increment of the functional:

$$\delta J(q) = J(q+\delta q) - J(q) = \int_0^l 2[u(x,T) - y(x)][\delta u(x,T)]\, dx + \int_0^l [\delta u(x,T)]^2\, dx. \quad (30)$$

Since $y(x) \in L_2(0,l)$, if we consider inequalities (22) and (27), we conclude that this increment satisfies the following continuity inequality on the set Q:

$$|\delta J(q)| \leq c_5 \left(\|\delta q\|_{W_2^1(0,l)} + \|\delta q\|^2_{W_2^1(0,l)} \right). \tag{31}$$

Here c_5 is independent of δq.

Thanks to this inequality, we can say that this functional is also lower semicontinuous and bounded from below on the set Q.

On the other hand, the set $W_2^1(0,l)$ is a uniformly convex Banach space [12], the set Q is a closed, bounded, and convex subset of $W_2^1(0,l)$, and $\beta = 2$.

Therefore the conditions of above theorem hold and optimal solution to the problem (18) is unique.

5. Adjoint Problem and Gradient of the Functional

In this section, we write the Lagrange functional used for finding adjoint problem, before we show the Frechet differentiability of the functional $J_\alpha(q)$ on the set Q. Lagrange functional to the problem is

$$\begin{aligned} L(u,q,\eta) = & \int_0^l [u(x,T;q) - y(x)]^2 \, dx \\ & + \alpha \|q - r\|^2_{W_2^1(0,l)} \\ & + \int_0^T \int_0^l [u_{tt} - u_{xx} + q(x) u] \eta \, dx \, dt. \end{aligned} \tag{32}$$

The first variation of this functional according to the function u is obtained such as

$$\begin{aligned} \delta L = & \int_0^l 2[u(x,T) - y(x) - \eta_t(x,T)] \delta u(x,T) \, dx \\ & + \int_0^l \int_0^T (\eta_{tt} - \eta_{xx} + q(x)\eta) \delta u \, dt \, dx = 0. \end{aligned} \tag{33}$$

By means of stationary condition $\delta L = 0$, the following adjoint boundary value problem is found:

$$\eta_{tt} - \eta_{xx} + q(x)\eta = 0 \tag{34}$$

$$\begin{aligned} \eta(x,T) &= 0, \\ \eta_t(x,T) &= 2[u(x,T) - y(x)] \end{aligned} \tag{35}$$

$$\begin{aligned} \eta(0,t) &= 0, \\ \eta(l,t) &= 0. \end{aligned} \tag{36}$$

For $\forall \gamma \in \overset{o}{W}{}_2^{1,1}(\Omega)$, the function $\eta \in C^1([0,T], L_2(0,l)) \cap C^0([0,T], W_2^1(0,l))$ which satisfies the following equality

$$\begin{aligned} & \int_0^T \int_0^l [-\eta_t \gamma_t + \eta_x \gamma_x + q(x) \eta \gamma] \, dx \, dt \\ & = \int_0^l \eta_t(x,0) \gamma(x,0) \, dx \\ & - \int_0^l 2[u(x,T) - y(x)] \gamma(x,T) \, dx \end{aligned} \tag{37}$$

is the solution of adjoint boundary value problem (34)-(36).

This solution satisfies the following inequality:

$$\|\eta\|_{L_2(0,l)} \leq c_6 \|u(x,T) - y(x)\|_{L_2(0,l)}, \quad \forall t \in [0,T]. \tag{38}$$

Now, we can pass the calculation of the gradient. In order to do this, we must evaluate the increment of the functional $J_\alpha(q)$. The increment can be written such as

$$\begin{aligned} \delta J_\alpha(q) &= J_\alpha(q + \delta q) - J_\alpha(q) \\ &= \int_0^l 2[u(x,T) - y(x)](\delta u) \, dx \\ &+ \int_0^l (\delta u)^2 \, dx + 2\alpha \langle q - r, \delta q \rangle_{W_2^1(0,l)}. \end{aligned} \tag{39}$$

The difference problem (24)-(26) and the adjoint problem (34)-(36) give together the equality of

$$\begin{aligned} & 2\int_0^l [u(x,T) - y(x)](\delta u) \, dx \\ & = \int_0^T \int_0^l [\delta q \delta u \eta + \delta q u \eta] \, dx \, dt. \end{aligned} \tag{40}$$

Inserting (40) in (39), we have

$$\begin{aligned} \delta J_\alpha(q) = & \int_0^T \int_0^l u \eta \delta q \, dx \, dt + \int_0^T \int_0^l \eta \delta u \delta q \, dx \, dt \\ & + \int_0^l (\delta u(x,T))^2 \, dx + 2\alpha \langle q - r, \delta q \rangle_{W_2^1(0,l)}. \end{aligned} \tag{41}$$

By (27) and (38), the second and third integrals of the above equality give the following inequality:

$$\begin{aligned} & \int_0^T \int_0^l \eta \delta u \delta q \, dx \, dt + \int_0^l (\delta u(x,T))^2 \, dx \\ & \leq c_7 \|\delta q\|^2_{W_2^1(0,l)}. \end{aligned} \tag{42}$$

The statement (41) can be rewritten as

$$\begin{aligned} \delta J_\alpha(q) = & \langle u\eta, \delta q \rangle_{L_2(\Omega)} + 2\alpha \langle q - r, \delta q \rangle_{W_2^1(0,l)} \\ & + o\left(\|\delta q\|^2_{W_2^1(0,l)} \right) \end{aligned} \tag{43}$$

or

$$\delta J_\alpha(q) = \left\langle \int_0^T u\eta\, dt, \delta q \right\rangle_{L_2(0,l)} + 2\alpha \langle q - r, \delta q\rangle_{W_2^1(0,l)} + o\left(\|\delta q\|^2_{W_2^1(0,l)}\right). \quad (44)$$

In order to pass the inner product in $W_2^1(0,l)$, we rearrange (44) such as

$$\delta J_\alpha(q) = \langle \xi + 2\alpha(q - r), \delta q\rangle_{W_2^1(0,l)} + o\left(\|\delta q\|^2_{W_2^1(0,l)}\right). \quad (45)$$

Here function $\xi(x)$ is the solution of the second adjoint problem:

$$\begin{aligned} -\xi'' + \xi &= \int_0^T u\eta\, dt \\ \xi'(0) &= 0, \\ \xi'(l) &= 0. \end{aligned} \quad (46)$$

Therefore, we have the following gradient:

$$J'_\alpha(q) = \xi + 2\alpha(q - r). \quad (47)$$

6. Lipschitz Continuity of the Gradient

In this section, we introduce a theorem about Lipschitz continuity of the gradient. By this means, we can express the necessary condition for optimal solution.

Theorem 2. *Gradient $J'_\alpha(q)$ satisfies the following Lipschitz inequality:*

$$\left\|J'_\alpha(q + \delta q) - J'_\alpha(q)\right\|^2_{W_2^1(0,l)} \le c_8 \|\delta q\|^2_{W_2^1(0,l)}. \quad (48)$$

Here c_8 is independent from δq.

Hence, it has been proven that the gradient $J'_\alpha(q)$ is continuous on the set Q and it can be seen that it holds the Lipschitz condition with constant $c_8 > 0$.

Proof. Increment of the functional $J'_\alpha(q)$ by giving the increment of δq to the control $q \in Q$ is obtained:

$$J'_\alpha(q + \delta q) - J'_\alpha(q) = \xi_\delta + 2\alpha(q + \delta q - r) - \xi + 2\alpha(q - r) = \delta\xi + 2\alpha\delta q \quad (49)$$

where the function $\delta\xi(x)$ is the solution of the increment problem:

$$\delta\xi''(x) - \delta\xi(x) = \int_0^T (u_\delta \delta\eta + \delta u\eta)\, dt. \quad (50)$$

Taking the norm of (49) in the space $W_2^1(0,l)$, we acquire the following inequality belonging to the functional $\delta J'_\alpha(q)$:

$$\left\|\delta J'_\alpha(q)\right\|^2_{W_2^1(0,l)} \le 2\|\delta\xi\|^2_{W_2^1(0,l)} + 8\alpha^2 \|\delta q\|^2_{W_2^1(0,l)}. \quad (51)$$

There is a solution of problem (50) in $W_2^1(0,l)$ and this solution satisfies the following inequality:

$$\|\delta\xi(x)\|_{W_2^1(0,l)} \le \left\| \int_0^T (u_\delta \delta\eta + \delta u\eta)\, dt \right\|_{L_2(0,l)}. \quad (52)$$

The function

$$\begin{aligned} \delta\eta(x,t) &= \eta_\delta(x,t) - \eta(x,t) \\ &= \eta(x,t; q + \delta q) - \eta(x,t;q) \end{aligned} \quad (53)$$

in the right hand side of inequality (52) is the solution of the following problem:

$$\begin{aligned} &\frac{\partial^2 \delta\eta}{\partial t^2} - \frac{\partial^2 \delta\eta}{\partial x^2} + (q(x) + \delta q(x))\delta\eta + \delta q(x)\eta = 0, \quad (x,t) \in \Omega \\ &\delta\eta(x,T) = 0, \\ &\delta\eta_t(x,T) = 2\delta u(x,T), \\ &\delta\eta(0,t) = \delta\eta(l,t) = 0 \end{aligned} \quad (54)$$

and this function holds the following inequality:

$$\max_{0 \le t \le T} \left(\|\delta\eta(.,t)\|^2_{L_2(0,l)}\right) \le c_9 \|\delta q\|^2_{W_2^1(0,l)}. \quad (55)$$

Here c_9 is independent of δq.

So, the function u_δ that takes place in the right hand side of (52) holds the same inequality given as follows:

$$\|u_\delta\|^2_{L_2(\Omega)} \le c_3. \quad (56)$$

Hence inequality (52) has the following property:

$$\begin{aligned} \|\delta\xi(x)\|^2_{W_2^1(0,l)} \le{}& 2\|u_\delta\|^2_{L_2(\Omega)} \max_{0 \le t \le T} \left(\|\delta\eta(.,t)\|^2_{L_2(0,l)}\right) \\ &+ 2\|\eta\|^2_{L_2(\Omega)} \max_{0 \le t \le T} \left(\|\delta u(.,t)\|^2_{L_2(0,l)}\right). \end{aligned} \quad (57)$$

If inequalities (27), (38), (55), and (56) about functions u_δ, $\delta\eta(.,t)$, η, and $\delta u(.,t)$ are written in (57), then the following assessment is obtained:

$$\|\delta\xi(x)\|^2_{W_2^1(0,l)} \le c_{10} \|\delta q\|^2_{W_2^1(0,l)}. \quad (58)$$

Here c_{10} is independent of δq.

Considering inequality (58), the following is written:

$$\begin{aligned} \left\|\delta J'_\alpha(q)\right\|^2_{W_2^1(0,l)} &\le 2\|\delta\xi(x)\|^2_{W_2^1(0,l)} + 8\alpha^2 \|\delta q(x)\|^2_{W_2^1(0,l)} \\ &\le 2c_{10} \|\delta q\|^2_{W_2^1(0,l)} + 8\alpha^2 \|\delta q\|^2_{W_2^1(0,l)} \\ &\le c_{11} \|\delta q\|^2_{W_2^1(0,l)}. \end{aligned} \quad (59)$$

So the following inequality for the gradient $J'_\alpha(q)$ is obtained:

$$\left\|J'_\alpha(q + \delta q) - J'_\alpha(q)\right\|^2_{W_2^1(0,l)} \le c_{11} \|\delta q\|^2_{W_2^1(0,l)}. \quad (60)$$

Once we take as $c_8 = c_{11}$, then the proof is obtained. □

7. The Necessary Condition for Optimal Solution

After showing Lipschitz continuity of the gradient, it can be said that the gradient $J'_\alpha(q)$ is continuous on the set Q and it holds the Lipschitz constant $c_8 > 0$. The fact that the functional $J_\alpha(q)$ is continuously differentiable on the set Q and the set Q is convex, in that case the following inequality is valid according to theorem in [13]:

$$\left\langle J'_\alpha\left(q^*\right), q - q^*\right\rangle_{W_2^1(0,l)} \geq 0, \quad \forall q \in Q. \tag{61}$$

Therefore, the following inequality is written for optimal control problem:

$$\left\langle \xi + 2\alpha\left(q^* - r\right), q - q^*\right\rangle_{W_2^1(0,l)} \geq 0, \quad \forall q \in Q. \tag{62}$$

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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