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*Research Article*

# The Existence of Strong Solutions for a Class of Stochastic Differential Equations

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In this paper, we will consider the existence of a strong solution for stochastic differential equations with discontinuous drift coefficients. More precisely, we study a class of stochastic differential equations when the drift coefficients are an increasing function instead of Lipschitz continuous or continuous. The main tools of this paper are the lower solutions and upper solutions of stochastic differential equations.

## 1. Introduction

There are many works [1–3] about the existence and uniqueness of strong or weak solutions for the following stochastic differential equation (denoted briefly by SDE):

$$dX_t = b(t, X_t)\,dt + \sigma(t, X_t)\,dW_t \quad t \geq 0, \tag{1}$$

where $b(t,x) : \mathbb{R}_+ \times \mathbb{R} \longrightarrow \mathbb{R}$ and $\sigma(t,x) : \mathbb{R}_+ \times \mathbb{R} \longrightarrow \mathbb{R}$ are called drift and diffusion coefficients, respectively. $W_t$ is standard Brownian motion. Usually, the drift and diffusion coefficients are Lipschitz or local Lipschitz continuous or at least are continuous with respect to $x$ when the existence and uniqueness of solutions are investigated. In fact, the solutions of stochastic differential equations may exist when their drift and diffusion coefficients are discontinuous with respect to $x$. Therefore, many authors discussed the existence of solutions for SDE with discontinuous coefficients. For example, L. Karatzas and S. E. Shreve [1] (Proposition 3.6 of §5.3) considered the existence of a weak solution when the drift coefficient of SDE need not be continuous with respect to $x$. A. K. Zvonkin [4] considered the following stochastic differential equation with a discontinuous diffusion coefficient:

$$X_t = \int_0^t \operatorname{sgn}(X_s)\,dW_s; \quad 0 \leq t < \infty, \tag{2}$$

where

$$\operatorname{sgn}(x) = \begin{cases} 1, & x > 0; \\ -1, & x \leq 0. \end{cases} \tag{3}$$

The weak solution of this stochastic differential equation exists, but there is not the strong solution. N. V. Ktylov [5] and N. V. Ktylov and R. Liptser [6] also discussed existence issues of SDE when their diffusion coefficients are discontinuous with respect to $x$. And many authors also considered the approximation solutions of SDE with discontinuous coefficients, such as [7–11].

In this paper, we will consider the existence of a strong solution of SDE (1) when the drift coefficient $b(t,x)$ is an increasing function but need not be continuous with respect to $x$ and the diffusion coefficient $\sigma(t, X_t)$ satisfies $(C_\sigma)$ condition. Section 1 is an introduction. In Section 2, we will show a comparison theorem by using the upper and lower solutions of SDE. We will prove our main result by using the above comparison theorem in Section 3.

## 2. The Setup and a Comparison Theorem

In our paper, we just consider a 1-dimensional case. We always assume that $(\Omega, \mathcal{F}, \mathbf{P})$ is a completed probability space, $W =: \{W_t : t \geq 0\}$ is a real-valued Brownian motion defined on

$(\Omega, \mathscr{F}, \mathbf{P})$, and $\{\mathscr{F}_t : t \geq 0\}$ is natural filtration generated by the Brownian motion $W$; i.e., for any $t \geq 0$

$$\mathscr{F}_t = \sigma \{W_s : s \leq t\}. \tag{4}$$

We consider SDE (1) with coefficients $b(t,x) : \mathbb{R}^+ \times \mathbb{R} \longrightarrow \mathbb{R}$ and $\sigma(t,x) : \mathbb{R}^+ \times \mathbb{R} \longrightarrow \mathbb{R}$, where $\mathbb{R}^+$ and $\mathbb{R}$ are a positive real number and real number, respectively. And we use $\|\cdot\|$ to denote norm of $\mathbb{R}$. The following is the definition of a strong solution for SDE.

*Definition 1.* An adapted continuous process $X_t$ defined on $(\Omega, \mathscr{F}, \mathbf{P})$ is said to be a strong solution for SDE (1) if it satisfies that

$$X_t = X_0 + \int_0^t b(s, X_s)\, ds + \int_0^t \sigma(s, X_s)\, dW_s, \quad t \geq 0, \tag{5}$$

holds with probability 1.

Moreover, $X_t$ and $\widetilde{X}_t$ are two strong solutions of SDE (1); then $P[X_t = \widetilde{X}_t;\ 0 \leq t < \infty] = 1$. Under this condition, the solution of SDE (1) is said to be unique.

The following is the conception of upper and lower solutions for stochastic differential equations, which are given by N. Halidias and P. E. Kloeden [12]. Many authors discussed the upper and lower solutions of the stochastic differential equation by using the other name which is the solutions of the stochastic differential inequality, for example, S. Assing and R. Manthey [13] and X. Ding and R. Wu [14].

*Definition 2.* An adapted continuous stochastic process $U_t$ (resp., $L_t$) is an upper (resp., lower) solution of SDE (1) if the inequalities

(1) $U_t \geq U_s + \int_s^t b(u, U_u) du + \int_s^t \sigma(u, U_u) dW_u,\ t \geq s \geq 0$;

(2) $L_t \leq L_s + \int_s^t b(u, L_u) du + \int_s^t \sigma(u, L_u) dW_u,\ t \geq s \geq 0$,

hold with probability 1.

*Remark 3.* It is not an easy thing to calculate the exact upper and lower solution of the general stochastic differential equations. However, one can discuss the existence of upper and lower solutions. S. Assing and R. Manthey [13] discussed the "maximal/minimal solution" of the stochastic differential inequality. They proved the existence of a "maximal/minimal solution" under some conditions. However, it is easy to show there exist the upper solutions of stochastic differential equations if the minimal solution of the stochastic differential inequality exists. In fact, the minimal solution is special upper solutions of stochastic differential equations. Similarly, we can show the existence of the lower solution by using the maximal solution of the stochastic differential inequality.

Usually, the existence and uniqueness of solutions of SDE (1) are investigated under the conditions in which the diffusion coefficient satisfies Lipschitz condition and liner growth condition. In fact, the Lipschitz condition can be generalized. In this paper, the diffusion coefficient satisfies the $(C_\sigma)$ condition.

$(C_\sigma)$: For $N > 0$, there exist an increasing function $\rho_N : \mathbb{R}_+ \longrightarrow \mathbb{R}_+$ and a predictable process $G_N(t,\omega)$ such that

$$\begin{gathered} |\sigma(t,\omega,x) - \sigma(t,\omega,y)| \leq G_N(t,\omega)\, \rho_N(|x-y|), \\ \int_0^t G_N(t,\omega)\, dt < \infty \ a.s., \\ \int_{0^+} \rho_N^{-2}(u)\, du = \infty, \end{gathered} \tag{6}$$

for all $t \geq 0$, and $x, y \in \mathbb{R}$ with $\|x\|, \|y\| \leq N$.

Note that the Lipschitz condition satisfies the $(C_\sigma)$ condition. The following lemma is an important tool of this paper and had to be proved in proposition 2.3 of X. Ding and R. Wu [14].

**Lemma 4.** *In SDE (1), we assume $\sigma$ satisfies $(C_\sigma)$ and $b$ satisfies that, for each $N > 0$, there exists a measurable process $L_N(t,\omega)$ such that*

$$\begin{gathered} \|b(t,\omega,x) - b(t,\omega,y)\| \leq L_N(t,\omega)\, \|x-y\|, \\ \int_0^t L_N(t,\omega)\, dt < \infty, \ a.s., \end{gathered} \tag{7}$$

*for all $t \geq 0$ and $x, y \in \mathbb{R}$ with $\|x\|, \|y\| \leq N$. Then SDE (1) has a unique local (explosion in the finite time) strong solution.*

*Remark 5.* Moreover, if $b$ and $\sigma$ satisfy the liner growth condition (cf. J. Jacod and J. Memin [15])

$$\|b(t,\omega,x)\| + \|\sigma(t,\omega)\| \leq H(t,\omega)(1 + \|x\|), \tag{8}$$

where $H(t,\omega)$, $t \geq 0$, is a predictable process such that $\int_0^t H^2(s,\omega) ds < \infty$, *a.s.* Then SDE (1) has a unique global strong solution.

The following theorem can be considered as a comparison theorem, and we will use it to arrive at our main result.

**Theorem 6.** *Let $b : \mathbb{R}^+ \times \Omega \longrightarrow \mathbb{R}$ be predictable such that $\int_0^t b^2(s,\omega) ds < \infty$, a.s. for any $t \geq 0$, and let $\sigma : \mathbb{R}^+ \times \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ be predictable. Suppose that $\sigma$ satisfies $(C_\sigma)$ and there exists a predictable process $H(t,\omega)$, $t \geq 0$ such that*

$$\|\sigma(t,\omega)\| \leq H(t,\omega)(1 + \|x\|), \tag{9}$$

*where $\int_0^t H^2(s,\omega) ds < \infty$, a.s. And suppose that $U_t$ and $L_t$ are upper and lower solutions of the following SDE:*

$$X_t = X_0 + \int_0^t b(s,\omega)\, ds + \int_0^t \sigma(s, X_s)\, dW_s, \quad t \geq 0, \tag{10}$$

*such that $L_0 \leq X_0 \leq U_0$, a.s.*

*Then there is a unique strong solution $X_t$ which satisfies that $L_t \leq X_t \leq U_t$ for any $t \geq 0$ holds with probability 1.*

*Proof.* Obviously, we have that SDE (10) has a unique strong solution $X_t$ by using Lemma 4 and Remark 5. In the following we will show

$$\mathbf{P}\{L_t \leq X_t \leq U_t,\ \forall t \geq 0\} = 1. \tag{11}$$

We only prove $\mathbf{P}\{X_t \le U_t,\ \forall t \ge 0\} = 1$, because we can prove $\mathbf{P}\{L_t \le X_t,\ \forall t \ge 0\} = 1$ by using the similar way.

Define the stopping time

$$T_N =: \inf\{t \in [0,\infty) : |X_t| \vee |L_t| \vee t > N\} \wedge N. \tag{12}$$

Obviously, $T_N \longrightarrow \infty$ when $N \longrightarrow \infty$. And define the stopping time $\tau =: \inf\{t \in [0,\infty) : X_t < L_t\}$. If $\mathbf{P}\{\tau < T_N\} = 0$ for $N \ge 1$, then $\mathbf{P}\{\tau < \infty\} = 0$; that is, $\mathbf{P}\{L_t \le X_t,\ \forall t \ge 0\} = 1$. Indeed, $\forall q \in Q^+$ and $N \ge 1$, we define $\alpha =: (\tau + q) \wedge T_N$ and $\Omega_\alpha =: \{X_\alpha < L_\alpha\}$. Note that

$$\mathbf{P}\{\Omega_\alpha\} = 0, \quad \forall q \in Q^+,\ N \ge 1 \Longrightarrow \mathbf{P}\{\tau < T_N\} = 0. \tag{13}$$

In fact, by $\mathbf{P}\{\Omega_\alpha\} = 0$ and $X, L$ being continuous and the denseness of the rational number in $\mathbb{R}$, we have

$$X_{(\tau+t)\wedge T_N} \ge L_{(\tau+t)\wedge T_N} \ a.s. \text{ on } \{\tau < T_N\} \tag{14}$$

for all $t \ge 0$. That is for *a.s.* $\omega \in \{\tau < T_N\}$ and $t \in [\tau(\omega), T_N(\omega)]$ one has $X_t \ge L_t$. However, by the definition of $\tau$ and $L_\tau \le X_\tau$, *a.s.* we have $\mathbf{P}\{\tau < T_N\} = 0$.

In the following we shall prove $\mathbf{P}\{\Omega_\alpha\} = 0$, $\forall q \in Q^+,\ N \ge 1$. Set $\beta =: \sup\{t \in [0,\alpha) : L_t \le X_t\}$. By continuity of $X$ and $L$ we have $X_\beta \ge L_\beta$, *a.s.* Obviously, $\{X_\alpha \ge L_\alpha\} = \{\beta = \alpha\}$. So, we have $\Omega_\alpha =: \{X_\alpha < L_\alpha\} = \{\beta < \alpha\}$. Hence, for $\omega \in \Omega_\alpha$ and $t \in (\beta(\omega), \alpha(\omega)]$ we have $X_t < L_t$. Using $L$ as a lower solution of SDE (10), we have

$$L_t - X_t \le \int_\beta^t [\sigma(s, L_s) - \sigma(s, X_s)]\, dW_s =: M_t. \tag{15}$$

Hence,

$$[L_t - X_t]\, I_{\Omega_\alpha} I_{(\beta,\alpha]}(t) \le M_t I_{\Omega_\alpha} I_{(\beta,\alpha]}(t). \tag{16}$$

Let us take $M^+ =: \max\{M, 0\}$. By the Tanaka formula (refer to [3]) we have

$$\begin{aligned} M_t^+ I_{\Omega_\alpha} &= M_\beta^+ I_{\Omega_\alpha} + I_{\Omega_\alpha} \int_\beta^t I_{\{M_s>0\}} dM_s \\ &\quad + \frac{1}{2} I_{\Omega_\alpha} \left[L_t^0(M) - L_\beta^0(M)\right], \end{aligned} \tag{17}$$

where $L_t^x(M)$ denotes local time at the point $x$ for $M$. By the definition of local time, one can prove easily that $L_t^0(M) - L_\beta^0(M) = 0$, for $t \in (\beta, \alpha]$ on $\Omega_\alpha$. So, by $M_\beta^+ I_{\Omega_\alpha} = 0$ (using the definition $M$) we have

$$\begin{aligned} M^+ I_{\Omega_\alpha} &= \int_\beta^t I_{\{M_s>0\}} I_{\Omega_\alpha} [\sigma(s, L_s) - \sigma(s, X_s)]\, dW_s \\ &=: N_t. \end{aligned} \tag{18}$$

Since for $\omega \in \Omega_\alpha$ and $t \in (\beta(\omega), \alpha(\omega)]$ we have $X_t < L_t$, by (18) we have

$$M^+ I_{\Omega_\alpha} \le N_t + \int_\beta^t I_{\{M_s>0\}} I_{\Omega_\alpha} [L_s - U_s]\, ds. \tag{19}$$

Using (16), we have

$$M^+ I_{\Omega_\alpha} \le N_t + \int_\beta^t I_{\Omega_\alpha} M^+ ds. \tag{20}$$

By the stochastic Gronwall inequality (e.g., Lemma 2.1 [14]), we have

$$I_{\Omega_\alpha} M_\alpha^+ e^{-t} \le N_\beta e^{-t} + \int_\beta^\alpha e^{-t} dN_s. \tag{21}$$

By $N_\beta = 0$ we have

$$E\left(I_{\Omega_\alpha} M_\alpha^+ e^{-t}\right) \le E \int_\beta^\alpha e^{-t} dN_s = 0. \tag{22}$$

So, using (16) once again we have

$$I_{\Omega_\alpha} [L_\alpha - X_\alpha] \le I_{\Omega_\alpha} M_\alpha^+ = 0 \ a.e. \tag{23}$$

That is $L_\alpha \le X_\alpha$ on $\Omega_\alpha$ a.s. Hence, $\mathbf{P}\{\Omega_\alpha\} = 0$. The proof is completed. □

## 3. Existence of Strong Solutions

In this section, we will show the existence of the solution for SDEs with discontinuous drift coefficients. The method of the proof of our main result is based on Amann's fixed point theorem (e.g., Theorem 11.D [16]), so we introduce it in the following.

**Lemma 7.** *Suppose that*

*(1) the mapping $f : X \longrightarrow X$ is monotone increasing on an ordered set $X$*

*(2) every chain in $X$ has a supremum*

*(3) there is an element $x_o \in X$ for which $x_0 \le f(x_0)$*

*Then $f$ has a smallest fixed point in the set $\{x \in X : x_0 \le x\}$.*

The following theorem is our main result.

**Theorem 8.** *Let $b, \sigma : \mathbb{R}^+ \times \Omega \times \mathbb{R} \longrightarrow \mathbb{R}$ be predictable. Suppose that $b$ is an increasing function in $x$ and $\sigma$ satisfies $(C_\sigma)$ and there exists a predictable process $H(t, \omega)$, $t \ge 0$, such that*

$$\|b(t, \omega, x)\| + \|\sigma(t, \omega, x)\| \le H(t, \omega)(1 + \|x\|), \tag{24}$$

*where $\int_0^t H^2(s, \omega) ds < \infty$, a.s. Moreover, suppose that $U_t$ and $L_t$ are upper and lower solutions of the SDE*

$$X_t = X_0 + \int_0^t b(s, X_s)\, ds + \int_0^t \sigma(s, X_s)\, dW_s, \quad t \ge 0, \tag{25}$$

*such that $L_0 \le X_0 \le U_0$, a.s.*

*Then there is at least a strong solution $X_t$ which satisfies that $L_t \le X_t \le U_t$ for $t \ge 0$ holds with probability 1.*

*Proof.* Let $\mathcal{X}$ be a space of adapted and continuous processes and define the order relation $\preceq$:

$$X \preceq Y \Longleftrightarrow \mathbf{P}\{X_t \le Y_t,\ \forall t \ge 0\} = 1, \tag{26}$$

for $X, Y \in \mathscr{X}$. We consider a subset of the space $(\mathscr{X}, \preceq)$

$$\begin{aligned}\mathscr{D} &=: [L, U] \\ &=: \{X \in \mathscr{X} : \mathbf{P}\{L_t \le X_t \le U_t,\ \forall t \ge 0\} = 1\}.\end{aligned} \tag{27}$$

For arbitrary fixed $Z \in \mathscr{D}$, we consider the following equation:

$$X_t = X_0 + \int_0^t b(s, Z_s)\, ds + \int_0^t \sigma(s, X_s)\, dW_s; \tag{28}$$

by Theorem 6 there exists a unique strong solution $X_t^*$. Define a mapping $S : \mathscr{D} \longrightarrow \mathscr{X}$ and $S(Z) = X^*$. To complete the proof it is enough to show $S$ has a fixed point.

Since $b$ is an increasing function and $U$ is an upper solution of SDE (25), we have that

$$U_t \ge U_s + \int_s^t b(u, Z_u)\, du + \int_s^t \sigma(u, U_u)\, dW_u \tag{29}$$

holds with probability 1 for $t \ge s \ge 0$. Then $U$ is also an upper solution of SDE (28). Similarly, we have that

$$L_t \le L_s + \int_s^t b(u, Z_u)\, du + \int_s^t \sigma(u, L_u)\, dW_u \tag{30}$$

holds with probability 1 for $t \ge s \ge 0$ such that $L$ is also a lower solution of SDE (28). Hence, using Theorem 6 we have

$$\mathbf{P}\{L_t \le S(Z_t) \le U_t,\ \forall t \ge 0\} = 1. \tag{31}$$

Since $Z$ is arbitrary, we have $S : \mathscr{D} \longrightarrow \mathscr{D}$ and $L \preceq S(L)$ and $S(U) \preceq U$. If $S$ is an increasing mapping, by Lemma 7 $S$ has a fixed point on $\mathscr{D}$. In fact, take $Z^1, Z^2 \in \mathscr{D}$ and $Z^1 \preceq Z^2$ and set $X^i =: S(Z^i)$; that is,

$$X_t^i = X_0 + \int_0^t b(s, Z_s^i)\, ds + \int_0^t \sigma(s, X_s^i)\, dW_s, \quad i = 1, 2. \tag{32}$$

Since $b$ is an increasing function, we have that

$$X_t^2 \ge X_s + \int_s^t b(u, Z_u^1)\, du + \int_s^t \sigma(u, X_u^2)\, dW_u \tag{33}$$

holds with probability 1 for $t \ge s \ge 0$. Hence $X^2$ is an upper solution of the following equation:

$$X_t = X_0 + \int_0^t b(s, Z_s^1)\, ds + \int_0^t \sigma(s, X_s)\, dW_s. \tag{34}$$

And by (29) $U$ is an upper solution of (34). Using Theorem 6 again, we have

$$\mathbf{P}\{S(Z_t^1) \le S(Z_t^2) \le U_t,\ t \ge 0\} = 1; \tag{35}$$

that is, $S(Z_t^1) \preceq S(Z_t^2)$. Hence $S$ is an increasing function. The proof is completed. □

*Example 9.* We consider the following SDE:

$$dX_t = \operatorname{sgn}(X_t)\, dt + dW_t, \quad \forall t \ge 0, \tag{36}$$

with initial value $X_0$. Obviously, $X_0 - t + W_t \le X_0 + \int_0^t \operatorname{sgn}(X_s) ds + W_t \le X_0 + t + W_t$. By Theorem 8, there exists at least one solution $X_t$ such that $X_0 - t + W_t \le X_t \le X_0 + t + W_t, t \ge 0$ holds with probability 1.

*Example 10.* We have the SDE

$$dX_t = f(X_t, t)\, dt + \sigma dW_t, \quad \forall t \ge 0, \tag{37}$$

with initial value $X_0$, where $f(x,t)$ is a bounded function and is defined as

$$f(x,t) = \begin{cases} M+1, & x \ge M; \\ x+1, & 0 \le x < M; \\ x-1, & -M \le x < 0; \\ -M-1, & x \le -M. \end{cases} \tag{38}$$

It is easy to show $X_t = X_0 - (M+1)t + \sigma W_t$ and $X_t = X_0 + (M+1)t + \sigma W_t$ are the lower solution and upper solution of (37), respectively. And $f(x,t)$ is an increasing function in $x$ but is not continuous in $x$, so we have that SDE (37) has a strong solution by using Theorem 8.

## Conflicts of Interest

The author declares that they have no conflicts of interest.


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