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Research Article

Existence of Positive Bounded Solutions of Semilinear Elliptic Problems

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This paper is concerned with the existence of bounded positive solution for the semilinear elliptic problem $\Delta u = \lambda p(x) f(u)$ in Ω subject to some Dirichlet conditions, where Ω is a regular domain in $\mathbb{R}^n$ $(n \geq 3)$ with compact boundary. The nonlinearity f is nonnegative continuous and the potential p belongs to some Kato class $K(\Omega)$. So we prove the existence of a positive continuous solution depending on λ by the use of a potential theory approach.

1. Introduction

In this paper, we study the existence of positive bounded solution of semilinear elliptic problem

$$\begin{gathered}
\Delta u = \lambda p(x) f(u) \quad \text{in } \Omega, \\
\frac{u}{\partial \Omega} = \alpha \varphi, \\
\lim_{|x| \to +\infty} u(x) = \beta \quad (\text{if } \Omega \text{ is unbounded}),
\end{gathered} \tag{P}$$

where Ω is a $C^{1,1}$-domain in $\mathbb{R}^n$ $(n \geq 3)$ with compact boundary, α and β are fixed nonnegative constants such that $\alpha + \beta > 0$, and $\beta = 0$ when Ω is bounded. The parameter λ is nonnegative, and the function φ is nontrivial nonnegative and continuous on $\partial\Omega$.

Numerous works treated semilinear elliptic equations of the type

$$\begin{gathered}\Delta u = F(x,u) \quad \text{in } D, \\ u(x) = \Phi(x) \quad \text{on } \partial D, \\ \lim_{|x|\to+\infty} u(x) = b > 0.\end{gathered} \tag{Q}$$

For the case of nonpositive function F, many results of existence of positive solutions are established in smooth domains or in $\mathbb{R}^n$, for instance, see [1–5] and the references therein.

In the case where F changes sign, many works can be cited, namely, the work of Glover and McKenna [6], whose used techniques of probabilistic potential theory for solving semilinear elliptic and parabolic differential equations in $\mathbb{R}^n$. Ma and Song [7] adapted the same techniques of those of Glover and McKenna to elliptic equations in bounded domains. More precisely, the hypotheses in [6, 7] require in particular that $F(x,u) = F(u)$ and on each compact, there is a positive constant A such that $-Au \le F(u) \le 0$.

In [8], Chen et al. used an implicit probabilistic representation together with Schauder's fixed point theorem to obtain positive solutions of the problem (Q). In fact, the authors considered a Lipschitz domain D in $\mathbb{R}^n$ $(n \ge 3)$, with compact boundary and imposed to the function F to satisfy on $D \times (0,b)$, $b \in (0,+\infty]$

$$-U(x)t \le F(x,t) \le V(x)f(t), \tag{1.1}$$

where f is nonnegative Borel measurable function defined on $(0,b)$ and the potentials U, V are nonnegative Green-tight functions in D. In particular, the authors showed the existence of solutions of (Q) bounded below by a positive harmonic function.

In [9], Athreya studied (Q) with the singular nonlinearity $F(x,t) = g(t) \le \max(1, t^{-\alpha})$, $0 < \alpha < 1$, in a simply connected bounded C^2-domain D in $\mathbb{R}^n$, $n \ge 3$. He showed the existence of solutions bounded below by a given positive harmonic function h_0, under the boundary condition $\Phi \ge (1 + A)h_0$, where A is a constant depending on h_0, α, and D.

Recently, Hirata [20] gave a Chen-Williams-Zhao type theorem for more general regular domains D. More precisely, the author imposed to the function F to satisfy

$$|F(x,u)| \le V(x)u^{-\mu}, \quad \mu > 0, \tag{1.2}$$

where the potential V belongs to a class of functions containing Green-tight ones. We remark that the class of functions introduced by Hirata coincides with the classical Kato class introduced for smooth domains in [10, 11].

In this paper, we will consider $F(x,u) = \lambda p(x) f(u)$. We impose to the potential p to be in a new Kato class $K(\Omega)$ (see Definition 1.1 below), which contains the Green-tight functions and the classical Kato class used by Hirata. More precisely, we will prove using potential theory's tools, the existence of positive solution for (P). Moreover, we will give global behaviour for the solution.

So, in the remainder of this introduction, we will give some results related on potential theory, and we will prove others. In the second section, we will give the main theorem and some examples of applications.

Let us recall that $\mathcal{B}(\Omega)$ is the set of Borel measurable functions in Ω and $C_0(\Omega)$ is the set of continuous ones vanishing at $\partial\Omega \cup \{\infty\}$. The exponent + means that only the nonnegative functions are considered.

We denote by $H_\Omega\varphi$ the bounded continuous solution of the Dirichlet problem

$$\begin{gathered}\Delta u = 0 \quad \text{in } \Omega,\\ u = \varphi \quad \text{on } \partial\Omega,\\ \lim_{|x|\to+\infty} u(x) = 0, \quad \text{if } \Omega \text{ is unbounded,}\end{gathered} \tag{1.3}$$

where the function φ is nontrivial nonnegative continuous on $\partial\Omega$. In the remainder of this paper, we denote $h = 1 - H_\Omega 1$, and we remark that $h = 0$ when Ω is bounded.

Let us recall some notations and notions concerning essentially the potential theory.

(i) For $f \in \mathcal{B}^+(\Omega)$, we denote by Vf the potential defined in Ω by

$$Vf(x) = \int_\Omega G(x,y)f(y)dy, \tag{1.4}$$

where G is the Green function of the Laplace operator Δ on Ω with Dirichlet conditions.

(ii) We recall that if $f \in L^1_{\text{loc}}(\Omega)$ and $Vf \in L^1_{\text{loc}}(\Omega)$, then we have $\Delta(Vf) = -f$ in Ω (in the sense of distributions), see [10, page 52].

(iii) Let $(X_t, t > 0)$ be the Brownian motion in $\mathbb{R}^n$ and P^x be the probability measure on the Brownian continuous paths starting at x. For $p \in \mathcal{B}^+(\Omega)$, we define the kernel V_p by

$$V_pf(x) = E^x\left(\int_0^{\tau_\Omega} e^{-\int_0^t p(X_s)ds} f(X_t)dt\right), \tag{1.5}$$

where E^x is the expectation on P^x and $\tau_\Omega = \inf\{t > 0 : X_t \notin \Omega\}$. If $p \in \mathcal{B}^+(\Omega)$ is such that $Vp < \infty$, the kernel V_p satisfies (see [10, 12])

$$V = V_p + V_p(pV) = V_p + V(pV_p). \tag{1.6}$$

So for, each $u \in \mathcal{B}(\Omega)$ such that $V(p|u|) < \infty$, we have

$$(u - V_p(pu))(u + V(pu)) = (u + V(pu))(u - V_p(pu)) = u. \tag{1.7}$$

(iv) We recall that a function $f : [0,\infty) \to \mathbb{R}$ is called completely monotone if $(-1)^n f^{(n)} \geq 0$, for each $n \in \mathbb{N}$. Moreover, if f is completely monotone on $[0,\infty)$, then by [13, Theorem 12a], there exists a nonnegative measure μ on $[0,\infty)$ such that

$$f(t) = \int_0^\infty \exp(-tx)d\mu(x). \tag{1.8}$$

So, using this fact and the Hölder inequality, we deduce that if f is completely monotone from $[0,\infty)$ to $(0,\infty)$, then $\mathrm{Log}(f)$ is a convex function.

(v) Let $f \in \mathcal{B}^+(\Omega)$ be such that $Vf < \infty$. From (1.5), it is easy to see that for each $x \in \Omega$, the function $t \to V_{tq}f(x)$ is completely monotone on $[0,\infty)$.

Now, we recall some properties relating to the Kato class $K(\Omega)$.

Definition 1.1 (see [14, 15]). A Borel measurable function p in Ω belongs to the class $K(\Omega)$ if p satisfies

$$\begin{gathered}\lim_{\alpha\to 0}\left(\sup_{x\in\Omega}\int_{\Omega\cap B(x,\alpha)}\frac{\rho(y)}{\rho(x)}G(x,y)|p(y)|dy\right) = 0,\\ \lim_{M\to\infty}\left(\sup_{x\in\Omega}\int_{\Omega\cap(|y|\geq M)}\frac{\rho(y)}{\rho(x)}G(x,y)|q(y)|dy\right) = 0 \quad (\text{if } \Omega \text{ is unbounded}),\end{gathered} \tag{1.9}$$

where $\rho(x) = \min(1,\delta(x))$ and $\delta(x)$ is the Euclidean distance between x and $\partial\Omega$.

Remark 1.2. When Ω is a bounded domain, then we can replace $\rho(x)$ by $\delta(x)$ and the condition (1.9) is superfluous.

Proposition 1.3 (See [14, 15]). *Let p be a nonnegative function in $K(\Omega)$. Then one has*

(i) $\|p\|_\Omega := \sup_{x\in\Omega}\int_\Omega \rho(y)/\rho(x)G(x,y)p(y)dy < \infty$,

(ii) *the potential $Vp \in C_0(\Omega)$.*

Proposition 1.4 (see [16, 17]). *Let p be a nonnegative function belonging to $K(\Omega)$. Then, one has*

(i)

$$\alpha_p = \sup_{x,y\in\Omega}\int_\Omega \frac{G(x,z)G(z,y)}{G(x,y)}p(z)dz < \infty, \tag{1.10}$$

(ii) *for any nonnegative superharmonic function v in Ω, one has*

$$\int_\Omega G(x,y)v(y)p(y)dy \leq \alpha_p v(x), \quad \forall x \in D. \tag{1.11}$$

Proposition 1.5. *Let v be a nonnegative superharmonic function in Ω and p be a nonnegative function in $K(\Omega)$. Then, for each $x \in \Omega$ such that $0 < v(x) < \infty$, one has*

$$\exp(-\alpha_p)v(x) \le v(x) - V_p(pv)(x) \le v(x). \tag{1.12}$$

Proof. Let v be a nonnegative superharmonic function, then by [18, Theorem 2.1, page 164], there exists a sequence $(v_k)_k$ of nonnegative measurable functions in Ω such that the sequence $(Vv_k)_k$ defined in Ω by

$$Vv_k(x) := \int_\Omega G(x,y)v_k(y)dy \tag{1.13}$$

increases to v.

Let $x \in \Omega$ such that $0 < v(x) < \infty$. Then, there exists $k_0 \in \mathbb{N}$ such that $0 < Vv_k(x) < \infty$, for $k \ge k_0$.

Now, for a fixed $k \ge k_0$, we consider the function $\kappa(t) = V_{tp}v_k(x)$. Since the function κ is completely monotone on $[0,\infty)$, then $\log(\kappa)$ is convex on $[0,\infty)$. Therefore,

$$\kappa(0) \le \kappa(1)\exp\left(-\frac{\kappa'(0)}{\kappa(0)}\right), \tag{1.14}$$

which means

$$Vv_k(x) \le V_p v_k(x)\exp\left(\frac{V(pVv_k)(x)}{Vv_k(x)}\right). \tag{1.15}$$

Hence, it follows from Proposition 1.4 (i) that

$$\exp(-\alpha_v)Vv_k(x) \le V_p v_k(x). \tag{1.16}$$

Consequently, from (1.6), we obtain that

$$\exp(-\alpha_p)Vv_k(x) \le Vv_k(x) - V_p(pVv_k(x))(x) \le Vv_k(x). \tag{1.17}$$

By letting $k \to +\infty$, we deduce the result. □

2. Main Result

In this section, we will give an existence result for the problem (P). Assume the following assumptions.

(A_1) The function p is nonnegative and belongs to $K(\Omega)$.

(A_2) The function f is a nonnegative, continuous on $[0,+\infty)$ and satisfies $\forall c > 0$, $\exists a \ge 0$ such that, $\forall 0 \le s < t \le c$, $f(t) - f(s) \le a(t-s)$.

(A_3) $\sigma_0 = \inf_{x\in\overline{\Omega}}(\alpha H_\Omega\varphi(x) + \beta h(x))/f(0)Vp(x) > 0$.

Remark 2.1. Let f be in $C^1([0,+\infty))$, then for $a := \max(\sup_{t\in[0,c]} f'(t), 0)$, the function f satisfies (A_2). In particular, if f is nonincreasing, then $a = 0$ holds.

Consider the function $\theta : \lambda \longrightarrow \lambda \exp(\lambda a \alpha_p)$, where α_p is the constant associated to the potential p defined by (1.10). It is obvious to see that θ is bijective from $[0,+\infty)$ to $[0,+\infty)$.

Theorem 2.2. *Assume that the hypotheses* (A_1)–(A_3) *are satisfied. Then, for each* $\lambda \in [0, \theta^{-1}(\sigma_0))$, *the problem* (P) *has a positive continuous bounded solution satisfying*

$$\left(1 - \frac{\theta(\lambda)}{\sigma_0}\right) \exp(-\lambda a \alpha_p)(\alpha H_\Omega \varphi + \beta h) \leq u \leq \alpha H_\Omega \varphi + \beta h. \tag{2.1}$$

Remark 2.3. We remark that if f satisfies the hypothesis (A_2) and $f(0) = 0$, we take $\sigma_0 = +\infty$, in this case for each $\lambda \in \mathbb{R}_+$, the problem (P) has a positive bounded solution satisfying

$$\exp(-\lambda a \alpha_p)(\alpha H_\Omega \varphi + \beta h) \leq u \leq \alpha H_\Omega \varphi + \beta h. \tag{2.2}$$

Now, let us give some examples of applications of the above theorem.

Example 2.4. Assume that (A_1) is satisfied. Let $\mu \geq 1$. Then, for each $\lambda \in \mathbb{R}_+$, the following problem

$$\begin{gathered} \Delta u = \lambda p(x) u^\mu \quad \text{in } \Omega, \\ u(x) = \alpha \varphi(x) \quad \text{on } \partial\Omega, \\ \lim_{|x| \to +\infty} u(x) = \beta \end{gathered} \tag{2.3}$$

admits a positive continuous bounded solution. Indeed, for each $c > 0$, one verifies that for $a = \mu c^{\mu-1}$, the function $f(t) = t^\mu$ satisfies (A_2).

Example 2.5. Let $\mu \geq 0$. Assume (A_1) and (A_3). Consider the following:

$$\begin{gathered} \Delta u = \lambda p(x)(1+u)^{-\mu} \quad \text{in } \Omega, \\ u(x) = \alpha \varphi(x) \quad \text{on } \partial\Omega, \\ \lim_{|x| \to +\infty} u(x) = \beta, \quad u > 0. \end{gathered} \tag{2.4}$$

Then, the function $f(t) = (1+t)^{-\mu}$ is in $C^1([0,+\infty))$ and decreasing. By Remark 2.1, the hypothesis (A_2) is satisfied for $a = 0$. So that for each $\lambda \in [0, \sigma_0)$, (2.4) has a positive continuous bounded solution satisfying

$$\left(1 - \frac{\lambda}{\sigma_0}\right)(\alpha H_\Omega \varphi + \beta h) \leq u \leq \alpha H_\Omega \varphi + \beta h. \tag{2.5}$$

Example 2.6. Let Ω be a $C^{1,1}$-bounded domain and suppose that the hypothesis (A_2) is satisfied. Let g be a nonnegative function in $L^q(\Omega)$ such that $q > n/2$ and suppose that $\mu < 1 - n/q$. Then,

$$\begin{gathered} \Delta u = \lambda \frac{g(x)}{\delta(x)^{\mu}} f(u) \quad \text{in } \Omega, \\ u(x) = \alpha\varphi(x) \quad \text{on } \partial\Omega \end{gathered} \tag{2.6}$$

has a positive continuous solution.

Let us verify the assumptions (A_1) and (A_3). From [16, Proposition 2.3], the function $p = g/\delta(\cdot)^{\mu} \in K(\Omega)$, and so the hypothesis (A_1) is satisfied. From [16, Proposition 2.7(iii)], there exists a constant $c_1 > 0$ such that we have for each $x \in \Omega$

$$Vp(x) \le c_1 \delta(x). \tag{2.7}$$

Now, since φ is nontrivial continuous function at $\partial\Omega$, then there exists $c_2 > 0$, such that one has on Ω

$$H_{\Omega}\varphi(x) \ge c_2 \|\varphi\|_1 \delta(x). \tag{2.8}$$

Thus, $\sigma_0 = \inf_{x\in\overline{\Omega}}(H_{\Omega}\varphi(x)/f(0)Vp(x)) > 0$ and so the assumption (A_3) is satisfied.

Example 2.7. Let $\Omega = \overline{B(0,1)}^{c}$ be the exterior of the unit ball in $\mathbb{R}^n$ $(n \ge 3)$. Suppose that the hypothesis (A_2) is satisfied. Let $\gamma,\ \mu \in \mathbb{R}$ such that $\gamma < 1 < 2 < n < \mu$. Then,

$$\begin{gathered} \Delta u = \lambda \frac{1}{|x|^{\mu-\gamma}(|x|-1)^{\gamma}} f(u) \quad \text{in } \Omega, \\ u(x) = \alpha\varphi(x) \quad \text{in } \partial\Omega, \\ \lim_{|x|\to+\infty} u(x) = \beta \end{gathered} \tag{2.9}$$

has a positive continuous solution.

From [14], the function $p(x) = 1/|x|^{\mu-\gamma}(|x|-1)^{\gamma} \in K(\Omega)$ and so the assumption (A_1) is satisfied. Moreover, from [14, Proposition 3.5], there exists a constant $c_1 > 0$ such that one has

$$Vp(x) \le c_1 \frac{|x|-1}{|x|^{n-1}}. \tag{2.10}$$

Now, from [19, page 258], there exists a constant $c_2 > 0$ such that one has on Ω

$$\alpha H_{\Omega}\varphi(x) + \beta h(x) \ge c_2 \frac{|x|-1}{|x|^{n-1}}. \tag{2.11}$$

Thus, $\sigma_0 = \inf_{x\in\overline{\Omega}}((\alpha H_\Omega\varphi(x) + \beta h(x))/f(0)Vp(x)) \ge c_2/c_1 > 0$ and so the assumption (A_3) is satisfied.

In the next, we will give the proof of Theorem 2.2.

Proof of the Main Theorem. Let $p \in K^+(\Omega)$ and put $w := \alpha H_\Omega\varphi + \beta h$. Let $c = \|\omega\|_\infty > 0$, then from (A_2), there exists $a \ge 0$, such that the function $\psi : t \to at + f(0) - f(t)$ is a nondecreasing function on $[0, c]$. Let σ_0 be the constant given by (A_3), and let $\lambda \in [0, \theta^{-1}(\sigma_0))$ where $\theta(\lambda) := \lambda\exp(\lambda a\alpha_p)$. Put $q := \lambda ap$. Consider the nonempty bounded convex set given by

$$\Lambda := \left\{u \in \mathcal{B}^+(\Omega) : \left(1 - \frac{\theta(\lambda)}{\sigma_0}\right)\exp(-\lambda a\alpha_p)w \le u \le w\right\}. \tag{2.12}$$

Let T be the operator defined on Λ by

$$Tu := w - V_q(qw) - \lambda f(0)V_q p + \lambda V_q(p\psi(u)). \tag{2.13}$$

We claim that the operator T maps Λ to itself. Indeed, by (A_2) and using the monotony of the function ψ, we have for $u \in \Lambda$

$$\begin{aligned}
Tu &= w - V_q(\lambda p(aw + f(0))) + \lambda V_q(p\psi(u)) \\
&= w - V_q(\lambda p(\psi(w) + f(w))) + \lambda V_q(p\psi(u)) \\
&\le w - \lambda V_q(pf(w)) + \lambda V_q(p(\psi(u) - \psi(w))) \\
&\le w.
\end{aligned} \tag{2.14}$$

On the other hand, by using Proposition 1.5 and (A_3), we have

$$\begin{aligned}
Tu &\ge \exp(-\lambda a\alpha_p)w - \lambda f(0)V_q p + \lambda V_q(p\psi(u)) \\
&\ge \exp(-\lambda a\alpha_p)w - \lambda f(0)Vp \\
&\ge \exp(-\lambda a\alpha_p)w - \frac{\lambda}{\sigma_0}w \\
&= \left(1 - \frac{\theta(\lambda)}{\sigma_0}\right)\exp(-\lambda a\alpha_p)w.
\end{aligned} \tag{2.15}$$

Hence, $T\Lambda \subset \Lambda$.

Next, we prove that the operator T is nondecreasing on Λ. Let $u_1, u_2 \in \Lambda$ such that $u_1 \le u_2$, then by hypothesis (A_2), we obtain

$$Tu_2 - Tu_1 = \lambda V_q p(\psi(u_2) - \psi(u_1)) \ge 0. \tag{2.16}$$

Now, consider the sequence $(u_k)_k$ defined by

$$u_0 = w - \lambda a V_q(pw) - \lambda f(0) V_q(p), \tag{2.17}$$

and $u_{k+1} = Tu_k$ for $k \in \mathbb{N}$.

It is obvious to see that $u_0 \in \Lambda$ and $u_1 = Tu_0 \geq u_0$. Thus, using the fact that Λ is invariant under T and the monotony of T, we deduce that

$$\left(1 - \frac{\theta(\lambda)}{\sigma_0}\right) \exp(-\lambda a \alpha_p) w \leq u_0 \leq u_1 \leq \cdots \leq u_k \leq w. \tag{2.18}$$

Hence, the sequence $(u_k)_k$ converges to a function $u \in \Lambda$.

Therefore, from the monotone convergence theorem and the fact that ψ is continuous, the sequence $(Tu_k)_k$ converges to Tu. So,

$$u := w - V_q(qw) - \lambda f(0) V_q p + \lambda V_q(p\psi(u)), \tag{2.19}$$

or equivalently

$$(u - V_q(qu)) = (w - V_q(qw)) - V_q(\lambda p f(u)). \tag{2.20}$$

Applying the operator $(I + V(q\cdot))$ to both sides of the above equality and using (1.6) and (1.7), we conclude that u satisfies

$$u = \alpha H_\Omega \varphi + \beta h - \lambda V(pf(u)). \tag{2.21}$$

Finally, let us verify that u is a solution of the problem (P). Using the fact that $p \in K^+(\Omega)$ and $f(u)$ is bounded on $[0, c]$, we obtain $pf(u) \in K^+(\Omega)$. So, Proposition 1.3 (ii) yields $V(pf(u)) \in C_0(\Omega)$ which implies with the continuity of the harmonic continuous function w that u is continuous on Ω. This completes the proof. □

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