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Research Article

A Fast Geometric Optics-Based Design Approach for Dielectric Rod Antennas

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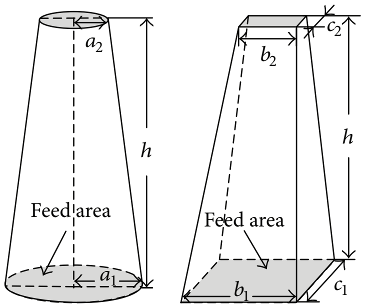
A simple and effective dielectric rod antenna design approach based on geometric optics and modal analysis is presented. The tapered antennas from microwave to terahertz can be readily designed with the equations presented in this paper. The radius of antenna is determined by the desired traveling wave mode inside the antenna. The field inside the antenna consists of the fundamental mode and the second mode. For the end-fire operation, there is only the fundamental mode. The length of the antenna is designed based on geometric optics theory so that most of the traveling wave will be converted to the radiating field towards the output plane, avoiding reflection inside the antenna. Such antennas can achieve wide bandwidth. The gain increases with the length of the antenna as long as the diameters and length satisfy the conditions given in this paper. A number of antenna design examples with high gain and wide bandwidth are presented. The measurement results of a 130 GHz rectangular antenna with a length of 4λ show a bandwidth of 60 GHz and a gain of 12 dB.

1. Introduction

Dielectric rod antennas have been investigated and developed for many years. There are generally two kinds of shapes for the dielectric rod antennas. One is the rectangular rod, and the other one is the cylindrical rod. Rectangular antennas for the frequency ranges of 81.5 GHz, 30 GHz, and 150 GHz have been presented, respectively, in [1–3]. Partially metallized rectangular rod antenna is described in [4]. Cylindrical rod antenna inside the metallic waveguide is designed and simulated in [5]. Wide bandwidth cylindrical rod antennas are also presented in [6, 7]. However there have not been clear equations for the design of high gain and wide bandwidth antennas. In [8], the author used simple formulas to design rectangular rod antennas based on the dielectric materials. The antennas with different lengths were measured and compared. Although it was expected that the gain would increase with the length, the gain of the antenna with length $L = 15\lambda_0$ is not higher than that of the antenna with $L = 8\lambda_0$. In [6, 7], even though bandwidths are wide, the gains are very low, and the design method has not been described. Dielectric resonator antenna (DRA) has also been studied for years, but its gain is lower than that of the dielectric rod antennas due to the nontravelling nature of DRA modes.

In this paper, the geometric optics is used for the first time to design dielectric rod antennas. Using the proposed method, most of the power contained in the traveling wave radiates through the output plane of the rod antenna, minimizing reflected power. The propagating modes inside the antenna are the fundamental mode and the second mode, and for the end-fire operation only the fundamental mode is excited. Accurate design equations based upon these principles are given. The design parameters obtained from these equations can be subsequently verified by a rigorous electromagnetic (EM) simulation software, such as HFSS. It is shown that only minor fine tuning/optimization is needed. Following this approach, design of dielectric rod antennas will be fast and straightforward as demonstrated



(a) Cylindrical antenna

(b) Rectangular antenna

Figure 1: Structures of dielectric rod antennas.

with a number of design examples. In addition, a 130 GHz rectangular antenna is fabricated and measured. The simulation and measurement results show a wide bandwidth of 60 GHz and 12 dB gain. It needs to be emphasized that with the proposed design method the dielectric antennas (microwave to terahertz) with extreme bandwidth can be readily designed.

2. Design of Tapered Dielectric Rod Antennas

2.1. The Feed Design. The dielectric antenna can be fed by a waveguide and is simply placed on the top of the waveguide. For cylindrical rod antennas, the waveguide can be either a rectangular waveguide or a circular one. For rectangular antenna, the waveguide should be rectangular. The waveguide dimension can be the same as the bottom plane or matched with the frequency. In our design, the waveguide dimension which corresponds to the frequency is bigger than the bottom plane. Substrate made of the same dielectric material is therefore added under the antennas.

2.2. Design of Cylindrical Rod Antennas. Cylindrical dielectric waveguides support non-TEM modes. The fundamental mode of a cylindrical dielectric waveguide, HE_{11}, has a zero cutoff frequency. The second mode, TE_{01} and TM_{01}, has a normalized cutoff frequency $B = 2.405$. The third mode, HE_{21}, has a normalized cutoff frequency $B = 2.909$.

The structure of a tapered cylindrical dielectric rod antenna is shown in Figure 1(a). The bottom radius is a_1, the upper radius is a_2, and the length of the antenna is h.

It was proposed that only single mode travels inside the rod antennas [7, 8], but the gains are very low. In order to improve the gain and keep wide bandwidth, the bottom radius a_1 is designed to support both the fundamental and second modes, and upper radius a_2 supports only the fundamental mode.

As a quasi-travelling antenna, the length of the rod should normally be larger than one wavelength in the dielectric material.

The circular dielectric waveguide cutoff frequencies, f_c, except for the fundamental mode are [9]

$$f_c = \frac{B \cdot c}{2\pi a \cdot \sqrt{\varepsilon_r - 1}}, \tag{1}$$

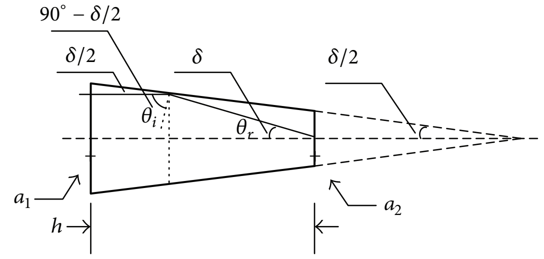


Figure 2: Cross section of a dielectric rod antenna.

where c is the speed of light and ε_r is the dielectric constant. If the antenna is fed by the fundamental and second modes, the bottom radius a_1 should be

$$\frac{2.405 \cdot c}{2\pi f_c \cdot \sqrt{\varepsilon_r - 1}} < a_1 < \frac{2.909 \cdot c}{2\pi f_c \cdot \sqrt{\varepsilon_r - 1}}. \tag{2}$$

If the upper section supports only the fundamental mode, the upper radius a_2 should be

$$0 < a_2 < \frac{2.405 \cdot c}{2\pi f_c \cdot \sqrt{\varepsilon_r - 1}}. \tag{3}$$

The cross section of the antenna is shown in Figure 2. Assume that the incident wave angle, θ_i, is parallel to radial direction, and the tapered angle of the antenna is $\delta/2$ as shown in Figure 2. The total reflection at the interface of the dielectric will happen for $\theta_i > \theta_c$, where θ_c is the critical angle:

$$\theta_c = \arcsin\left(\frac{\varepsilon_1}{\varepsilon_2}\right). \tag{4}$$

Here ε_1 is the dielectric constant of air and ε_2 is the dielectric constant of the antenna.

In order to avoid the resonance in the antenna, the attempt is made to ensure that all of the singly reflected ray fields will be transmitted through the upper plane (tip) of the antenna and radiate, and θ_r should be smaller than critical angle θ_c. If the total transmission happens at the end of the antenna, the antenna will have much higher efficiency and gain.

From Figure 2, we obtain

$$\begin{gathered} a_1 \tan\left(90^\circ - \delta\right) > h, \\ \tan\frac{\delta}{2} = \frac{a_1 - a_2}{h}, \\ \delta < \theta_c. \end{gathered} \tag{5}$$

From (4) and (5), the relationship among length h and the radiuses a_1 and a_2 should be

$$a_2 > a_1 - h \tan\left(\frac{90^\circ - \arctan\left(h/a_1\right)}{2}\right) \tag{6}$$

$$2 \arctan\left(\frac{a_1 - a_2}{h}\right) < \arcsin\left(\frac{\varepsilon_1}{\varepsilon_2}\right). \tag{7}$$

However, according to the plane wave ray theory, only if the wave incident is at Brewster angle and is polarized parallel to the plane of incidence, the total transmission will happen. For cylindrical dielectric antenna this situation is impossible because the base mode is HE_{11}, which includes perpendicularly polarized wave component as well. Nonetheless, (6) and (7) can serve as accurate design guidelines, as will be shown in a number of design examples.

In addition, proper design of the upper radius can increase the bandwidth and make the gain higher. The upper radius should be small enough to suppress the second mode so that only the fundamental mode, HE_{11}, can reach the end of the antenna.

Generally speaking, increasing length will enhance the gain. However, if the conditions in (6) and (7) are not satisfied, increasing h will cause the gain to decrease.

2.3. Design of Rectangular Rod Antennas. The structure of a tapered rectangular dielectric rod antenna is shown in Figure 1(b). The bottom diameters are b_1 and c_1, the top diameters are b_2 and c_2, and the length of the antenna is h.

For rectangular dielectric rod antenna, the bottom dimensions should satisfy the following condition [10]:

$$\begin{gathered}\frac{c}{f_r\sqrt{\varepsilon_r}}\left(\frac{1}{2}-\frac{1}{\pi\sqrt{\varepsilon_r}\sqrt{\varepsilon_r-1}}\right)<b_1\left(c_1\right)\\<\frac{c}{f_r\sqrt{\varepsilon_r}}\left(1-\frac{\sqrt{\varepsilon_r}}{\pi\sqrt{\varepsilon_r-1}}\right),\end{gathered}\tag{8}$$

where f_r is the center frequency of the antennas.

The top dimensions should satisfy the following condition:

$$\begin{gathered}\frac{c}{f_r\sqrt{\varepsilon_r}}\left(\frac{1}{2}-\frac{\sqrt{\varepsilon_r}}{\pi\sqrt{\varepsilon_r-1}}\right)<b_2\left(c_2\right)\\<\frac{c}{f_r\sqrt{\varepsilon_r}}\left(\frac{1}{2}-\frac{1}{\pi\sqrt{\varepsilon_r}\sqrt{\varepsilon_r-1}}\right).\end{gathered}\tag{9}$$

When the bottom plane and top plane are rectangle, the dimensions are calculated using (8) and (9).

Decreasing the top dimensions b_2 or c_2 can increase the bandwidth and shift the frequency band up due to cutoff frequency reduction [10]. Increasing the bottom diameters b_1 or c_1 will shift the frequency band down; conversely, reducing the bottom diameters will make the frequency band shift up.

Because the cutoff frequencies are derived from the approximate boundary conditions, the antenna dimensions can be tuned based on (8) and (9).

The relationship of bottom dimensions, top dimensions, and the length is similar to cylindrical antennas as shown below:

$$\begin{gathered}b_1\left(c_1\right)<b_2\left(c_2\right)+h\tan\left(\frac{90^\circ-\arctan\left(h/b_1\left(c_1\right)\right)}{2}\right),\\2\arctan\left(\frac{b_1\left(c_1\right)-a_2\left(c_2\right)}{h}\right)<\arcsin\left(\frac{\varepsilon_1}{\varepsilon_2}\right).\end{gathered}\tag{10}$$

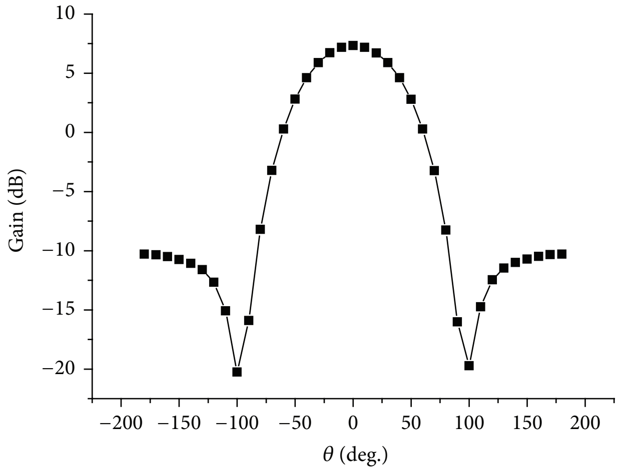


Figure 3: The gain of the antenna with $f = 32$ GHz and $h = 0.75\lambda$.

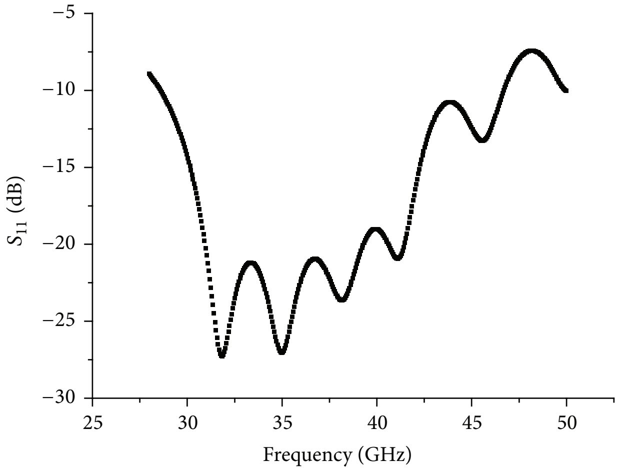


Figure 4: The return loss of the antenna with $f = 32$ GHz and $h = 0.75\lambda$.

3. The Examples of Antenna Design and Discussion

3.1. Design Description. Design examples of both cylindrical and rectangular rod antennas from millimeter wave to terahertz will be presented in this section. All simulations in the following are performed using HFSS. The dielectric material is silicon with dielectric constant $\varepsilon_r = 11.9$.

Figures 3 and 4 show the simulation results of a rectangular antenna with a center frequency of 32 GHz.

According to (8), (9), and (10), the dimensions are $b_1 = c_1 = 1.1$ mm, $b_2 = c_2 = 2$ mm, and $h = 0.75\lambda$. The simulation results show that a 7.3 dB gain can be obtained with only 0.75λ antenna length, and the bandwidth is 18 GHz with the return loss $S_{11} \leq -10$ dB.

Figures 5 and 6 show the simulation results of a rectangular antenna with a center frequency of 1 THz.

According to (8), (9), and (10), the dimensions are $b_1 = c_1 = 0.07$ mm, $b_2 = c_2 = 0.036$ mm, and $h = 3\lambda$. The simulation results show that a broadband gain of 12.5 dB can be obtained from a 3λ antenna length.

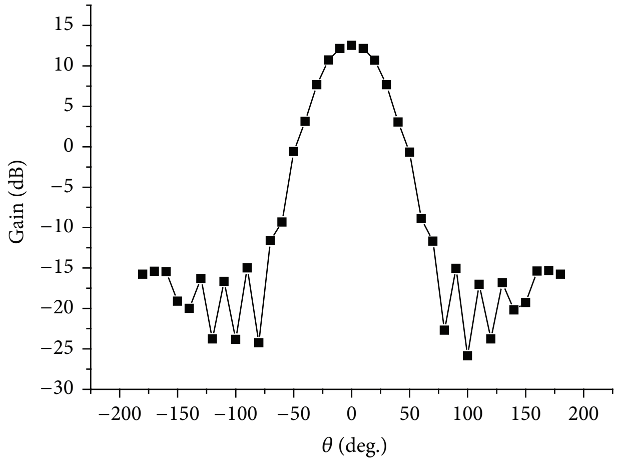


FIGURE 5: The gain of the antenna with $f = 1$ THz and $h = 3\lambda$.

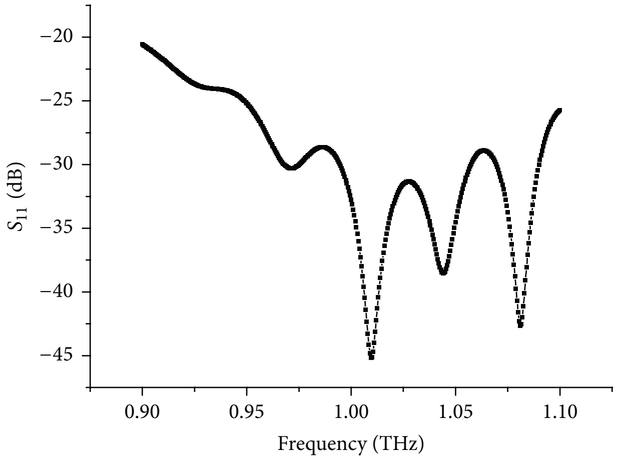


FIGURE 6: The return loss of the antenna with $f = 1$ THz and $h = 3\lambda$.

The gain will increase as the length increases if the diameters and length satisfy the conditions given in the previous section. For rectangular rod antenna with $f = 300$ GHz, if the bottom plane and top plane are all square with $b_1 = c_1 = 0.2$ mm and $b_2 = c_2 = 0.13$ mm, the far field patterns with different length are shown in Figure 7. In Figure 7, with a length of $h = 2.5$ mm, the antenna gain is 11.2 dB. The gain increases to 16.6 dB for $h = 10$ mm and 20.6 dB for $h = 40$ mm. All of the three antennas have wide bandwidth.

For rectangular rod antennas, the diameters can be tuned to be slightly different from the calculated values. However if the dimensions are too large or too small, the antenna gain and the bandwidth will decrease. As an example, the different design results for a 100 GHz antenna are shown in Figures 8 and 9.

For the 100 GHz antenna, according to (7) and (9), the diameters should be

$$\begin{gathered} 0.408\text{ mm} < b_1\left(c_1\right) < 0.579\text{ mm}, \\ 0.144\text{ mm} < b_2\left(c_2\right) < 0.408\text{ mm}. \end{gathered} \tag{11}$$

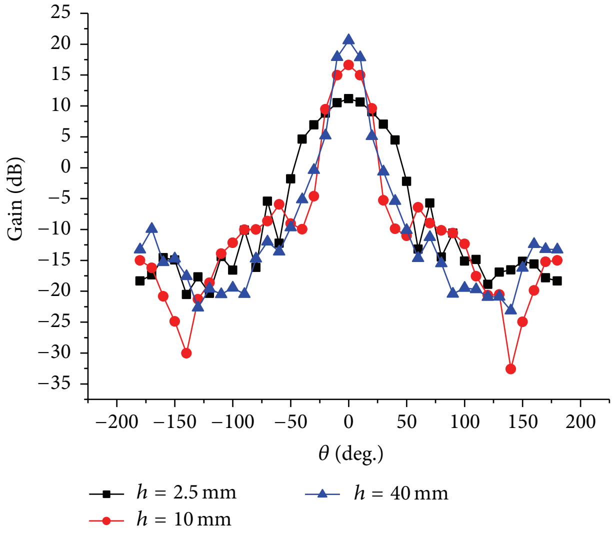


FIGURE 7: The gain of the 300 GHz antenna for different lengths.

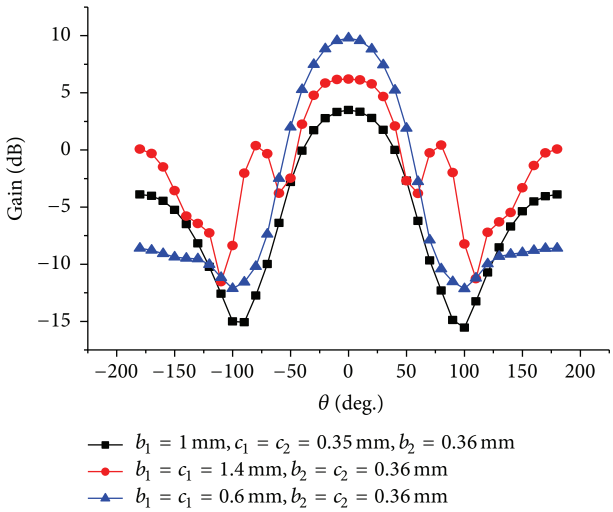


FIGURE 8: The gain of the 100 GHz rectangular antennas ($h = 1.3\lambda$).

When we choose the bottom diameters $b_1 = c_1 = 0.6$ mm, top diameters $b_2 = c_2 = 0.36$ mm, and the length $h = 4$ mm $= 1.3\lambda$, the gain is 9.8 dB, and the bandwidth is 55 GHz as shown in Figures 8 and 9.

With the same top dimensions and the same length, if the bottom dimensions are $b_1 = c_1 = 1.4$ mm, which is too large compared to the calculated value of 0.579 mm, the antenna gain is only 6.2 dB and the bandwidth is very narrow. Significant reduction in the antenna dimensions has similar effects. If c_1 is reduced to 0.35 mm for the bottom rectangle, even with b_1 increased to 1 mm, the antenna gain is only 3.5 dB with very narrow bandwidth as shown in Figures 8 and 9.

3.2. Fabrication and Measurement. A rectangular rod antenna is designed and fabricated. The center frequency is 130 GHz, and the dielectric is also silicon. The bottom plane is square, and $b_1 = b_2 = c_1$.

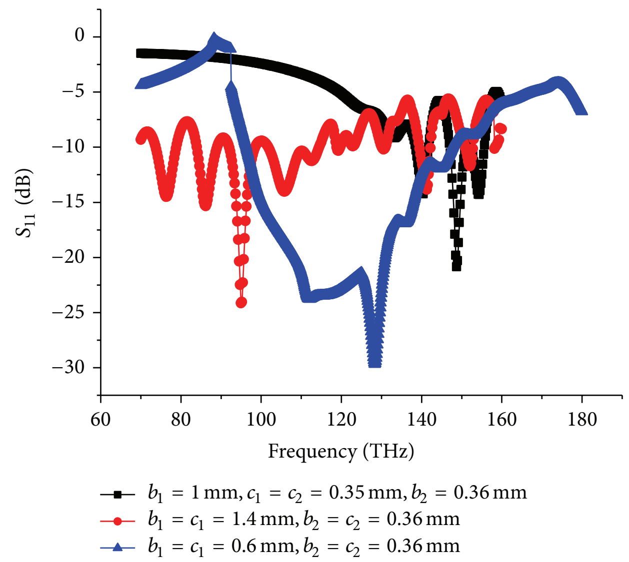


Figure 9: The return loss of 100 GHz rectangular antennas ($h = 1.3\lambda$).

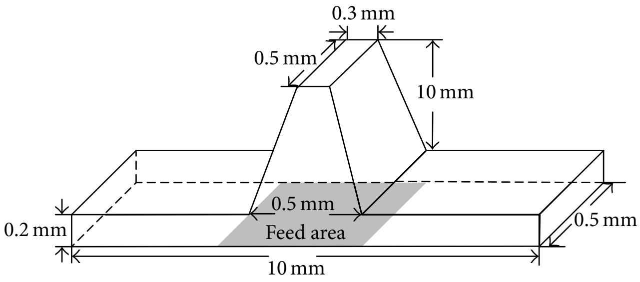


Figure 10: The antenna structure for fabrication.

From (8), the bottom width is $0.3158\text{ mm} < c_1 < 0.4464\text{ mm}$. From (9), we obtain $0.1119\text{ mm} < c_2 < 0.3158\text{ mm}$.

The length of antenna is $h = 10\text{ mm}$ (4λ). Because the modal equations are approximate, the antenna dimensions derived from (8), (9), and (10) are tuned slightly. The optimized dimensions are $b_1 = b_2 = c_1 = 0.5$ mm and $c_2 = 0.3$ mm as shown in Figure 10.

The size of the bottom plane is smaller than the metal waveguide. In order to measure the antenna, a pedestal made from the same dielectric is added, as shown in Figure 10. The pedestal length has less effect on the results. To facilitate measurement, the pedestal length is chosen to be 10 mm. The pedestal thickness is 0.2 mm and width is 0.5 mm.

A laser machine is used to cut the silicon wafer to fabricate the antenna. The antenna is placed on top of a waveguide for measurement. The photograph of the silicon antenna and waveguide is shown in Figure 11.

The antenna is attached to the metal rectangular waveguide simply using a sticking tape as shown in Figure 11. The inside waveguide dimension shown in Figure 11 is 1.7 mm × 0.83 mm, and the resulting TE_{10} mode cutoff frequency is 90.85 GHz. The operating frequency range of this waveguide is from 110 GHz to 170 GHz.

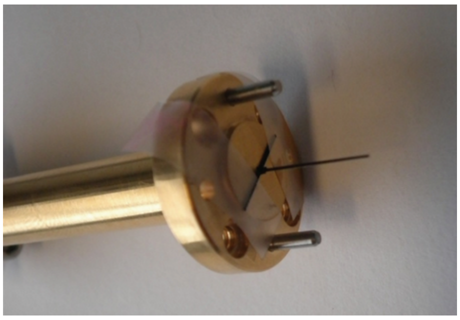


Figure 11: The photograph of silicon antenna and waveguide.

The dominant electric fields profiles of the metal rectangular waveguide and the rectangular dielectric waveguide are shown in Figure 12. The TE_{10} mode of the metal waveguide is transformed into the rectangular dielectric. As indicated in Figure 12, the waveguide is well suited to excite the rectangular antenna because the electric fields of the two components are approximately oriented in the same direction.

The simulation and measurement results of the return loss and antenna pattern are shown in Figures 13 and 14, respectively.

The return loss measurement matches well with simulation as shown in Figure 13. The simulated return loss is −54.5 dB at the center frequency 130 GHz. The lowest return loss of measurement is −51.2 dB at 133 GHz. From 110 GHz to 170 GHz, the return loss is less than −10 dB. The bandwidth of the antenna with the center frequency $f_r = 130$ GHz is greater than 60 GHz ($S_{11} \leq -10$ dB) as shown in Figure 13, which is close to 50% of the center frequency. The difference between the simulation and the measurement is due to fabrication tolerance, which has a relatively large effect for such high frequency.

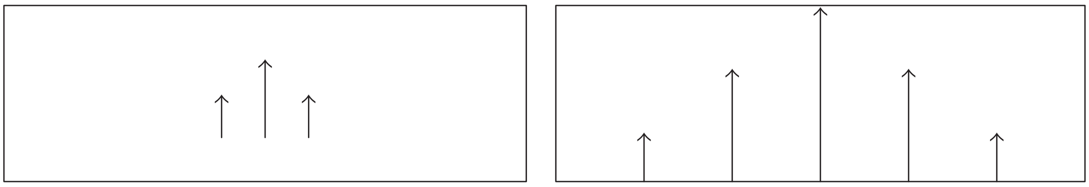
The gain measurement curve matches well with the simulation curve in Figure 14. The simulated gain is 12.85 dB, and the measured gain is 12.25 dB along the antenna boresight direction.

Note that the antenna length in this example is chosen to be relatively short for the ease of fabrication. In fact the gain can be increased with an increase in the length, as long as the conditions in (6) and (7) or (10) are satisfied.

4. Conclusion

The bandwidth of the tapered rod dielectric antenna is shown to be significantly enhanced by optimally combining the propagating modes inside the antenna. Through the comparison of various modes combination, we conclude that the optimal combination is to have the fundamental and second modes inside and only the fundamental mode for end-fire. The antenna radius can be subsequently calculated satisfying this condition.

The geometric optics is used for the first time for an approximate analysis of the electromagnetic wave propagation inside the antenna. Simple and accurate design equations are developed for the design of dielectric rod antennas.



(a) Rectangular dielectric antenna

(b) Rectangular waveguide

FIGURE 12: Dominant modal electric field profiles (a) in rectangular dielectric and (b) in rectangular waveguide.

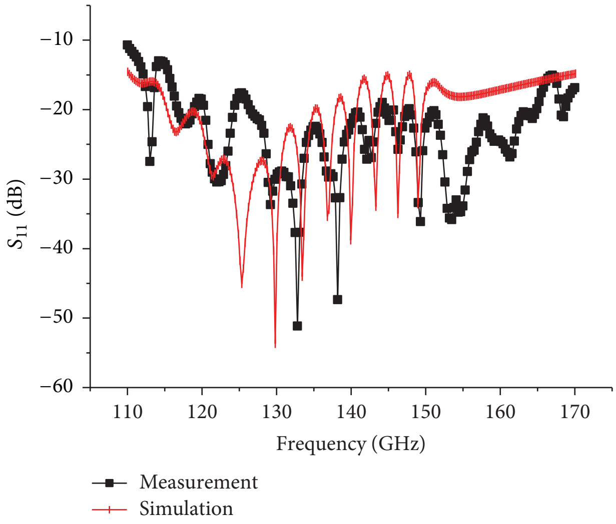


FIGURE 13: The simulation and measurement results of the antenna return loss.

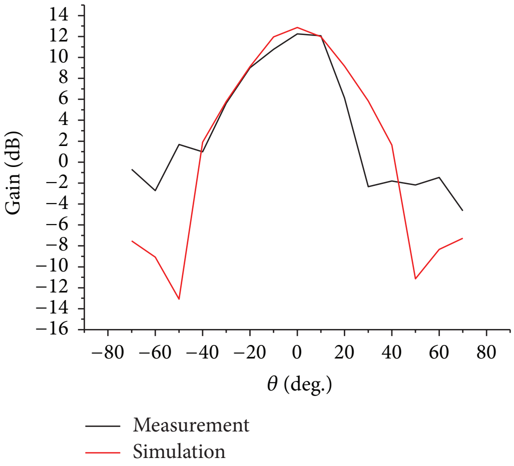


FIGURE 14: The simulation and measurement of the gain of the antenna ($h = 4\lambda$).

The design approach is easy to follow. The dependencies of the operational bandwidth, center frequency, and the gain on the design parameters have been described. The simulation and measurement results of a number of millimeter wave to terahertz rod antennas with high gain and wide bandwidth demonstrate the efficiency of the proposed design method.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

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