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Review Article

Zagreb Connection Numbers for Cellular Neural Networks

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Neural networks in which communication works only among the neighboring units are called cellular neural networks (CNNs). These are used in analyzing 3D surfaces, image processing, modeling biological vision, and reducing nonvisual problems of geometric maps and sensory-motor organs. Topological indices (TIs) are mathematical models of the (molecular) networks or structures which are presented in the form of numerical values, constitutional formulas, or numerical functions. These models predict the various chemical or structural properties of the under-study networks. We now consider analogous graph invariants, based on the second connection number of vertices, called Zagreb connection indices. The main objective of this paper is to compute these connection indices for the cellular neural networks (CNNs). In order to find their efficiency, a comparison among the obtained indices of CNN is also performed in the form of numerical tables and 3D plots.

1. Introduction

A neural system that consists of a multidimensional cluster of neurons and neighborhood-connected associations between the cells is called a cellular neural network (CNN) as shown in Figure 1. This kind of system presented in [1] is a consistent time network in the form of an $n \times m$ rectangular matrix array having n rows and m columns (see Figures 1–3 for some values of m and n).

A component of the rectangular array corresponds to a cell in a neural arrangement. But it is noted that the geometry exhibited requires not only to be rectangular, but also such shapes can be triangles or hexagons [2]. Multiple clusters can be represented with a proper interconnected structure to construct a multilayered cell neural system (Figure4).

A cell $C(h;k)$, where $1 \leq h \leq n$ and $1 \leq j \leq m$ with its lth neighborhood, can be presented as $Nr(h;k)$ and is described as the set of cells $C(p;q)$, where $1 \leq p \leq n$ and $1 \leq q \leq m$, such that $|p-h| \leq l$ and $|q-k| \leq l$. The cells in lth neighborhood of a cell $C(h,k)$ are directly interconnected with cell $C(h,k)$ through $A(p,q,h,k)$, $A(h,k,p,q)$, $B(p,q,h,k)$, and $B(h,k,p,q)$, where $A(p,q,h,k)$ and $A(h,k,p,q)$ are known as the feedback weights and $B(p,q,h,k)$ and $B(h,k,p,q)$ known as the feedforward weights. The index pair (p,q,h,k) describes the direction of signal from $C(h,k)$ to $C(p,q)$. The cell $C(h,k)$ is connected directly with its adjacent cells $C(p,q) \in Nr(h,k)$. Since every $C(p,q)$ has its adjacent cells, the cell $C(h,k)$ can also be linked with all other cells indirectly as shown in Figure 5.

The CNN has a lot of applications that are indicated by their spatial dynamics. The filtering image processing is one of the good applications of CNN [3]. For more related works about CNN and PNN, one can consult the references [4–13].

Thoroughly, we take the graph $G = (V, E)$ which does not contain loops and multiple or directed edges, where the sets V and $E \subseteq V \times V$ are of vertices and edges, respectively. The length of the shortest path from u to v (denoted by $d(u,v)$) is called its distance and $d_u = |N|$ is known by the degree of u, where $N = \{v \in V: d(v,u) = 1\}$. A topological index (TI) defined with the help of the degrees of nodes of the (molecular) network is a class of indices which are used to find out and model the certain properties of the chemical compounds of the (molecular) networks (see [14–16]). In particular, the degree-based topological properties for the CNN are studied in [17].

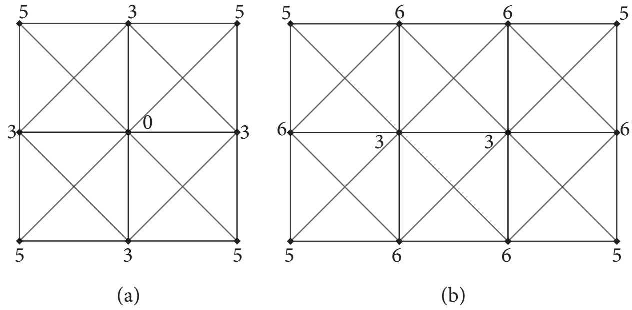


Figure 1: Cellular neural network CNN (3, 3) (a) and CNN (4, 3) (b).

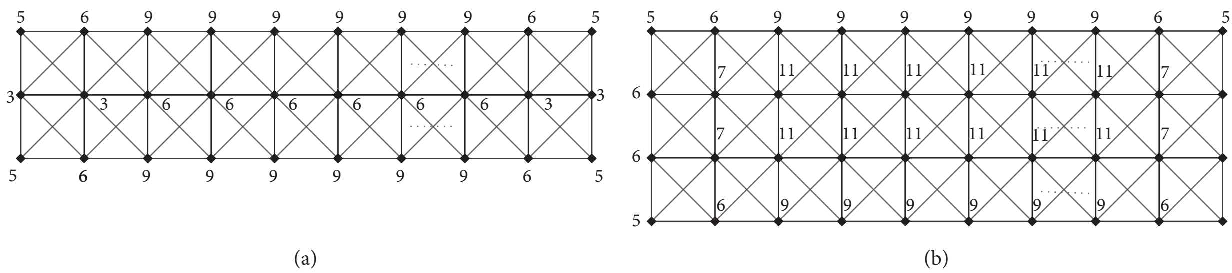


Figure 2: Cellular neural network CNN $(m, 3)$ (a) and CNN $(m, 4)$ (b).

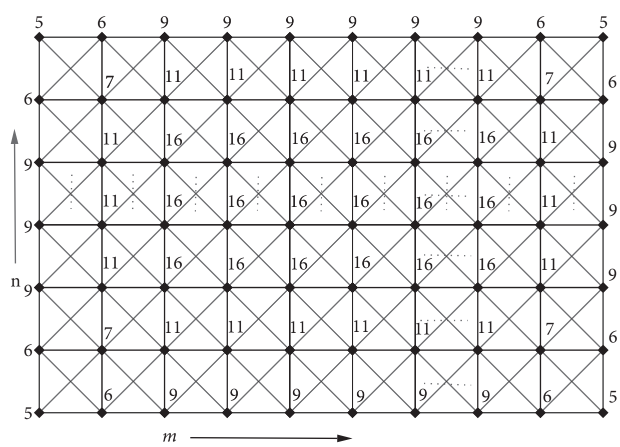


Figure 3: Cellular neural network CNN (m, n).

The first Zagreb index is studied for the total π-electron energy [18], and the second Zagreb index appeared to compute molecular branching [19]; they are denoted by M_1 and M_2, respectively:

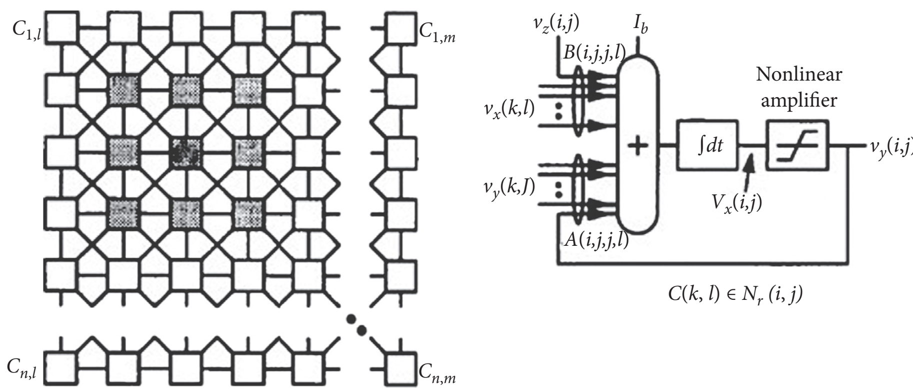
$$M_1(G) = \sum_{u \in V(G)} [d_u]^2, \qquad M_2(G) = \sum_{u \in E(G)} d_u d_v. \tag{1}$$

In relation to the above equations, the first and second Zagreb connection indices (ZCIs) have been put forward in [20, 21] independently:

$$ZC_1(G) = \sum_{u \in V(G)} (\tau_u)^2, \qquad ZC_2(G) = \sum_{uv \in E(G)} \tau_u \tau_v, \qquad ZC_1^*(G) = \sum_{u \in V(G)} d_u \tau_u, \tag{2}$$

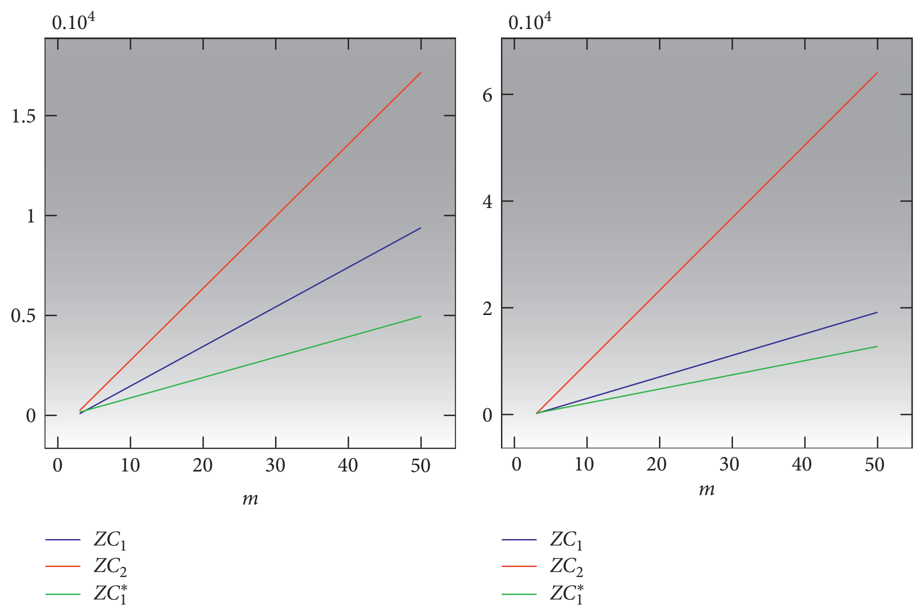
where τ_u denotes the number of vertices $v \in G$ such that $d(u, v) = 2$. It has been proved by Ali and Trinajstic [20] that the topological index ZC_1^* can be written as

$$ZC_1^*(G) = \sum_{uv \in E(G)} (\tau_u + \tau_v). \tag{3}$$



(a) (b)

Figure 4: Cellular neural network.



(a) (b)

Figure 5: Differences of indices for $CNN(m,3)$ (a) and for $CNN(m,4)$ (b).

In [17], the authors checked the chemical applicability of these three Zagreb connection indices on the set of octane isomers, and they found that $ZC_1^*(G)$ has better correlating ability than the other two Zagreb connection indices in the cases of entropy, enthalpy of vaporization, standard enthalpy of vaporization, and acentric factor. Basavanagoud and Jakkannavar checked the chemical applicability of ZC_1 and found that the index has a very good correlation with physical properties of chemical compounds such as boiling point, entropy, enthalpy of evaporation, standard enthalpy of vaporization, and acentric factor (see [23]).

Ali and Trinajstic [20] checked the chemical applicability of ZC_1^*, and they found that this TI correlates well with the entropy and acentric factor of octane isomers. A large number of networks has been studied with the help of connection number-based TIs such as T-sum networks [24], resultant networks [25, 26], connected networks [27, 28], alkanes [22, 29, 30], dendrimer nanostars [31], trees, and unicyclic networks [32] and subdivided and semitotal point networks [33].

2. Main Results and Discussion

Let $c_k(G)$ denote the number of vertices in G with connection number k and $m_{k,l}(G)$ denote the number of edges in G whose vertices have connection numbers k and l.

The following formulas for the ZCIs are equivalent to the previous definitions:

$$ZC_1(G) = \sum_{0 \le k \le n-2} c_k(G) k^2, \quad (4)$$

$$ZC_2(G) = \sum_{0 \le l \le k \le n-2} m_{k,l}(G)(k.l), \quad (5)$$

$$ZC_1^*(G) = \sum_{0 \le l \le k \le n-2} m_{k,l}(G)(k+l). \quad (6)$$

From Figure 1 and definition of the ZCIs, we have the following:

(1) For $m = 3$ and $n = 3$,

(a) $ZC_1(\mathrm{CNN}(3,3)) = 100$
(b) $ZC_2(\mathrm{CNN}(3,3)) = 156$
(c) $ZC_1^*(\mathrm{CNN}(3,3)) = 88$

(2) For $m = 4$ and $n = 3$,

(a) $ZC_1(\mathrm{CNN}(4,3)) = 280$
(b) $ZC_2(\mathrm{CNN}(4,3)) = 555$
(c) $ZC_1^*(\mathrm{CNN}(4,3)) = 258$

(3) For $m = 4$ and $n = 4$,

(a) $ZC_1(\mathrm{CNN}(4,4)) = 584$
(b) $ZC_2(\mathrm{CNN}(4,4)) = 1634$
(c) $ZC_1^*(\mathrm{CNN}(4,4)) = 524$

Theorem 1. *Let $m \ge 5$ and $CNN(m,3)$ be the CNN. Then,*

(1) $ZC_1(CNN(m,3)) = 198m - 512$
(2) $ZC_2(CNN(m,3)) = 360m - 840$
(3) $ZC_1^*(CNN(m,3)) = 102m - 144$

Proof. In order to prove our result, we will compute c_k, the number of vertices of connection number k, and $y_{k,l}(G)$ is the edge of $\mathrm{CNN}(m,3)$ whose vertices have connection numbers k and l. It is easy to see from the structure of $\mathrm{CNN}(m,n)$ that $c_3 = 4, c_5 = 4, c_6 = m,$ and $c_9 = 2m - 8$. Thus, from equation (4), we have the following:

$$\begin{aligned} ZC_1(\mathrm{CNN}(m,3)) &= \sum_{0 \le k \le n-2} c_k(G)k^2 = c_3\left(3^2\right) + c_5(5)^2 \\ &\quad + c_6(6)^2 + c_9(9)^2 \\ &= 36 + 100 + 36m + 81(2m-8) \\ &= 198m - 512. \end{aligned} \quad (7)$$

The edge set of $\mathrm{CNN}(m,3)$ can be partitioned into different classes depending upon the edge types of $y_{k,l}(G)$ as listed in Table 1.

From the definition of the second ZCI and substitution of $y_{k,l}$ from Table 1 in (5), it follows that

$$\begin{aligned} ZC_2(\mathrm{CNN}(m,3)) &= y_{3,5}(3\times 5) + y_{3,6}(3\times 6) + y_{3,9}(3\times 9) \\ &\quad + y_{5,6}(5\times 6) + y_{6,6}(6\times 6) + y_{6,9}(6\times 9) \\ &= 8(15) + 8(18) + 4(27) + 4(30) \\ &\quad + (m-1)(36) + (6m-24)(54) \\ &= 360m - 840. \end{aligned} \quad (8)$$

Similarly, from substitution of $y_{k,l}(G)$ from Table 1 in (6), we have

$$\begin{aligned} ZC_1^*(\mathrm{CNN}(m,3)) &= y_{3,5}(3+5) + y_{3,6}(3+6) + y_{3,9}(3+9) \\ &\quad + y_{5,6}(5+6) + y_{6,6}(6+6) + y_{6,9}(6+9) \\ &= 8(8) + 8(9) + 4(12) + 4(11) \\ &\quad + (m-1)(12) + (6m-24)(15) \\ &= 102m - 144. \end{aligned} \quad (9)$$

Theorem 2. *Let $m \ge 5$ and $CNN(m,4)$ be the CNN. Then,*

(1) $ZC_1(CNN(m,4)) = 404m - 1032$
(2) $ZC_2(CNN(m,4)) = 1361m - 3940$
(3) $ZC_1^*(CNN(m,4)) = 266m - 540$

Proof. It is easy to see from the structure of $\mathrm{CNN}(m,4)$ that $c_5 = 4, c_6 = 8, c_7 = 4, c_9 = 2m - 8,$ and $c_{11} = 2m - 8$. Thus, from equation (4), we have the following:

$$\begin{aligned} ZC_1(\mathrm{CNN}(m,4)) &= \sum_{0 \le k \le n-2} c_k(G)k^2 = c_5(5)^2 + c_6(6)^2 \\ &\quad + c_7(7)^2 + c_9(9)^2 + c_{11}(11)^2 \\ &= 4(25) + 8(36) + 4(49) + (2m-8)(81) \\ &\quad + (2m-8)(121) = 404m - 1032. \end{aligned} \quad (10)$$

The edge set of $\mathrm{CNN}(m,4)$ can be partitioned into different classes depending upon the edge types of $y_{k,l}(G)$ as listed in Table 2.

From the definition of the second ZCI and substitution of $y_{k,l}$ from Table 2 in (5), it follows that

$$\begin{aligned} ZC_2(\mathrm{CNN}(m,4)) &= y_{5,6}(5\times 6) + y_{5,7}(5\times 7) + y_{6,6}(6\times 6) + y_{6,7}(6\times 7) + y_{6,9}(6\times 9) \\ &\quad + y_{6,11}(6\times 11) + y_{7,7}(7\times 7) + y_{7,9}(7\times 9) + y_{7,11}(7\times 11) + y_{9,9}(9\times 9) + y_{9,11}(9\times 11) + y_{11,11}(11\times 11) \\ &= 8(30) + 4(35) + 6(36) + 12(42) + 4(54) + 4(66) + 2(49) + 4(63) + 8(77) + (2m-10)(81) \\ &\quad + (6m-28)(99) + (5m-24)(121) = 1361m - 3940. \end{aligned} \quad (11)$$

Table 1: Partition of edge set of CNN$(m,3)$.

Edges of type $y_{k,l}$	Number of edges
$y_{3,5}$	8
$y_{3,6}$	8
$y_{3,9}$	4
$y_{5,6}$	4
$y_{6,6}$	$m-1$
$y_{6,9}$	$6m-24$

Table 2: Partition of edge set of CNN$(m,4)$.

Edges of type $y_{k,l}$	Number of edges
$y_{5,6}$	8
$y_{5,7}$	4
$y_{6,6}$	6
$y_{6,7}$	12
$y_{6,9}$	4
$y_{6,11}$	4
$y_{7,7}$	2
$y_{7,9}$	4
$y_{7,11}$	8
$y_{9,9}$	$2m-10$
$y_{9,11}$	$6m-28$
$y_{11,11}$	$5m-24$

Similarly, from substitution of $y_{k,l}(G)$ from Table 2 in (6). we have

$$\begin{aligned} ZC_1^*(\mathrm{CNN}(m,4)) &= y_{5,6}(5+6)+y_{5,7}(5+7) \\ &\quad + y_{6,6}(6+6)+y_{6,7}(6+7) \\ &\quad + y_{6,9}(6+9)+y_{6,11}(6+11) \\ &\quad + y_{7,7}(7+7)+y_{7,9}(7+9) \\ &\quad + y_{7,11}(7+11)+y_{9,9}(9+9) \\ &\quad + y_{9,11}(9+11)+y_{11,11}(11+11) \\ &= 8(11)+4(12)+6(12)+12(13) \\ &\quad + 4(15)+4(17) \\ &\quad + 2(14)+4(16)+8(18) \\ &\quad + (2m-10)(18)+(6m-28)(20) \\ &\quad + (5m-24)(22) \\ &= 266m-540. \end{aligned} \tag{12}$$

(1) $ZC_1(\mathrm{CNN}(m,n)) = 256mn-700m-700n+1448$

(2) $ZC_2(\mathrm{CNN}(m,n)) = 1024mn-2808m-2808n+7474$

(3) $ZC_1^*(\mathrm{CNN}(m,n)) = 128mn-246m-246n-100$

Table 3: Partition of edge set of CNN(m,n).

Edges of type $y_{k,l}$	Number of edges
$y_{5,6}$	8
$y_{5,7}$	4
$y_{6,6}$	4
$y_{6,7}$	8
$y_{6,9}$	8
$y_{6,11}$	8
$y_{7,9}$	8
$y_{7,11}$	8
$y_{7,16}$	4
$y_{9,9}$	$2m+2n-20$
$y_{9,11}$	$6m+6n-56$
$y_{11,11}$	$2m+2n-16$
$y_{11,16}$	$6m+6n-56$
$y_{16,16}$	$4mn-19m-19n+90$

Theorem 3. *Let $m,n \geq 5$ and CNN(m,n) be the CNN. Then,*

Proof. In order to prove our result, we will compute c_k, the number of vertices of connection number k, and $y_{k,l}(G)$ is the edge of CNN(m,n) whose vertices have connection numbers k and l. It is easy to see from the structure of CNN(m,n) that $c_5=4, c_6=8, c_7=4, c_9=2m+2n-16, c_{11}=2m+2n-16$, and $c_{16}=(m-4)(n-4)$. Thus, from equation (4), we have the following:

$$\begin{aligned} ZC_1(\mathrm{CNN}(m,n)) &= c_5(5)^2+c_6(6)^2+c_7(7)^2+c_9(9)^2 \\ &\quad + c_{11}(11)^2+c_{16}(16)^2 \\ &= 4(25)+8(36)+4(49) \\ &\quad + (2m+2n-16)(81)+(m-4) \\ &\quad \cdot (n-4)(256)+(2m+2n-16)(121) \\ &= 256mn-700m-700n+1448. \end{aligned} \tag{13}$$

The edge set of CNN(m,n) can be partitioned into different classes depending upon the edge types of $y_{k,l}(G)$ as listed in Table 3.

From the definition of the second ZCI and substitution of $y_{k,l}$ from Table 3 in (5), it follows that

$$\begin{aligned} Z_2(\mathrm{CNN}(m,n)) &= y_{5,6}(5\times 6)+y_{5,7}(5\times 7)+y_{6,6}(6\times 6)+y_{6,7}(6\times 7)+y_{6,9}(6\times 9) \\ &\quad + y_{6,11}(6\times 11)+y_{7,9}(7\times 9)+y_{7,11}(7\times 11)+y_{7,16}(7\times 16)+y_{9,9}(9\times 9)+y_{9,11}(9\times 11) \\ &\quad + y_{11,11}(11\times 11)y_{11,16}(11\times 16)+y_{16,16}(16\times 16)=8(30)+4(35)+4(36)+8(42)+8(54) \\ &\quad + 8(66)+8(63)+8(77)+4(112)+(2m+2n-20)81+99(6m+6n-56)+121(2m+2n-16) \\ &\quad + (6m+6n-56)(176)+(4mn-19m-19n+90)(256)=1024mn-2808m-2808n+7474. \end{aligned} \tag{14}$$

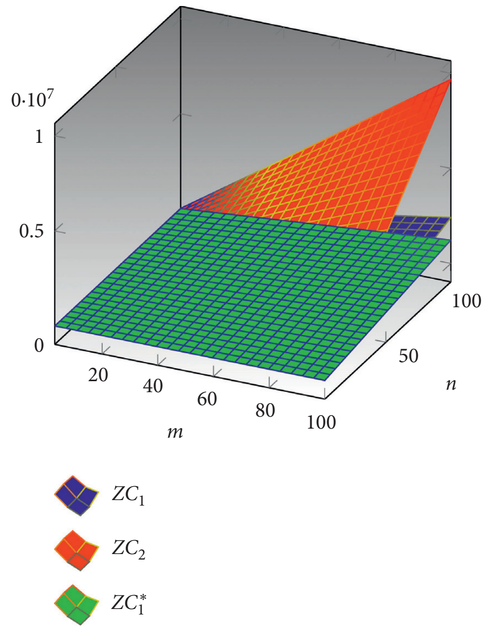


Figure 6: Differences of indices for CNN (m, n).

Table 4: Differences of indices for CNN $(m, 3)$.

m	ZC_1	ZC_2	ZC_1^*
5	478	960	366
6	676	1320	468
7	874	1680	570
8	1072	2040	672
9	1270	2400	774
10	1468	2760	876
11	1666	3120	978
12	1864	3480	1080
13	2062	3840	1182
14	2260	4200	1284
15	2458	4560	1386

Table 5: Differences of indices for CNN $(m, 4)$.

m	ZC_1	ZC_2	ZC_1^*
5	988	2865	790
6	1392	4226	1056
7	1796	5587	1322
8	2200	6948	1588
9	2604	8309	1854
10	3008	9670	2120
11	3412	11031	2386
12	3816	12392	2652
13	4220	13753	2918
14	4624	15114	3184
15	5028	16475	3450

Table 6: Differences of indices for CNN (m, n).

(m, n)	ZC_1	ZC_2	ZC_1^*
(5, 5)	848	4994	640
(6, 5)	1428	7306	1034
(7, 5)	2008	9618	1428
(8, 5)	2588	11930	1822
(9, 5)	3168	14242	2216
(10, 5)	3748	16554	2610
(6, 6)	2264	10642	1556
(7, 6)	3100	13978	2078
(8, 6)	3936	17314	2600
(9, 6)	4772	20650	3122
(10, 6)	5608	23986	3644

Similarly, from substitution of $y_{k,l}(G)$ from Table 3 in (6), we have

$$\begin{aligned} Z_1^*(\mathrm{CNN}(m,n)) &= y_{5,6}(5+6) + y_{5,7}(5+7) + y_{6,6}(6+6) + y_{6,7}(6+7) + y_{6,9}(6+9) + y_{6,11}(6+11) \\ &\quad + y_{7,9}(7+9) + y_{7,11}(7+11) + y_{7,16}(7+16) + y_{9,9}(9+9) + y_{9,11}(9+11) \\ &\quad + y_{11,11}(11+11) + y_{11,16}(11+16) \\ &\quad + y_{16,16}(16+16) = 8(11) + 4(12) + 4(12) + 8(13) + 8(15) + 8(17) + 8(16) + 8(18) + 4(23) \\ &\quad + (2m+2n-20)18 + 20(6m+6n-56) + 22(2m+2n-16) + 27(6m+6n-56) \\ &\quad + 32(4mn-19m-19n+90) \\ &= 128mn - 246m - 246n - 100. \end{aligned} \tag{15}$$

3. Numerical and Graphical Comparisons

In this section, we will give numerical and graphical comparisons of the Zagreb connection indices with respect to the cellular neural network. Maple software is used to construct a simple comparison of the Zagreb connection indices related to the cellular neural network into 3D plots (Figures 5 and 6). The numerical comparison is given in Tables 4–6. We can see from the 3D plots and numerical tables that the second Zagreb index is always greater than the other two indices.

4. Conclusion

The Zagreb connection indices for the cellular neural system on a rectangular grid have been computed. Later on, the obtained results for the Zagreb connection indices, has an application; with the help of numerical tables and 3D plots, the determination of detailed comparisons among these indices of CNN has been outlined. It is notable that the obtained results for these networks are all quadratic in terms of the order of the network, which showed that one can build efficient graph algorithms to compute the indices within polynomial time.

Data Availability

All the data are included within this paper. However, the reader may contact the corresponding author for more details of the data.

Conflicts of Interest

The authors declare no conflicts of interest.

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