

Book Review

Complex Economic Dynamics Volume I: An introduction to dynamical systems and market mechanism by Richard H. Day

Richard Day was one of the first economists who recognized the importance of nonlinear dynamics, in particular the occurrence of chaotic fluctuations in simple deterministic models for economic modelling. Amongst sometimes skeptical colleagues, Day has been advocating and applying this intriguing branch of mathematics, and he has been developing simple models of economic behaviour in macro, micro and finance for more than 15 years. The present volume I of *Complex Economic Dynamics* is based upon Day's work over the last two decades or so; another volume II will appear in the near future.

The book consists of three parts – I: Purpose and Method, II: Dynamical Systems and III: Market Mechanisms. In part one, the philosophy behind nonlinear modelling and chaos and its relevance to economic growth and economic change is discussed extensively. Basic concepts such as dynamic models of economic change, steady states, cycles, bifurcations, chaos and a statistical description of chaos are discussed at a very elementary level, without going into technical details. In particular, Day emphasizes what he calls a *causal effect reversal*, that is, a nonmonotonic relationship between two variables. In one-dimensional nonlinear models, this is the key nonlinearity which leads to complicated dynamics. Whenever this causal effect reversal becomes sufficiently strong a bifurcation route to chaos sets in. The intuitive approach

in part I should help and motivate economists with little prior knowledge of nonlinear dynamics.

The second part is an introduction into the mathematics of nonlinear dynamics in one-dimensional discrete time models $x_{t+1} = f(x_t)$. Basic notions such as the stability and instability of steady states and cycles, chaos, sensitive dependence on initial conditions and sufficient conditions for chaos are introduced and illustrated by simple examples. Also more advanced material in ergodic theory, that is, the statistical description of nonlinear dynamics, is introduced, such as the law of large numbers for chaotic systems, invariant measures, absolutely continuous measures, etc. Many simple and intuitive numerical examples should help economists in grasping this sometimes difficult subject matter. Day emphasizes the idea of 'multiple phase dynamics' where a switching between different modes or regimes occurs. Chaotic fluctuations can describe an irregular switching between phases of structural growth, sudden decline, and evolutionary change.

Part III contains economic applications of simple one-dimensional nonlinear dynamic models. In particular, tâtonnement process, intermediate price adjustment processes and simple financial market models with different types of traders are introduced and discussed extensively. To me, one of the most fascinating aspects is

how the simple chaotic financial feedback model can produce an irregular switching between “bear” and “bull” markets, so frequently observed in real financial markets.

Day’s book does a good job in explaining methods and techniques, in the analysis of nonlinear one-dimensional dynamic models, to economists. Indeed, most of the material should become part of the standard toolbox of anyone with an interest in economic dynamics. The simple, stylized economic models nicely illustrate how ‘chaos theory’ can help to uncover nonlinear market forces that may be responsible for part of the fluctuations observed in real markets. I therefore recommend Day’s book to any economist with an interest in economic dynamics. But I also would like to make one critical remark, concerning possible directions of future research in economic dynamics. Day’s book concentrates entirely on one-dimensional systems. In such models, strong causal effect reversals, perhaps even unrealistically strong nonlinearities, are needed to produce complicated dynamics. On the other hand, in higher dimensional systems weak

nonlinearities can already lead to complicated dynamics. The vast majority of the research in nonlinear (chaotic) economic dynamics is in fact an application of one-dimensional models. Only few two or higher dimensional examples have been introduced in the economic literature. I hope that Day’s book will also motivate economists to go one step further and start exploring two and higher dimensional nonlinear economic models. These higher dimensional models really seem to be needed in order to arrive at Day’s goal, as set on p.7: ‘*to build mathematical models based on economic forces that can generate these facts*’. If economists want to generate (a significant part of the) facts, both qualitatively and quantitatively, by simple deterministic models, I believe that the dimension of the models should be higher than one. Day’s book gives a very useful start, but it is now time to go beyond the one-dimensional case.

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