Hindawi
Advances in Tribology
Volume 2020, Article ID 8151069, 8 pages
https://doi.org/10.1155/2020/8151069

Research Article

The Performance of Squeeze Film between Parallel Triangular Plates with a Ferro-Fluid Couple Stress Lubricant

Akbar Toloian,[1] Maghsood Daliri ,[1] and Nader Javani[2]

[1]*Department of Mechanical Engineering, Islamic Azad University, Bonab Branch, Bonab, Iran*
[2]*Faculty of Mechanical Engineering, Yildiz Technical University, Besiktas, Istanbul, Turkey*

Correspondence should be addressed to Maghsood Daliri; dmaghsood@yahoo.com

Received 19 August 2019; Revised 1 December 2019; Accepted 27 December 2019; Published 30 January 2020

Academic Editor: GunHee Jang

The present study aims at investigating a couple stress ferrofluid lubricant effects on the performance of the squeezed film when a uniform external magnetic field is applied. For this purpose, Shliomis ferrohydrodynamic and couple stress fluid models are employed. The considered geometry is parallel triangular plates. The effects of couple stress, volume concentration, and Langevin parameters on squeeze film characteristics including time vs. height relationship and load-carrying capacity are investigated. According to the results, employing couple stress ferrofluid lubricant in the presence of the magnetic field leads to an increased performance of the squeeze film.

1. Introduction

The squeeze film technology has numerous applications in different fields of industry, ranging from disc clutches to rotating devices including almost all types of power plant where turbomachines are used. In order to overwhelm the related issues and making improvements, magnetic field base ferrofluid couple stress fluids are employed. The present study aims at investigating the couple stress ferrofluid lubricant effects on the performance of the squeezed film once a uniform external magnetic field is available. For this purpose, Shliomis ferrohydrodynamic and couple stress fluid models are employed. The considered geometry is parallel triangular plates. The effects of couple stress, volume concentration, and Langevin parameters on squeeze film characteristics including time-height relationship and load-carrying capacity are investigated.

Daniel et al. [1] applied a numerical method to analyze the performance of an elastohydrodynamic journal bearing which is lubricated with ferrofluids, using couple stresses. By increasing the couple stresses and magnetism, temperature in the bearing increases. Also, the effect of Prandtl and Eckert numbers on temperature is investigated in their research. Daniel et al. [2] also analyzed magneto-electro-hydrodynamics by deriving the Reynolds equation through employing continuum and momentum equations. Pressure distribution in the journal bearing was obtained which indicated an increase in the pressure by enhancement of magnetism and couple stresses. Daliri and Javani [3] investigated a squeezing motion in conical plates where the lubricant was ferrofluid couple stress along with the effect of convective fluid inertia. They concluded that an enhancement in convective fluid inertia increases the performance of squeezing film. On the contrary, Maxwell equations can be applied in this case. Shah and Bhat [4] and Lin [5] also developed a procedure to derive the governing equation which has been followed in the current study. Daliri et al. [6] investigated the characteristics of a squeezed film in conjunctions of parallel rectangular plates. A couple stress incompressible fluid was considered once a magnetic field is applied. Stokes microcontinuum in a couple-stress fluid along with the modified Reynolds equation were combined to obtain an analytical solution. They also showed that magnetohydrodynamic couple stress fluids perform better in a steady load rather than a transient load conditions. For lubricated parallel annular discs, combined with squeeze film and shear motions, Daliri et al. [7] showed that contact load-carrying capacity increases by the presence of magnetohydrodynamic couple stress fluids. For the same geometry of parallel annular plates, however, increased rotational inertia can diminish the squeeze film characteristics [8]. Parallel disc lubrication film geometry, with rough surfaces where the lubricant is piezoviscous couple stress, is also studied by Daliri and Jalali-Vahid [9]. They showed that surface roughness pattern and the effect of pressure on viscosity improve the squeeze film characteristics. Another geometry of interest is parallel-stepped plates with porous surfaces where couple stress fluids are employed for the lubrication purpose. Biradar [10] derived the modified Reynolds equation using Stokes modeling couple stress fluid with additives in the squeeze film. Results of his study show an increase in the squeeze film load-carrying capacity and its performance, once the respond time decreases compared with a classical Newtonian lubricant. For a narrow journal bearing with transversal rough porous surface, the effect of a slip on the hydromagnetic squeeze film is investigated elsewhere [11] insisting the importance of the slip minimization in order to improve the performance of these bearings. Recently, Hanumagowda et al. [12] studied the effects of viscosity-pressure dependency on the non-Newtonian squeeze film performance between parallel-stepped circular plates considering effects of surface roughness. They concluded that the using of couple stress fluid and considering the viscosity-pressure dependency increased the squeeze film characteristics. It was also found that considering the surface roughness effects has significant effect on the performance of the squeeze film.

Parallel circular disc lubrication film geometry with rough surfaces considering effects of rotational inertia, where the lubricant is ferrofluid couple stress, is also studied by Daliri [13]. He showed that using of both ferrofluid and couple stress lubricants improve the squeeze film characteristics. It was also found that with increasing rotational inertia, the squeeze film performance is decreased. In a recent study of triangular plates, the effects of viscosity-pressure dependency together with couple stress fluid on the squeeze film characteristics were investigated by AminKhani and Daliri [14]. According to their research, it was concluded that the dependency of lubricant viscosity on pressure increases the squeeze film characteristics. On the contrary, it was found that the couple stress fluid improves the squeeze film performance compared with Newtonian fluid.

According to the earlier studies, analysis of squeeze film characteristics between triangular plates with ferrofluid couple stress lubricant has not been investigated previously. The motivation of the present study is due to the application of triangular plates' geometry in industry. Friction discs used in wet clutches normally have some kind of groove pattern (triangular shape) on the friction surface to allow the lubricant to flow over the surface and cool down the clutch pack. Therefore, one of the practical situations where the geometry of triangular plates could be used is wet clutch.

In this research, squeeze film performance is investigated between parallel triangular plates which are lubricated by a couple stress ferrofluid lubricant in the presence of a uniform external magnetic field (H_o) to obtain the characteristics of the squeeze film. The load-carrying capacity and the relation between time and height are studied for different couple stress values along with the Langevin and volume concentration parameters as outcomes of squeeze film characteristics.

2. Governing Equations

The geometry of triangular plates which are lubricated under a couple stress ferro-fluid is shown in Figure 1. Transverse magnetic field is dominant in the system. Two plates are approaching to each other at a relative velocity of $-dh/dt$ in which the upper plates is moving down. Considering the available geometry and dimensions, it is reasonable to apply the prevalence of thin-film lubrication theory. In order to formulate the x-direction component of the velocity (u) and y-direction velocity component (v), Shliomis [15, 16] model of ferro-hydrodynamic and Stokes micro-continuum theory [17] are employed. Therefore, continuity and momentum equations are expressed as follows:

$$0 = -\frac{\partial p}{\partial x} + \eta \frac{\partial^2 u}{\partial z^2} + \eta\tau \frac{\partial^2 u}{\partial z^2} - \eta_c \frac{\partial^4 u}{\partial z^4}, \quad (1)$$

$$0 = -\frac{\partial p}{\partial y} + \eta \frac{\partial^2 v}{\partial z^2} + \eta\tau \frac{\partial^2 v}{\partial z^2} - \eta_c \frac{\partial^4 v}{\partial z^4}, \quad (2)$$

$$\frac{\partial p}{\partial z} = 0. \quad (3)$$

The equation of continuity is

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \quad (4)$$

where

$$\tau = \frac{3}{2}\Phi \frac{\xi - \tanh \xi}{\xi + \tanh \xi}. \quad (5)$$

Here, τ represents the rotational viscosity, η is the suspension viscosity, and η_c is a constant related to the couple stress fluid property. Also, Φ and ξ represent the volumetric concentration of the magnetic particles and Langevin parameter. w is the z-direction component of the velocity; the suspension viscosity is approximated as follows [16]:

$$\eta = \eta_0 (1 + 2.5\Phi). \quad (6)$$

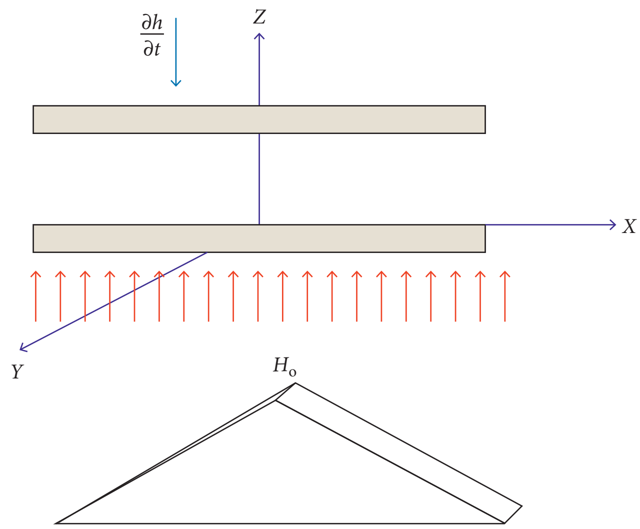

Figure 1: Squeeze film geometry between parallel triangular plates.

The main liquid velocity is η_0, and the boundary conditions of velocity components can be formulated as follows:

$$z = 0, \quad u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad v = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0, \quad w = 0, \tag{7}$$

$$z = h. \quad u = 0, \quad \frac{\partial^2 u}{\partial z^2} = 0, \quad v = 0, \quad \frac{\partial^2 v}{\partial z^2} = 0, \quad w = \frac{\mathrm{d}h}{\mathrm{d}t}. \tag{8}$$

Considering no-slip condition, at $z=0$ and $z=h$, the velocity (u) is equal to zero. At the same time, no-couple stress conditions at the surface implies that $\partial^2 u/\partial z^2 = 0$ and $\partial^2 v/\partial z^2 = 0$ for $z=0$ and $z=h$ [3]. Since the lower plate is not moving then for $z=0$, both v and w velocities are zero. Upper plate squeezing motion will lead to $v = 0$ at $z=h$ and $w = \mathrm{d}h/\mathrm{d}t$.

Equations (1) and (2) will lead to the velocity components of u and v under the boundary conditions described in (7) and (8):

$$u = \frac{1}{2\eta_0(1+\tau)(1+2.5\Phi)}\frac{\partial p}{\partial x}\left\{\left(z^2 - hz\right) + \frac{2l_c^2}{(1+\tau)(1+2.5\Phi)} \cdot \left[1 - \frac{\cosh\left(\left((2z-h)\sqrt{(1+\tau)(1+2.5\Phi)}\right)/2l_c\right)}{\cosh\left(\left(h\sqrt{(1+\tau)(1+2.5\Phi)}\right)/2l_c\right)}\right]\right\},$$

$$v = \frac{1}{2\eta_0(1+\tau)(1+2.5\Phi)}\frac{\partial p}{\partial y}\left\{\left(z^2 - hz\right) + \frac{2l_c^2}{(1+\tau)(1+2.5\Phi)} \cdot \left[1 - \frac{\cosh\left(\left((2z-h)\sqrt{(1+\tau)(1+2.5\Phi)}\right)/2l_c\right)}{\cosh\left(\left(h\sqrt{(1+\tau)(1+2.5\Phi)}\right)/2l_c\right)}\right]\right\}, \tag{9}$$

where $l_c = \eta_c/\eta_0$ shows characteristics length of the additives responsible for couple stress fluid.

Continuity equation integration over the thickness of the lubricant film will give the following expression:

$$\frac{\partial}{\partial x}\left(\int_0^h u\mathrm{d}z\right) + \frac{\partial}{\partial y}\left(\int_0^h v\mathrm{d}z\right) = \frac{\partial h}{\partial t}. \tag{10}$$

Once u and v are substituted in equation (10), Reynolds-type coupled stress ferro-fluid squeeze-film can be obtained as follows:

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial y^2} = \frac{-2\eta_0 A^2(\mathrm{d}h/\mathrm{d}t)}{f(l_c, \tau, \Phi, h)}, \tag{11}$$

where $A = \sqrt{(1+\tau)(1+2.5\Phi)}$ $f(l_c,\tau,\Phi,h) = -(h^3/6) + (2l_c^2/A^2)[h - (2l_c/A)\tanh(Ah/2l_c)]$

The boundary conditions for pressure field are as follows:

$$p(x_1, y_1) = 0, \quad (12)$$

where

$$(x_1 - a)(x_1 - \sqrt{3}\,y_1 + 2a)(x_1 + \sqrt{3}\,y_1 + 2a) = 0. \quad (13)$$

The equilateral triangle side, a, is expressed as

$$(x - a)(x - \sqrt{3}\,y + 2a)(x + \sqrt{3}\,y + 2a) = 0. \quad (14)$$

The intersection point of the triangle medians is selected to be the origin, and thereafter equations (11) and (12) can be solved to obtain the pressure equation as follows:

$$p = \frac{+2\eta_0 A^2(\mathrm{d}h/\mathrm{d}t)}{f(l_c,\tau,\Phi,h)}\frac{a^2}{3}\left(1 - \frac{3}{4a^2}x^2 - \frac{3}{4a^2}y^2 - \frac{1}{4a^3}x^3 + \frac{3}{4a^3}xy^2\right). \quad (15)$$

Dimensionless form of the pressure distribution will have the following format:

$$p^* = \frac{-ph_o^3}{\eta_0(\mathrm{d}h/\mathrm{d}t)3\sqrt{3}a^2} = \frac{-2A^2}{9\sqrt{3}f^*(l^*,\tau,\Phi,h^*)} \cdot \left\{(1-x^*)\left(1+\frac{\sqrt{3}}{2}y^* + \frac{x^*}{2}\right)\left(1-\frac{\sqrt{3}}{2}y^* + \frac{x^*}{2}\right)\right\}, \quad (16)$$

where

$$l^* = \frac{l_c}{h_o},$$

$$x^* = \frac{x}{a},$$

$$y^* = \frac{y}{a},$$

$$h^* = \frac{h}{h_o},$$

$$f^*(l^*,\tau,\Phi,h^*) = \frac{f(l_c,\tau,\Phi,h)}{h_o^3},$$

$$f^*(l^*,\tau,\Phi,h^*) = -\frac{h^{*3}}{6} + \frac{2l^{*2}}{A^2}\left[h^* - \frac{2l^*}{A}\tanh\left(\frac{Ah^*}{2l^*}\right)\right], \quad (17)$$

where h_o is the initial film thickness.

The expression for the load-carrying capacity is obtained by

$$w = \int_{-2a}^{a}\int_{-(2a+x_1)/\sqrt{3}}^{(2a+x_1)/\sqrt{3}} p\,\mathrm{d}y_1\mathrm{d}x_1,$$
$$w = \frac{9\sqrt{3}\eta_0 A^2(\mathrm{d}h/\mathrm{d}t)a^4}{10 f(l_c,\tau,\Phi,h)}. \quad (18)$$

Table 1: Rheological parameters of lubricant.

Initial film thickness: $h_o = 12\times10^{-5}$ m
Triangle geometry: $a = 0.2$ m
Viscosity of main liquid: $\eta_0 = 2.7\times10^{-5}$ pa·s
Couple stress material constants: $\eta_c = 3.888\times10^{-15}$ N·s
Squeezing velocity: $V = 3.2$ m/s
Characteristic length of the additives: $l_c = 12\times10^{-6}$ m
Couple stress parameter: $l^* = 0.1$
The free space permeability: $\mu_p = 4\pi\times10^{-7}$ kg·m·s^{-2}·Amp^{-2}
Magnetic moment of a particle: $m = 0.04$
Magnetic field: $H_o = 1.6363\times10^{-5}$ Amp·m^{-1}
The Boltzmann constant: $K_B = 1.38\times10^{-23}$ kg·m^2·s^{-2}·k^{-1}
The absolute temperature: $T = 298$k
The Langevin parameter: $\xi = 10$

The expression for the load-carrying capacity in dimensionless form is

$$w^* = \frac{-wh_o^3}{27\eta_0(\mathrm{d}h/\mathrm{d}t)a^4} = \frac{-\sqrt{3}A^2}{30f^*(l^*,\tau,\Phi,h^*)}. \quad (19)$$

Introducing the dimensionless response time

$$w^* = \frac{wh_o^2}{\eta_0 a^4}t. \quad (20)$$

The time-height relationship can be derived in the dimensionless form as follows:

$$\frac{\mathrm{d}h^*}{\mathrm{d}t^*} = -\frac{1}{W^*}. \quad (21)$$

Equation (21) is an ordinary but nonlinear differential equation. To obtain an accurate result, 4th order Runge–Kutta method is applied through employing the following initial condition:

$$h^* = 1, \quad \text{at}\, t^* = 0. \quad (22)$$

3. Results and Discussion

From the above analysis, the couple stress ferrofluid-lubricated squeeze film performances are characterized by three parameters: the non-Newtonian couple stress parameter, l^*; the volume concentration of particles, ϕ; and the Langevin parameter ξ. The special case can be obtained from specific values of these parameters.

(i) $\phi = 0$ and $\xi = 0$: the couple stress nonferrofluid case without magnetic fields. The nondimensional load capacity in equation (19) reduces to

(ii) $\lim_{\phi\longrightarrow0,\,\xi\longrightarrow0} w^* = \sqrt{3}/(5h^{*3} - 60l^{*2}h^* + 120l^{*3}\tanh(h^*/2l^*))$

(iii) This is the derivation of Fathima et al. [18] when neglecting the MHD effects(the Hartmann number tends to zero).

The geometry of parallel triangular plates with a ferrofluid lubricant of couple stress is investigated. The results of the study, three parameters including the non-Newtonian couple stress parameter, l^*; the volume concentration of particles, Φ; and Langevin parameter ξ are employed to

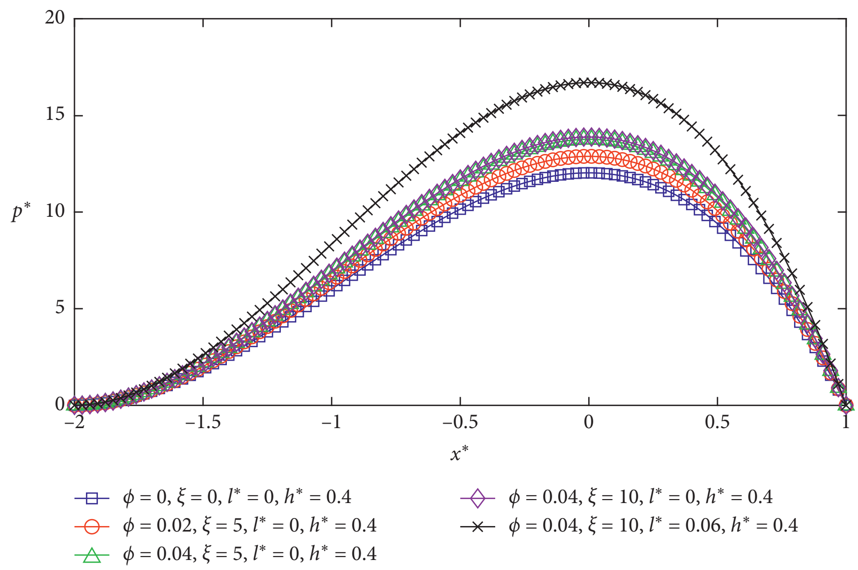

Figure 2: Pressure distribution for different values of Φ, ξ, and l^* at $h^* = 0.4$.

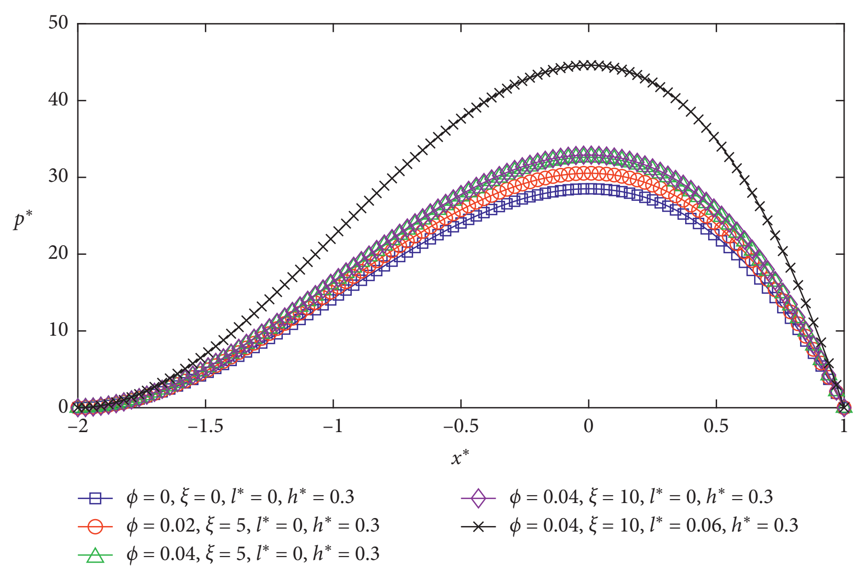

Figure 3: Pressure distribution for different values of Φ, ξ, and l^* at $h^* = 0.3$.

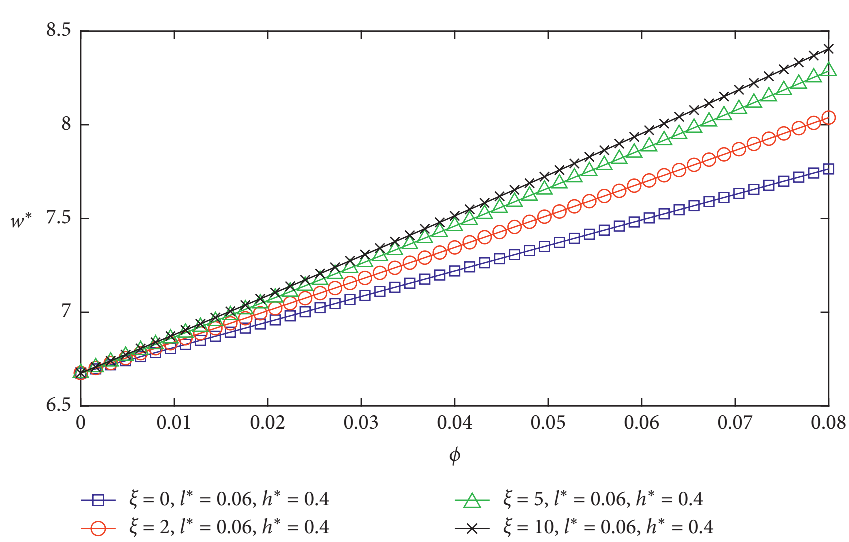

Figure 4: Variations of dimensionless load capacity versus volume concentration Φ for different values of ξ at $l^* = 0.06$ and $h^* = 0.4$.

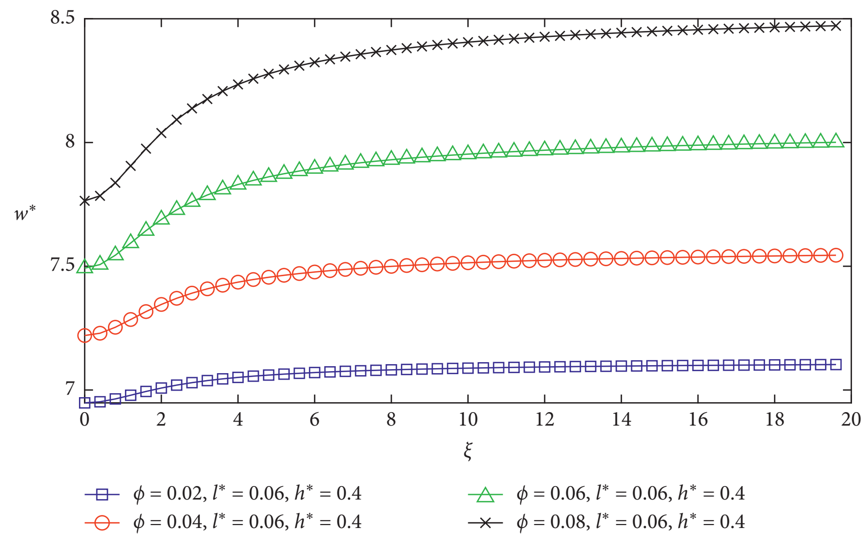

FIGURE 5: Variations of dimensionless load capacity versus Langevin parameter ξ for different values of Φ at $l^* = 0.06$ and $h^* = 0.4$.

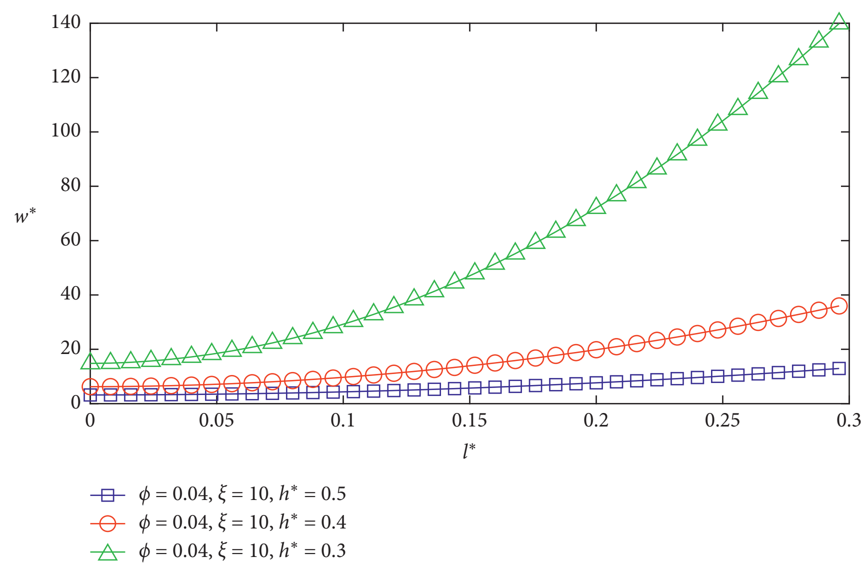

FIGURE 6: Dimensionless load capacity versus couple stress parameter l^* for different values of h^* at $\xi = 10$ and $\Phi = 0.04$.

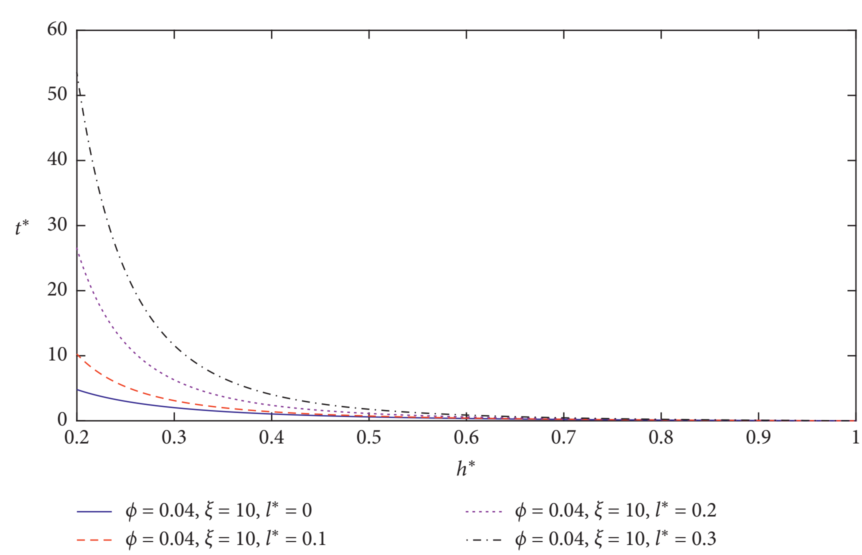

FIGURE 7: Dimensionless lubricant film thickness variations as a function of dimensionless time at s of 0.04 and $\xi = 10$ for different values of l^*.

characterize the squeeze film performance. Table 1 shows the rheological parameters of lubricant. At $h^* = 0.4$ and $h^* = 0.3$, lubricant film pressure distribution for altered values of Φ, ξ are shown in Figures 2 and 3, respectively. By increasing the values of Φ, ξ and l^*, there will occur an enhancement in the maximum pressure of lubricant.

Dimensionless load-carrying capacity W^* variation with respect to the volumetric concentration (Φ), Langevin (ξ), and couple stress l^* are shown in Figures 4–6. Knowing that these parameters lead to a higher film pressure, it can be seen that the integrated load-carrying capacity will increase subsequently. The load-carrying capacity can be wisely considered as a function of Φ, ξ, and l^* which is shown in Figures 4–6 where increasing Φ, ξ, and l^* will lead to an increase in load-carrying capacity which refers to the least load-carrying capacity in a Newtonian fluid.

Dimensionless film thickness (h^*) changes versus dimensionless response time (t^*) at altered coupled stresses (l^*) is illustrated in Figure 7. It is also concluded that increasing l^* will lead to an increase in the response time.

4. Conclusion

In this research, the effects of couple stress ferrofluid lubricant in the presence of magnetic field on the squeeze film performance are investigated. Shliomis ferrofluid and Stokes couple stress fluid models are used, and the considered geometry is triangular plates. By solving Reynolds equation, the pressure distribution is obtained and also with integrating on pressure in the film region, load-carrying capacity is derived. A 4^{th} order Runge–Kutta method is used to solve the non linear differential equation between lubricant film thickness and time. Furthermore, the variations of lubricant film thickness with the time are presented. According to the results, it is found that using both ferrofluid and couple stress fluid as a lubricant will increase characteristics of the squeeze film such as load-carrying capacity, pressure distribution, and triangular plates moving time, significantly. The obtained results are beneficial in designing of the bearings in the engineering applications.

Nomenclature

a: The equilateral triangle side
u, v, w: Velocity component in the Cartesian coordinate system
Φ: The volume concentration of particles
W^*: Dimensionless load-carrying capacity, $Wh_o^3/27\eta_0 a^4 V$
t, t^*: Response time, $(Wh_o^2/\eta_0 a^4)t$
η_0: Viscosity of main liquid
l_c: Characteristic length of the additives, (η_c/η_0)
x, y, z: Cartesian coordinates
l^*: The couple stress parameter, l_c/h_o
H_o: Applied magnetic field
m: Magnetic moment of a particle
T: The absolute temperature
ξ: The Langevin parameter, $\xi = \mu_p m H_o/K_B T$
h_o: Initial lubricant film thickness
x^*, y^*: Dimensionless coordinates, $x^* = x/a$, $y^* = y/a$
V: Squeezing velocity, $-dh/dt$
p, p^*: Squeeze film pressure, $ph_o^3/\eta_0 V 3\sqrt{3}a^2$
τ: Rotational viscosity parameter
η_c: Couple stress fluid index
h, h^*: Film thickness, h/h_o(dimensionless)
η: Viscosity of the suspension
μ_P: The free space permeability
K_B: The Boltzmann constant.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare no conflicts of interest.

References

[1] K. Daniel, K. Mathew, and K. Mark, "Investigation of Temperature effects for a finite elasto-hydrodynamic journal bearing lubricated by ferro fluids with couple stresses," *Journal of Computations & Modelling*, vol. 5, no. 3, pp. 79–90, 2015.
[2] K. Daniel, K. Mathew, and K. Mark, "Investigation of pressure distributions for a finite elastohydrodynamic journal bearing lubricated by Ferro fluids with couple stresses," *American Journal of Applied Mathematics*, vol. 2, no. 4, pp. 135–140, 2014.
[3] M. Daliri and N. Javani, "Squeeze film characteristics of ferro-coupled stress inertial fluids in conical plates," *Industrial Lubrication and Tribology*, vol. 70, no. 5, pp. 872–877, 2018.
[4] R. C. Shah and M. V. Bhat, "Ferrofluid squeeze film between curved annular plates including rotation of magnetic particles," *Journal of Engineering Mathematics*, vol. 51, no. 4, pp. 317–324, 2005.
[5] J.-R. Lin, "Derivation of ferrofluid lubrication equation of cylindrical squeeze films with convective fluid inertia forces and application to circular disks," *Tribology International*, vol. 49, pp. 110–115, 2012.
[6] M. Daliri, D. Jalali-Vahid, and H. Rahnejat, "Squeeze film lubrication of coupled stress electrically conducting inertial fluids in wide parallel rectangular conjunctions subjected to a magnetic field," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 228, no. 3, pp. 288–302, 2014.
[7] M. Daliri, D. Jalali-Vahid, and H. Rahnejat, "Magneto-hydrodynamics of couple stress lubricants in combined squeeze and shear in parallel annular disc viscous coupling systems," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 229, no. 5, pp. 578–596, 2015.
[8] M. Daliri and D. Jalali-Vahid, "Investigation of combined effects of rotational inertia and viscosity–pressure dependency on the squeeze film characteristics of parallel annular plates lubricated by couple stress fluid," *Journal of Tribology*, vol. 137, no. 3, Article ID 031702, 2015.
[9] M. Daliri and D. Jalali-Vahid, "Combined effects of piezo—viscous coupled stress lubricant and rotational inertia upon squeeze film performance of rough circular discs," *Industrial Lubrication and Tribology*, vol. 67, no. 6, pp. 564–571, 2015.

[10] T. V. Biradar, "Squeeze film lubrication between porous parallel stepped plates with couple stress fluids," *Tribology Online*, vol. 8, no. 5, pp. 278–284, 2013.

[11] S. Shukla and G. Deheri, "Hydromagnetic squeeze film in rough porous narrow journal bearing: a study of slip effect," *Tribology Online*, vol. 12, no. 4, pp. 177–186, 2017.

[12] B. Hanumagowda, B. Raju, J. Santhosh Kumar, and K. Vasanth, "Combined effect of surface roughness and pressure-dependent viscosity over couple-stress squeeze film lubrication between circular stepped plates," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, vol. 232, no. 5, pp. 525–534, 2018.

[13] M. Daliri, "Investigation of squeeze film performance in rough parallel circular discs by ferro-fluid couple stress lubricant considering effects of rotational inertia," *Industrial Lubrication and Tribology*, vol. 70, no. 7, pp. 1201–1208, 2018.

[14] H. Aminkhani and M. Daliri, "Effects of piezo-viscous-coupled stress lubricant on the squeeze film performance of parallel triangular plates," *Proceedings of the Institution of Mechanical Engineers, Part J: Journal of Engineering Tribology*, 2019.

[15] M. I. Shliomis, "Effective viscosity of magnetic suspensions," *Soviet Physics, Journal of Experimental and Theoretical Physics*, vol. 34, no. 6, pp. 1291–1294, 1972.

[16] M. I. Shliomis, "Magnetic fluids," *Soviet Physics Uspekhi*, vol. 17, no. 2, pp. 153–169, 1974.

[17] V. K. Stokes, "Couple stresses in fluids," *Physics of Fluids*, vol. 9, no. 9, pp. 1709–1715, 1966.

[18] S. T. Fathima, N. B. Naduvinamani, B. N. Hanumagowda, and J. Santhosh Kumar, "Modified Reynolds equation for different types of finite plates with the combined effect of MHD and couple stresses," *Tribology Transactions*, vol. 58, no. 4, pp. 660–667, 2015.