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Research Article

A Particle Swarm Optimization Algorithm for Solving Pricing and Lead Time Quotation in a Dual-Channel Supply Chain with Multiple Customer Classes

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The combination of traditional retail channel with direct channel adds a new dimension of competition to manufacturers' distribution system. In this paper, we consider a make-to-order manufacturer with two channels of sale, sale through retailers and online direct sale. The customers are classified into different classes, based on their sensitivity to price and due date. The orders of traditional retail channel customers are fulfilled in the same period of ordering. However, price and due date are quoted to the online customers based on the available capacity as well as the other orders in the pipeline. We develop two different structures of the supply chain: centralized and decentralized dual-channel supply chain which are formulated as bilevel binary nonlinear models. The Particle Swarm Optimization algorithm is also developed to obtain a satisfactory near-optimal solution and compared to a genetic algorithm. Through various numerical analyses, we investigate the effects of the customers' preference of a direct channel on the model's variables.

1. Introduction

The rapidly expanding Internet provides an opportunity for organizations to distribute their products via both direct channel and traditional retail channel. Some personal computer manufacturers (like Dell company), apparel retailers, and automotive industries (like General Motors) are examples of companies that use hybrid of both direct and retailer channels.

In the direct channel, the firm interacts with consumers directly through Internet. There are a number of benefits from direct channel distribution such as controlling the distribution and pricing directly, providing a broader product selection, and improving firms' visibility [1].

Despite hybrid channel's benefits such as capturing a larger share of the market, combining the retail distribution channel with direct channel may pose some challenges, including pricing policies, distribution strategies, and conflicting demands placed on internal company resources such as capital, personal, products, and technology by multiple channels [2].

To overcome these challenges, supply chain's members negotiate the retail as well as wholesale price to cooperate with each other. In addition to the product price, there are other factors such as product availability and service quality that contribute to consumer preference of the direct channel [3].

Delivery lead time is one of the important factors that can affect customers' demand [3]. This is the reason many e-retailers, such as Amazon.com, BestBuy.com, Walmart.com, and FYE.com, try to offer competitively quoted lead times [4].

This paper is focused on competitive pricing strategy as well lead time decisions in a supply chain for a manufacturer that sells the products through two channels. One channel is the traditional one in which the firm uses an intermediary to reach final consumers, while the other is a direct channel in which the customer places direct orders through the Internet. It is assumed that there are multiple classes of customers in the market. The demand of each class depends on the price and lead time. A finite planning horizon is considered, where the production capacity in each period is finite but varies. The manufacturer has to respond to the customers quickly, based on the available capacity and the orders in pipeline. Due to the finite production capacity and the demand sensitivity to price and lead time changes, the manufacturer needs to decide on the selling price and the lead time of direct customers, the contract terms with the retailer, and the wholesale price as well as the production schedule to maximize his own profit.

We consider two different cases, centralized and decentralized dual-channel supply chain. We propose a model to determine suitable lead time and price, simultaneously for each case. In a decentralized dual-channel supply chain, the two members interact within the framework of Stackelberg game. Under this framework, the manufacturer, as the leader, determines the wholesale price for retailer and also a price and lead time for direct sale in each period. The intermediary then reacts by choosing a retail price to maximize its own profit. On the other hand, for a centralized dual-channel supply chain, the direct sale price, the traditional retail price, and the quoted lead time in the direct channel are determined by a vertically integrated manufacturer.

The resulting models happen to be binary nonlinear programs. These kinds of problems with large dimensions are not usually solved by exact algorithms. Therefore, an alternative method for solving the models is to use metaheuristic algorithms to obtain a near-optimal solution with reasonable computational time. Thus, an algorithm of Particle Swarm Optimization (PSO) is developed to solve the models. The results will be compared, in terms of solution quality and computational time, with genetic algorithm (GA).

The remainder of this paper is structured as follows. Section 2 reviews the more relevant literature. We formulate the models in Section 3. The principles of the PSO metaheuristic and our algorithm are explained in Section 4. Section 5 presents some numerical examples of our model to examine the effects of customer preference of a direct channel on the pricing strategies and lead time decisions. Our conclusions are summarized in Section 6.

2. Literature Review

This paper focuses on competition in a dual-channel supply chain. A comprehensive review of multichannel models can be found in [1, 5].

Several researchers and practitioners have focused on dual-channel supply chains during the last decade. Different aspects of this chain are investigated. Most of the papers that formulated the dual-channel supply chain focused on the competition context in the supply chain and pricing optimization issues in each channel [6–9]. The researchers combined the pricing issues with other aspects of supply chain. In this regard, we can refer to sales effort determining and service management in each channel [10–13], contract optimization [6, 14], disruption management [15–17], product variety in supply chain [18, 19], and multiperiod model [20].

Besides parameters such as price, services, and quality that can affect the demand process, the lead time (or due date) is also an important parameter. Some researchers have considered that the quoted lead times (or due dates) also affect customers' decisions on placing an order (e.g., Duenyas and Hopp [21]; So and Song [22]; Easton and Moodie [23]; Keskinocak et al. [24]; Webster [25]; Watanapa and Techanitisawad [26]; Charnsirisakskul et al. [27]; Mustafa et al. [28]; Chaharsooghi et al. [29]; and Chaharsooghi et al. [30]). However, the above papers have not addressed the dual-channel distribution issue. The papers by Chen et al. [31]; Hua et al. [3]; Xu et al. [32]; Batarfi et al. [33]; Yang et al. [4]; and Modak and Kelle [34] are some examples that investigated lead time optimization in the dual-channel supply chain.

Xu et al. [32] considered the unit delivery cost (m/t) with $m > 0$ if the product is to be delivered with lead time t. Batarfi et al. [33] considered the combination of two production approaches: make-to-order and make-to-stock in direct online channel and indirect offline channel, respectively. The demand depends on the prices, the quoted delivery lead time, and the product differentiation.

Yang et al. [4] modeled delivery lead time optimization for perishable products in a dual-channel supply chain. Modak and Kelle [34] considered that the demand not only is dependent on price and delivery time but also is stochastic.

The lead times in the above papers are determined exogenously (determined by the sales department, without knowing the actual production schedule). In addition, the production capacity is unlimited. Table 1 illustrates the major literature review with our paper included.

Based on the above literature, this paper investigates the joint decision on production, pricing, and lead time in a dual-channel distribution system where the lead times are assigned internally by the scheduling model. Aside from considering the above literature, we address a new model in which the production capacity in each period is limited. We also consider multiple customer classes that differ in their arrival (commitment) times, quantities demanded, and sensitivity to price and lead time.

3. Problem Description

We consider a dual-channel supply chain in which a manufacturer sells to retailers as well as directly to end customers. The planning horizon is assumed to be finite and divided into periods of equal length. The capacity of the manufacturer is limited but may vary in different periods. Customers can be classified with respect to their sensitivity to price and lead time. Furthermore, the attributes of customers such as arrival (commitment) times, quantities demanded, unit production, and holding costs are different.

In each period, the manufacturer must set a wholesale price for the traditional retail channel as well as setting both

Table 1: Related literature.

Research paper	Production type		Pricing decision	Due date quotation		Capacity constraint	Production planning	Customer classes	Competition approach
	Make-to-stock	Make-to-order		Internally	Exogenously				
Tsay and Agrawal [13]	*		*						Nash
Chiang et al. [7]	*		*						Nash
Yao and Liu [9]	*		*						Bertrand and Stackelberg
Cai et al. [6]	*		*						Stackelberg, Nash
Dan et al. [10]	*		*						Stackelberg
Huang et al. [15]	*		*				*		Stackelberg
Soleimani et al. [17]	*		*						Nash
Roy et al. [12]	*		*						Stackelbarg
Hua et al. [3]	*		*		*				Nash
Xu et al. [32]	*		*		*				Nash
Batarfi et al. [33]	*	*	*		*				-
Yang et al. [4]	*		*		*				Nash
Rofin and Mahanty [18]	*		*						Nash
Pi et al. [16]	*		*						Stackelberg
Modak and Kelle [34]	*		*		*				Stackelberg
This paper		*	*	*		*	*	*	Stackelberg

price and lead time (or due date) for the customers of direct channel. For the customers of traditional retail channel, the production must be scheduled at the same period, at which the order arrives. However, the manufacturer can quote a lead time (due date) to the customers of direct channel. The production for these customers is scheduled within any period between the arrival time of order and the quoted due date. A holding cost occurs for any order of direct channel which is completed before the quoted due date.

As mentioned before, in order to evaluate the effects of the delivery lead time and customer's preference of the direct channel on the pricing decisions of the manufacturer and retailer, we consider two different dual-channel supply chains, centralized and decentralized systems.

3.1. Notation. We use the following notation.

3.1.1. Sets

$\Psi = \{1, \ldots, N\}$: set of customer classes, based on sensitivity to price and due date

$T = \{1, \ldots, T\}$: set of planning periods

3.1.2. Parameters

$e(i)$: arrival time of customer of class $i \in \Psi$

Ch_i: third party holding cost per time period per unit of customer of class $i \in \Psi$

$Cp^1_{i,t}$: production cost of customer of class $i \in \Psi$ in the direct channel in period $t \in T$

$Cp^2_{i,t}$: production cost of customer of class $i \in \Psi$ in the retail channel in period $t \in T$

Cr_i: operational cost in the retail channel for customer of class $i \in \Psi$

Ce_i: operational cost in the direct channel for customer of class $i \in \Psi$

K_t: production capacity available in period $t \in T$

$D^r_{i,j}$: demand (in terms of production capacity units) for customer of class $i \in \Psi$ in the retail channel, quoted due date $j \in [e(i), \ldots, T]$

$D^s_{i,j}$: demand (in terms of production capacity units) for customer of class $i \in \Psi$ in the direct channel, quoted due date $j \in [e(i), \ldots, T]$

3.1.3. Decision Variables

$Z_{i,j}$: 1 if the due date j, $j \in [e(i), \ldots, T]$, is selected (quoted) for customer of class $i \in \Psi$ in direct channel; 0 otherwise

P^r_i: price in the retail channel for customer of class $i \in \Psi$

P^s_i: price in the direct channel for customer of class $i \in \Psi$

W_i: wholesale price charged for customer of class $i \in \Psi$ in the retail channel

$Y_{i,t}$: total production (in units of capacity) for customer of class $i \in \Psi$ in the direct channel in period $t \in T$

H_i: total inventory of customer of class $i \in \Psi$

3.2. Demand Function. In line with Kurata et al. [35], Cai et al. [6], and Hua et al. [3], we assume that the demand is a linear function of the self- and cross-price and lead time, as follows:

$$D_i^s = a_i \cdot \theta_i - b_i^s \cdot P_i^s + \alpha_i^r \cdot P_i^r - c_i^s \cdot L_i^s, \tag{1}$$

$$D_i^r = a_i \cdot (1-\theta_i) - b_i^r \cdot P_i^r + \alpha_i^s \cdot P_i^s + c_i^r L_i^s, \tag{2}$$

where D_i^s and D_i^r denote the consumer demand to the manufacturer and the consumer demand to the retailer, respectively, a_i denotes the base level of industry demand or the demand rate for customer of class i, and $\theta_i\,(0<\theta_i<1)$ represents the initial portion of customers of class i that prefer the direct channel if the price and lead time are zero. Thus, $1-\theta_i$ represents the portion of customers of class i that prefer purchasing from the retailer. The coefficients b_i^s and b_i^r are the coefficients of the price elasticity in the direct channel and retail channel demand functions, respectively. The cross-price sensitivities α_i^r and α_i^s reflect the substitution's degree of the two channels, and c_i^s is the lead time sensitivity of the demand in the direct channel. If the lead time L_i^s increases by one unit, c_i^r units of demand will transfer to the retail channel and c_i^s units of direct channel's demand will be decreased. The total demand of the two channels should have a negative slope with respect to the retailer's price, the direct sale price, and the quoted lead time. Thus, we have $\alpha_i^r < b_i^r$, $\alpha_i^s < b_i^s$, and $c_i^s > c_i^r$.

Following Hua et al. [3], it is assumed that the manufacturer uses dual channels to sell their goods, and the base level of industry demand or demand rate in both channels is very large; i.e., θ_i should not be unreasonably small or large.

3.3. Model of the Decentralized Dual-Channel Supply Chain. In this section, we study a decentralized dual-channel supply chain. In this model both the manufacturer and the retailer make their own decisions separately to maximize their profits. The manufacturer, as the Stackelberg leader, first determines the wholesale price W_i, the direct sale price P_i^s, and the direct sale quoted lead time L_i^s. Then, the retailer chooses his own optimal retail price P_i^r based on the manufacturer's decisions.

3.3.1. Manufacturer's Best Response. The goal of the manufacturer as a leader is to maximize his profit, considering the capacity constraints and demand constraints in the systems and constraints determined by the retailer optimization problem. The problem is formulated within the framework of bilevel programming (BLP), first level (the manufacturer model called the leader) and second level (the retailer model called a follower). In BLP model, each decision maker tries to optimize its own objective function without considering the objective of the other party. However, the decision of each party affects the objective value of the other one as well as the decision space.

3.3.2. First-Level Model: Manufacturer Model.

$$\text{Max}\quad \Pi_s = \left(\sum_{i=1}^{N}\sum_{j=e(i)}^{T}\left(p_i^s \times D_{i,j}^s \times Z_{i,j}\right) - \sum_{i=1}^{N}\sum_{t=e(i)}^{T}\left(Y_{i,t} \times \left(Cp_{i,t}^1 - Ce_i\right)\right) + \sum_{i=1}^{N}\sum_{j=e(i)}^{T}\left(W_i - Cp_{i,e(i)}^2\right)\left(D_{i,j}^r \times Z_{i,j}\right) - \sum_{i=1}^{N} Ch_i \times H_i\right), \tag{3}$$

S.T.

$$W_i \le P_i^s, \quad \forall i \in \Psi, \tag{4}$$

$$\sum_{j=e(i)}^{T} Z_{i,j} = 1, \quad \forall i \in \Psi, \tag{5}$$

$$\sum_{t=e(i)}^{T} Y_{i,t} = \sum_{j=e(i)}^{T} D_{i,j}^s Z_{i,j}, \quad \forall i \in \Psi, \tag{6}$$

$$\sum_{i\in\Psi|e(i)\le t} Y_{i,t} + \sum_{i\in\Psi|e(i)=t}\sum_{j=e(i)}^{T} D_{i,j}^r Z_{i,j} \le K_t,\ t=1,\ldots,T, \tag{7}$$

$$\sum_{t=j}^{T} Y_{i,t} \le M\left(1 - Z_{i,j}\right), \quad \forall i \in \Psi,\ j = e(i),\ldots,T, \tag{8}$$

$$H_i \ge \sum_{t=e(i)}^{j} (j-t)\left(Y_{i,t}\right) + M\left(Z_{i,j} - 1\right), \quad \forall i \in \Psi,\ j = e(i),\ldots,T, \tag{9}$$

$$D^s_{i,j} = a_i \cdot \theta_i - b^s_i \leq P^s_i + \alpha^r_i \cdot P^r_i - c^s_i \cdot (j - e(i) + 1),\ \forall i \in \Psi,\ j = e(i), \ldots \infty, T. \tag{10}$$

3.3.3. Second-Level Model: Retailer Model.

$$\underset{P^r_i}{\text{Max}} \quad \Pi_r = \sum_{i\in\psi} (P^r_i - W_i - Cr_i) \cdot \left(a_i \cdot (1-\theta_i) - b^r_i \cdot P^r_i + \alpha^s_i \cdot P^s_i + c^r_i \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} \right), \tag{11}$$

S.T.

$$\sum_{i|t=e(i)} a_i \cdot (1-\theta_i) - b^r_i \cdot P^r_i + \alpha^s_i \cdot P^s_i + c^r_i \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} \leq K_t,\ t = 1, \ldots, T, \tag{12}$$

$$a_i \cdot (1-\theta_i) - b^r_i \cdot P^r_i + \alpha^s_i \cdot P^s_i + c^r_i \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} \geq 0,\ \forall i \in \Psi, \tag{13}$$

$$\begin{gathered} Y_{i,t}, H_i, D^s_{i,j}, P^r_i, P^s_i, W_i \geq 0,\ \forall i \in \Psi,\ t = e(i), \ldots, T,\ j = e(i), \ldots, T, \\ Z_{i,j} \in \{0,1\},\ \forall i \in \Psi,\ j = e(i), \ldots, T. \end{gathered} \tag{14}$$

The first and second terms in the first-level objective represent the total revenue and production cost of direct sales. The third term represents the total profit obtained by sales to the retailer and the fourth one is the carrying cost.

Constraint (4) indicates that the wholesale price cannot be higher than the direct channel price; otherwise, the retailer may purchase from the direct channel at a lower price. Constraints (5) ensure that only one due date is chosen for each direct channel customer order. Constraint (6) ensures that if due date j is selected for customer of class i in the direct channel, then exact $D^s_{i,j}$ units must be produced and delivered, where $D^s_{i,j}$ depends on the selected price and due date. Constraint (7) is a capacity constraint that ensures that the production capacity in each period is not exceeded. An order of a retail channel customer is produced in the customer's arrival time period and delivered instantaneously in the same period, whereas an order of a direct channel customer can be produced in any period between its arrival period and the quoted due date. The first term in the left hand side of constraint (7) indicates the total production for direct channel customers and the second term is the total production for retail channel customers. Constraint (8) indicates that an order of a direct channel customer can be produced in any period between its arrival period and the quoted due date.

The required inventory for orders is scheduled in any period prior to its commitment, and the negotiated due date is controlled by constraint (9), where M is a sufficiently large number. Constraint (10) is demand functions for customer orders in the direct channel. The term $j - e(i) + 1$ is a time interval between the arrival time of an order and the quoted due date, which is called the lead time. The objective function of second-level optimization problem for the retailer channel is represented by (11). Constraints (12) and (13) control feasibility as well as demand restrictions.

Without considering constraints (12)–(14) for retailer, its best response to the wholesale price can be defined in Proposition 1.

Proposition 1. *The retailer's best response to the wholesale price W_i, the direct sale price P^s_i, and the direct quoted lead time L^s_i set by the manufacturer is as follows:*

$$P^r_i = \frac{a_i \cdot (1-\theta_i) + W_i \cdot b^r_i + \alpha^s_i P^s_i + c^r_i L^s_i + Cr_i b^r_i}{2b^r_i}, \quad \forall i \in \Psi. \tag{15}$$

The Proof of Proposition 1 as well as the other propositions is given in the appendix.

The BLP model (4)–(14) can be formulated as a single level mixed binary problem. This is achieved by replacing the lower level problem (12)–(14) with its Kuhn–Tucker conditions, which we name Model I, as follows.

3.3.4. Model I.

$$\text{Max} \quad \Pi_s = \left(\sum_{i=1}^{N} \sum_{j=e(i)}^{T} \left(p^s_i \times D^s_{i,j} \times Z_{i,j}\right) - \sum_{i=1}^{N} \sum_{t=e(i)}^{T} \left(Y_{i,t} \times Cp^1_{i,t}\right) + \sum_{i=1}^{N} \sum_{j=e(i)}^{T} \left(W_i - Cp^1_{i,e(i)}\right)\left(D^r_{i,j} \times Z_{i,j}\right) - \sum_{i=1}^{N} Ch_i \times H_i \right), \tag{16}$$

S.T.

$$W_i \le P_i^s, \tag{17}$$

$$\sum_{t=e(i)}^{T} Y_{i,t} = \sum_{j=e(i)}^{T} D_{i,j}^s Z_{i,j}, \ \forall i \in \Psi, \tag{18}$$

$$\sum_{j=e(i)}^{T} Z_{i,j} = 1, \ \forall i \in \Psi, \tag{19}$$

$$\sum_{i\in\Psi|e(i)\le t} Y_{i,t} + \sum_{i\in\Psi|e(i)=t} \sum_{j=e(i)}^{T} D_{i,j}^r Z_{i,j} \le K_t, \ t = 1,\ldots,T, \tag{20}$$

$$\sum_{t=j+1}^{T} Y_{i,t} \le M\left(1 - Z_{i,j}\right), \ \forall i \in \Psi, \ j = e(i),\ldots,T-1, \tag{21}$$

$$H_i \ge \sum_{t=e(i)}^{j} (j-t)\left(Y_{i,t}\right) + M\left(Z_{i,j} - 1\right), \ \forall i \in \Psi, \ j = 1,\ldots,T, \tag{22}$$

$$D_{i,j}^s = a_i \cdot \theta_i - b_i^s \cdot P_i^s + \alpha_i^r \cdot P_i^r - c_i^s \cdot (j - e(i) + 1), \ \forall i \in \Psi, \ j = e(i),\ldots,T, \tag{23}$$

$$a_i\left(1-\theta_i\right) - 2P_i^r \cdot b_i^r + \alpha_i^s P_i^s + c_i^s \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} + W_i \cdot b_i^r + Cr_i \cdot b_i^r + \lambda_{e(i)} \cdot b_i^r - \lambda_i' \cdot b_i^r + \nu_i = 0, \ \forall i \in \Psi, \tag{24}$$

$$\sum_{i|e(i)=t} \left(a_i \cdot \left(1-\theta_i\right) - b_i^r \cdot P_i^r + \alpha_i^s \cdot P_i^s + c_i^r \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} \right) + S_t = K_t, \ t = 1\ldots T, \tag{25}$$

$$a_i \cdot \left(1-\theta_i\right) - b_i^r \cdot P_i^r + \alpha_i^s \cdot P_i^s + c_i^r \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} - S_i' = 0, \tag{26}$$

$$\lambda_t \cdot S_t + \lambda_i' \cdot S_i' + \nu_i \cdot P_i^r = 0, \tag{27}$$

$$\begin{gathered} Y_{i,t}, H_i, D_{i,j}^s, P_i^r, P_i^s, W_i, \lambda_i, \lambda_i', S_i', S_i, \nu_i \ge 0, \ \forall i \in \Psi, \ t = e(i),\ldots,T, \ j = e(i),\ldots,T, \\ Z_{i,j} \in \{0,1\}, \ \forall i \in \Psi, \ j = e(i),\ldots,T. \end{gathered} \tag{28}$$

Constraints (24)–(27) indicate the optimality condition for the retailer to convert the bilevel problem into a single level one.

Proposition 2. *For any given due date j, the wholesale price W_i, and the direct sale price P_i^s for customer class i, the optimal retail price P_i^r is always positive.*

As a result of Proposition 2, it can be concluded that ν_i is always equal to zero, because $\nu_i \cdot P_i^r = 0$ in Model I.

Proposition 3. *Consider an unlimited production capacity in each period for a decentralized dual-channel supply chain. If $8b_i^r \cdot b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 6\alpha_i^s \cdot \alpha_i^r > 0$, the dual channel Π_s is strictly jointly concave in W_i and P_i^s.*

From Proposition 3, if $\alpha_i^s = \alpha_i^r$, then $8b_i^r \cdot b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 6\alpha_i^s \cdot \alpha_i^r$ is always greater than zero. Thus, in this case, the manufacturer profit Π_s is always strictly jointly concave in W_i and P_i^s. Thus, for the sake of analytical tractability, we will assume that the cross-price effects are symmetric and $\alpha_i^s = \alpha_i^r$.

Proposition 4. *Let the production capacity be unlimited in each period for a decentralized dual-channel supply chain and suppose there is a $L_i^0 \in [L_i^l, L_i^u]$ with*

$$L_i^0 = \frac{-a_i\alpha_i(1-\theta_i) - a_i\theta_i(b_i^r - \alpha_i) + a_i b_i^s(1-\theta_i) - Cr_i\left(b_i^r b_i^s - (\alpha_i)^2\right)}{c_i^r(b_i^s - \alpha_i) + c_i^s(b_i^r - \alpha_i)}. \quad (29)$$

Then, the optimal lead time $L_i^s \in [L_i^l, L_i^u]$ can be found from L_i^l, L_i^u, and L_i^0 by comparing their Π_s values; the one with the largest Π_s value is the optimal lead time.

3.4. Model of the Centralized Dual-Channel Supply Chain. In a centralized dual-channel supply chain, a vertically integrated manufacturer controls the wholesale price W_i, the direct sale price P_i^s, the direct sale quoted lead time L_i^s, and the retailer sale price P_i^r. The model, called Model II, is as follows.

3.4.1. Model II.

$$\text{Max} \quad \Pi_c = \Pi_s + \Pi_r = \left(\sum_{i=1}^{N}\sum_{j=e(i)}^{T}\left(p_i^s \times D_{i,j}^s \times Z_{i,j}\right) - \sum_{i=1}^{N}\sum_{t=e(i)}^{T}\left(Y_{i,t} \times Cp_{i,t}^1\right) + \sum_{i=1}^{N}\sum_{j=e(i)}^{T}\left(p_i^r - Cp_{i,e(i)}^2 - Cr_i\right)\left(D_{i,j}^r \times Z_{i,j}\right) - \sum_{i=1}^{N} Ch_i \times H_i\right), \quad (30)$$

S.T.

$$\sum_{j=e(i)}^{T} Z_{i,j} = 1, \quad \forall i \in \Psi, \quad (31)$$

$$\sum_{t=e(i)}^{T} Y_{i,t} = \sum_{j=e(i)}^{T} D_{i,j}^s Z_{i,j}, \quad \forall i \in \Psi, \quad (32)$$

$$\sum_{i\in\Psi|e(i)\le t} Y_{i,t} + \sum_{i\in\Psi|e(i)\le t}\sum_{j=e(i)}^{T} D_{i,j}^r Z_{i,j} \le K_t, \; t = 1,\ldots,T, \quad (33)$$

$$\sum_{t=j+1}^{T} Y_{i,t} \le M\left(1 - Z_{i,j}\right), \quad \forall i \in \Psi, \; j = e(i),\ldots,T-1, \quad (34)$$

$$H_i \ge \sum_{t=e(i)}^{j} (j-t)\left(Y_{i,t}\right) + M\left(Z_{i,j} - 1\right), \quad \forall i \in \Psi, \; j = e(i),\ldots,T, \quad (35)$$

$$D_{i,j}^s = a_i \cdot \theta_i - b_i^s \cdot P_i^s + \alpha_i^r \cdot P_i^r - c_i^s \cdot (j - e(i) + 1), \quad \forall i \in \Psi, \; j = e(i),\ldots,T, \quad (36)$$

$$D_{i,j}^r = a_i \cdot (1-\theta_i) - b_i^r \cdot P_i^r + \alpha_i^s \cdot P_i^s + c_i^r \cdot (j - e(i) + 1), \quad \forall i \in \Psi, \; j = e(i),\ldots,T, \quad (37)$$

$$Y_{i,t}, H_i, D_{i,j}^s, P_i^r, P_i^s \ge 0, \quad \forall i \in \Psi, \; t = e(i),\ldots,T, \; j = e(i),\ldots,T, \quad Z_{i,j} \in \{0,1\}, \; \forall i \in \Psi, \; j = e(i),\ldots,T. \quad (38)$$

The objective function (30) maximizes the total profit of the centralized supply chain, which is the sum of manufacturer's profit and retailer's profit. The remaining constraints are the same as those in Model I.

Proposition 5. *Consider an unlimited production capacity in each period for a centralized dual-channel supply chain. If $4b_i^r \cdot b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 2\alpha_i^s \cdot \alpha_i^r > 0$, then the optimal lead time $L_i^s \in [L_i^l, L_i^u]$ can be found from L_i^l and L_i^u by comparing their Π_c values; the largest Π_c value is the optimal lead time.*

4. Solution Approach

Models I and II are 0-1 nonlinear programs with continuous price and production decision variables and binary variables for due date selection. It is well known that the problem of nonlinear 0-1 programming is NP-hard [36]. Thus, these problems cannot be solved easily, and exact algorithms can be time consuming when the number of 0-1 variables is large. An alternative for solving these models is to use approximate algorithms or heuristic methods with reasonable computational times.

In this paper, PSO metaheuristic is used and compared to the GA for modeling and optimization of the pricing and due date setting problems in a dual-channel supply chain.

PSO and GA can handle the whole MINLP easily and naturally, and it is easy to apply it to various problems for comparison with conventional methods.

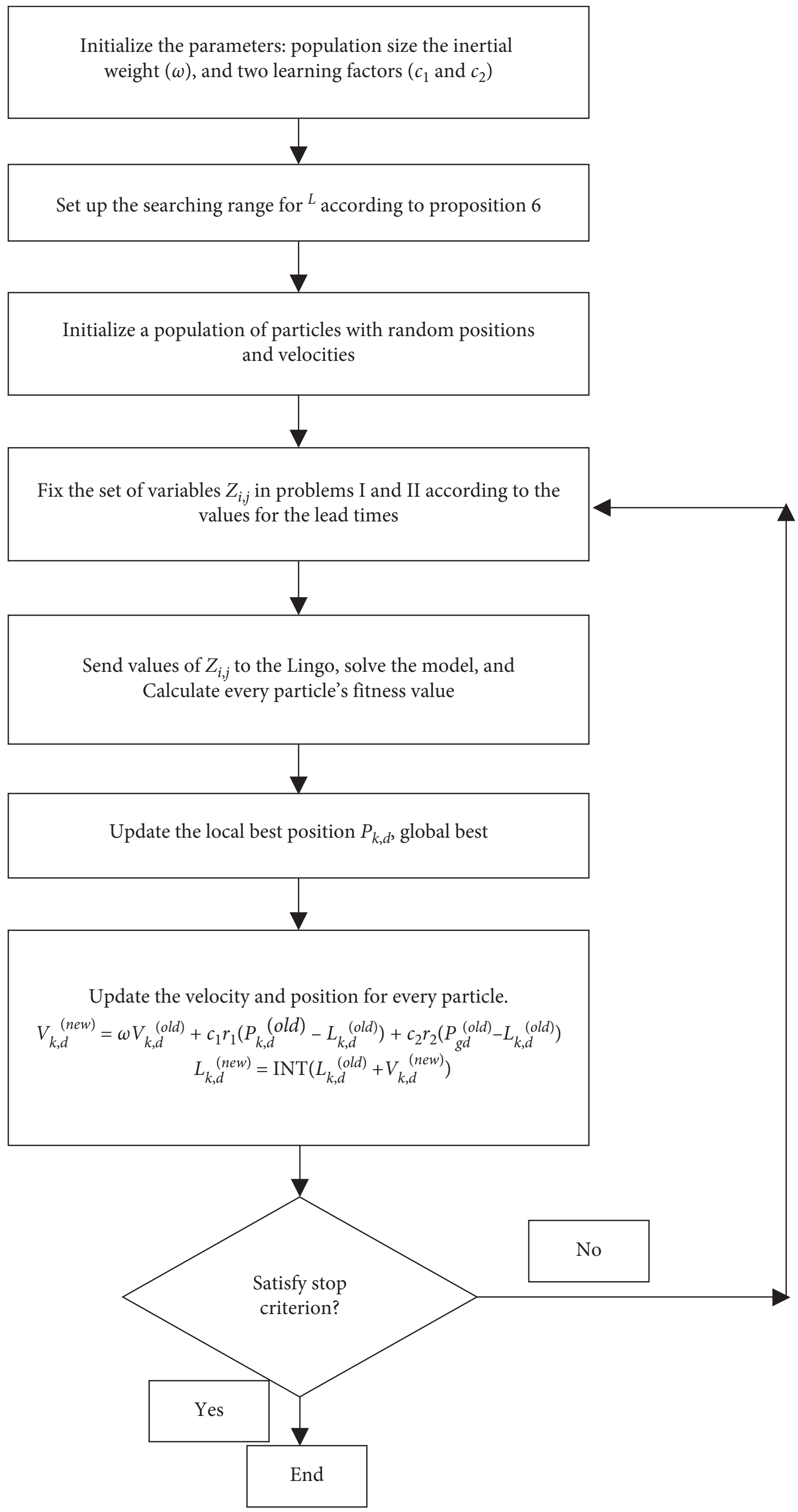

Figure 1: PSO flowchart for pricing and due date quotation.

If we fix the set of binary variables Z in the models, then both models are converted to quadratic programs that can be solved by the nonlinear programming solver LINGO.

4.1. Particle Swarm Optimization (PSO). The PSO algorithm was first proposed by Kennedy and Eberhart [37]. This method is based on the information shared among members of a species and then used for evolution.

Simple structure, ease of implementation, speed of acquiring solutions, and robustness are the advantages of the PSO that persuade us to use it in solving the models.

Recently, PSO algorithms were successfully applied to a wide range of applications. A comprehensive survey of PSO algorithms and applications can be found in the paper by Kennedy et al. [38].

In the proposed PSO algorithm, particle k is represented as $L_k = \{L_{k,1}, L_{k,2}, \ldots, L_{k,N}\}$, which denotes the due dates quoted to N customers. For each $L_{k,i}$ in N-dimensional space, the $Z_{iL_{k,i}}$ is set to one in Models I and II, and the other variables $Z_{i,j}$, $j = e(i), \ldots T$, and $j \neq L_{k,i}$, are fixed to zero for customer order $i \in \Psi$.

This section will present the application of the PSO algorithm for Models I and II. The PSO algorithm for solving these models is illustrated in Figure 1.

The key point in a constrained optimization process is to deal with the constraints. We must modify the original PSO method for constrained optimization. Many methods were proposed for handling constraints, such as methods based on preserving the feasibility of solutions, methods based on penalty functions, methods that make a clear distinction between feasible and infeasible solutions, and other hybrid methods [39]. The most straightforward method is the one based on preserving the feasibility of solutions. In this method, each particle searches the whole space but only keeps track of the feasible solutions. To accelerate the process, we derive an upper bound on the search range by finding the largest lead times such that both demand in retail and direct channel will be nonnegative; i.e., $D^s_{i,j} \geq 0\ D^r_{i,j} \geq 0$. The upper bounds on lead times are given in the following proposition.

Proposition 6. *The best due date for each customer $i \in \Psi$ is restricted to a range $L^l_i \leq L_i \leq L^u_i$. The lower bound of the range is the arrival time of the customer, i.e., $e(i)$, and the upper bound of the range is*

$$L^u_j = \frac{a_i\left(\theta_i \cdot b^r_i - \theta_i \cdot \alpha^r_i + \alpha^r_i\right)}{c^s_i \cdot b^r_i - c^r_i \cdot \alpha^r_i} + e(i). \qquad (39)$$

4.2. Genetic Algorithm (GA). GA is a well-known metaheuristic optimization technique based on Darwin's theory of the "survival of the fittest," proposed by Holland [40]. The GA starts with a group of individuals created randomly. The individuals in the population are then evaluated using a fitness value. Two individuals are then selected based on their fitness. These individuals "reproduce" to create one or more offspring by crossover and mutation operators. The one-point crossover and swap mutation are used in this article. According to the description provided in PSO algorithm, each solution can be represented by an integer string with length N (number of customers). Each gene represents the due date quoted to each customer.

The GA algorithm for solving these models is illustrated in Figure 2:

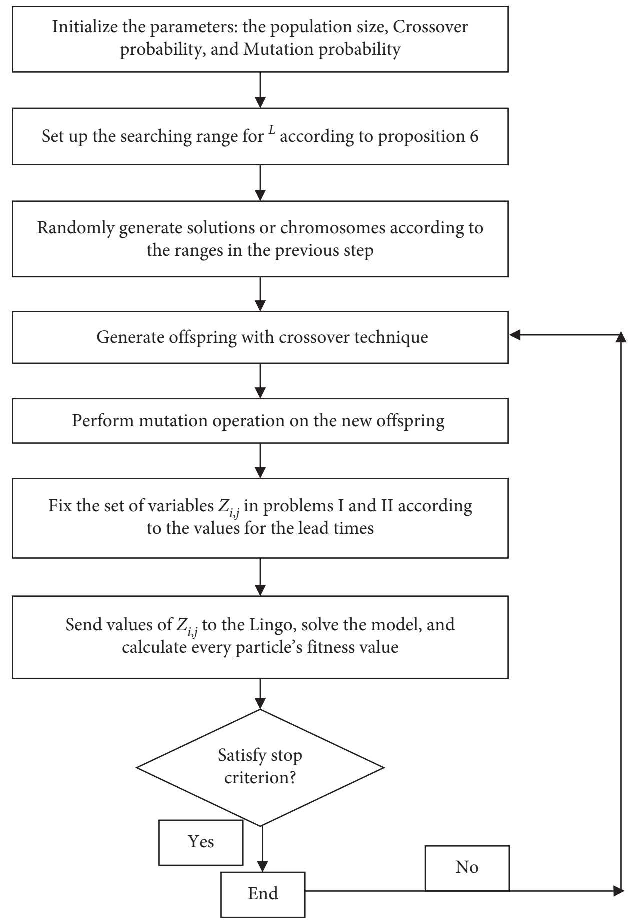

Figure 2: GA flowchart for pricing and due date quotation.

5. Numerical Studies

In this section, we investigate the relation between the wholesale price, the direct sale price, and the retail price in the decentralized and centralized supply chain. The pricing strategies and quoted lead time decisions are compared for the two models using numerical experiments.

The parameters of the proposed PSO algorithm are selected based on parameter tuning. With testing different values for PSO' parameters, the ones selected are those that gave the best results. The best values of the computational experiments are as follows: (1) the values of a population of 30 individuals are used for both GA and PSO, (2) the initial inertia weight is set to 0.9, and (3) the values of the acceleration constants c1 and c2 are fixed to 0.9. The maximum velocity is set as the difference between the upper and the lower bounds, which ensures that the particles are able to fly across the problem-specific constraints region. The other parameters used in GA are crossover rate of 0.80 and mutation rate of 0.3.

Each instance was run for 50 iterations, and 30 replications were conducted for each instance. The performance of PSO algorithm was compared with MINLP solver GAMS and GA algorithm by testing on a set of 8 small size instances in the decentralized (Model I) and centralized (Model II) supply chain. Table 2 reports the comparison between the

Table 2: A comparison between the GAMS solutions, the presented PSO algorithm, and the GA algorithm.

Problem number	N	T	Model I			Model II		
			PSO	GAMS solver	GA	PSO	GAMS solver	GA
Test1	6	12	104202.90	99659.64	103960.30	131059	126182.57	131190.62
Test2	6	12	191024.60	179887.90	186743.10	204614.50	192186.88	203760.16
Test3	12	12	700107.10	656490.40	687800.42	786710.60	745207.09	784619.42
Test4	12	12	125705.30	116692.20	118314.32	135097.20	125816.00	135266.22
Test5	12	12	160890.30	139234.50	161420.60	185068.60	164681.83	186343.64
Test6	12	20	123685.50	107878.50	119659.70	129199.40	119983.29	129934.9
Test7	20	20	138419.10	No feasible	139520.10	146737.70	82849.83	147780.33
Test8	30	20	149965.6	No feasible	148832.40	156788.81	No feasible	157613.04

N: number of customer classes; T: planning periods.

Table 3: Specifications of different groups of problems.

Problem number	K	(Ce/Cr)	PV	LV
1	M	M	No	No
2	H	M	No	No
3	M	M	No	Yes
4	H	M	No	Yes
5	M	M	Yes	No
6	H	M	Yes	No
7	M	H	No	No
8	H	H	No	No
9	M	H	Yes	No
10	H	H	Yes	No
11	M	H	No	Yes
12	H	H	No	Yes
13	M	L	No	No
14	H	L	No	No
15	M	L	Yes	No
16	H	L	Yes	No
17	M	L	No	Yes
18	H	L	No	Yes

GAMS solutions and the presented PSO algorithm. The sum of manufacturer's and retailer's profit is shown in Table 2. All the results show that PSO algorithm can achieve better results than those obtained by GAMS solver.

Comparing the results of the GA and PSO methods, we can see that there is no conclusive winner. In some cases, the GA method results in a better solution and in some cases the PSO method does. Due to the simplicity of the PSO structure, we use it for our sensitivity analysis.

To analyze the effects of models' parameters on prices and profits, we considered 18 different problem groups to carry out this numerical study. For each group of problems, the production capacity of each period (K) is categorized as high and low capacity. The ratio of operational costs in direct channel and retailer (Ce/Cr) is categorized as high, medium, and low. We can have single class customer problems where the price and lead time sensitivities are the same across the customers and multiclass customer problems based on the variability of these sensitivities. With this assumption, the additional categories are based on price sensitivity variability (PV) and lead time sensitivity variability (LV). Table 3 presents the characteristics of different groups of problems.

The other parameters are considered as follows:

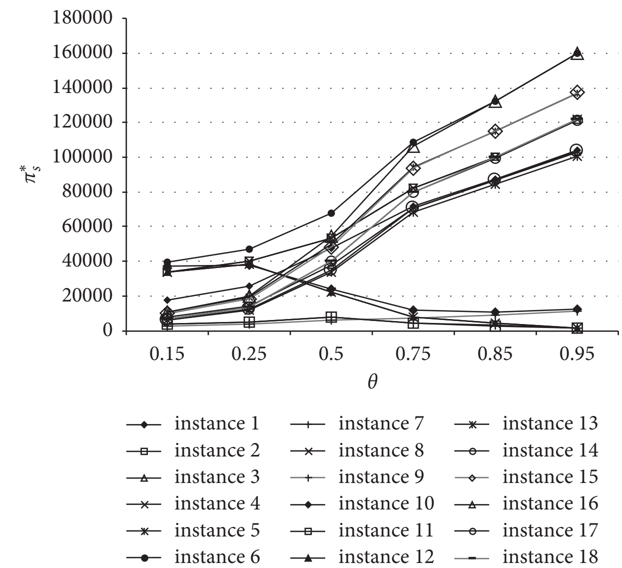

Figure 3: Manufacturer' profit in the decentralized dual-channel supply chain for different instances.

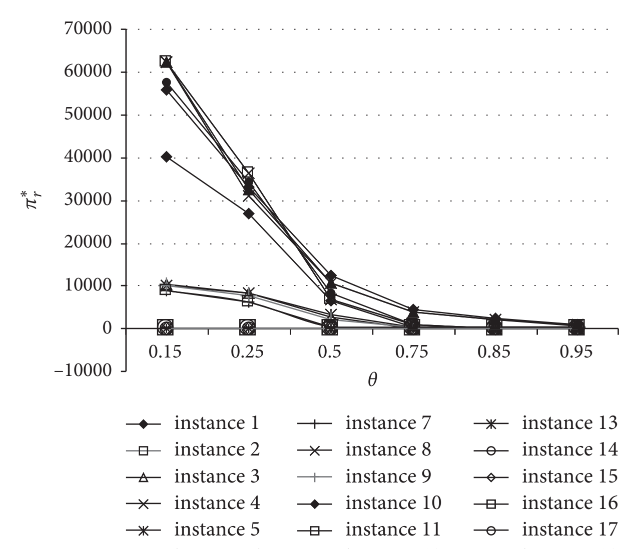

Figure 4: Retailer' profit in the decentralized dual-channel supply chain for different instances.

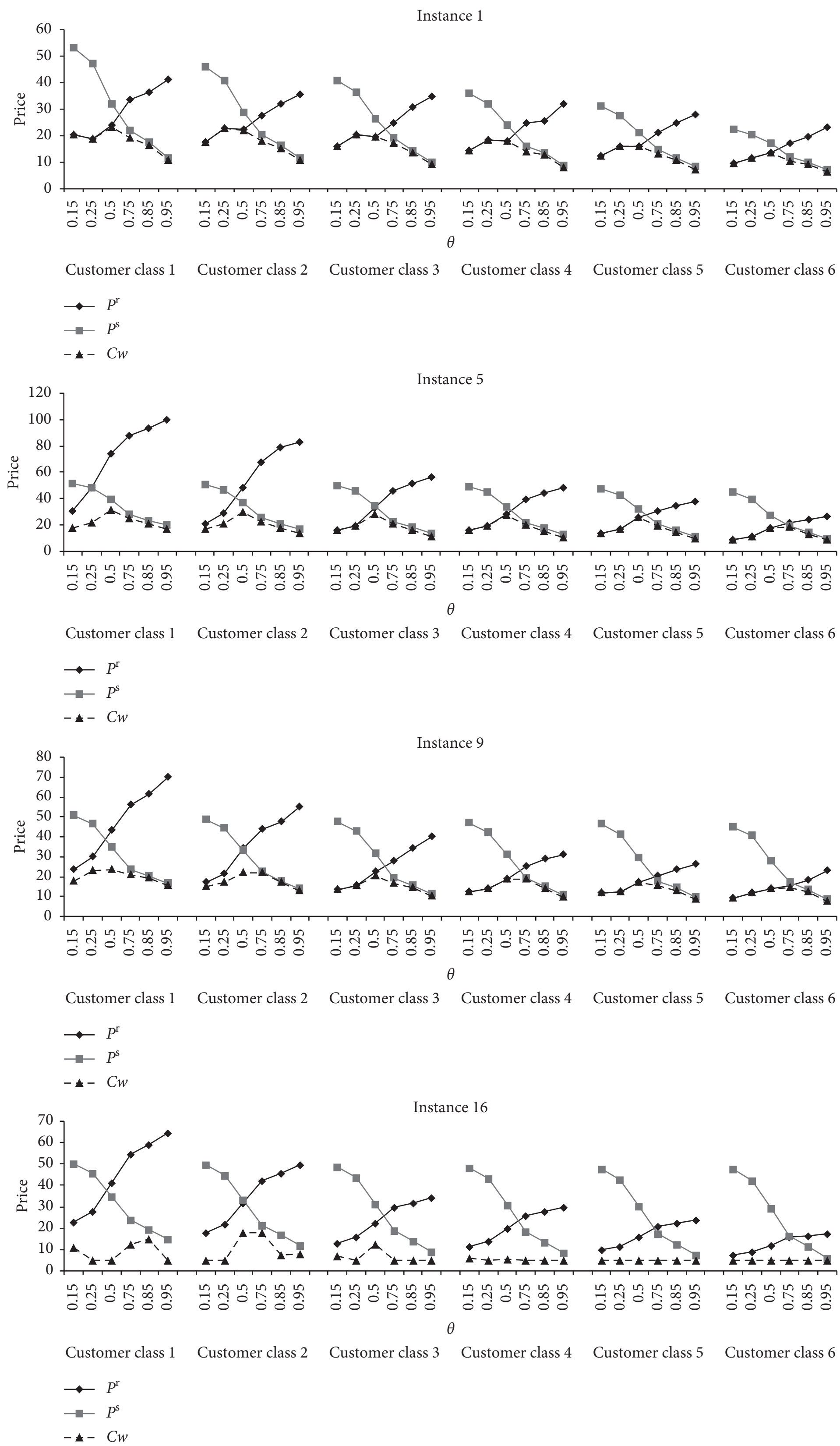

Figure 5: Comparison of the retail price, direct sale price, and wholesale price in decentralized dual-channel supply chain for different instances.

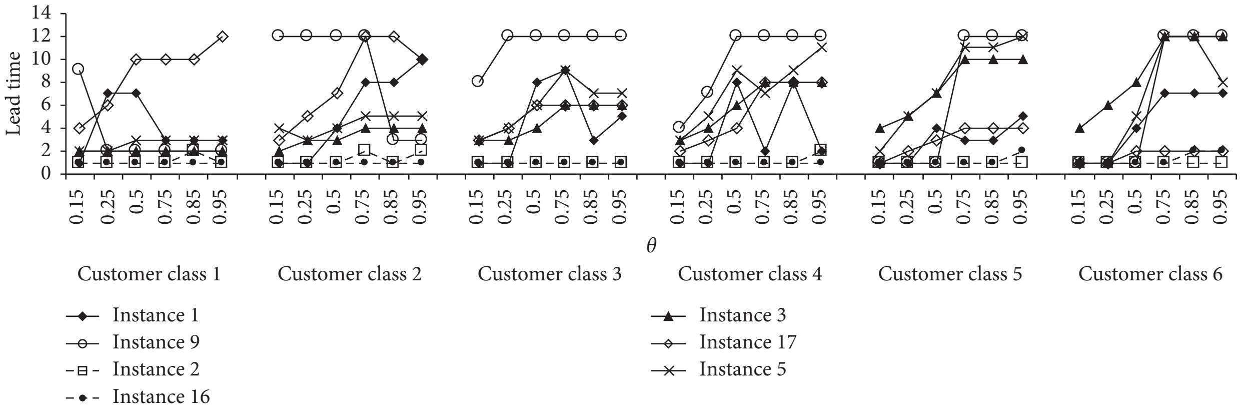

FIGURE 6: Comparison of the optimal quoted lead time in decentralized dual-channel supply chain for different instances.

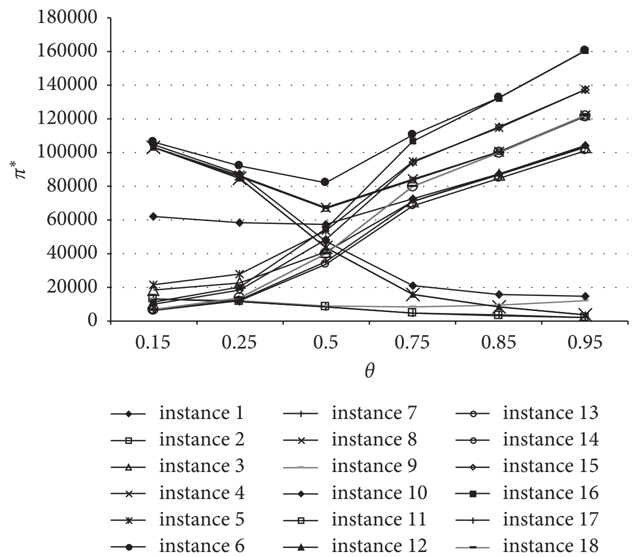

FIGURE 7: Supply chain profit in the centralized dual-channel supply chain for different instances.

The holding costs per unit of time for each customer of class i are $Ch_i = 5$, and the production cost in each period t for each customer of class i is $Cp_t^i = 10$. The demand rate for customer of class i, a_i, is generated randomly from $a_i \in [500, 3000]$. We considered a planning horizon with 12 time periods and 6 customer classes arriving during the planning horizon for all problems. Two types of arrival time distributions are considered: the arrival times near the beginning of planning horizon (B) ($e(i) \sim$ uniform $[1, 4]$) and the uniform distribution along planning horizon (U) ($e(i) \sim$ uniform $[1, 12]$) [29, 33].

In all instances, we investigated the effect of customer preference of the direct channel, θ, on pricing and lead time decisions. The results of some carried instances are summarized in Figures 3–9 and Tables 4 and 5. Tables 4 and 5 show the differences between average values of retailer prices ($\overline{P}_{\text{II}}^r - \overline{P}_{\text{I}}^r$) and direct prices ($\overline{P}_{\text{II}}^s - \overline{P}_{\text{I}}^s$) obtained for customer classes and the profits ($\Pi_{\text{II}} - \Pi_{\text{I}}$) under both decentralized (Model I) and centralized (Model II) supply chains. In the decentralized supply chain, as illustrated in Figures 3 and 4, in some examples, the manufacturer's profit increases as the customer preference of the direct channel increases, whereas the profit decreases for others. Almost all the examples with decreasing profit functions are consequence of high operational cost in the direct channel. The retailer's profit is decreasing with customer preference of the direct channel.

As we can see from Figures 5 and 8, in both the centralized and decentralized dual-channel supply chain, when customer preference of the direct channel θ is below a certain level, the retail price is higher than the direct sale price; conversely, when θ exceeds that level, the direct price becomes higher than the retail sale price. This result shows that if the base level of demand or demand rate in one channel is relatively higher than a certain threshold, the sale price in that channel should be set higher than the one in the other channel.

From Figure 5, in some examples, when θ is relatively low, the direct sale price should be set equal to the wholesale price, and when θ is relatively high, the direct sale

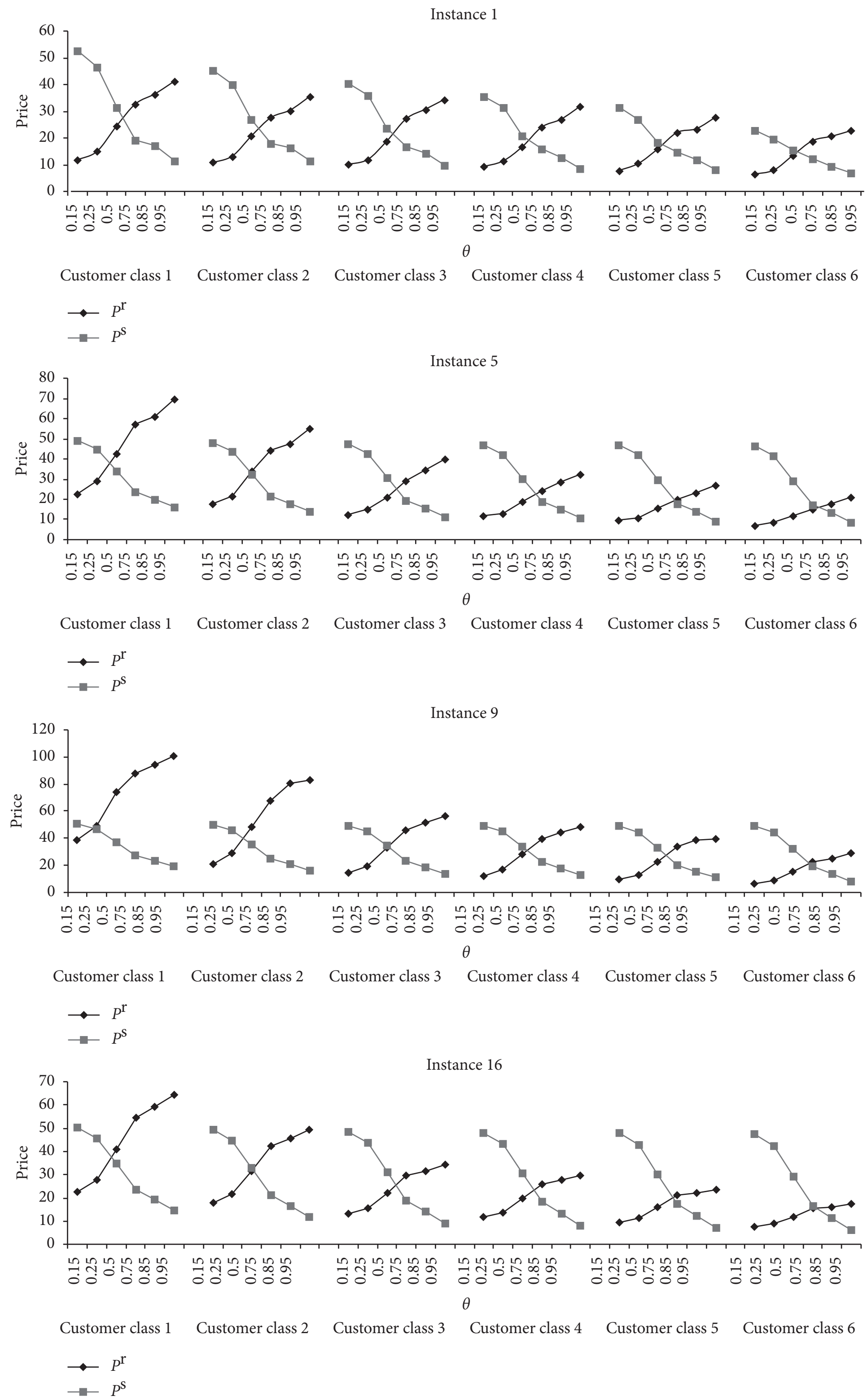

Figure 8: Comparison of the retail price and direct sale price in centralized dual-channel supply chain for different instances.

price should be set higher than the wholesale price. In other words, when θ is lower than a certain threshold, the base level of demand in the retail channel is high. Thus, the retail price can also be set to be high, but according to the low base demand in the direct channel, the direct sale price is set to be low. The wholesale price must be less than or equal to the direct sale price. Therefore, the direct sale price is equal to the wholesale price, and the retail price is higher than the wholesale price; conversely, when θ exceeds the threshold, the base level of demand in the retail channel is

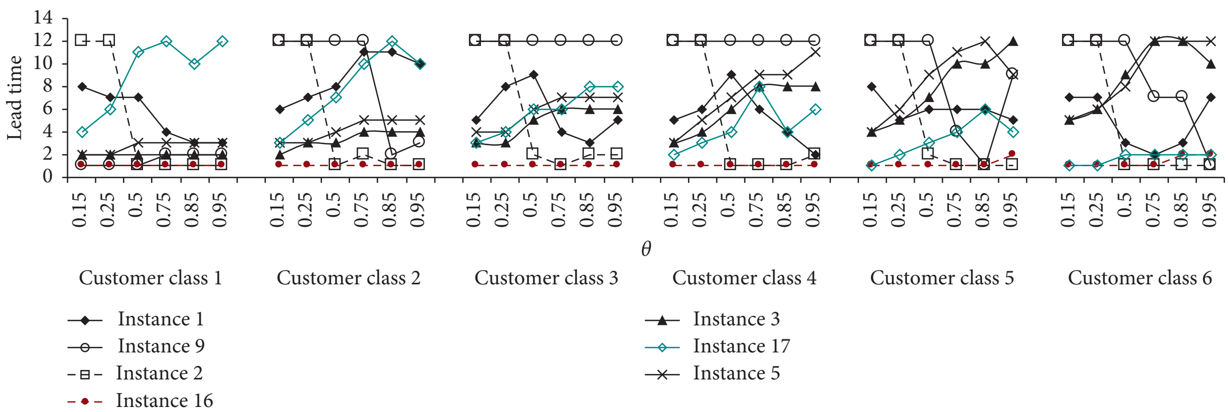

Figure 9: Comparison of the optimal quoted lead time in centralized dual-channel supply chain for different instances.

Table 4: Comparison of retailer and direct prices under both decentralized and centralized supply chain.

Problem number	$\overline{P}^r_{II} - \overline{P}^r_{I}$						$\overline{P}^s_{II} - \overline{P}^s_{I}$					
	$\theta = 0.15$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$	$\theta = 0.15$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
1	−5.20	−5.99	−0.23	0.88	0.15	0.00	0.06	−0.36	−1.94	−0.93	−0.06	0.00
2	−5.24	−5.74	0.15	0.65	0.37	0.02	−0.19	−4.24	−5.18	−1.39	−0.36	−0.01
3	−0.45	−0.44	−0.97	0.05	0.00	−0.05	−0.05	−0.08	−0.12	−0.05	0.00	−0.01
4	−5.64	−4.52	0.19	−0.02	0.37	0.01	−0.26	−4.64	−4.92	−2.16	−0.36	−0.01
5	−0.82	−1.11	−0.86	0.44	0.01	−3.49	0.03	−0.02	−0.07	−0.14	0.00	−0.62
6	−4.47	−3.76	−0.34	−0.01	0.27	−3.81	−1.53	−5.06	−5.06	−2.18	−0.61	−0.69
7	−4.22	−4.22	−1.53	0.00	0.00	0.00	1.28	1.28	0.47	0.00	0.00	0.00
8	−5.24	−6.21	−1.32	−0.79	−0.58	−0.37	−0.19	−5.31	−6.99	−4.20	−3.08	−1.97
9	−0.78	−1.42	−1.07	0.68	1.29	−5.67	0.67	0.55	0.46	−0.28	−0.29	−1.25
10	−5.31	−4.60	−1.82	−0.74	−1.42	−6.64	−1.74	−5.44	−7.01	−4.46	−3.20	−2.89
11	−4.99	−4.99	−2.63	0.00	0.00	−0.76	1.14	1.14	0.29	0.00	0.00	−0.11
12	−5.24	−5.96	−1.32	−0.79	−0.58	−1.41	−0.19	−5.34	−6.99	−4.20	−3.08	−1.94
13	0.00	0.00	0.00	0.00	0.00	0.45	0.00	0.00	0.00	0.00	0.00	−0.10
14	0.00	0.00	0.00	0.00	0.00	0.02	0.00	0.00	0.00	0.00	0.00	−0.01
15	0.00	0.00	0.07	0.09	−0.01	−3.96	0.00	0.00	0.01	0.02	0.00	−0.69
16	0.00	0.00	0.00	0.00	0.00	−3.81	0.00	0.00	0.00	0.00	0.00	−0.69
17	0.00	0.00	−0.01	0.05	0.00	0.28	0.00	0.00	0.03	0.01	0.00	−0.14
18	0.00	0.00	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00	−0.01

Table 5: Comparison of obtained profits under both decentralized and centralized supply chain.

Problem number	$\Pi_{II} - \Pi_{I}$					
	$\theta = 0.15$	$\theta = 0.25$	$\theta = 0.5$	$\theta = 0.75$	$\theta = 0.85$	$\theta = 0.95$
1	3884.36	5657.56	3091.33	136.57	91.42	16.27
2	6589.72	9055.78	6495.73	839.46	75.91	0.00
3	75.40	98.32	336.52	8.66	0.00	−167.17
4	6713.26	9965.70	7062.69	908.76	75.91	0.00
5	440.87	582.24	502.25	272.85	−438.03	609.73
6	8812.77	10719.97	5635.36	965.10	123.80	0.00
7	385.00	385.00	140.00	0.00	0.00	67.57
8	6589.72	15270.77	10452.74	3773.87	2034.84	971.90
9	566.32	612.44	912.49	101.92	96.21	108.02
10	11481.49	16019.67	10816.05	4407.86	2449.25	1022.60
11	354.53	354.53	81.87	3.20	3.20	54.06
12	6603.00	13601.09	10455.94	3777.07	2038.04	1062.36
13	0.00	0.00	0.00	0.00	0.00	0.00
14	0.00	0.00	0.00	0.00	0.00	0.00
15	0.00	0.00	111.02	130.35	147.26	−185.62
16	0.00	0.00	0.00	0.00	0.00	0.00
17	0.00	0.00	−47.52	156.25	−10.91	166.67
18	0.00	0.00	0.00	0.00	0.00	0.00

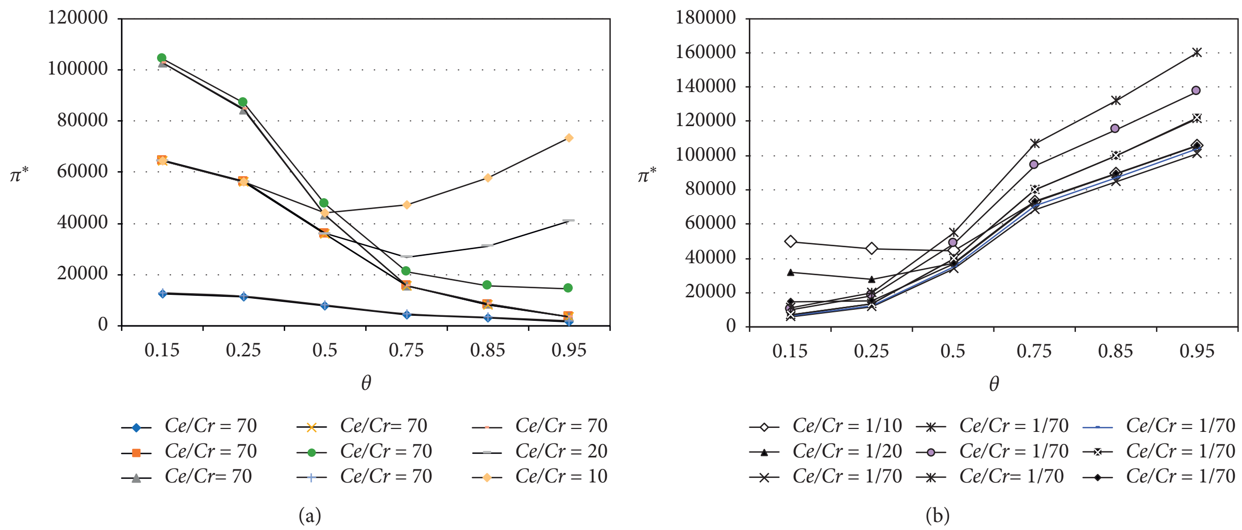

FIGURE 10: Comparison of the profitability of the firm according to different direct and retail channel operational costs.

TABLE 6: The percentage of profit increases over P2 policy with no lead time flexibility.

Problem number	Capacity (K)	Lead time sensitivity variability (LV)	Gap = $((P1 - P2)/P2) * 100$		
			$b_i^s > b_i^r$	$b_i^s = b_i^r$	$b_i^s < b_i^r$
1	M	Yes	19.74	19.81	22.86
2	M	No	17.20	15.98	18.34
3	H	Yes	1.08	1.64	0.11
4	H	No	0	0.80	0

low, and the retail price is generally also set to be low. We know that the wholesale price must be lower than the retail price. Therefore, with increasing θ, the retail price and consequently the wholesale price decrease. As we can see from Figures 5 and 8, for some values of θ, both the retail price and the direct price in the decentralized dual-channel supply chain should be set higher than those in the centralized one, while the total profit in the decentralized dual-channel supply chain is lower than that in the centralized one. The negative value of gap in price differences in Table 4 and the positive value of gap for profits in Table 5 show this fact. This shows that when each partner in the decentralized dual-channel supply chain maximizes his own profit, this leads to double marginalization. Double marginalization means that the retailer and the manufacturer independently set their price to maximize their profit margins; as a result, the price is higher and the sales volume and profits are lower than those of a vertically integrated channel [3].

In instances with low (Ce/Cr) or with high Cr, the direct sale price in the centralized setting tends to equal the direct sale price in decentralized settings. This trend is also true for retail prices in the two settings. Moreover, when Cr is very large, the retail channel has a small impact on the manufacturer's profits, and in both settings, the manufacturer has a main role in determining prices. Therefore, the prices defined in the two settings become close.

As illustrated in Figure 7, in the centralized dual-channel supply chain, when customer preference of the direct channel θ is lower than a certain threshold, the total profit decreases with increasing θ; conversely, when θ exceeds the threshold, the total profit increases with increasing θ. This threshold differs from one instance to another. In examples with high direct channel operational cost, the threshold is near 1, meaning the function seems as decreasing function. To compare the profitability changes of the firm according to different direct and retail channel operational costs, we carried some more examples by varying these two parameters. Figure 10 shows the results. As we can see from Figure 10, under high value of direct channel operational cost, the profit decreases with increasing θ. Therefore, it is more profitable to encourage customers buying from retail channel.

As we can see from Figures 6 and 9, the high lead time changes depending on θ in two centralized and decentralized setting show that the delivery lead time has a strong effect on the manufacturer's and the retailer's pricing strategies and profits. We can see the results of Propositions 4 and 5 in instances 2 and 16, where the unlimited capacity is considered. The quoted lead times for the customers are equal to the lower bound or the upper bound of the lead time.

We perform a numerical study to compare how the lead time flexibility affects the profitability of the firm.

We consider two policies of lead time flexibility: P1 (lead time flexibility), P2 (no lead time flexibility). With lead time flexibility, we quote different lead times to different customers. When there is no lead time flexibility, a single (fixed) lead time is quoted to all customers.

We have generated the problems in three situations: the customers' sensitivity to price in direct channel is 1- greater, 2- equal, and 3- lower than customers' sensitivity to price in retail channel. We have considered the same price sensitivities for all customers. The categorization for production capacity and variability of customers' lead time sensitivity is shown in Table 3. The percentage of profit increases over P2 policy with no lead time flexibility under each policy and category is represented in Table 6. The main conclusions one can draw from Table 6 are as follows:

(1) The effect of production capacity: comparing instances 2 and 4 (instance 4 with high production capacity), we can see that more capacity makes the manufacturer able to charge equal lead times to all customers, and manufacturers usually determine a lead time equal to that of the first period for all customers.

(2) The effect of variability of customers' sensitivity to lead time: comparing instances 1 and 2 (instance 1 with high variability in customers' sensitivity), we can see that if there is high variability in customers' lead time sensitivities, the manufacturer can obtain more profit from lead time flexibility. The manufacturer can charge the high lead time for customer with lower sensitivity to lead time and reserve the capacity of the first period for customers with higher sensitivity to lead time.

6. Conclusion

In this paper, we presented a pricing and due date setting model for a manufacturer with a dual sale channel, an online direct channel, and a traditional retail channel. We see that a large number of e-retailers, such as Amazon.com, BestBuy.com, and Walmart.com, attempt to offer a proper delivery lead time (Yang et al. [4]). Also the model can be developed for 's auto industry which is the second largest industry in Iran after oil and gas. Currently, Iran's auto industry has an online sale and the proposed list price is sometimes based on delivery time.

We assumed a finite planning horizon divided into periods of equal length. We considered the limit production capacity in each period and multiple classes of customers arriving during these periods. We developed a Stackelberg game to model the decentralized dual-channel supply chain. Under this game framework, the supplier, as the leader, announces the wholesale price to an intermediary in addition to the direct channel sales price and lead time. The intermediary then reacts by choosing a retail price to maximize its own profit. Bilevel programming was developed to model this situation. For a centralized dual-channel supply chain, the traditional retail price, the direct sale price, and the quoted lead time in the direct channel are determined in an integrated manner. Considering the lead time selection for each customer order, mixed binary integer nonlinear programming models for the design of the centralized and decentralized dual-channel supply chains were used.

Because exact algorithms for solving the proposed models can be expensive and time consuming for instances with large numbers of 0-1 variables, the PSO algorithm solving model is adopted to get a satisfactory near-optimal solution efficiently.

Through numerical analyses, we have examined the effects of the customer preference of a direct channel on the manufacturers' and retailers' pricing behaviors. We also compared the optimal lead times, prices, and profits in the two settings of a centralized and decentralized dual-channel supply chain.

Numerical examples have shown that the retail price is decreasing with increasing customer preference of the direct channel and that the direct sale price is increasing with increasing customer preference of the direct channel. Therefore, when the customer preference of the direct channel is low, the retail price is higher than the direct sale price; conversely, when the customer preference of the direct channel is high, the retail price is lower than the direct sale price. Comparing the two settings of dual-channel supply chains, we found that when θ is relatively low and, thus, the base level demand in the retail channel is high, both the retail price and the direct price in the decentralized dual-channel supply chain should be set to be higher than those in the centralized supply chain, while the total profit in the decentralized dual-channel supply chain is lower than that in the centralized supply chain. This shows double marginalization in the decentralized dual-channel supply chain which each member maximizes his own profit.

The high changes in the lead time as a function of θ in the centralized and decentralized settings show that the delivery lead time strongly influences the manufacturers' and the retailers' pricing strategies and profits. Therefore, considering a constant value for the lead time in all scenarios leads to a decrease in profits.

This research can be extended in several directions in future work. First, our model is deterministic, and we can study a model with stochastic demand. Second, we can consider competition among several manufacturers and retailers. Third, it is worth investigating the coordination of the dual-channel supply chain by contracts when the price and lead time are both considered in the models.

Appendix

Proof of Proposition 1. For given W_i, P_i^s, and L_i^s, the retailer's profit is determined by

$$\underset{P_i^r}{\text{Max}}\,\Pi_r = \sum_{i\in\psi}\left(P_i^r - W_i - Cr_i\right)\cdot\left(a_i\cdot\left(1-\theta_i\right) - b_i^r\cdot P_i^r + \alpha_i^s\cdot P_i^s + c_i^r\sum_{j=e(i)}^{T}\left(j-e(i)+1\right)\cdot Z_{i,j}\right). \quad (A.1)$$

Obviously, Π_r is a concave quadratic function of P_i^r. Using the partial first-order condition, we get

$$P_i^r = \frac{a_i \cdot (1-\theta_i) + W_i \cdot b_i^r + \alpha_i^s P_i^s + c_i^r L_i^s + Cr_i b_i^r}{2b_i^r}, \quad \forall i \in \Psi. \tag{A.2}$$

Proof of Proposition 2. If $P_i^r = 0$, then we can simplify (24)–(26) as follows:

$$a_i^r (1-\theta_i) + \alpha_i^s P_i^s + c_i^s \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} + W_i \cdot b_i^r + Cr_i \cdot b_i^r + \lambda_i \cdot b_i^r - \lambda_i' \cdot b_i^r + \nu_i = 0, \quad \forall i \in \Psi, \tag{A.3}$$

$$a_i \cdot (1-\theta_i) + \alpha_i^s \cdot P_i^s + c_i^r \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} + S_t = K_t t = e(i), \quad \forall i \in \Psi,\ j = 1, \dots, T, \tag{A.4}$$

$$a_i \cdot (1-\theta_i) + \alpha_i^s \cdot P_i^s + c_i^r \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} = S_i'. \tag{A.5}$$

Hence, $S_i' > 0$. Based on complementary conditions (27), the $\lambda_i' = 0$. Thus, the constraint (A.3) can be simplified as follows:

$$-\left(a_i^r (1-\theta_i) + \alpha_i^s P_i^s + c_i^s \sum_{j=e(i)}^{T} (j - e(i) + 1) \cdot Z_{i,j} + W_i \cdot b_i^r + Cr_i \cdot b_i^r + \lambda_i \cdot b_i^r \right) = \nu_i, \quad \forall i \in \Psi. \tag{A.6}$$

Therefore, the ν_i is always negative which is a contradiction.

Proof of Proposition 3. The retailer's best response to the wholesale price W_i, the direct sale price P_i^s, and the direct sale quoted lead time L_i^s set by the manufacturer is given by (3).

According to unlimited capacity in each period, the manufacturer's profit is determined by

$$\Pi_s = \sum_{i=1}^{N} \left(p_i^s - Cp_i^2 \right) \times D_i^s + \sum_{i=1}^{N} \left(W_i - Cp_i^1 \right) \times D_i^r. \tag{A.7}$$

Substituting (15) into (A.7) and simplifying it, we get

$$\begin{aligned} \Pi_s = {} & \sum_{i=1}^{N} \left(p_i^s - Cp_i^2 \right) \times \left(a_i \cdot \theta_i - b_i^s \cdot P_i^s + \alpha_i^r \cdot \left(\frac{a_i \cdot (1-\theta_i) + W_i \cdot b_i^r + \alpha_i^s P_i^s + c_i^r L_i^s + Cr_i b_i^r}{2b_i^r} \right) - c_i^s \cdot L_i^s \right) \\ & + \sum_{i=1}^{N} \left(W_i - Cp_i^1 \right) \times \left(a_i \cdot (1-\theta_i) - b_i^r \cdot \left(\frac{a_i \cdot (1-\theta_i) + W_i \cdot b_i^r + \alpha_i^s P_i^s + c_i^r L_i^s + Cr_i b_i^r}{2b_i^r} \right) + \alpha_i^s \cdot P_i^s + c_i^r L_i^s \right). \end{aligned} \tag{A.8}$$

Taking the second-order partial derivatives of Π_s with respect to P_i^s and W_i, we have the Hessian matrix as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2 \Pi_s}{\partial (p_i^s)^2} & \dfrac{\partial^2 \Pi_s}{\partial (p_i^s) \partial (W_i)} \\ \dfrac{\partial^2 \Pi_s}{\partial (W_i) \partial (p_i^s)} & \dfrac{\partial^2 \Pi_s}{\partial (W_i)^2} \end{bmatrix} = \begin{bmatrix} -2b_i^s + \dfrac{\alpha_i^r \alpha_i^s}{b_i^r} & \dfrac{\alpha_i^r + \alpha_i^s}{2} \\ \dfrac{\alpha_i^r + \alpha_i^s}{2} & -b_i^r \end{bmatrix}. \tag{A.9}$$

From assumption $8b_i^r \cdot b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 6\alpha_i^s.\alpha_i^r > 0$, Π_s is strictly jointly concave in P_i^s and W_i, since $(\partial^2\Pi_s/\partial(p_i^s)^2) = -2b_i^s + (\alpha_i^r\alpha_i^s/b_i^r) < 0$ and

$$\begin{vmatrix} -2b_i^s + \dfrac{\alpha_i^r\alpha_i^s}{b_i^r} & \dfrac{\alpha_i^r + \alpha_i^s}{2} \\ \dfrac{\alpha_i^r + \alpha_i^s}{2} & -b_i^r \end{vmatrix} = \frac{8b_i^r.b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 6\alpha_i^s \cdot \alpha_i^r}{4} > 0. \tag{A.10}$$

Proof of Proposition 4. As expressed in Proposition 3, under unlimited production capacity, the manufacturer's best price is obtained from

$$\begin{aligned} \text{Max} \quad \Pi_s = {} & \sum_{i=1}^{N} (p_i^s - Cp_i^1) \times \left(a_i \cdot \theta_i - b_i^s \cdot P_i^s + \alpha_i^r \cdot \left(\frac{a_i \cdot (1-\theta_i) + W_i \cdot b_i^r + \alpha_i^s P_i^s + c_i^r L_i^s + Cr_i b_i^r}{2b_i^r} \right) - c_i^s \cdot L_i^s \right) + \\ & \sum_{i=1}^{N} (W_i - Cp_i^2) \times \left(a_i \cdot (1-\theta_i) - b_i^r \cdot \left(\frac{a_i \cdot (1-\theta_i) + W_i \cdot b_i^r + \alpha_i^s P_i^s + c_i^r L_i^s + Cr_i b_i^r}{2b_i^r} \right) + \alpha_i^s \cdot P_i^s + c_i^r L_i^s \right) \\ \text{S.T.} \quad & W_i \le p_i^s, \end{aligned} \tag{A.11}$$

and the KKT conditions for this model are

$$2a_i\theta_i b_i^r + a_i\alpha_i(1-\theta_i) - p_i^s\left(4b_i^r b_i^s - 2(\alpha_i)^2\right) + \alpha_i b_i^r Cr_i + L_i^s(\alpha_i c_i^r - 2c_i^s b_i^r) + 2\alpha_i b_i^r W_i + Cp_i^2\left(2b_i^r b_i^s - (\alpha_i)^2\right) - Cp_i^1\alpha_i b_i^r + 2\eta_i b_i^r = 0, \tag{A.12}$$

$$2\alpha_i p_i^s - \alpha_i Cp_i^2 + a_i(1-\theta_i) - b_i^r Cr_i + c_i^r L_i^s - 2b_i^r W_i + b_i^r Cp_i^1 - 2\eta_i = 0, \tag{A.13}$$

$$\eta_i(W_i - p_i^s) = 0, \tag{A.14}$$

$$W_i \le p_i^s, \tag{A.15}$$

$$\eta_i \ge 0. \tag{A.16}$$

Case 1. When $\eta_i = 0$, we obtain $W_i^*(L_i^s)$ and $p_i^{s*}(L_i^s)$ from ((A.12) and (A.13)). Notice that $W_i^*(L_i^s) \le p_i^{s*}(L_i^s)$. Therefore, we have

$$\begin{gathered} W_i^*(L_i^s) - p_i^{s*}(L_i^s) = \frac{-a_i\alpha_i(1-\theta_i) - a_i\theta_i(b_i^r - \alpha_i) + a_i b_i^s(1-\theta_i) - Cr_i\left(b_i^r b_i^s - (\alpha_i)^2\right) + c_i^r L_i^s(b_i^s - \alpha_i) + c_i^s L_i^s(b_i^r - \alpha_i)}{2\left(b_i^r b_i^s - (\alpha_i)^2\right)} \le 0, \\ \text{or } L_i^s \ge \frac{-a_i\alpha_i(1-\theta_i) - a_i\theta_i(b_i^r - \alpha_i) + a_i b_i^s(1-\theta_i) - Cr_i\left(b_i^r b_i^s - (\alpha_i)^2\right)}{c_i^r(b_i^s - \alpha_i) + c_i^s(b_i^r - \alpha_i)} = L_i^0. \end{gathered} \tag{A.17}$$

The second-order partial derivative of Π_s with respect to L_i^s is $(\partial^2\Pi_s/\partial(L_i^s)^2) = 0$ which indicates that the Π_s is a linear function of L_i^s. Therefore, in this case, the optimal value of L_i^s will be equal to its upper bound (L_i^u) or lower bound (L_i^0).

Case 2. When $\eta_i > 0$, then $W_i^*(L_i^s) = p_i^{s*}(L_i^s)$. When $L_i^s < L_i^0$, we have $\eta_i > 0$. In this case, the second-order partial derivative of Π_s with respect to L_i^s is also $(\partial^2\Pi_s/\partial(L_i^s)^2) = 0$ and we only need to check the upper bound (L_i^0) and lower bound (L_i^L).

Overall, the optimal value L_i^s can be found from L_i^l, L_i^u, and L_i^0 by comparing the Π_s values at them, and the one at which Π_s is the largest is the optimal.

Proof of Proposition 5. According to unlimited capacity in each period, the profit of the centralized supply chain is

$$\Pi_c = \Pi_s + \Pi_r = \left(\sum_{i=1}^{N}\left(p_i^s - Cp_i^1\right)\times\left(a_i\cdot\theta_i - b_i^s\cdot P_i^s + \alpha_i^r\cdot p_i^r - c_i^s\cdot L_i^s\right)\right) + \sum_{i=1}^{N}\left(p_i^r - Cp_i^2 - Cr_i\right)\times\left(a_i\cdot(1-\theta_i) - b_i^r\cdot p_i^r + \alpha_i^s\cdot P_i^s + c_i^r L_i^s\right). \tag{A.18}$$

Taking the second-order partial derivatives of Π_c with respect to P_i^r, P_i^s, and L_i^s we have the Hessian matrix as follows:

$$H = \begin{bmatrix} \dfrac{\partial^2\Pi_c}{\partial(p_i^s)^2} & \dfrac{\partial^2\Pi_c}{\partial(p_i^s)\partial(p_i^r)} & \dfrac{\partial^2\Pi_c}{\partial(p_i^s)\partial(L_i^s)} \\ \dfrac{\partial^2\Pi_c}{\partial(p_i^r)\partial(p_i^s)} & \dfrac{\partial^2\Pi_c}{\partial(p_i^r)^2} & \dfrac{\partial^2\Pi_c}{\partial(p_i^r)\partial(L_i^s)} \\ \dfrac{\partial^2\Pi_c}{\partial(L_i^s)\partial(p_i^s)} & \dfrac{\partial^2\Pi_c}{\partial(L_i^s)\partial(p_i^r)} & \dfrac{\partial^2\Pi_c}{\partial(L_i^s)^2} \end{bmatrix} = \begin{bmatrix} -2b_i^s & \alpha_i^r + \alpha_i^s & -c_i^s \\ \alpha_i^r + \alpha_i^s & -2b_i^r & c_i^r \\ -c_i^s & c_i^r & 0 \end{bmatrix}. \tag{A.19}$$

From assumption $4b_i^r\cdot b_i^s - (\alpha_i^s)^2 - (\alpha_i^r)^2 - 2\alpha_i^s\cdot\alpha_i^r > 0$, Π_c is strictly jointly concave in P_i^s and P_i^r, since $(\partial^2\Pi_c/\partial(p_i^s)^2) = -2b_i^s < 0$ and

$$\begin{vmatrix} -2b_i^s & \alpha_i^r + \alpha_i^s \\ \alpha_i^r + \alpha_i^s & -2b_i^r \end{vmatrix} = 4b_i^r\cdot b_i^s - \left(\alpha_i^s\right)^2 - \left(\alpha_i^r\right)^2 - 2\alpha_i^s\cdot\alpha_i^r > 0. \tag{A.20}$$

The second-order partial derivative of Π_c with respect to L_i^s is $(\partial^2\Pi_c/\partial(L_i^s)^2) = 0$ which indicates that the Π_c is a linear function of L_i^s. According to the above properties, the optimal value of L_i^s will be equal to its upper bound or lower bound.

Proof of Proposition 6. From $D_i^s \geq 0$, $D_i^r \geq 0$ and constraints (1) and (2), we obtain

$$\begin{aligned} P_i^r &\leq \frac{a_i(1-\theta_i) + \alpha_i^s\cdot P_i^s + c_i^r\cdot L_i^s}{b_i^r}, \quad \forall i\in\Psi, \\ P_i^r &\geq \frac{-a_i\cdot\theta_i + b_i^s\cdot P_i^s + c_i^s\cdot L_i^s}{\alpha_i^r}, \quad \forall i\in\Psi. \end{aligned} \tag{A.21}$$

To have a feasible space for P_i^s and P_i^r, the upper bound obtained for P_i^r must be greater than or equal to the lower bound. Consequently, we have

$$\begin{aligned} &\frac{a_i(1-\theta_i) \leq + \alpha_i^s\cdot P_i^s + c_i^r\cdot L_i^s}{b_i^r} \geq \frac{-a_i\cdot\theta_i + b_i^s\cdot P_i^s + c_i^s\cdot L_i^s}{\alpha_i^r}, \quad \forall i\in\Psi, \\ &L_i^s \leq \frac{a_i\left(\theta_i\cdot b_i^r - \theta_i\cdot\alpha_i^r + \alpha_i^r\right)}{b_i^r\cdot c_i^s - \alpha_i^r\cdot c_i^r} - P_i^s\left(\frac{b_i^s\cdot b_i^r - \alpha_i^s\cdot\alpha_i^r}{b_i^r\cdot c_i^s - \alpha_i^r\cdot c_i^r}\right), \quad \forall i\in\Psi. \end{aligned} \tag{A.22}$$

Substituting $P_i^s = 0$ into the above inequality, we can derive (39) which is upper bound for L_i^s.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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