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Research Article

Existence and Large Time Behavior of Entropy Solutions to One-Dimensional Unipolar Hydrodynamic Model for Semiconductor Devices with Variable Coefficient Damping

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In this paper, we investigate the global existence and large time behavior of entropy solutions to one-dimensional unipolar hydrodynamic model for semiconductors in the form of Euler-Possion equations with time and spacedependent damping in a bounded interval. Firstly, we prove the existence of entropy solutions through vanishing viscosity method and compensated compactness framework. Based on the uniform estimates of density, we then prove the entropy solutions converge to the corresponding unique stationary solution exponentially with time. We generalize the existing results to the variable coefficient damping case.

1. Introduction

The present paper is concerned with the one-dimensional isentropic Euler-Possion model for semiconductor devices with damping:

$$\begin{cases} \rho_t + m_x = 0, \\ m_t + \left(\dfrac{m^2}{\rho} + P(\rho)\right)_x = \rho E + H(x,t)m, \\ E_x = \rho - b(x), \end{cases} \tag{1}$$

where space variable $x \in [L_1, L_2]$(L_1 and L_2 are two positive constants) and time variable $t \in [0, T)(T > 0)$. Here, $\rho \geq 0$, m, $H(x,t)$, $P(\rho)$, and E stand for electron density, electron current density, damping coefficient, pressure, and electric filed, respectively. We assume the damping coefficient $H(x, t)$ is bounded, and the pressure function is given by $P(\rho) = p_0\rho^\gamma$, where $p_0 = \theta^2/\gamma$ and $\theta = (\gamma-1)/2$. Here, γ presents the adiabatic coefficient, and $\gamma > 1$ corresponds to the isentropic case. The doping profile $b(x) \geq 0$ stands for the density of fixed, positively charged background ions. In this paper, we assume

$$b(x) \in C[L_1, L_2], 0 < b_* \leq b(x) \leq b^*, \tag{2}$$

where b_* and b^* are two positive constants. The initial-boundary value conditions of system (1) are

$$\begin{aligned} (\rho, m)(x, 0) &= (\rho_0(x), m_0(x)), L_1 < x < L_2, \\ m(L_1, t) &= m(L_2, t) = 0, t \geq 0, \\ E(L_1, t) &= E_-, t \geq 0, \end{aligned} \tag{3}$$

where $\rho_0(x)$ satisfies

$$\int_{L_1}^{L_2} (\rho_0(x) - b(x))dx = 0. \tag{4}$$

Firstly, let us survey the related mathematical results. In 1990, Degond and Markowich [1] firstly proved the existence and uniqueness of the steady-state to (1) in subsonic case, which is characterized by a smallness assumption on the current flowing through the device. It was proved that the existence of local smooth solution to the time-dependent problem by using Lagrangian mass coordinates in [2]. However, Chen-Wang in [3] had studied the smooth solution would blow up in finite time; therefore, it is worthwhile considering the existence and other properties of weak solutions. As for weak solutions, Zhang [4] and Marcati-Natalini [5] proved the global existence of entropy solutions to the initial-boundary value and Cauchy problems for $\gamma > 1$, respectively. Li [6] and Huang et al. [7] proved the existence of L^∞ entropy solution of (1) with $\gamma = 1$ on a bounded interval and the whole space by using a fractional Lax-Friedrichs scheme. It is worth noting that the L^∞ estimates of entropy solution, especially the estimate of density, in all of the above works [4–7] depend on time t, which restricted us to consider their large time behavior further. We refer [8–10] for more results on this model and topic. In this paper, for $1 < \gamma \le 3$ and variable coefficient damping, we shall first verify the assumption in [11], where the density is assumed to be uniformly bounded with respect to space x and time t and then use the entropy inequality to consider the large time behavior of the obtained solutions.

Based on the related results in [12–16], we are convinced that the method developed in this paper can be used to bipolar Euler-Poisson system with time depended damping. We will investigate this problem in next papers.

To start our main theorem, we define the entropy solution of system (1) as.

Definition 1. For every $T > 0$, a pair of bounded measurable functions $v(x,t) = (\rho(x,t), m(x,t), E(x,t))$ is called a L^∞ weak solution of (1) with initial-boundary condition (3) if

$$\begin{cases} \int_0^T \int_{L_1}^{L_2} (\rho\varphi_t + m\varphi_x)dxdt + \int_{L_1}^{L_2} \rho_0\varphi(x,0)dx = 0, \\ \int_0^T \int_{L_1}^{L_2} \left(m\varphi_t + \left(\frac{m^2}{\rho} + P(\rho)\right)\varphi_x\right)dxdt + \int_0^T \int_{L_1}^{L_2} (\rho E + H(x,t)m)\varphi dxdt \\ +\int_{L_1}^{L_2} m_0\varphi(x,0)dx = 0, \\ E(x,t) = \int_{L_1}^{x} (\rho - b(s))ds + E_-, \end{cases} \tag{5}$$

holds for any test function $\varphi \in C_0^\infty([L_1, L_2] \times [0,T))$, and the boundary condition is satisfied in the sense of divergence-measure field [17]. Furthermore, we call the weak solution $(\rho, m, E)(x,t)$ to be an entropy solution if the entropy inequality

$$\eta_t + q_x \le \eta_m(\rho E + H(x,t)), \tag{6}$$

satisfies in the sense of distribution for any weak convex entropy pairs $(\eta(\rho,m), q(\rho,m))$.

Definition 2. The stationary solution of problems (1) and (3) is the smooth solution of

$$\begin{cases} P(\tilde{\rho})_x = \tilde{\rho}\tilde{E}, \\ \tilde{E}_x = \tilde{\rho} - b(x), \end{cases} \tag{7}$$

with the boundary condition

$$\tilde{E}(L_1) = \tilde{E}(L_2) = 0. \tag{8}$$

Our main results in this paper are as follows.

Theorem 3 (Existence). *Let $1 < \gamma \le 3$, we assume that the initial data and the damping coefficient satisfy*

$$0 \le \rho_0(x) \le M_0,\ |m_0(x)| \le M_0\rho_0(x),\ |H(x,t)| \le M_1, \tag{9}$$

for some positive constants M_0 and M_1. Then, there exists a global entropy solution $(\rho, m, E)(x,t)$ of the initial-boundary value problems (1) and (3) satisfying

$$\begin{aligned} 0 \le \rho(x,t) \le C,\ |m(x,t)| \le C\rho(x,t),\ |E(x,t)| \\ \le C,\ (x,t) \in [L_1, L_2] \times [0,T), \end{aligned} \tag{10}$$

where C is independent of t.

Remark 4. To get the global existence of the L^∞ weak solution, we only need $H(x,t)$ is bounded. However, to get the large time behavior of the obtained solution, the uniform negative upper bound is necessary.

Theorem 5 (Large time behavior). *Suppose there exists a positive constant $\delta_0 > 0$, such that the damping coefficient*

$$H(x,t) < -\delta_0 \text{ and } H_t(x,t) > -2b_*, \tag{11}$$

for any $[L_1, L_2] \times \mathbb{R}^+$. Denote $(\rho, m, E)(x,t)$ is the global entropy solution of (1) and (3) obtained in Theorem 3, and $(\tilde{\rho}, \tilde{E})$ is the stationary solution; then, it holds that

$$\int_{L_1}^{L_2} \left((\rho - \tilde{\rho})^2 (E - \tilde{E})^2 + m^2\right)(x,t)dx \le Ce^{-Ct}, \tag{12}$$

for some positive constant C.

Remark 6. Theorems 3 and 5 are generalizations of the corresponding theorem of [18], in which the damping coefficient $H(x,t) = -1$. Suppose α, β, and λ are three positive constants, then $H(x,t) = -\alpha/(1+t)^\lambda - \beta$ satisfies all the assumptions of $H(x,t)$ in Theorems 3 and 5.

2. Preliminary and Formulation

We consider the homogeneous system

$$\begin{cases} \rho_t + m_x = 0, \\ m_t + \left(\dfrac{m^2}{\rho} + P(\rho)\right)_x = 0. \end{cases} \tag{13}$$

Firstly, we use r_1 and r_2 to denote the right eigenvectors corresponding to the eigenvalues λ_1 and λ_2. After simple calculation, we have

$$\begin{gathered} \lambda_1 = \frac{m}{\rho} - \theta\rho^\theta, \quad \lambda_2 = \frac{m}{\rho} + \theta\rho^\theta, \quad \theta = \frac{\gamma - 1}{2}, \\ r_1 = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix}, \quad r_2 = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix}. \end{gathered} \tag{14}$$

The Riemann invariants (w, z) are given by

$$w = \frac{m}{\rho} + \rho^\theta, \quad z = \frac{m}{\rho} - \rho^\theta, \tag{15}$$

satisfying $\nabla w \cdot r_1 = 0$ and $\nabla z \cdot r_2 = 0$, where $\nabla = (\partial_\rho, \partial_m)$ is the gradient with respect to $U = (\rho, m)$.

A pair of functions (η, q): $\mathbb{R} \times \mathbb{R}^+ \mapsto \mathbb{R}^2$ is called an entropy-entropy flux of system (13) if it satisfies

$$\nabla q(U) = \nabla \eta(U) \nabla \begin{bmatrix} m \\ \dfrac{m^2}{\rho} + P(\rho) \end{bmatrix}. \tag{16}$$

Furthermore, if for any fixed $m/\rho \in (-\infty, +\infty)$, η vanishes on the vacuum $\rho = 0$; then, η is called a weak entropy. For example, the mechanical energy-energy flux pair

$$\eta_e = \frac{m^2}{2\rho} + \frac{p_0\rho^\gamma}{\gamma - 1}, \quad q_e = \frac{m^3}{2\rho^2} + \frac{p_0\rho^{\gamma-1}}{\gamma - 1} m, \tag{17}$$

should be a strictly convex entropy pair. We approximate the equations in (1) by adding artificial viscosity to get the smooth approximate solutions $(\rho^\varepsilon, m^\varepsilon)$, that is,

$$\begin{cases} \rho_t + m_x = \varepsilon\rho_{xx}, \\ m_t + \left(\dfrac{m^2}{\rho} + P(\rho)\right)_x = \varepsilon m_{xx} + \rho E - 2M\varepsilon\rho_x + H(x,t)m, \\ E(x,t) = \displaystyle\int_{L_1}^{x} (\rho - b(s))\,ds + E_-, \end{cases} \tag{18}$$

with initial-boundary value conditions

$$\begin{aligned} (\rho, m)(x, 0) &= (\rho_0^\varepsilon(x),\ m_0^\varepsilon(x)) = (\rho_0(x) + \varepsilon,\ m_0(x)) * m^\varepsilon, \quad L_1 < x < L_2, \\ m(L_1, t) &= m(L_2, t) = 0,\ \rho(L_1, t) = \rho_0^\varepsilon(L_1),\ \rho(L_2, t) = \rho_0^\varepsilon(L_2), \quad t \ge 0, \end{aligned} \tag{19}$$

where M in (18) is a big enough constant to be determined later and m^ε in (19) is the standard mollifier with small parameter ε.We shall prove that the viscosity solutions of (18) and (19) are uniformly bounded with respect to time t.

3. Viscosity Solutions and A Priori Estimates

For any fixed $\varepsilon > 0$, we denote the solution of (18) and (19) by $(\rho^\varepsilon, m^\varepsilon, E^\varepsilon)$, since $E^\varepsilon(x,t)$ is uniquely determined by $\rho^\varepsilon(x,t)$, $b(x)$, and E_-; then, the system (18) may be seen as one system with the unknowns ρ^ε and m^ε. Regarding the proof of local existence of approximate solution, the techniques used in this article are similar to those used in [19]. To extend the local solution to global one, the key point is to obtain the uniform upper bound of $\rho^\varepsilon, |m^\varepsilon|$ and the lower bound of density ρ^ε. The following theorem gives the uniform bound of $(\rho^\varepsilon, m^\varepsilon)$.

Lemma 7. *For any $T > 0$, let $(\rho^\varepsilon, m^\varepsilon)(x,t) \in C^1([0,T], C^2[L_1, L_2])$ to be the smooth solution of (18) and (19). Then*

$$0 \le \rho^\varepsilon(x,t) \le C, \quad |m^\varepsilon(x,t)| \le C\rho^\varepsilon(x,t), \tag{20}$$

where C is a positive constant independent of time t.

Proof. (For simplicity of notation, the superscript of ρ^ε and m^ε will be omitted as (ρ, m).) By the formulas of Riemann invariants (15), we can decouple the viscous perturbation equation (18) as

$$\begin{cases} w_t + \lambda_2 w_x = \varepsilon w_{xx} + 2\varepsilon(w_x - M)\dfrac{\rho_x}{\rho} - \varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E + H(x,t)\dfrac{m}{\rho}, \\ z_t + \lambda_1 z_x = \varepsilon z_{xx} + 2\varepsilon(z_x - M)\dfrac{\rho_x}{\rho} + \varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E + H(x,t)\dfrac{m}{\rho}. \end{cases} \tag{21}$$

We set the control functions (φ, ψ) as

$$\begin{gathered} \varphi = M(M + x), \\ \psi = M(M - x). \end{gathered} \tag{22}$$

A direct calculation tells us

$$\begin{gathered} \varphi_t = 0,\ \varphi_x = M,\ \varphi_{xx} = 0, \\ \psi_t = 0,\ \psi_x = -M,\ \psi_{xx} = 0. \end{gathered} \tag{23}$$

Define the modified Riemann invariants $(\bar{w}, \bar{z})$ as:

$$\bar{w} = w - \varphi,\ \bar{z} = z + \psi. \tag{24}$$

Then, inserting the above formulas into (21) yields the decoupled equations for $\bar{w}$ and $\bar{z}$:

$$\begin{cases} \bar{w}_t + \lambda_2 \bar{w}_x = \varepsilon \bar{w}_{xx} + 2\varepsilon \bar{w}_x \dfrac{\rho_x}{\rho} - \varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E - \lambda_2 \varphi_x + H(x,t)\dfrac{m}{\rho}, \\ \bar{z}_t + \lambda_1 \bar{z}_x = \varepsilon \bar{z}_{xx} + 2\varepsilon \bar{z}_x \dfrac{\rho_x}{\rho} + \varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E + \lambda_1 \psi_x + H(x,t)\dfrac{m}{\rho}. \end{cases} \tag{25}$$

We rewrite (25) into

$$\begin{cases} \bar{w}_t + \left(\lambda_2 - 2\varepsilon \dfrac{\rho_x}{\rho}\right)\bar{w}_x = \varepsilon \bar{w}_{xx} + a_{11}\bar{w} + a_{12}\bar{z} + R_1, \\ \bar{z}_t + \left(\lambda_1 - 2\varepsilon \dfrac{\rho_x}{\rho}\right)\bar{z}_x = \varepsilon \bar{z}_{xx} + a_{21}\bar{w} + a_{22}\bar{z} + R_2, \end{cases} \tag{26}$$

with

$$\begin{aligned} a_{11} &= -\left(\frac{1+\theta}{2}\right)M + \frac{1}{2}H(x,t),\ a_{12} = \left(\frac{\theta-1}{2}\right)M + \frac{1}{2}H(x,t), \\ a_{21} &= \left(\frac{\theta-1}{2}\right)M + \frac{1}{2}H(x,t),\ a_{22} = -\left(\frac{1+\theta}{2}\right)M + \frac{1}{2}H(x,t), \\ R_1 &= -\varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E - \theta M^3 - M^2 x + MH(x,t)x, \\ R_2 &= \varepsilon\theta(\theta+1)\rho^{\theta-2}\rho_x^2 + E + \theta M^3 - M^2 x + MH(x,t)x. \end{aligned} \tag{27}$$

In above calculation, we have used the relations:

$$\lambda_1 = \frac{w+z}{2} - \theta\frac{w-z}{2}, \quad \lambda_2 = \frac{w+z}{2} + \theta\frac{w-z}{2}, \quad \frac{m}{\rho} = \frac{w+z}{2}. \tag{28}$$

Noting $0 < \theta \le 1$, $|H(x,t)| \le M_1$, and choosing $M \ge (2/(1-\theta))M_1$, we have

$$a_{12} \le 0,\ a_{21} \le 0. \tag{29}$$

On the other hand, (27) tells us

$$\begin{aligned} R_1 &\le E - \theta M^3 - M^2 x + MH(x,t)x, \\ R_2 &\ge E + \theta M^3 - M^2 x + MH(x,t)x. \end{aligned} \tag{30}$$

And use the same calculations in [18], we estimate the approximate electric fields and obtain

$$|E(x,t)| \le M_2, \tag{31}$$

where M_2 depends only on initial data. Thus, taking M big enough, we have

$$\begin{aligned} R_1 &\le M_2 - \theta M^3 + MM_1 L_2 \le 0, \\ R_2 &\ge -M_2 + \theta M^3 - M^2 L_2 - MM_1 L_1 \ge 0, \end{aligned} \tag{32}$$

and the initial-boundary value conditions satisfy

$$\begin{aligned} \bar{w}(x,0) &= w(x,0) - \varphi(x,0) = \frac{m_0}{\rho_0} + \rho_0^\theta - M^2 - Mx \le 0, \\ \bar{z}(x,0) &= z(x,0) + \psi(x,0) = \frac{m_0}{\rho_0} - \rho_0^\theta + M^2 - Mx \ge 0, \\ &\bar{w}(L_1,t) \le 0,\ \bar{z}(L_1,t) \ge 0,\ \bar{w}(L_2,t) \le 0,\ \bar{z}(L_2,t) \ge 0. \end{aligned} \tag{33}$$

Basing on the above discussion, using Lemma 7 of [18], we have

$$\bar{w}(x,t) \le 0, \bar{z}(x,t) \ge 0, \quad \forall (x,t) \in [L_1, L_2] \times [0,T). \tag{34}$$

Therefore,

$$\begin{aligned} w(x,t) &\le \varphi(x,t) \le M^2 + Mx \le M^2 + ML_2, \\ z(x,t) &\ge -\psi(x,t) \ge -M^2 + Mx \ge -M^2. \end{aligned} \tag{35}$$

By (35), we have

$$\rho \le \left(\frac{w-z}{2}\right)^{1/\theta} \le \left(\frac{3}{2}M^2\right)^{1/\theta}, \tag{36}$$

and Lemma 7 is completed.

From (20), the velocity $u = m/\rho$ is uniformly bounded, i.e., $|u| < C$. Then, following the same way of [20], we could obtain

$$\rho(x,t) \ge \delta(t,\varepsilon) > 0. \tag{37}$$

Based on the local existence of smooth solution, the uniform upper estimates (Lemma 7) and the lower bound estimate of density (37), we derive the following lemma.

Lemma 8. *For any time $T > 0$, there exists a unique global classical solution $(\rho^\varepsilon, m^\varepsilon)(x,t) \in C^1([0,T], C^2[L_1, L_2])$ to the initial-boundary value problems (18) and (19) satisfying*

$$0 \le \delta(t,\varepsilon) \le \rho^\varepsilon(x,t) \le C, |m^\varepsilon(x,t)| \le C\rho^\varepsilon(x,t), \tag{38}$$

where C is independent of ε and T.

Through Lemma 8 and the compensated compactness framework theory established in [19, 21–23], we can prove that there has a subsequence of $(\rho^\varepsilon, m^\varepsilon)$ (still denoted by

$(\rho^{\varepsilon}, m^{\varepsilon})$), so that

$$\left(\rho^{\mathscr{E}}, m^{\mathscr{E}}\right) \longrightarrow (\rho, m), \text{ in } L^{p}_{loc}([L_1, L_2] \times [0, T)) \tag{39}$$

Furthermore, it is clear for us that (ρ, m) is an entropy solution of initial-boundary value problems (1) and (3). We complete the proof of Theorem 3.

4. Large Time Behavior of Weak Solutions

This section is devoted to the proof of Theorem 5. Firstly, for stationary solution, from the result in [24], we have the following argument:

Lemma 9. *Under the assumption (2) of $b(x)$, there exists a unique solution $(\tilde{\rho}, \tilde{E})$ to problems (7) and (8) satisfying*

$$0 < b_* \leq \tilde{\rho}(x) \leq b^*, \left|\tilde{\rho}'(x)\right| \leq C, \left|\tilde{\rho}''(x)\right| \leq C, x \in [L_1, L_2], \tag{40}$$

where C only depends on γ, b^* and b_*.

Now, we shall derive that the entropy solution (ρ, m, E) acquired in Theorem 3 converges strongly to the corresponding stationary solution $(\tilde{\rho}, \tilde{E})$ in the norm of L^2 with exponential decay rate. From (7) and (8), we see that

$$\int_{L_1}^{L_2} (\tilde{\rho}(x) - b(x))dx = \int_{L_1}^{L_2} \tilde{E}_x dx = \tilde{E}(L_2) - \tilde{E}(L_1) = 0. \tag{41}$$

Give the definition of the new function as follows

$$\begin{aligned} y(x,t) &= -\int_{L_1}^{x} (\rho(s,t) - \tilde{\rho}(s))ds \\ &= -\left(E - \tilde{E}\right), (x,t) \in [L_1, L_2] \times [0, \infty). \end{aligned} \tag{42}$$

Obviously, we observe that

$$y_x = -(\rho - \tilde{\rho}), y_t = m, y(L_1) = y(L_2). \tag{43}$$

From (1) and (7), we have

$$y_{tt} + \left(\frac{m^2}{\rho}\right)_x + (p(\rho) - p(\tilde{\rho}))_x - H(x,t)y_t = -\tilde{\rho}y - \tilde{E}y_x + yy_x. \tag{44}$$

Multiplying y with (44) and integrating from L_1 to L_2, we have

$$\begin{aligned} &\frac{d}{dt}\int_{L_1}^{L_2} \left(yy_t - \frac{1}{2}y^2 H\right)dx + \int_{L_1}^{L_2} \frac{1}{2}y^2 H_t dx \\ &\quad + \int_{L_1}^{L_2} (P(\rho) - P(\tilde{\rho}))(\rho - \tilde{\rho}) + \left(\tilde{\rho} - \frac{\tilde{E}_x}{2}\right)y^2 dx \\ &\leq \int_{L_1}^{L_2} y_t^2 dx + \int_{L_1}^{L_2} \frac{y_t^2}{\rho} y_x dx = \int_{L_1}^{L_2} \frac{\tilde{\rho}}{\rho} y_t^2 dx. \end{aligned} \tag{45}$$

Lemma 7 of [25] tells us there exist two nonnegative constants $\tilde{C}_1$ and $\tilde{C}_2$ such that

$$\tilde{C}_2(\rho - \tilde{\rho})^2 \geq (P(\rho) - P(\tilde{\rho}))(\rho - \tilde{\rho}) \geq \tilde{C}_1(\rho - \tilde{\rho})^2 = \tilde{C}_1 y_x^2. \tag{46}$$

Putting (46) into (45), we have

$$\begin{aligned} &\frac{d}{dt}\int_{L_1}^{L_2} \left(yy_t - \frac{1}{2}y^2 H\right)dx + \int_{L_1}^{L_2} \frac{1}{2}y^2 H_t dx \\ &\quad + \tilde{C}_1 \int_{L_1}^{L_2} y_x^2 dx + \int_{L_1}^{L_2} b_* y^2 dx \leq \int_{L_1}^{L_2} \frac{\tilde{\rho}}{\rho} y_t^2 dx. \end{aligned} \tag{47}$$

Additionally, denote the relative entropy-entropy flux by

$$\begin{aligned} \eta_* &= \eta_e - \frac{p_0 \tilde{\rho}^{\gamma}}{\gamma - 1} - \frac{p_0 \gamma}{\gamma - 1}\tilde{\rho}^{\gamma - 1}(\rho - \tilde{\rho}), \\ q_* &= q_e - \frac{p_0 \gamma}{\gamma - 1}\tilde{\rho}^{\gamma - 1} m. \end{aligned} \tag{48}$$

From the entropy inequality (16), we have the following inequality holds in the sense of distribution:

$$\begin{aligned} \eta_t^* + q_x^* &= \eta_{et} + q_{ex} - \frac{p_0\gamma}{\gamma - 1}\tilde{\rho}^{\gamma - 1}(\rho - \tilde{\rho})_t - \frac{p_0\gamma}{\gamma - 1}\left(\tilde{\rho}^{\gamma - 1}m\right)_x \\ &\leq mE + \frac{Hm^2}{\rho} - \frac{p_0\gamma}{\gamma - 1}\tilde{\rho}^{\gamma - 1}(\rho - \tilde{\rho})_t - \frac{p_0\gamma}{\gamma - 1}\left(\tilde{\rho}^{\gamma - 1}m\right)_x \\ &= mE + \frac{Hm^2}{\rho} - p_0\gamma\tilde{\rho}^{\gamma - 2} y_t \tilde{\rho}_x. \end{aligned} \tag{49}$$

We notice that

$$\begin{aligned} mE &= m\tilde{E} + m\left(E - \tilde{E}\right) = y_t \tilde{E} - yy_t \\ &= \frac{P(\tilde{\rho})_x}{\tilde{\rho}} y_t - yy_t = p_0\gamma\tilde{\rho}^{\gamma - 2} y_t \tilde{\rho}_x - yy_t, \end{aligned} \tag{50}$$

and use the theory of divergence-measure fields [17] to arrive at

$$\frac{d}{dt}\int_{L_1}^{L_2} \left(\eta_* + \frac{1}{2}y^2\right)dx - \int_{L_1}^{L_2} \frac{Hy_t^2}{\rho} dx \leq 0. \tag{51}$$

Let Λ sufficiently big so that $\Lambda > b^*/\delta_0 + \|\rho\|_{L^\infty} + 1$.

Multiply (51) by Λ and add the result to (47), we have

$$\frac{d}{dt}\int_{L_1}^{L_2}\left(\Lambda\eta_* + \frac{\Lambda y^2}{2} + yy_t - \frac{y^2}{2}H\right)dx + \int_{L_1}^{L_2}\left[\left(\frac{1}{2}H_t + b_*\right)y^2 + \tilde{C}_1 y_x^2 + \frac{-\Lambda H - \tilde{\rho}}{\rho}y_t^2\right]dx \le 0. \tag{52}$$

Since

$$-\Lambda H - \tilde{\rho} > \left(\frac{b^*}{\delta_0} + \|\rho\|_{L^\infty}\right)\delta_0 - b^* > \|\rho\|_{L^\infty}\delta_0, \tag{53}$$

then there exists $\tilde{C}_3 > 0$ such that

$$\frac{d}{dt}\int_{L_1}^{L_2}\left(\Lambda\eta_* + \frac{\Lambda y^2}{2} + yy_t - \frac{y^2}{2}H\right)dx + \tilde{C}_3\int_{L_1}^{L_2}\left(y^2 + y_x^2 + \frac{y_t^2}{\rho}\right)dx \le 0. \tag{54}$$

Since $\|\rho(x,t)\|_{L\infty} \le C$ and

$$\eta_* \sim y_x^2 + \frac{y_t^2}{\rho}, \tag{55}$$

we can directly conclude that

$$\Lambda\eta_* + \frac{\Lambda y^2}{2} + yy_t - \frac{y^2}{2}H \sim y^2 + y_x^2 + \frac{y_t^2}{\rho}. \tag{56}$$

Now from (54), the Gronwall inequality implies Theorem 5.

Data Availability

This paper uses the method of theoretical analysis.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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