Hindawi
Advances in Mathematical Physics
Volume 2020, Article ID 8815363, 8 pages
https://doi.org/10.1155/2020/8815363

Research Article

Exact Analytical Solutions of Nonlinear Fractional Liouville Equation by Extended Complex Method

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Received 1 September 2020; Revised 25 October 2020; Accepted 4 November 2020; Published 19 November 2020

Academic Editor: Soheil Salahshour

The extended complex method is investigated for exact analytical solutions of nonlinear fractional Liouville equation. Based on the work of Yuan et al., the new rational, periodic, and elliptic function solutions have been obtained. By adjusting the arbitrary values to the constants in the constructed solutions, it can describe the physical phenomena to the traveling wave solutions, since traveling wave has significant value in applied sciences and engineering. Our results indicate that the extended complex technique is direct and easily applicable to solve the nonlinear fractional partial differential equations (NLFPDEs).

1. Introduction

The idea of FPDEs has been an area of focus not only among mathematicians but also among physicists and engineers. FPDEs have played an important task in numerous fields like physics, biology, biogenetics, and fluid mechanics [1–3].

Because of their several applications, numerous techniques have been evolved to attain analytical and numerical solutions for FPDEs, for example, modified extended tanh method [4], differential transform method [5], Ansatz method [6], Sine-Gordon expansion method [7], unified method [8], Q-homotopy analysis Sumudu transform technique [9], implicit Riesz wavelet-based method [10], first integral method [11], Backlund transformation [12], Chebyshev wavelet operational matrix [13], F-expansion method [14], homogeneous balance method [15], a generalization of truncated M-fractional derivative [16], fractional natural decomposition method [17], iterative method [18], and Adam's-type predictor-corrector method [19].

The nonlinear evolution equations have been solved symbolically and numerically by using various methods, for example, Durur et al. have applied $(m+1/G')$-expansion method to attain analytical solutions of the hyperbolic nonlinear Schrödinger′s equation (NLSE) [20]. Taher has applied simple equation method to the Kadomtsev-Petviashvili (KP) equation [21]. Miao and Zhang have introduced modified (G'/G)-expansion method to attain the exact analytical solutions of perturbed nonlinear Schrödinger's equation [22]. Khatera et al. have used tanh method to investigate analytical solutions of nonlinear reaction-diffusion equation [23]. Zhang et al. have used first integral method to attain the exact analytical solutions of nonlinear Boussinesq wave packet mode and $(2+1)$-dimensional Zoomeron equation [24]. Wazwaz has introduced Sine-cosine method to explore analytical solutions of Benjamin-Bona-Mahony equation and Phi-four equation [25].

El-Danaf et al. have applied the new numerical treatment to generalized long-wave equation system [26]. Khalid et al. have introduced the nonpolynomial spline method to attain numerical solutions of Coupled Burgers equations [27]. Lakshmi and Ashish have attained numerical solutions of Burgers equation by using cubic B-spline method [28], and Raslan et al. have applied the B-spline collocation method to find numerical solutions of coupled-BBM system [29].

Recently, Hadi et al. have introduced analytical methods to solve the nonlinear evolution equations [30–32]. Nestor

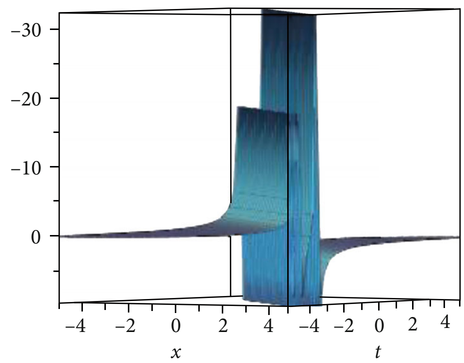

FIGURE 1: 3D graph of $V_{r,1}(z)$ for the fixed values $\nu = 1$, $z_0 = 0.5$, and $z_1 = 1$ represent the exact traveling wave solutions.

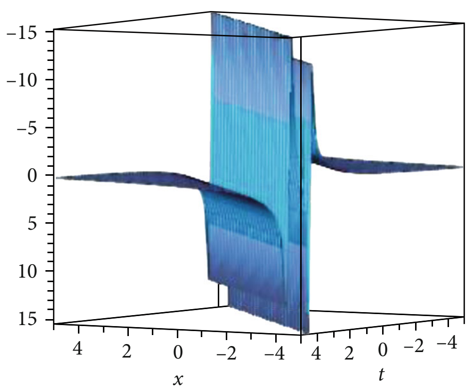

FIGURE 2: 3D graph of $V_{r,1}(z)$ for the fixed values $\nu = 1$, $z_0 = -0.5$, and $z_1 = 1$ represent the exact traveling wave solution.

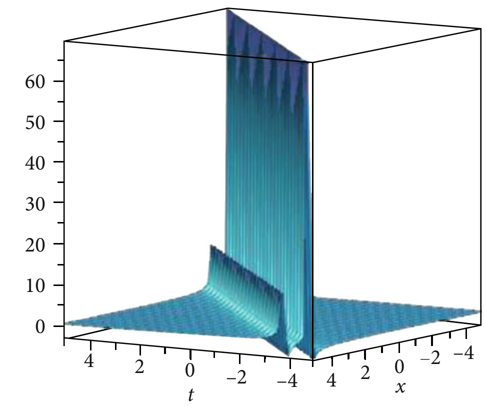

FIGURE 3: 3D graph of $V_{r,2}(z)$ for the fixed values $\nu = 1$, $z_0 = 0.2$, and $z_1 = 1$ represent the exact traveling wave solutions.

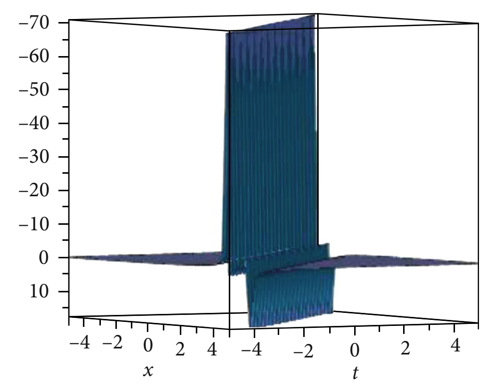

FIGURE 4: 3D graph of $V_{r,2}(z)$ for the fixed values $\nu = 1$, $z_0 = -0.2$, and $z_1 = 1$ represent the exact traveling wave solutions.

et al. have applied modified Kudryashov method to solve perturbed nonlinear Schrödinger's equation [33]. Nauman et al. have attained analytical solutions for various nonlinear evolution equations [34, 35]. The general form of nonlinear fractional Liouville equation [36] is given by

$$\frac{\partial^{2\alpha}\nu}{\partial X^{2\alpha}} + \frac{\partial^{2\beta}\nu}{\partial T^{2\beta}} = e^{2\nu}, \quad 0 < \alpha, \beta \le 1. \tag{1}$$

This equation plays an important task in different scientific applications such as solid-state physics, nonlinear optic, and chemical kinetics. Where ν is a function of X and T. The conformable derivative of order α and β in nonlinear fractional Liouville equation is defined as a conformable derivative sense, introduced by Khalil et al. [37]. The main advantages of conformable derivatives is to satisfy the rules of ODE like quotient, product, and chain rules as well as other definitions fail to satisfy these rules, and it can be solved exactly and numerically of other FPDEs. The conformable order definition can be expressed as follows.

Definition 1. Let e: $[0,\infty) \to R$ be a function, then conformable order can be expressed as

$$D_\beta e(x) = \lim_{x\to 0} \frac{e\left(x + \delta x^{1-\beta}\right) - e(x)}{\delta}, \tag{2}$$

where $h^\beta(0) = \lim_{x\to 0} h^{(\beta)}(x)$. From the above conformable integral function k defined as

$$I^l_\beta e(x) = \int_l^x \frac{e(t)}{t^{1-\beta}} dt, \quad \ge 0, \ \beta \in (0, 1]. \tag{3}$$

The significant properties of the conformable derivatives are introduced as below:

(1) $D_\beta(au + bv) = aD_\beta(u) + bD_\beta(v), \forall a, b \in \mathbb{R}$

(2) $D_\beta(x^p) = px^{p-\beta}, \forall p \in \mathbb{R}$

(3) $D_\beta(A) = 0, \forall u(x) = A(\text{constant functions})$

$$D(uv) = uD_\beta(v) + vD_\beta(u) \tag{4}$$

(4) $D_\beta(u/v) = vD_\beta(u) - uD_\beta(v)/v^2$, moreover, if the function u is differentiable, then

(5) $D_\beta(u)(x) = x^{1-\beta} du/dx$

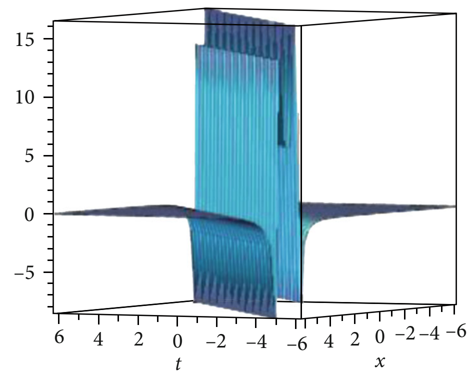

Figure 5: 3D graph of $V_{s,1}(z)$ for the fixed values $\nu = 1$, $z_0 = 1/6$, $z_1 = 1$, and $\alpha = 1$ represent traveling wave solutions.

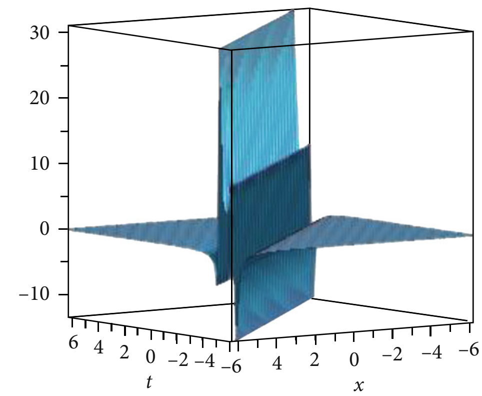

Figure 6: 3D graph of $V_{s,1}(z)$ for the fixed values $\nu = 1$, $z_0 = -1/6$, $z_1 = 1$, and $\alpha = 1$ represent traveling wave solutions.

In present work, our goal is to solve the nonlinear fractional Liouville equation by using the extended complex method based on the work of Yuan et al. [38–43]. This is a beneficial technique to attain exact analytical solutions. This method's application can be used in the discipline of mathematical physics and engineering. The whole article is organized as mention below: In Section 2, methods and materials are explained. In Section 3, analytical solutions for the given problem are computed. Section 4 graphical structures of our results are given, and we have mentioned the conclusions in Section 5.

2. Methods and Materials

In this section, we introduce the extended complex method for solving FPDE. Let us consider that the general FPDE form is expressed as

$$f\left(v, D_T^{\alpha}v, D_{X_1}^{\alpha}v, D_{X_2}^{\beta}v, D_{X_3}^{\gamma}v, D_{X_4}^{\delta}v\cdots\cdots\right) = 0, 0 < \alpha, \gamma, \delta, \beta \le 1. \tag{5}$$

Where v is an unknown function of $X_1, X_2, X_3 \cdots\cdots X_n$ and T, f is a polynomial of v and its fractional partial derivatives. The FPDE has been solved by the following steps.into Eq. (8), respectively, then the systems of algebraic equations are calculated by equating the coefficient to zero. These algebraic equations are solved with the help of maple packages. Finally, elliptic function solutions, rational function solutions, and simply periodic solutions with the pole at $z = 0$ are determined. As γ_{-ij} are obtained by (9), $E_i^2 = 4F_i^3 - g_2F_i - g_3$ and $\sum_{i=1}\gamma_{-i1} = 0$ and $V(z), V(e^{\alpha z})(\alpha \in \mathbb{C})$ have $r(\le p)$ distinct poles of multiplicity q.

Step 1. The transformation $z = (T^{\alpha}/\alpha) + (X_1^{\beta}/\beta) + \cdots\cdots z_0$ is applied to Eq. (5), it becomes NPDE and this equation can be expressed as

$$s\left(v, v_t, v_x, v_y, v_z, v_{tt}, v_{xx}\cdots\cdots\cdots\right). \tag{6}$$

Step 2. A transformation $T : v(x,t) \rightarrow V(z)$ is applied, (x,t) can be explained in different criteria. By this way, we have defined the following transformation

$$v(x,t) = V(z), z = x + \nu t. \tag{7}$$

Step 3. This transformation changes Eq. (6) into nonlinear ODE:

$$S\left(V, V', V'', V''', \cdots\right) = 0, \tag{8}$$

where V prime are the derivatives in Eq. (8) with respect to z. Equation (8) can be reduced by integrating with respect to z.

Step 4. The weak $\langle p,q\rangle$ condition is determined. Let $p, q \in \mathbb{Z}$, and let us consider the meromorphic solutions V of Eq. (8) have at least one pole. Inserting the Laurent series

$$V(z) = \sum_{k=-q}^{\infty} B_k z^k, q > 0, B_{-q} \neq 0, \tag{9}$$

into Eq. (8), if it has been found out p distinct Laurent singular parts

$$\sum_{k=-q}^{-1} B_k z^k, \tag{10}$$

then the weak $\langle p,q\rangle$ condition of Eq. (8) holds. Weierstrass elliptic function $\wp(z) := \wp(z, k_2, k_3)$ with double periods of the following equation as mentioned below:

$$\left(\wp'(z)\right)^2 = 4\wp(z)^3 - k_2\wp(z) - k_3. \tag{11}$$

Step 5. Substitute the indeterminate formulas

$$\begin{aligned} V(z) = &\sum_{i=1}^{r-1}\sum_{j=2}^{q}\frac{(-1)^j\gamma_{-ij}}{(j-1)!}\frac{d^{j-2}}{dz^{j-2}}\left(\frac{1}{4}\left[\frac{\wp'(z)+E_i}{\wp(z)-F_i}\right]^2 - \wp(z)\right) \\ &+ \sum_{i=1}^{r-1}\frac{\gamma_{-i1}}{2}\frac{\wp'(z)+E_i}{\wp(z)-F_i} + \sum_{j=2}^{q}\frac{(-1)^j\gamma_{-rj}}{(j-1)!}\frac{d^{j-2}}{dz^{j-2}}\wp(z) + \gamma_0, \end{aligned} \tag{12}$$

$$V(z) = \sum_{i=1} \sum_{j=1} \frac{\gamma_{ij}}{(z - z_i)^j} + \gamma_0, \tag{13}$$

$$V(e^{\mu z}) = \sum_{i=1}^{s} \sum_{j=1}^{q} \frac{\gamma_{ij}}{(e^{\mu z} - e^{\mu z_i})^j} + \gamma_0, \tag{14}$$

Step 6. Meromorphic solutions are obtained with an arbitrary pole, then the inverse transformation T^{-1} is inserted into meromorphic solutions, and exact analytical solutions of FPDE are explored.

3. Application of the Method

In this section, we will seek exact analytical solutions of nonlinear fractional Liouville equation by the extended complex technique. Suppose nonlinear fractional Liouville equation is given

$$\frac{\partial^{2\alpha} v}{\partial X^{2\alpha}} + \frac{\partial^{2\beta} v}{\partial T^{2\beta}} = e^{2v}, \quad 0 < \alpha, \quad \beta \le 1. \tag{15}$$

By applying the transformation (16) into Eq. (15)

$$x = \frac{X^{\alpha}}{\alpha}, \quad t = \frac{T^{\beta}}{\beta}, \tag{16}$$

then, we have

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial t^2} = e^{2v}, \tag{17}$$

if we consider $u = e^v$, it represents Eq. (17) into

$$(u_x)^2 + (u_t)^2 - uu_{xx} - vv_{tt} + u^4 = 0, \tag{18}$$

using wave transformation $v(x, t) = V(z)$, $z = x + vt$, Eq. (18) is reduced to

$$\left(1 + v^2\right) V V'' - \left(1 + v^2\right) \left(V'\right)^2 - V^4 = 0. \tag{19}$$

Substituting (9) into (19), then we have $p = 2$ and $q = 1$ and then the weak $\langle 2, 1 \rangle$ condition of (19) holds. By the weak $\langle 2, 1 \rangle$ and (13), we have defined the forms of the rational solutions

$$V_r(z) = \frac{\gamma_{12}}{z} + \frac{\gamma_{11}}{z - z_1} + \gamma_{10}, \tag{20}$$

with at $z = 0$. Putting the $V_r(z)$ into the Eq. (19), then we have

$$\sum_{i=1}^{9} c_{1i} z^{-i-3} (z - z_1)^{-4} = 0, \tag{21}$$

where

$$c_{11} = \gamma_{12}^4 z_1^4 - \gamma_{12}^2 v^2 z_1^4 - \gamma_{12}^2 z_1^4,$$

$$\begin{aligned} c_{12} = {} & 4\gamma_{12}^3 \gamma_{10} z_1^4 - 2\gamma_{12}\gamma_{10} v^2 z_1^4 - 4\gamma_{12}^4 z_1^3 - 4\gamma_{12}^3 \gamma_{11} z_1^3 + 4\gamma_{12}^2 v^2 z_1^3 \\ & + 2\gamma_{12}\gamma_{11} v^2 z_1^3 - 2\gamma_{12}\gamma_{10} z_1^4 + 4\gamma_{12}^2 z_1^3 + 2\gamma_{12}\gamma_{11} z_1^3, \end{aligned}$$

$$\begin{aligned} c_{13} = {} & 6\gamma_{12}^2 \gamma_{10}^2 z_1^4 - 16\gamma_{12}^3 \gamma_{11} z_1^3 - 12\gamma_{12}^2 \gamma_{11}\gamma_{10} z_1^3 + 8\gamma_{12}\gamma_{10} v^2 z_1^3 \\ & + 6\gamma_{12}^4 z_1^2 + 12\gamma_{12}^3 \gamma_{11} z_1^2 + 6\gamma_{10}^2 \gamma_{11}^2 z_1^2 - 6\gamma_{12}^2 v^2 z_1^2 \\ & - 4\gamma_{12}\gamma_{11} v^2 z_1^2 + 8\gamma_{12}\gamma_{10} z_1^3 - 6\gamma_{12}^2 z_1^2 - 4\gamma_{12}\gamma_{11} z_1^2, \end{aligned}$$

$$\begin{aligned} c_{14} = {} & 4\gamma_{12}\gamma_{11}^3 z_1^4 - 24\gamma_{12}^2 \gamma_{10}^2 z_1^3 - 12\gamma_{12}\gamma_{11}\gamma_{10}^2 z_1^3 + 24\gamma_{12}^3 \gamma_{10} z_1^2 \\ & + 36\gamma_{12}^2 \gamma_{11}\gamma_{10} z_1^2 + 12\gamma_{12}\gamma_{11}^2 \gamma_{10} z_1^2 - 12\gamma_{12}\gamma_{10} v^2 z_1^2 \\ & - 4\gamma_{12}^4 z_1 - 12\gamma_{12}^3 \gamma_{11} z_1 - 12\gamma_{12}^2 \gamma_{11}^2 z_1 + 4\gamma_{12}^2 v^2 z_1 \\ & - 4\gamma_{12}\gamma_{11}^3 z_1 + 4\gamma_{12}\gamma_{11} v^2 z_1 - 12\gamma_{12}\gamma_{10} z_1^2 + 4\gamma_{12}^2 z_1 \\ & + 4\gamma_{12}\gamma_{11} z_1, \end{aligned}$$

$$\begin{aligned} c_{15} = {} & \gamma_{10}^4 z_1^4 - 16\gamma_{12}\gamma_{10}^3 z_1^3 - 4\gamma_{11}\gamma_{10}^3 z_1^3 + 36\gamma_{12}^2 \gamma_{10} z_1^2 \\ & + 36\gamma_{12}\gamma_{11}\gamma_{10}^2 z_1^2 + 6\gamma_{11}^2 \gamma_{10}^2 z_1^2 - 16\gamma_{12}^3 \gamma_{10} z_1 \\ & - 36\gamma_{12}^2 \gamma_{11}\gamma_{10} z_1 - 24\gamma_{12}\gamma_{11}^2 \gamma_{10} z_1 + 8\gamma_{12}\gamma_{10} v^2 z_1 \\ & - 4\gamma_{11}^3 \gamma_{10} z_1 + 2\gamma_{11}\gamma_{10} v^2 z_1 + \gamma_{12}^4 + 4\gamma_{12}^3 \gamma_{11} + 6\gamma_{12}^2 \gamma_{11}^2 \\ & - \gamma_{12}^2 v^2 + 4\gamma_{12}\gamma_{11}^3 - 2\gamma_{12}\gamma_{11} v^2 + \gamma_{11}^4 - \gamma_{11}^2 v^2 + 8\gamma_{12}\gamma_{10} z_1 \\ & + 2\gamma_{11}\gamma_{10} z_1 - \gamma_{12}^2 - 2\gamma_{12}\gamma_{11} - \gamma_{11}^2, \end{aligned}$$

$$\begin{aligned} c_{16} = {} & -4\gamma_{10}^4 z_1^3 + 24\gamma_{12}\gamma_{10}^3 z_1^2 + 12\gamma_{11}\gamma_{10}^3 z_1^2 - 24\gamma_{12}^2 \gamma_{10}^2 z_1 \\ & - 36\gamma_{12}\gamma_{11}\gamma_{10}^2 z_1 - 12\gamma_{11}^2 \gamma_{10}^2 z_1 + 4\gamma_{12}^3 \gamma_{10} + 12\gamma_{12}^2 \gamma_{11}\gamma_{10} \\ & + 12\gamma_{12}\gamma_{11}^2 \gamma_{10} - 2\gamma_{12}\gamma_{10} v^2 + 4\gamma_{11}^3 \gamma_{10} - 2\gamma_{12}\gamma_{10} v^2 \\ & - 2\gamma_{12}\gamma_{10} - 2\gamma_{11}\gamma_{10}, \end{aligned}$$

$$\begin{aligned} c_{17} = {} & 6\gamma_{10}^4 z_1^2 - 16\gamma_{12}\gamma_{10}^3 z_1 - 12\gamma_{11}\gamma_{10}^3 z_1 + 6\gamma_{12}^2 \gamma_{10}^2 \\ & + 12\gamma_{12}\gamma_{11}\gamma_{10}^2 + 6\gamma_{11}^2 \gamma_{10}^2, \end{aligned}$$

$$c_{18} = -4\gamma_{10}^4 z_1 + 4\gamma_{12}\gamma_{10}^3 + 4\gamma_{11}\gamma_{10}^3,$$

$$c_{19} = \gamma_{10}^4. \tag{22}$$

Setting the coefficients of the identical powers about z in Eq. (21) to zero, then attain the number of following equations:

$$c_{1i} = 0, \, (i = 1, 2, \cdots, 9). \tag{23}$$

By solving the following equations, we attain

$$\gamma_{12} = \sqrt{v^2+1}, \gamma_{10} = 0, \gamma_{11} = \sqrt{v^2+1}, \\ \gamma_{12} = v+1-\sqrt{v^2+1}, \gamma_{11} = v+1-\sqrt{v^2+1}, \tag{24}$$

then

$$V_{r10}(z) = \frac{\sqrt{v^2+1}}{z} + \frac{\sqrt{v^2+1}}{z-z_1}, \\ V_{r20}(z) = \frac{v+1-\sqrt{v^2+1}}{z} + \frac{v+1-\sqrt{v^2+1}}{z-z_1}, \tag{25}$$

where $\gamma_{12}/a - 1 = v$ is the first case and $\gamma_{11}/a - 1 = v$ is the second case. Applying $V(z) = R(\eta)$ into Eq. (19), then

$$\left(1+v^2\right)\alpha^2 R\left(\eta R' + \eta^2 R''\right) - \left(1+v^2\right)\alpha^2\left(R'\eta\right)^2 - R^4 = 0, \tag{26}$$

inserting

$$V_s(\eta) = \frac{\gamma_{12}}{\eta-1} + \frac{\gamma_{11}}{\eta-\eta_1} + \gamma_{10}, \tag{27}$$

into the Eq. (26), we attain that

$$\sum_{i=1}^{9} \frac{c_{2i}\alpha^2\eta^i}{(\eta-1)^4(\eta-\eta_1)^4} = 0, \tag{28}$$

where

$$\begin{aligned} c_{21} = {} & \gamma_{12}^4\eta_1^4 - 4\gamma_{12}^3\gamma_{10}\eta_1^4 + 6\gamma_{12}^2\gamma_{10}^2\eta_1^4 - 4\gamma_{12}\gamma_{10}^3\eta_1^4 + \gamma_{10}^4\eta_1^4 \\ & + 4\gamma_{12}^3\gamma_{11}\eta_1^3 - 12\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^3 + 12\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^3 \\ & - 4\gamma_{11}\gamma_{10}^3\eta_1^3 + 6\gamma_{10}^2\gamma_{11}^2\eta_1^2 - 12\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1^2 + 6\gamma_{11}^2\gamma_{10}^2 z_1^2 \\ & + 4\gamma_{12}\gamma_{11}^3\eta_1 - 4\gamma_{11}^3\gamma_{10}\eta_1 + \gamma_{11}^4, \end{aligned}$$

$$\begin{aligned} c_{22} = {} & -\gamma_{12}^2\alpha^2\eta_1^4 v^2 + \gamma_{12}\alpha^2\gamma_{10}\eta_1^4 v^2 - \gamma_{12}\alpha^2\gamma_{11}\eta_1^3 v^2 + 4\gamma_{12}^3\gamma_{10}\eta_1^4 \\ & - \gamma_{12}^2\alpha^2\eta_1^4 - 12\gamma_{12}^2\gamma_{10}^2\eta_1^4 - \gamma_{12}\alpha^2\gamma_{11}\eta_1^2 v^2 + \gamma_{12}\alpha^2\gamma_{10}\eta_1^4 \\ & + 12\gamma_{12}\gamma_{10}^3\eta_1^4 + \alpha^2\gamma_{11}\gamma_{10}\eta_1^2 v^2 - 4\gamma_{10}^4\eta_1^4 - 4\gamma_{12}^4\eta_1^3 \\ & - 4\gamma_{12}^3\gamma_{11}\eta_1^3 + 16\gamma_{12}^3\gamma_{10}\eta_1^3 + 24\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^3 - 24\gamma_{12}^2\gamma_{10}^2\eta_1^3 \\ & - \gamma_{12}\alpha_1^2\gamma_{11}\eta_1^3 - 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^3 + 16\gamma_{12}\gamma_{10}^3\eta_1^3 - \alpha^2\gamma_{11}^2\eta_1 v^2 \\ & + 16\gamma_{11}\gamma_{10}^3\eta_1^3 - 4\gamma_{10}^4\eta_1^3 - 12\gamma_{10}^3\gamma_{11}\eta_1^2 - 12\gamma_{12}^2\gamma_{11}^2\eta_1^2 \\ & + 36\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^2 - \gamma_{12}\alpha^2\gamma_{11}\eta_1^2 + 36\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1^2 \\ & - 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^2 + \alpha^2\gamma_{11}\gamma_{10}\eta_1^2 - 24\gamma_{11}^2\gamma_{10}^2\eta_1^2 \\ & + 12\gamma_{11}\gamma_{10}^3\eta_1^2 - 12\gamma_{12}^2\gamma_{11}^2\eta_1 - 12\gamma_{12}\gamma_{11}^3\eta_1 \\ & + 24\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1 - \alpha^2\gamma_{11}^2\eta_1 + 16\gamma_{11}^3\gamma_{10}\eta_1 - 12\gamma_{11}^2\gamma_{10}^2\eta_1 \\ & - 4\gamma_{12}\gamma_{11}^3 - 4\gamma_{11}^4 + 4\gamma_{11}^3\gamma_{10}, \end{aligned}$$

$$\begin{aligned} c_{23} = {} & 4\gamma_{12}^2\alpha^2\eta_1^3 v^2 - 4\gamma_{12}\alpha^2\gamma_{10}\eta_1^3 v^2 + 6\gamma_{12}^2\gamma_{10}^2\eta_l^4 + 8\gamma_{12}\alpha^2\gamma_{11}\eta_1^2 v^2 \\ & - 12\gamma_{12}\gamma_{10}^3\eta_1^4 - 4\alpha^2\gamma_{11}\gamma_{10}\eta_1^2 v^2 + 6\gamma_{10}^4\eta_1^4 - 16\gamma_{12}^3\gamma_{10}\eta_1^3 \\ & + 4\gamma_{10}^2\alpha^2\eta_1^3 - 12\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^3 + 48\gamma_{12}^2\gamma_{10}^2\eta_1^3 - 4\gamma_{12}\alpha^2\gamma_{10}\eta_1^3 \\ & + 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^3 - 48\gamma_{12}\gamma_{10}^3\eta_1^3 + 4\alpha^2\gamma_{11}^2\eta_1 v^2 \\ & - 24\gamma_{11}\gamma_{10}^3\eta_1^3 + 16\gamma_{10}^4\eta_1^3 + 6\gamma_{12}^4\eta_1^2 + 12\gamma_{12}^3\gamma_{11}\eta_1^2 \\ & - 24\gamma_{12}^3\gamma_{10}\eta_1^2 + 6\gamma_{12}^2\gamma_{11}^2\eta_1^2 - 72\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^2 + 36\gamma_{12}^2\gamma_{10}^2\eta_1^2 \\ & + 8\gamma_{12}\alpha^2\gamma_{11}\eta_1^2 - 36\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1^2 + 108\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^2 \\ & - 24\gamma_{12}\gamma_{10}^3\eta_1^2 - 4\alpha^2\gamma_{11}\gamma_{10}\eta_1^2 + 36\gamma_{11}^2\gamma_{10}^2\eta_1^2 - 48\gamma_{11}\gamma_{10}^3\eta_1^2 \\ & + 6\gamma_{10}^4\eta_1^2 + 12\gamma_{12}^3\gamma_{11}\eta_1 + 24\gamma_{12}^2\gamma_{11}^2\eta_1 - 36\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1 \\ & + 12\gamma_{12}\gamma_{11}^3\eta_1 - 72\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1 + 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1 \\ & + 4\alpha^2\gamma_{11}^2\eta_1 - 24\gamma_{11}^3\gamma_{10}\eta_1 + 48\gamma_{11}^2\gamma_{10}^2\eta_1 - 12\gamma_{11}\gamma_{10}^3\eta_1 \\ & + 6\gamma_{12}^2\gamma_{11}^2 + 12\gamma_{12}\gamma_{11}^3 - 12\gamma_{12}\gamma_{11}^2\gamma_{10} + 6\gamma_{11}^4 - 16\gamma_{11}^3\gamma_{10} \\ & + 6\gamma_{11}^2\gamma_{10}^2, \end{aligned}$$

$$\begin{aligned} c_{24} = {} & -\gamma_{12}\alpha^2\gamma_{10}\eta_1^4 v^2 + \gamma_{12}\alpha^2\gamma_{11}\eta_1^3 v^2 - 6\gamma_{12}^2\alpha^2\eta_1^2 v^2 \\ & - 7\gamma_{12}\alpha^2\gamma_{11}\eta_1^2 v^2 - \gamma_{12}\alpha^2\gamma_{10}\eta_1^4 + 6\gamma_{12}\alpha^2\gamma_{10}\eta_1^2 v^2 \\ & + 4\gamma_{12}\gamma_{10}^3\eta_1^4 + 6\alpha^2\gamma_{11}\gamma_{10}z_1^2 v^2 - 4\gamma_{10}^4\eta_1^4 - 24\gamma_{12}^2\gamma_{10}^2\eta_1^3 \\ & + \gamma_{12}\alpha^2\gamma_{11}\eta_1^3 - 7\gamma_{12}\alpha^2\gamma_{11}\eta_1 v^2 - 12\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^3 \\ & + 48\gamma_{12}\gamma_{10}^3\eta_1^3 - 6\alpha^2\gamma_{11}^2\eta_1 v^2 + 16\gamma_{11}\gamma_{10}^3\eta_1^3 - 24\gamma_{10}^4\eta_1^3 \\ & + 24\gamma_{12}^3\gamma_{10}\eta_1^2 - 6\gamma_{12}^2\alpha^2\eta_1^2 + 36\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1^2 - 72\gamma_{12}^2\gamma_{10}^2\eta_1^2 \\ & - 7\gamma_{12}\alpha^2\gamma_{11}\eta_1^2 + \gamma_{12}\alpha^2\gamma_{11}v^2 + 6\gamma_{12}\alpha^2\gamma_{10}\eta_1^2 \\ & + 12\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1^2 - 108\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^2 + 72\gamma_{12}\gamma_{10}^3\eta_1^2 \\ & + 6\alpha^2\gamma_{11}\gamma_{10}\eta_1^2 - \alpha^2\gamma_{11}\gamma_{10}v^2 - 24\gamma_{11}^2\gamma_{10}^2\eta_1^2 + 72\gamma_{11}\gamma_{10}^3\eta_1^2 \\ & - 24\gamma_{10}^4\eta_1^2 - 4\gamma_{12}^4\eta_1 - 12\gamma_{12}^3\gamma_{11}\eta_1 + 16\gamma_{12}^3\gamma_{10}\eta_1 \\ & - 12\gamma_{12}^2\gamma_{11}^2\eta_1 + 72\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1 - 24\gamma_{12}^2\gamma_{10}^2\eta_1 \\ & - 7\gamma_{12}\alpha^2\gamma_{11}\eta_1 - 4\gamma_{12}\gamma_{11}^3\eta_1 + 72\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1 \\ & - 108\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1 + 16\gamma_{12}\gamma_{10}^3\eta_1 - 6\alpha^2\gamma_{11}^2\eta_1 + 16\gamma_{11}^3\gamma_{10}\eta_1 \\ & - 72\gamma_{11}^2\alpha^2\eta_1 + 48\gamma_{11}\gamma_{10}^3\eta_1 - 4\gamma_{10}^4\eta_1 - 4\gamma_{12}^3\gamma_{11} - 12\gamma_{12}^2\gamma_{11}^2 \\ & + 12\gamma_{12}^2\gamma_{11}\gamma_{10} + \gamma_{12}\alpha^2\gamma_{11} - 12\gamma_{12}\gamma_{11}^3 + 36\gamma_{12}\gamma_{11}^2\gamma_{10} \\ & - 12\gamma_{12}\gamma_{11}\gamma_{10}^2 - \alpha^2\gamma_{11}\gamma_{10} - 4\gamma_{11}^4 + 24\gamma_{11}^3\gamma_{10} - 24\gamma_{11}^2\gamma_{10}^2 + 4\gamma_{11}\gamma_{10}^3, \end{aligned}$$

$$\begin{aligned} c_{25} = {} & 4\gamma_{12}\alpha^2\gamma_{10}\eta_1^3 v^2 - 4\alpha^2\gamma_{11}\gamma_{10}\eta_1^2 v^2 + \gamma_{10}^4\eta_1^4 + 4\gamma_{12}^2\alpha^2\eta_1 v^2 \\ & + 8\gamma_{12}\alpha^2\gamma_{11}\eta_1 v^2 + 4\gamma_{12}\alpha^2\gamma_{11}\eta_1^3 - 4\gamma_{12}\alpha^2\gamma_{11}\eta_1 v^2 \\ & - 16\gamma_{12}\gamma_{10}^3\eta_1^3 + 4\alpha^2\gamma_{11}^2\eta_1 v^2 - 4\gamma_{11}\gamma_{10}^3\eta_1^3 + 16\gamma_{10}^4\eta_1^3 \\ & + 36\gamma_{12}^2\gamma_{10}^2\eta_1^2 + 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1^2 - 72\gamma_{12}\gamma_{10}^3\eta_1^2 - 4\alpha^2\gamma_{11}\gamma_{10}\eta_1^2 \\ & + 4\alpha^2\gamma_{11}\gamma_{10}v^2 + 6\gamma_{11}^2\gamma_{10}^2\eta_1^2 - 48\gamma_{11}\gamma_{10}^3\eta_1^2 + 36\gamma_{10}^4 z_1^2 \\ & - 16\gamma_{12}^3\gamma_{10}\eta_1 + 4\gamma_{12}^2\alpha^2\eta_1 - 36\gamma_{12}^2\gamma_{11}\gamma_{10}\eta_1 + 48\gamma_{12}^2\gamma_{10}^2\eta_1 \\ & + 8\gamma_{12}\alpha^2\gamma_{11}\eta_1 - 4\gamma_{12}\alpha^2\gamma_{10}\eta_1 - 24\gamma_{12}\gamma_{11}^2\gamma_{10}\eta_1 \\ & + 108\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1 - 48\gamma_{12}\gamma_{10}^3\eta_1 + 4\alpha^2\gamma_{11}^2\eta_1 - 4\gamma_{11}^3\gamma_{10}\eta_1 \\ & + 48\gamma_{12}^2\gamma_{10}^2\eta_1 - 72\gamma_{11}\gamma_{10}^3\eta_1 + 16\gamma_{10}^4\eta_1 + \gamma_{12}^4 + 4\gamma_{12}^3\gamma_{11} \\ & - 4\gamma_{12}^3\gamma_{10} + 6\gamma_{12}^2\gamma_{11}^2 - 24\gamma_{12}^2\gamma_{11}\gamma_{10} + 6\gamma_{12}^2\gamma_{10}^2 + 4\gamma_{12}\gamma_{11}^3 \\ & - 36\gamma_{12}\gamma_{11}^2\gamma_{10} + 36\gamma_{12}\gamma_{11}\gamma_{10}^2 - 4\gamma_{12}\gamma_{10}^3 + 4\alpha^2\gamma_{11}\gamma_{10} + \gamma_{11}^4 \\ & - 16\gamma_{11}^3\gamma_{10} + 36\gamma_{11}^2\gamma_{10}^2 - 16\gamma_{11}\gamma_{10}^3 + \gamma_{10}^4, \end{aligned}$$

$$\begin{aligned}c_{26} = &-6\gamma_{12}\alpha^2\gamma_{10}\eta_1^2\nu^2 + \alpha^2\gamma_{11}\gamma_{10}\eta_1^2\nu^2 - \gamma_{12}\alpha^2\gamma_{11}\eta_1\nu^2 \\ &- \alpha^2\gamma_{11}^2\eta_1\nu^2 - 4\gamma_{10}^4\eta_1^3 - \gamma_{12}^2\alpha^2\nu^2 - \gamma_{12}\alpha^2\gamma_{11}\nu^2 \\ &- 6\gamma_{12}\alpha^2\gamma_{10}\eta_1^2 + \gamma_{12}\alpha^2\gamma_{10}\nu^2 + 24\gamma_{12}\gamma_{10}^3\eta_1^2 + \alpha^2\gamma_{11}\gamma_{10}\eta_1^2 \\ &- 6\alpha^2\gamma_{11}\gamma_{10}\nu^2 + 12\gamma_{11}\gamma_{10}^3\eta_1^2 - 24\gamma_{10}^4\eta_1^2 - 24\gamma_{12}^2\gamma_{10}^2\eta_1 \\ &- \gamma_{12}\alpha^2\gamma_{11}\eta_1 - 36\gamma_{12}\gamma_{11}\gamma_{10}^2\eta_1 + 48\gamma_{12}\gamma_{10}^3\eta_1 - \alpha^2\gamma_{11}^2\eta_1 \\ &- 12\gamma_{11}^2\gamma_{10}^2\eta_1 + 48\gamma_{11}\gamma_{10}^3\eta_1 - 24\gamma_{10}^4\eta_1 + 4\gamma_{12}^3\gamma_{10} - \gamma_{12}^2\alpha^2 \\ &+ 12\gamma_{12}^2\gamma_{11}\gamma_{10} - 12\gamma_{12}^2\gamma_{10}^2 - \gamma_{12}\alpha^2\gamma_{11} + \gamma_{12}\alpha^2\gamma_{10} \\ &+ 12\gamma_{12}\gamma_{11}^2\gamma_{10} - 36\gamma_{12}\gamma_{11}\gamma_{10}^2 + 12\gamma_{12}\gamma_{10}^3 - 6\alpha^2\gamma_{11}\gamma_{10} \\ &+ 4\gamma_{11}^3\gamma_{10} - 24\gamma_{11}^2\gamma_{10}^2 + 24\gamma_{12}\gamma_{10}^3 - 4\gamma_{10}^4,\end{aligned}$$

$$\begin{aligned}c_{27} = &\,4\gamma_{12}\alpha^2\gamma_{10}\eta_1\nu^2 + 4\alpha^2\gamma_{11}\gamma_{10}\nu^2 + 6\gamma_{10}^4\eta_1^2 + 4\gamma_{12}\alpha^2\gamma_{10}\eta_1 \\ &- 16\gamma_{12}\gamma_{10}^3\eta_1 - 12\gamma_{11}\gamma_{10}^3\eta_1 + 16\gamma_{10}^4\eta_1 + 6\gamma_{12}^2\gamma_{10}^2 \\ &+ 12\gamma_{12}\gamma_{11}\gamma_{10}^2 - 12\gamma_{12}\gamma_{10}^3 + 4\alpha^2\gamma_{11}\gamma_{10} + 6\gamma_{11}^2\gamma_{10}^2 \\ &- 16\gamma_{11}\gamma_{10}^3 + 6\gamma_{10}^4,\end{aligned}$$

$$\begin{aligned}c_{28} = &-\gamma_{12}\alpha^2\gamma_{10}\nu^2 - \alpha^2\gamma_{11}\gamma_{10}\nu^2 - 4\gamma_{10}^4\eta_1 - \gamma_{12}\alpha^2\gamma_{10} + 4\gamma_{12}\gamma_{10}^3 \\ &- \alpha^2\gamma_{11}\gamma_{10} + 4\gamma_{11}\gamma_{10}^3 - 4\gamma_{10}^4,\end{aligned} \tag{29}$$

Setting the coefficients of the identical powers about η in Eq. (28) to zero, then obtain the number of following equations:

$$c_{2i} = 0, (i = 1, 2, \cdots, 9). \tag{30}$$

Solve the number of following equations, then attain

$$\gamma_{12} = \sqrt{\nu^2 + 1}\alpha, \gamma_{11} = \sqrt{\nu^2 + 1}\alpha, \tag{31}$$

where $\eta = e^{\alpha z}(\alpha \in \mathbb{C})$.

$$V_s(e^{\alpha z}) = \frac{\sqrt{\nu^2 + 1}\alpha}{e^{\alpha z} - 1} + \frac{\sqrt{\nu^2 + 1}\alpha}{e^{\alpha z} - e^{\alpha z_1}}, \tag{32}$$

hence, we get the simply periodic solutions of Eq. (19) with pole at $z = 0$

$$V_{s10}(z) = \frac{\sqrt{\nu^2 + 1}\alpha}{2}\left(\coth\frac{\alpha}{2}z - \coth\frac{\alpha}{2}(z - z_1)\right), \tag{33}$$

where $\gamma_{12}/a - 1 = \nu$ is the first case and $\gamma_{11}/a - 1 = \nu$ is the second case.

According to the weak $\langle 2.1 \rangle$ condition, we introduce here elliptic solutions of (12) with $z = 0$ pole.

$$V_{d,1}(z) = \pm\frac{\sqrt{\nu^2 + 1}\alpha}{2}\frac{\wp'(z) + E_1}{\wp(z) - F_1} + \gamma_{10}, \tag{34}$$

where $E_1^2 = 4F_1^3 - k_2F_1 - k_3$. We can rewrite the $V_{d,1}(z)$ and considering the above mentioned results, we have $\gamma_{10} = 0$, $k_3 = 0$, and $E_1 = F_1 = 0$. So, we have

$$V_{d,1}(z) = \pm\frac{\sqrt{\nu^2 + 1}\alpha}{2}\frac{\wp'(z)}{\wp(z)}, \tag{35}$$

where $k_3 = 0$, hence, the elliptic function solutions of Eq. (19) are

$$V_{d,1}(z) = \pm\frac{\sqrt{\nu^2 + 1}\alpha}{2}\frac{\wp'(z - z_0 : k_2, 0)}{\wp(z - z_0 : k_2, 0)}, \tag{36}$$

where z_0 and k_2 are arbitrary constants. By further process, we attain meromorphic solutions of Eq. (19) with an arbitrary pole as follows:

$$\begin{aligned}V_{r,1}(z) &= \frac{\sqrt{\nu^2 + 1}}{z - z_0} + \frac{\sqrt{\nu^2 + 1}}{z - z_1 - z_0}, \\ V_{r,2}(z) &= \frac{\nu + 1 - \sqrt{\nu^2 + 1}}{z - z_0} + \frac{\nu + 1 - \sqrt{\nu^2 + 1}}{z - z_1 - z_0}, \\ V_{s,1}(z) &= \frac{\sqrt{\nu^2 + 1}\alpha}{2}\left(\coth\frac{\alpha}{2}(z - z_0) - \coth\frac{\alpha}{2}(z - z_1 - z_0)\right),\end{aligned} \tag{37}$$

where $\gamma_{12}/a - 1 = \nu$ is the first case and $\gamma_{1a}/a - 1 = \nu$ is the second case.

4. Description about Figures

In this section, we represent traveling wave solutions for $V_{r,1}(z)$, $V_{r,2}(z)$ and $V_{s,1}(z)$ by 3D graphs as in Figures 1–6. These figures are displayed by maple software to justify our main results physically.

Figures 1 and 2 show exact traveling wave solutions for $V_{r,1}(z)$, set as fixed values $\nu = 1$, $z_0 = 0.5$, and $z_1 = 1$ and $\nu = 1$, $z_0 = -0.5$, and $z_1 = 1$.

Figures 3 and 4 show exact traveling wave solutions for $V_{r,2}(z)$, set as fixed values $\nu = 1$, $z_0 = 0.2$, and $z_1 = 1$ and $\nu = 1$, $z_0 = -0.2$, and $z_1 = 1$.

Figures 5 and 6 show exact traveling wave solutions for $V_{s,1}(z)$, set as fixed values $\nu = 1$, $z_0 = 1/6$, $z_1 = 1$, and $\alpha = 1$ and $\nu = 1$, $z_0 = -1/6$, $z_1 = 1$, and $\alpha = 1$.

5. Comparison and Conclusion

Biswas et al. [36] introduced (G'/G)-expansion technique to explore the analytical solutions of nonlinear fractional Liouville equation. The exact analytical solutions attained by this proposed method show the inner structure of physical phenomena in applied science. The results indicate the different types of traveling wave solutions of this proposed method. For this reason, we compare (G'/G)-expansion method with extended complex technique.

We have applied the extended complex technique to attain the exact analytical solutions of the nonlinear fractional Liouville equation. The 3D graphs are represented by adjusting the values of arbitrary parameters, and these graphs have demonstrated the different physical structures such as Figures 1 and 2 represent multisolitary traveling wave solutions, Figures 3 and 4 represent soliton type traveling wave solutions, and Figures 5 and 6 show solitary traveling wave solutions. Figures 1–6 depict that traveling wave solutions are attained for smaller values of z_0, whereas other parameters are fixed. The extended complex method has been solved by means of maple packages. The proposed technique is an effective analytical approach in comparison with other analytical techniques via tanh method and first integral method [23, 24] because it gives different new traveling wave solutions which have been shown in terms of rational, periodic, and elliptic function solutions. The results that are attained by the proposed method play a significant task to show the deep mechanism of physical phenomena and give tedious solutions of higher degree NFPDEs.

By extended complex technique, we could find meromorphic solutions of numerous DEs which do not satisfy Briot-Bouquet equation. Therefore, in the future, this proposed approach plays an important task in mathematical physics. The intermediate forms of the solutions are applied to explore meromorphic solutions $V(z)$ for the DEs with a pole at $z = 0$. After that solve meromorphic solutions for DEs with an arbitrary pole.

Data Availability

The data applied to support the finding of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflict of interests.

Acknowledgments

This work is supported by the NSF of China (11901111) and Guangdong University Key Field Project (2020ZDZX3019) and also dedicated to Chern Institute of Mathematics at Nankai University. Here, the authors work as a visiting scholar. The authors want to give gratitude to the Chern Institute of Mathematics for giving us a useful opportunity to explore our ideas.

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