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Research Article

Degeneracy in Magneto-Active Dense Plasma

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Degenerate dense plasmas are of great interest due to their important applications in modern technology and astrophysics. Such plasmas have generated a lot of interest in the last decade owing to their importance in many areas of physics such as semiconductors, metals, microelectronics,carbon nanotubes, quantum dots, and quantum wells.Besides, degenerate plasmas present very interesting features for fusion burning waves' ignition and propagation. In this paper, we investigated the effects of static magnetic field on energy states and degeneracy of electrons in dense plasma. Using perturbation theory, two cases are considered, strongly and weakly magnetized electrons. Strong magnetic field will not eliminate completely the degeneracy, but it functions to reduce degeneracy. Perturbed energy eigenvalues ΔE are calculated to high accuracy. Besides, regardless of whether the perturbed state is degenerate or not, the energy ΔE is given by considering the average of orbital and spin coupling $W_s = \aleph(r)\overrightarrow{L} \cdot \overrightarrow{S}$ with respect to the eigenfunction Ψ_{n,l,m,m_s}. Here $\overrightarrow{L}$ is the angular momentum vector, $\overrightarrow{S}$ is the spin vector of electrons, and $\aleph(r)$ is the energy of spin orbit coupling in plasma, which plays a crucial role in the study of energy states and degeneracy of plasma electrons.

1. Introduction

When the plasma density is increased sufficiently, quantum effects become very interesting. This includes degeneracy effects, which becomes important when $T << T_F$, T is the plasma temperature, $T_F = E_F/k_B$ is the Fermi temperature, defined in Fermi energy as $E_F = (\hbar^2/2m)(3\pi^2 n_0)^{2/3}$, k_B is Boltzmann's constant, $\hbar$ is the reduced Planck constant. In this model, particle dispersive effects tend to be important for short scale-lengths (comparable to the characteristic de-Broglie length) when $\hbar\omega_p/k_B T_F \approx 1$, ω_p is the plasma frequency. In quantum kinetic theory, these effects can be well modeled using the perturbation theory.

In plasmas, in order to further improve these models, the electron spin is taken into account, which introduces a magnetic dipole force, spin precession, and spin magnetization currents into the picture [1–4].

In fact, there has been an increasing interest in plasmas of low-temperature and high densities, where quantum properties tend to be important [5–8]. Promising applications include quantum wells [9], spintronics [10], and plasmonics [11]. Quantum plasma effects can also be of interest in experiments with solid density targets [12]. Important classifications of dense plasmas include whether they are strongly or weakly coupled, and whether they are degenerate or nondegenerate [1]. Several works [5, 13–15] have applied quantum plasma effects, for example, in X-ray Thomson scattering in high energy density plasmas provide experimental techniques for accessing narrow bandwidth spectral lines [2], so as to detect frequency shifts due to quantum effects [13], and the next generation intense laser-solid density plasma interaction experiments [14].

Besides, quantum or degenerate plasmas are of great interest due to their important applications in modern technology and astrophysics. Such plasmas have generated a lot of interest in the last decade owing to their importance in many areas of physics such as semiconductors, metals, microelectronics [16] carbon nanotubes, quantum dots, and quantum wells [17–19]. Degenerate plasmas also play an important role in dense astrophysical objects like plasmas in the interior of stars and neutron stars [20]. The effect of trapping in a degenerate investigated in a plasma comprises degenerate electrons and nondegenerate ions in the presence of a quantizing magnetic field [4].

The usual perturbative treatment of magnetic effects like Zeeman splitting of atomic energy levels in a strong field regime does not apply in such a situation, but instead, the Coulomb forces act as a perturbation to the magnetic forces.

Owing to the extreme confinement of electrons in the transverse direction, the Coulomb force becomes much more effective in binding the electrons along the magnetic field direction [21]. As is well known, electron gas magnetization in a weak magnetic field has two independent parts; (i) paramagnetic, and (ii) the diamagnetic parts. The intrinsic or spin magnetic moment of electrons gives rise to Pauli paramagnetism. The diamagnetic part is due to the fact that the orbital motion of electrons becomes quantized in a magnetic field.

As a fact, the field of quantum plasma physics is becoming of an increasing current interest [22–25], motivated by its potential applications in modern technology (e.g., metallic and semiconductor nanostructures-such as metallic nanoparticles, metal clusters, thin metal films, spintronics, nanotubes, quantum well and quantum dots, nano-plasmonic devices, quantum X-ray free-electron lasers, etc.). In dense quantum plasmas and in the Fermi gas of metals, the number densities of degenerate electrons are extremely high so that their wave functions overlap, and therefore electrons obey the Fermi-Dirac statistics. The Fermi degenerate dense plasma may also arise when a pellet of hydrogen is compressed to many times the solid density in the fast ignition scenario for inertial confinement fusion (ICF) [26, 27].

Our work is of high current interest in experiments and theory-experiment comparisons are becoming possible, e.g., via Thomson scattering using free electron lasers, e.g., [5]. The increasing accuracy of these experiments will be a driving force for theory developments in the near future.

In the present work, we limit ourselves to considering only weakly coupled degenerate plasmas, where effects of ion viscosity are not considered because as ion viscosities can normally be neglected as long as the wave period is much larger than the time scale of the ion correlations and the damping rate due to the viscosities is much smaller than the work frequency of the wave [28].

We investigated the effects of static magnetic field on energy states and degeneracy of electrons in dense plasma. Using perturbation theory, two cases are considered, strongly and weakly magnetized electrons. Perturbed energy eigenvalues ΔE are calculated to high accuracy. The energy of spin orbit coupling in plasma, which plays a crucial role in the study of energy states of plasma electrons, is also calculated.

2. Basic Set of Equations

In degenerate plasmas, physical parameters like density, magnetic field, and temperature vary over a wide range of values. For example, the degenerate electron number density may exceed the solid matter density by many orders of magnitude in white dwarfs, neutron stars, and in the next generation of inertially compressed materials in intense laser-solid target interaction experiments.

The theory presented here is of most interest for systems where at least one of the parameters $\hbar\omega_p/mc^2$ or $\mu B/mc^2$ is not too small. Examples include e.g., laser-plasma interactions, solid state plasmas, and strongly magnetized systems.

The following parameters may be used for experimental applications. The number density and the magnetic field have the values of the order of $10^{26}\,\mathrm{cm}^{-3}$ and $10^{10}G$, respectively [4]. These numbers used to calculate the Fermi energy and the Fermi temperature as $T_F = 9.14108 \times 10^8$K and have taken the electron temperature $T << T_F$ [29].

We assume electrons in plasmas to be of a quantum medium of many body system. Such an assumption is due to the fact that the solutions obtained have characteristic sizes of atomic order.

Let us assume that we have a magnetic field $\overrightarrow{B}$ applied to unperturbed quantum plasma system with Hamiltonian given by:

$$H \equiv \frac{P^2}{2m} + V(r) + \aleph(r)\overrightarrow{L}\cdot\overrightarrow{S}, \qquad (1)$$

where, $\overrightarrow{L}$ - the angular momentum vector, $\overrightarrow{S}$ - the spin vector of electrons, and $\aleph(r)$—the energy of spin orbit coupling in plasmas, which has an essential role in the study of energy levels of plasma electrons. $V(r)$ is a type of potential in the system.

In the presence of a magnetic field, we have to introduce to (1) both (i) $W_{\mu B}$ as the interaction energy between the orbital magnetic moment $\overrightarrow{\mu}$ of the electrons and the magnetic induction,

$$W_{\mu B} = -\overrightarrow{\mu}\cdot\overrightarrow{B}, \quad \overrightarrow{\mu} = -\frac{e}{2m}\overrightarrow{L}, \qquad (2)$$

and (ii) W_{SB} as the interaction energy between the spin magnetic moment $\overrightarrow{\mu}_S$ and the magnetic induction,

$$W_{SB} = -\overrightarrow{\mu}_S\cdot\overrightarrow{B}, \quad \overrightarrow{\mu} = -\frac{e}{m}\overrightarrow{S}. \qquad (3)$$

For simplicity, let us consider a static magnetic field directed to z-direction $\left(\overrightarrow{B} = \overrightarrow{e}_z B\right)$.

The total Hamiltonian (1) for spinning electrons reads:

$$H \equiv H_0 + W_S + W_B, \qquad (4)$$

where, $W_S = \aleph(r)\overrightarrow{L}\cdot\overrightarrow{S}$, $W_B = (1/2)\overrightarrow{\omega}_{ce}\cdot\left(\overrightarrow{L} + 2\overrightarrow{S}\right)$, $\omega_{ce} = (eB/m)$ is the electron cyclotron frequency, and $H_0 = \left(P^2/2m\right) + V(r)$.

W_S and W_B are now considered as two perturbed terms, and let us now determine the energy levels of the eigenvalue equation for the Hamiltonian (4) using perturbation theory by considering and specify the relative values between W_S and W_B.

Two cases will be considered, i.e., (i) strongly magnetized electrons $W_B >> W_S$, and (ii) weakly magnetized electrons $W_B << W_S$.

3. Strongly Magnetized Plasma

If the magnetic field $\overrightarrow{B}$ is very strong such that $W_B >> W_S$, it is justifiable to consider only W_S as a perturbed quantity, and the unperturbed Hamiltonian reads:

Table 1: Degenerate, and nondegenerate electron states in magnetized plasma.

1	$E^0_{211(1/2)} = E^0_{21} + \hbar\omega_{ce}$	4	$E^0_{211(\overline{1/2})} = E^0_{21}$
2	$E^0_{210(1/2)} = E^0_{21} + \frac{\hbar}{2}\omega_{ce}$	5	$E^0_{210(\overline{1/2})} = E^0_{21} - \frac{\hbar}{2}\omega_{ce}$
3	$E^0_{21\bar{1}(1/2)} = E^0_{21}$	6	$E^0_{21\bar{1}(\overline{1/2})} = E^0_{21} - \hbar\omega_{ce}$

$$H_{01} \equiv H_0 + \frac{1}{2}\omega_{ce}\left(L_z + 2S_z\right). \tag{5}$$

It is easy to check that the operators H_0, L^2, S^2, L_z, and S_z are all commutes with the unperturbed Hamiltonian (5), and hence the eigenfunction corresponding to (5) may be represented as Ψ_{n,l,m,m_s}.

Accordingly, Schrodinger equation may have the form:

$$H_{01}\Psi_{n,l,m,m_s} \equiv E^0_{nlmm_s}\Psi_{n,l,m,m_s}, \tag{6}$$

where,

$$E^0_{nlmm_s} = E^0_{nl} + \frac{\hbar}{2}\omega_{ce}\left(m + 2m_s\right), \tag{7}$$

is the eigenvalue of the unperturbed Hamiltonian H_{01}.

For values $m_s = -(1/2)$; m and $m_s = (1/2)$; $m - 2$, then $m + 2m_s = m - 1$ for both cases and we have degeneracy of order two. For fixed states $n = 2$; $l = 1$, we have six possible states corresponding to $m = (-1, 0, 1)$; $m_s = (-(1/2), (1/2))$. These six states define six possible different eigenfunctions, two of them are degenerate, and the rest are nondegenerate as indicated in Table 1.

From (7), it is clear that, using strong magnetic field—to confine the plasma—will not eliminate completely the degeneracy but it functioning to reduce the degeneracy.

Let us consider now the perturbation theory to calculate the electron energy levels in plasmas, it will be very important to know if the state under investigation is degenerate or not. This is because calculation methodology is different for both cases.

The perturbed energy eigenvalues of the four nondegenerate states is given by

$$\Delta E = \left\langle \Psi_{n,l,m,m_s} \middle| W_s \middle| \Psi_{n,l,m,m_s} \right\rangle. \tag{8}$$

For degenerate states, let us make use of the above case.

Let $m_1 = m$; $m_2 = m - 2$ and $s_1 = -(1/2)$; $s_2 = (1/2)$ and assume the degenerate states

$$\Psi_1 = \Psi_{n,l,m_1,m_{s1}}, \tag{9}$$

$$\Psi_2 = \Psi_{n,l,m_2,m_{s2}}. \tag{10}$$

Besides, we define

$$E_{ij} = \left\langle \Psi_{n,l,m_i,m_{si}} \middle| W_s \middle| \Psi_{n,l,m_j,m_{sj}} \right\rangle, \quad i, j = 1, 2. \tag{11}$$

For the degenerate case**,** the perturbation theory requires the vanishing of the determinant of the unperturbed Hamiltonian W_s, which represents a matrix formed of the different eigen functions of the unperturbed terms corresponding to same energy, i.e.,

$$\begin{bmatrix} E_{11} - \Delta E & E_{12} \\ E_{21} & E_{22} - \Delta E \end{bmatrix} = 0. \tag{12}$$

Relation (11) determines the possible values of ΔE for degenerate state. The functions used are eigenfunctions for J_z, i.e., W_s has the eigenfunctions

$$\Psi_1 \Rightarrow \Psi_{l-(1/2)}, \tag{13}$$

$$\Psi_2 \Rightarrow \Psi_{l+(1/2)}. \tag{14}$$

Accordingly, the off-diagnoal terms of (11) vanish due to the orthognality of the wave functions, i.e.,

$$(\Delta E)_1 = \left\langle \Psi_1 \middle| W_s \middle| \Psi_1 \right\rangle, \tag{15}$$

or

$$(\Delta E)_2 = \left\langle \Psi_2 \middle| W_s \middle| \Psi_2 \right\rangle. \tag{16}$$

Relation (13) shows that perturbation theory in the presence of strong magnetic field $(W_B >> W_s)$ has eliminated the plasma electron's degeneracy. Besides, whenever the perturbed state is degenerate or not; ΔE is given by the average value of $W_s = \aleph(r)\overrightarrow{L} \cdot \overrightarrow{S}$ in the state Ψ_{n,l,m,m_s}.

It is clear from above that the perturbation theory, the presence of external magnetic field, has succeeded to eliminate completely the degeneracy, i.e., only one eigenfunction Ψ_i for each eigenvalue $(\Delta E)_i$. Besides, regardless the perturbed state is degenerate or not, the energy ΔE is given by considering the average of $W_s = \aleph(r)\overrightarrow{L} \cdot \overrightarrow{S}$ with respect to the eigenfunction Ψ_{n,l,m,m_s}.

From (13) it is easy to evaluate $(\Delta E)_{1,2}$ as

$$(\Delta E)_{1,2} = m_{(1,2)}m_{s(1,2)}\hbar^2\aleph_{nl}(r), \tag{17}$$

where,

$$\aleph_{nl}(r) = \left\langle \Psi_{n,l,m_{(1,2)},m_{s(1,2)}} \middle| \aleph(r) \middle| \Psi_{n,l,m_{(1,2)},m_{s(1,2)}} \right\rangle. \tag{18}$$

Now, the perturbed energy of (14) should be added to (7), i.e.,

$$E_{nlmm_s} = E^0_{nl} + \frac{\hbar}{2}\omega_{ce}\left(m + 2m_s\right) + mm_s\hbar^2\aleph_{nl}(r), \tag{19}$$

which shows the complete nondegeneracy of the final state due the third term on right hand side of (16).

4. Weakly Magnetized Plasma

In this case the electron spin orbit coupling W_s is assumed to be much greater than W_B, $W_B << W_s$ and the perturbed Hamiltonian reads:

$$H \equiv \frac{1}{2}\omega_{ce}\left(L_z + 2S_z\right), \tag{20}$$

while the unperturbed Hamiltoninan reads:

$$H \equiv H^0 + \aleph(r)\overrightarrow{L} \cdot \overrightarrow{S}. \tag{21}$$

From Pauli's spin theory, which has the same Hamiltonian (18), the constants of motion are H, L^2, S^2, J^2, J_z and the eigenfunctions and eigenvalues of (18) will have the form:

$$\Psi = \Psi_{nljm_j}, \tag{22}$$

$$E_{nljm_j} \Rightarrow \begin{matrix} E_{nl}^0 + (l+1)\hbar^2 \aleph_{nl}(r), & j = l + \dfrac{1}{2}, \\ E_{nl}^0 - (l+1)\hbar^2 \aleph_{nl}(r), & j = l - \dfrac{1}{2}. \end{matrix} \tag{23}$$

Both eigenvalues in (20) has degeneracy of order $(2j+1)$, and since the operators J_z.

Commutes with the Hamiltonian, therefore, the allowed eigenvalues of J_z in the state function Ψ_{nljm_j} will have the same energy given by (20). Besides, J_z commutes with H^0, W_B, W_s and therefore the perturbation matrix which determines the perturbed energy ΔE will be diagonal in this case also. Accordingly, the vanishing its elements yields:

$$\begin{aligned} \Delta E = \frac{1}{2}\omega_{ce}\left\langle \Psi_{nljm_j} \middle| L_z + 2S_z \middle| \Psi_{nljm_j} \right\rangle &= \frac{1}{2}\omega_{ce}\left\langle \Psi_{nljm_j} \middle| J_z + S_z \middle| \Psi_{nljm_j} \right\rangle \\ &= \frac{1}{2}\omega_{ce}\left[\hbar m_j + \left\langle \Psi_{nljm_j} \middle| S_z \middle| \Psi_{nljm_j} \right\rangle\right]. \end{aligned} \tag{24}$$

It is clear that the appearance of first term on the right hand side of (21), $(1/2)\omega_{ce}\hbar m_j$, is due to the external static magnetic field, functioning to remove completely the degeneracy.

To calculate $\left\langle \Psi_{nljm_j} \middle| S_z \middle| \Psi_{nljm_j} \right\rangle$, we note that Ψ_{nljm_j} is not an eigenfunction for S_z orL_z. Therefore, we can use the following mathematical rule:

$$\left\langle \gamma JM \middle| \overrightarrow{A} \middle| \gamma JM \right\rangle = \left\langle \gamma JM \middle| \overrightarrow{J} \middle| \gamma JM \right\rangle \cdot \frac{\left\langle \gamma JM \middle| \overrightarrow{A} \cdot \overrightarrow{J} \middle| \gamma JM \right\rangle}{\left\langle \gamma JM \middle| \overrightarrow{J}^2 \middle| \gamma JM \right\rangle}, \tag{25}$$

where, $\overrightarrow{A}$ is an arbitrary operator, γ any additional quantum number, and$|\gamma JM\rangle$ is an eigenket of the operators that J^2 and J_z.

Taking the z-component of $\overrightarrow{A}$ as S_z, then

$$\begin{aligned} \langle \gamma JM | S_z | \gamma JM \rangle &= \langle \gamma JM | J_z | \gamma JM \rangle \cdot \frac{\left\langle \gamma JM \middle| \overrightarrow{S} \cdot \overrightarrow{J} \middle| \gamma JM \right\rangle}{\left\langle \gamma JM \middle| \overrightarrow{J}^2 \middle| \gamma JM \right\rangle} \\ &= \frac{m_j}{j(j+1)\hbar} \left\langle \gamma JM \middle| \overrightarrow{S} \cdot \overrightarrow{J} \middle| \gamma JM \right\rangle. \end{aligned} \tag{26}$$

Set the scatter product $\overrightarrow{S} \cdot \overrightarrow{J} = (1/2)\left(J^2 + S^2 - L^2\right)$ into (23), we get

$$\langle \gamma JM | S_z | \gamma JM \rangle = m_j \hbar \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}. \tag{27}$$

Set (24) into (21) we obtain ΔE the shift in energy levels as:

$$\Delta E = \frac{\hbar}{2}\omega_{ce} m_j G, \tag{28}$$

$$G = 1 + \frac{j(j+1) + s(s+1) - l(l+1)}{2j(j+1)}, \tag{29}$$

where, G is the well known electron Lande's factor or the spectroscopic factor (lies between 1 and 2), which measures the plasma electrons energy levels. G, plays a crucial role in degenerate plasmas when considering fusion burning waves' ignition and propagation.

However, the outcome of fusion burning waves in nondegenerate plasmas is limited by the strength of ion-electron Coulomb collisions and subsequent energy loss mechanisms as electron heat conduction and radiation emission (Bremsstrahlung).

Relation (26) is in agreement with (37) as per [30].

It is clear that the shift in energy levels ΔE depends on the quantum number m_j, which removes the degeneracy as mentioned before. ΔE Also strongly depend on the Lande's factor G, hence the quantum numbers of the plasma electron states.

For instants, let us consider the following two cases, i.e.,

$$j = \left(l - \frac{1}{2}\right) \Rightarrow G = \frac{2l}{2l+1}, \quad j = \left(l + \frac{1}{2}\right) \Rightarrow G = \frac{2(l+1)}{2l+1}. \tag{30}$$

Therefore, in the presence of weak magnetic field $W_B << W_s$ we have the following energy states:

$$E = E_{nl}^0 + \frac{\hbar}{2}\omega_{ce} m_j \frac{2l}{2l+1} - \hbar(l+1)\aleph_{nl}(r) \quad \text{for } j = \left(l - \frac{1}{2}\right), \tag{31}$$

$$E = E_{nl}^0 + \frac{\hbar}{2}\omega_{ce} m_j \frac{2(l+1)}{2l+1} + \hbar l \aleph_{nl}(r) \quad \text{for } j = \left(l + \frac{1}{2}\right). \tag{32}$$

In case of neglecting electron spin, ($\overrightarrow{S} = 0$), the interaction Hamiltonian is reduced to $\approx (1/2)\overrightarrow{\omega}_{ce} \cdot \overrightarrow{L}$ and finally the plasma electron's energy E is reduced to,

$$E = E_{nl}^0 + \frac{\hbar}{2}\omega_{ce} m_j. \tag{33}$$

5. Results and Conclusions

In this work, we have investigated the effects of static magnetic field on the energy states and degeneracy of electrons in quantized dense plasma. Using perturbation theory, two cases are considered, strongly and weakly magnetized electrons. Perturbed energy eigenvalues $\Delta E$are calculated analytically to high accuracy in both cases. Let us summarize major results obtained in this work:

5.1. In Strong Magnetic Field

(i) The eigenvalues of the unperturbed Hamiltonian H_{01}, relation (7). Strong magnetic field will not eliminate completely the degeneracy but it functioning to reduce it.

(ii) Six states define six possible different eigenfunctions, two of them are degenerate, and the rest are nondegenerate as indicated in Table 1.

(iii) The perturbed energy eigenvalues of the four nondegenerate states is given by (13), (14).

(iv) Total final energy state for nondegenerate and degenerate states is given by (16).

5.2. In Weak Magnetic Field

(i) The eigenfunctions and eigenvalues of the unperturbed Hamiltonian (18) are given by (19) and (20), respectively. Both eigenvalues in (20) has degeneracy of order $(2j+1)$.
(ii) The perturbed energy ΔE is calculated and given by (21).
(iii) The appearance of first term on right hand side of (21) functioning to removes completely the degeneracy.
(iv) The perturbed energy ΔE is simply calculated as per (25), which strongly proportional to Lande's factor.
(v) Relations (28), (29) give the calculated energy states. Neglecting electron spin, both energy states are equal and given by (30).

When the plasma electrons become degenerate, the electron de Broglie wavelength becomes large compared with the mean interparticle spacing and quantum mechanical considerations are of paramount importance.

From (7), it is clear that, using strong magnetic field—to confine the plasma—will not eliminate completely the degeneracy, but it functions to reduce the degeneracy, while from (13), the perturbation theory has eliminated the plasma electron's degeneracy for eigenfunctions

$$\Psi_1 \Rightarrow \Psi_{l-(1/2)}, \tag{34}$$

$$\Psi_2 \Rightarrow \Psi_{l+(1/2)}. \tag{35}$$

Besides, regardless of whether the perturbed state is degenerate or not, the energy ΔE is given by considering the average of $W_S = \aleph(r)\overrightarrow{L} \cdot \overrightarrow{S}$ with respect to the eigenfunction Ψ_{n,l,m,m_s}.

It is clear from above that the perturbation theory, in the presence of external magnetic field, has succeeded to eliminate completely the degeneracy, i.e., only one eigenfunction Ψ_i for each eigenvalue $(\Delta E)_i$.

For weakly magnetized plasma, the energy correction ΔE depends on the quantum number m_j, which removes the degeneracy. Besides, ΔEalso strongly depends on the Lande's factor G, hence the quantum numbers of the plasma electron state.

The theory presented here is of most interest for systems where at least one of the parameters $\hbar\omega_p/mc^2$ or $\mu_s B/mc^2$ is not too small. Examples include e.g., laser-plasma interactions, astrophysical objects, solid state plasmas, and strongly magnetized systems.

As we have seen above, the addition of a magnetic field results in two extra terms in the Hamiltonian. The first arises because the electron is charged. The second term that arises from a magnetic field is the coupling to the spin. Combining the two terms linear in B gives the so-called plasma Zeeman Hamiltonian.

By using the results presented here, a unified treatment of the Zeeman effect becomes possible over the entire range of magnetic fields presently employed in e.g., fusion plasma, where the influence of the Zeeman effect on the plasma temperature measurements has been demonstrated to be significant in many cases.

Authors are very interested to investigate, in due course, the application of their methods to

(i) A relativistic dense plasma immersed in oscillating inhomogeneous magnetic field.
(ii) The plasma degeneracy in the presence of an electric field.

Data Availability

The data used to support the findings of this study are included within the article (see references). The more data used to support the findings of this study are available from the corresponding author (Sh. M. Khalil, Mohamed_khalil@post.com) upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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