

## Research Article

# Adaptive Control Design for Arneodo Chaotic System with Uncertain Parameters and Input Saturation

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In this paper, tracking controller and synchronization controller of the Arneodo chaotic system with uncertain parameters and input saturation are considered. An adaptive tracking control law and an adaptive synchronization control law are proposed based on backstepping and Lyapunov stability theory. The adaptive laws of the unknown parameters are derived by using the Lyapunov stability theory. To handle the effect caused by the input saturation, an auxiliary system is used to compensate the tracking error and synchronization error. The proposed adaptive tracking control and synchronization schemes ensure the effects of tracking and synchronization. Several examples have been detailed to illuminate the design procedure.

## 1. Introduction

Although chaotic systems are extremely complex nonlinear systems highly sensitive to initial value and parametric uncertainties [1], they have been well known owing to their potential applications in communications, information, chemical reactions, lasers, biological systems, etc. [2]. Synchronization and tracking control in chaotic systems has received more and more attention since several studies on suppression of chaotic motions [3–6]. In the past decades, lots of powerful control approaches have been proposed for chaotic synchronization and chaotic tracking, for example, backstepping control [7, 8], impulsive control [3, 9], sliding mode control [10, 11], and adaptive control [12, 13].

A challenge in the chaotic synchronization and tracking control is the fact that the chaotic systems have mostly uncertain parameters or dynamics. In reference [14], the strict-feedback form chaotic system with unknown parameter is studied by using adaptive backstepping. In reference [15], the adaptive backstepping technique is adopted to realize the synchronization between two chaotic systems with uncertainties. The adaptive synchronization for two different chaotic systems with uncertainty is given in reference [16]. To deal with the unknown parameters in chaotic system,

T-S fuzzy system is used for modeling of chaotic systems [17], and fuzzy neural network is used for modeling of chaotic systems [18]. A class of chaotic systems with time-varying unknown bounded parameters is stabilized by a novel robust adaptive controller [19]. LMIs and Barbalat's lemma is adopted to synchronize chaotic systems with uncertainty [20]. The synchronization between two different chaotic systems with unknown parameters and external disturbances is realized by a robust adaptive sliding mode controller [21]. In reference [22], the output feedback adaptive robust controller for uncertain chaotic systems is studied. Considering the unmeasured states and unknown parameters, a novel neural network-based adaptive observer and an adaptive controller have been designed [23]. To handle the disturbance, a sliding mode RBF neural network controller is presented by using the disturbance observer [24].

Unfortunately, input saturation has not been considered in most of the abovementioned works. However, in practical physical systems, there exist limitations for input, known as input saturation problem. Moreover, the limitations of input can cause serious influence on performance and stability. Tracking control for the chaotic systems with input nonlinearities via variable structure design is studied [25], and synchronization of the chaotic systems with input nonlinearities

is realized by an adaptive sliding mode controller [26]; however, the limitations of input are not considered. An adaptive neural synchronization control with Nussbaum-type function is developed for chaotic system, which has unknown control directions and input saturation [27]. An adaptive controller based on fuzzy neural is given for uncertain chaotic systems, in which the auxiliary system is used to deal with saturation [28]. However, owing to the adaptive neural network, the abovementioned works are complex in computation.

Motivated by the above works, both tracking and synchronization control for the Arneodo chaotic system with unknown parameters and input saturation is developed in this paper. To handle the input saturation, an auxiliary system has been constructed, similar to [29]. The unknown parameter adaptive law is obtained based on the Lyapunov stability analysis. With the proposed schemes, the output of the Arneodo chaotic system with uncertain parameters can track the expected trajectory. Furthermore, the synchronization of two chaotic systems with different initial states can be realized. Theoretical analysis and simulations demonstrate the effectiveness of the proposed method. Compared with the above works, the main merits of this paper are listed as follows. (a) A systematic design scheme is presented for both synchronization and tracking control of chaotic systems. (b) The transient performance is adjustable by choosing proper design parameters and can also be adjusted by choosing the initial value. (c) Both unknown parameters and input saturation are considered in this paper; the auxiliary system is designed to deal with saturation problem.

The rest of the brief is organized as follows. In Section 2, system description and problem are presented. In Section 3, the design procedure of the adaptive control is given. In Section 4, adaptive synchronization for chaotic systems is given. Simulation results are included in Section 5. Finally, some concluding remarks are included in Section 6.

## 2. System Description

Consider the Arneodo system ([5]) in the following form of

$$\begin{aligned}\dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u(v), \\ y &= x_1,\end{aligned}\tag{1}$$

where  $x_1, x_2, x_3$  are the system states,  $y \in R$  is the output, and  $a, b$  are unknown parameters;  $v \in R$  is the control input.  $u(v) \in R$  denotes the magnitude of the plant input, which can be described by

$$u(v) = \text{sat}(v) = \begin{cases} \text{sign}(v)u_M, & |v| \geq u_M, \\ v, & |v| < u_M. \end{cases}\tag{2}$$

When there is no input and  $a, b > 0$ , the Arneodo system (1) is unstable at the origin. And the Arneodo system (1) undergoes *chaotic* behavior, when  $a = 7.5, b = 3.8$ .

The goal is to design adaptive tracking and synchronization controller for the Arneodo system (1) such that the closed loop system is globally stable; meanwhile, the tracking error and synchronization error are adjustable by the designed parameters.

## 3. Adaptive Tracking Control

In this section, an adaptive tracking controller is designed for stabilizing the uncertain Arneodo system with input saturation. Define the tracking error vectors as the following:

$$\begin{aligned}e_1 &= y_1 - y_r = x_1 - y_r, \\ e_2 &= x_2 - x_{2d}, \\ e_3 &= x_3 - x_{3d},\end{aligned}\tag{3}$$

where  $\mathbf{x} = [x_1, x_2, x_3]^T \in R^3$  are the Arneodo system states,  $y_r$  is the reference trajectory, and the three-order derivative of  $y_r$  exists.  $x_{2d}, x_{3d}$  are the visual controllers in the following backstepping design.

*Remark 1.* The three-order derivative of  $y_r$  means  $y_r, \dot{y}_r, \ddot{y}_r$  is bounded in a compact set.

To compensate the effect of the saturation, the auxiliary signals  $\lambda \in R^3$  are generated by the following system:

$$\begin{aligned}\dot{\lambda}_1 &= \lambda_2 - c_1 \lambda_1, \\ \dot{\lambda}_2 &= \lambda_3 - c_2 \lambda_2, \\ \dot{\lambda}_3 &= -c_3 \lambda_3 + \Delta u,\end{aligned}\tag{4}$$

where  $c_1, c_2, c_3$  are design parameters,  $\Delta u = u(v) - v$ .

*Remark 2.* When  $c_1, c_2, c_3 > 0$ , the plant is BIBO, i.e., bounded input bounded output stable. And the error  $\Delta u$  has no effect on  $e_i$ , because it is the input of the constructed system (4).

Then, the tracking error vectors (3) can be compensated as follows:

$$\begin{aligned}z_1 &= e_1 - \lambda_1, \\ z_2 &= e_2 - \lambda_2, \\ z_3 &= e_3 - \lambda_3.\end{aligned}\tag{5}$$

The design scheme of the adaptive backstepping control is given.

*Step 1.* Starting as derivative the compensated tracking error (5), then we obtained

$$\dot{z}_1 = \dot{x}_2 - \dot{y}_r - \lambda_2 + c_1 \lambda_1 = z_2 + x_{2d} - \dot{y}_r + c_1 \lambda_1.\tag{6}$$

Choose the following virtual control:

$$x_{2d} = -c_1(x_1 - y_r) + \dot{y}_r, \quad (7)$$

where  $c_1 > 0$  is a positive constant to be designed. A Lyapunov function  $V_1$  is defined as

$$V_1 = \frac{1}{2}z_1^2, \quad (8)$$

Combining with formula (7), the derivative of  $V_1$  along (6) is given as

$$\dot{V}_1 = -c_1z_1^2 + z_1z_2. \quad (9)$$

*Step 2.* For  $z_2 = x_2 - x_{2d} - \lambda_2$ , we can design the virtual control law  $x_{3d}$  as

$$x_{3d} = -c_2(x_2 - x_{2d}) + \dot{x}_{2d} - z_1, \quad (10)$$

where  $c_2 > 0$  is a positive constant to be designed. With formula (10), the derivative of  $z_2$  can be written as

$$\dot{z}_2 = z_3 - c_2z_2 - z_1. \quad (11)$$

Then, the Lyapunov function can be chosen as

$$V_2 = V_1 + \frac{1}{2}z_2^2. \quad (12)$$

The derivative of  $V_2$  along (11) can be given as

$$\dot{V}_2 = -c_1z_1^2 - c_2z_2^2 + z_2z_3. \quad (13)$$

*Step 3.* We can obtain the derivative of  $z_3$  as follows:

$$\dot{z}_3 = ax_1 - bx_2 - x_3 - x_1^2 - \dot{x}_{3d} + c_3\lambda_3 + v. \quad (14)$$

Then, the adaptive control law  $v$  can be designed as follows:

$$v = -c_3(x_3 - x_{3d}) - z_2 - \hat{a}x_1 + \hat{b}x_2 + x_3 + x_1^2 + \dot{x}_{3d}, \quad (15)$$

where  $c_3 > 0$  is a positive constant to be designed and  $\hat{a}, \hat{b}$  are the estimate of  $a, b$ . Choose Lyapunov function as

$$V_3 = V_2 + \frac{1}{2}z_3^2. \quad (16)$$

Combining with formula (15), the derivative of  $V_3$  along (14) is given as

$$\dot{V}_3 = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2 + \tilde{a}x_1z_3 - \tilde{b}x_2z_3, \quad (17)$$

where  $\tilde{a} = a - \hat{a}$ ,  $\tilde{b} = b - \hat{b}$ . The parameter update laws can be designed as

$$\begin{aligned} \dot{\hat{a}} &= \frac{1}{\Gamma_a}x_1z_3, \\ \dot{\hat{b}} &= -\frac{1}{\Gamma_b}x_2z_3, \end{aligned} \quad (18)$$

where  $\Gamma_a, \Gamma_b$  are positive-designed parameters. We define the Lyapunov function  $V$  as

$$V = V_3 + \frac{\Gamma_a}{2}\tilde{a}^2 + \frac{\Gamma_b}{2}\tilde{b}^2. \quad (19)$$

Then, the derivative of  $V$  along with (18) is given as

$$\dot{V} = -c_1z_1^2 - c_2z_2^2 - c_3z_3^2, \quad (20)$$

which is a negative definite function, and it manifests that  $V$  is bounded. Thus,  $z_i$ ,  $i = 1, 2, 3$ , and  $\hat{a}, \hat{b}, \tilde{a}, \tilde{b}$  are bounded. From Remark 1 and Remark 2, we have found that  $x_i$ ,  $i = 1, 2, 3$ , are bounded. Therefore, the boundedness of  $x_{2d}, x_{3d}$  and the control signal  $v$  can be obtained from (7), (10), and (15). Thus,  $\Delta u = u(v) - v$  is also bounded. Therefore, all signals in the closed loop system are bounded, which is stated in the following theorem.

**Theorem 3.** For the uncertain chaotic system (1) with input saturation, the control signal  $v$  (15) can ensure the boundedness of all signals and the following statements:

$$\|y - y_r\|_2 \leq \sqrt{\Gamma_a\tilde{a}(0)^2 + \Gamma_b\tilde{b}(0)^2} + \frac{1}{\sqrt{c_0}}\|\Delta u\|. \quad (21)$$

*Proof.* From equation (20), we have the following inequation:

$$\dot{V} \leq -2c_{\min}V, \quad (22)$$

where  $c_{\min} = \min(c_1, c_2, \tau_a, \tau_b)$ , then we can have

$$\|z_1\|_2^2 = \|y - y_r - \lambda_1\|_2^2 \leq 2V \leq 2V(0)e^{-2c_{\min}t} \leq 2V(0). \quad (23)$$

From the auxiliary system (4), we define the positive Lyapunov function  $V_\lambda = 1/2\sum_{i=1}^3\lambda_i^2$ . Then, the derivative of  $V_\lambda$  along the auxiliary system (4) is given as

$$\begin{aligned} \dot{V}_\lambda &= -c_1\lambda_1^2 + \lambda_1\lambda_2 - c_2\lambda_2^2 + \lambda_2\lambda_3 - c_3\lambda_3^3 + \lambda_3\Delta u \\ &\leq -\sum_{i=1}^3\tilde{c}_i\lambda_i^2 + \Delta u^2 \\ &\leq -c_0\|\lambda\|^2 + \Delta u^2, \end{aligned} \quad (24)$$

where  $\bar{c}_1 = c_1 - (1/2)$ ,  $\bar{c}_2 = c_2 - 1$ ,  $\bar{c}_3 = c_3 - (3/4)$ ,  $c_0 = \min_{1 \leq i \leq 3} \bar{c}_i$ . Integrating both sides of (24), we can obtain the following:

$$\|\lambda_1\|_2^2 \leq \|\lambda\|_2^2 \leq \frac{1}{c_0} \left[ V_\lambda(0) + \int_0^\infty (\Delta u)^2 dt \right]. \quad (25)$$

By setting initial parameters  $\lambda_i = 0$ ,  $e_i(0) = 0$ ,  $i = 1, 2, 3$ , combine formulas (23) and (25), we have

$$\|y - y_r\|_2 \leq \sqrt{\Gamma_a \tilde{a}(0)^2 + \Gamma_b \tilde{b}(0)^2} + \frac{1}{\sqrt{c_0}} \|\Delta u\|. \quad (26)$$

From Theorem 3, we can draw the following conclusions.

*Remark 4.* The initial estimate error and the designed parameters determine the transient performance. The smaller the initial estimate error, the better the transient performance.

*Remark 5.* The bound of  $\|y - y_r\|_2$  depends on the bound of  $\Delta u$ , and increasing parameter  $c_0$  can improve the system transient performance. If  $\Delta u \rightarrow 0$  as  $t \rightarrow \infty$ ,  $\lambda_1 \rightarrow 0$ . Therefore,  $\lim_{t \rightarrow \infty} [y - y_r] = 0$ . It implies that no input saturation as  $t \rightarrow \infty$ , perfect tracking is ensured.

*Remark 6.* When there exists a bounded external disturbance  $|w(t)| \leq D$ , where  $D$  is the upper bound. The chaotic system (1) can be described as follows:

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2 + u(v) + w(t). \end{aligned} \quad (27)$$

Then, the adaptive control law (15) can be modified as follows:

$$\begin{aligned} v &= -c_3(x_3 - x_{3d}) - z_2 - \hat{a}x_1 + \hat{b}x_2 \\ &\quad + x_3 + x_1^2 + \dot{x}_{3d} - D \text{sign}(z_3), \end{aligned} \quad (28)$$

where  $-D \text{sign}(z_3)$  is a robust term to handle the external disturbance. As a result, the derivative of (19) along with (18) is given as

$$\dot{V} \leq -c_1 z_1^2 - c_2 z_2^2 - c_3 z_3^2. \quad (29)$$

#### 4. Adaptive Synchronization

As the master system, we take the Arneodo dynamics described by

$$\begin{aligned} \dot{x}_1 &= x_2, \\ \dot{x}_2 &= x_3, \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 - x_1^2, \end{aligned} \quad (30)$$

where  $a, b$  are unknown parameters. As the slave system, we consider the controlled Arneodo dynamics described by

$$\begin{aligned} \dot{y}_1 &= y_2, \\ \dot{y}_2 &= y_3, \\ \dot{y}_3 &= ay_1 - by_2 - y_3 - y_1^2 + u(v), \end{aligned} \quad (31)$$

where  $v \in R$  is the control input.  $u(v) \in R$  denotes the constrained input described in (2). Define the synchronization error as

$$e_i = y_i - x_i, \quad (i = 1, 2, 3). \quad (32)$$

Then, the error dynamics is obtained as

$$\begin{aligned} \dot{e}_1 &= e_2, \\ \dot{e}_2 &= e_3, \\ \dot{e}_3 &= ae_1 - be_2 - e_3 - (y_1 + x_1)e_1 + u(v). \end{aligned} \quad (33)$$

Combining the auxiliary system (4), we can define the compensatory synchronization error

$$\begin{aligned} \hat{e}_1 &= e_1 - \lambda_1, \\ \hat{e}_2 &= e_2 - e_{2d} - \lambda_2, \\ \hat{e}_3 &= e_3 - e_{3d} - \lambda_3, \end{aligned} \quad (34)$$

where  $e_{2d}, e_{3d}$  are the virtual controllers in the backstepping design. The control design procedure is omitted in here, which is similar to Section 3. The main results can be described in the following theorem.

**Theorem 7.** *The slave Arneodo system with unknown parameters (31) can synchronize with the master system by the following controller:*

$$\begin{aligned} e_{2d} &= -c_1 e_1, \\ e_{3d} &= -c_2(e_2 - e_{2d}) + \dot{e}_{2d} - \hat{e}_1, \\ v &= -c_3(e_3 - e_{3d}) - \hat{e}_2 + e_3 + (y_1 + x_1)e_1 \\ &\quad + \dot{e}_{3d} - \hat{a}e_1 + \hat{b}e_2, \end{aligned} \quad (35)$$

where  $\hat{a}, \hat{b}$  are estimates of the unknown parameters  $a, b$  and the parameter update laws are given by

$$\begin{aligned} \dot{\hat{a}} &= \frac{1}{\Gamma_a} \hat{e}_3 e_1, \\ \dot{\hat{b}} &= -\frac{1}{\Gamma_b} \hat{e}_3 e_2. \end{aligned} \quad (36)$$

*Proof.* Similar with Section 3, we define the Lyapunov function

$$V = \frac{1}{2} \sum_{i=1}^3 \widehat{e}_i^2 + \frac{\Gamma_a}{2} \widehat{a}^2 + \frac{\Gamma_b}{2} \widehat{b}^2. \quad (37)$$

Differentiating  $V$  along the derivative of the compensatory synchronization error system (34), we get

$$\dot{V} = -c_1 \widehat{e}_1^2 - c_2 \widehat{e}_2^2 - c_3 \widehat{e}_3^2, \quad (38)$$

which is a negative definite function. Hence, similar to the analysis in Section 3, we can obtain that the slave Arneodo system synchronizes with the master system.

*Remark 8.* When there exists a bounded external disturbance  $|w(t)| \leq D$ , where  $D$  is the upper bound. Then, the adaptive control law  $v$  in equation (35) can be modified as follows:

$$v = -c_3(e_3 - e_{3d}) - \widehat{e}_2 + e_3 + (y_1 + x_1)e_1 + \dot{e}_{3d} - \widehat{a}e_1 + \widehat{b}e_2 - D \text{sign}(\widehat{e}_3). \quad (39)$$

As a result, the derivative of (37) along with (36) is given as

$$\dot{V} \leq -c_1 \widehat{e}_1^2 - c_2 \widehat{e}_2^2 - c_3 \widehat{e}_3^2. \quad (40)$$

## 5. Numerical Simulations

*5.1. Tracking Control Example.* We take the reference trajectory as  $y_r = \sin(t)$ , and the true parameters in the chaotic system are taken as  $a = 7.5, b = 3.8$ . We choose  $\Gamma_a = \Gamma_b = 5$  for the adaptive and update laws. Suppose that the initial values of the estimated parameters are  $\widehat{a}(0) = 2, \widehat{b}(0) = 5$ . The parameters in the auxiliary system are set as  $c_1 = c_2 = c_3 = 5$ , and the initial states of the auxiliary system are  $\lambda_1(0) = \lambda_2(0) = \lambda_3(0) = 0$ . The initial state of the system (1) is selected as  $x_1 = 3, x_2 = 8, x_3 = -1$  and the control input is constrained by  $u_M = 20$ . Figures 1–4 show simulation results for the Arneodo system (1) with the control law (15) and the parameter update law (18). Figure 1 shows that the output of the Arneodo system (1) can asymptotically converge to the reference trajectory. Figure 2 shows the control law constrained by  $u_M = 20$ .

From Figures 1 and 2, the proposed control can achieve that the output of the Arneodo system (1) converge to the reference trajectory with input saturation. The time response of the parameter estimates  $\widehat{a}, \widehat{b}$  is shown in Figure 3. The time response of the parameter estimation errors  $\widetilde{a}, \widetilde{b}$  is displayed in Figure 4.

When the external disturbance is assumed to be  $w(t) = \sin(x_1 t)$  or a random noise less than 1, then the parameter  $D$  can be designed as  $D = 1$ . The other parameters are designed as above, and then, the simulation results are displayed in Figures 5–8.

From Figures 5–8, it can be concluded that whatever the external disturbance is, as long as it has an upper bound, the control method presented in this paper is applicable.

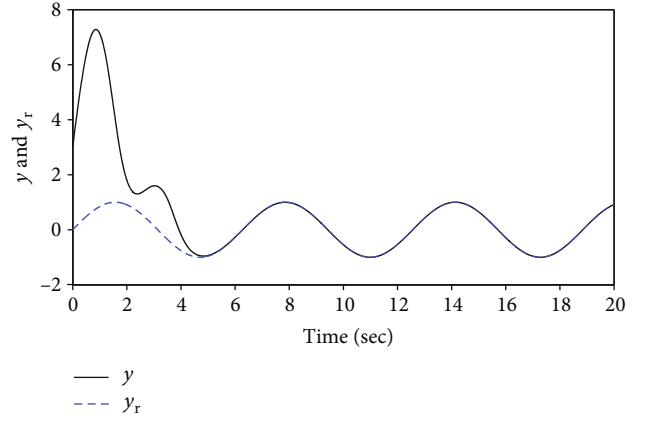


FIGURE 1: Tracking performance.

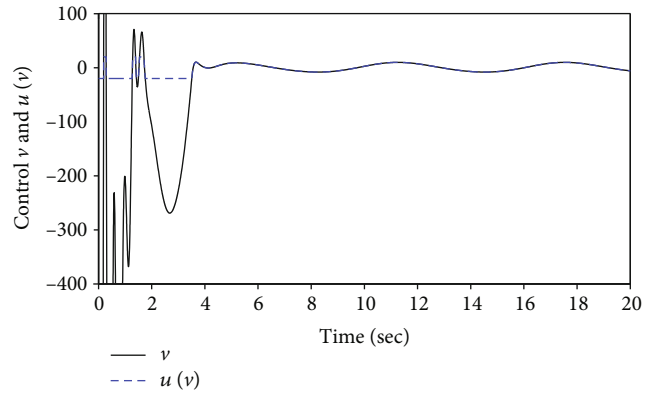


FIGURE 2: Control signal.

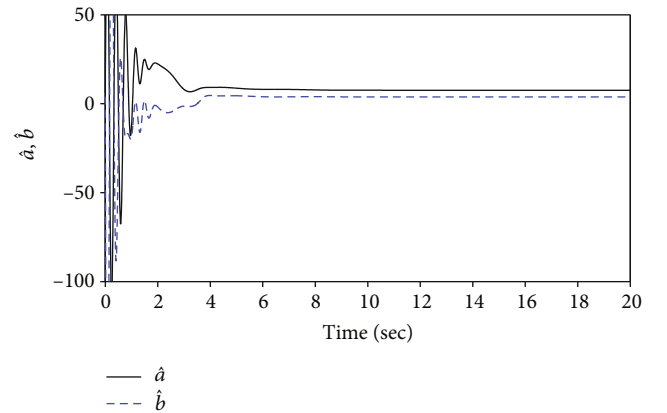


FIGURE 3: Time response of the parameter estimates.

*5.2. Synchronization Example.* The initial parameters can be selected as  $c_1 = c_2 = c_3 = 1, \Gamma_a = \Gamma_b = 1, u_M = 20, \lambda_1(0) = \lambda_2(0) = \lambda_3(0) = 0$ , and the true parameters in the chaotic system are taken as  $a = 7.5, b = 3.8$ .

The initial state of the system (30) is selected as  $x_1 = 3, x_2 = 8, x_3 = -1$ , and the initial state of the system (31) is selected as  $x_1 = 1, x_2 = 3, x_3 = 2$ . The external disturbance is

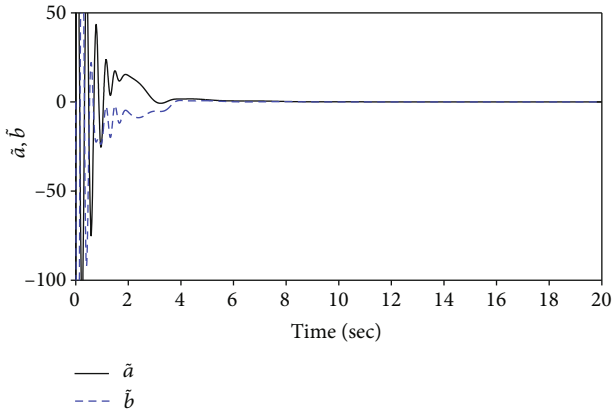


FIGURE 4: Time response of the parameter estimation errors.

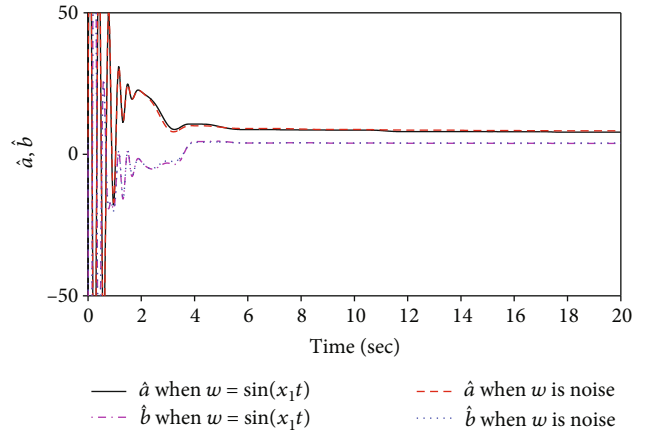


FIGURE 7: Time response of the parameter estimates.

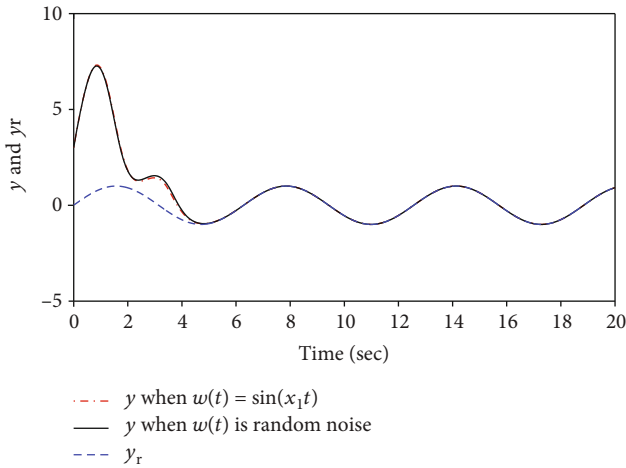


FIGURE 5: Tracking performance.

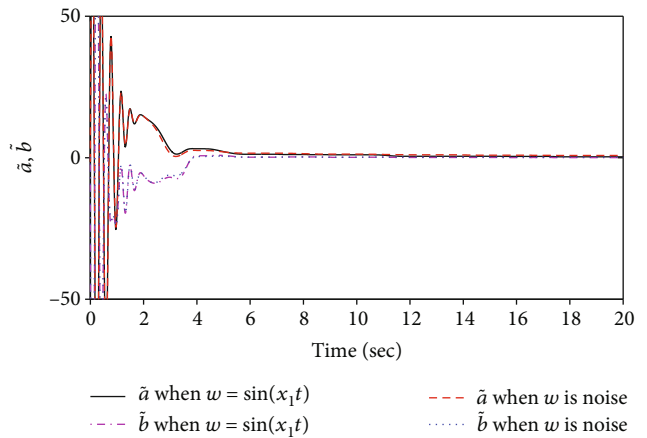


FIGURE 8: Time response of the parameter estimation errors.

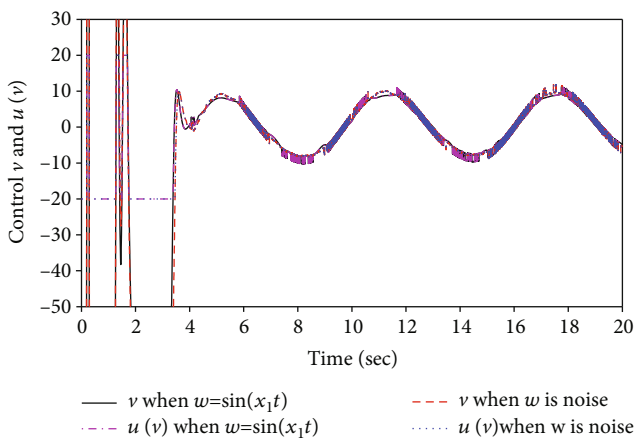


FIGURE 6: Control signal.

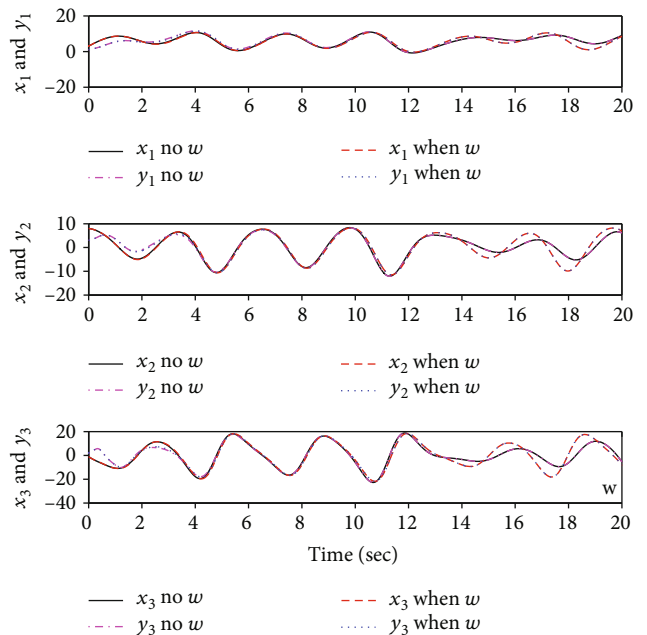


FIGURE 9: Synchronization of states.

assumed to be  $w(t) = \sin(x_1 t)$ . From Figures 9 and 10, we can see that the slave system and the master system achieve the synchronization by the proposed method even with the outside disturbance, although there exist input saturation and uncertain parameters. Figure 11 shows the adaptive

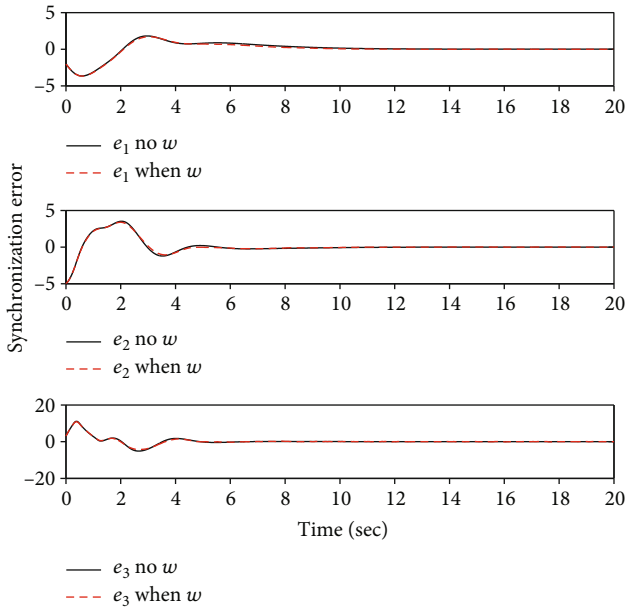


FIGURE 10: Synchronization error of states.

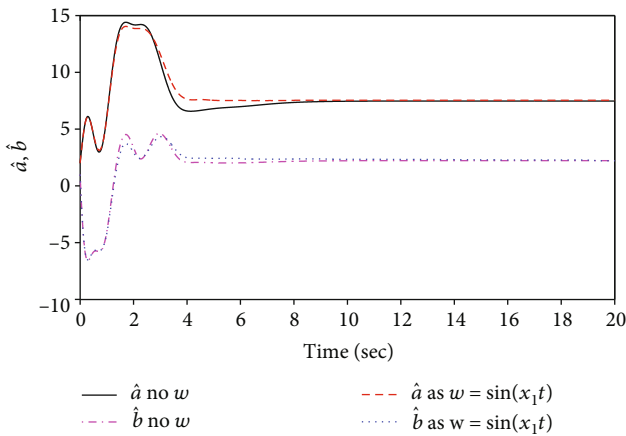


FIGURE 11: Time response of the parameter estimates.

estimation of parameters  $a, b$ . The control law constrained by  $u_M = 20$  is shown in Figure 12.

### 6. Conclusions

In this paper, we developed adaptive tracking and synchronization control design method for the Arneodo chaotic system with input saturation and unknown parameters. To handle the effect caused by the input saturation, an auxiliary system has been constructed to compensate the tracking error and synchronization error. Then, an adaptive tracking control and adaptive synchronization control were proposed based on backstepping. To handle the external disturbance, a robust term is added to the control. The main results derived in this paper were proved via the Lyapunov theorem analysis. Simulation results demonstrate that the tracking and syn-

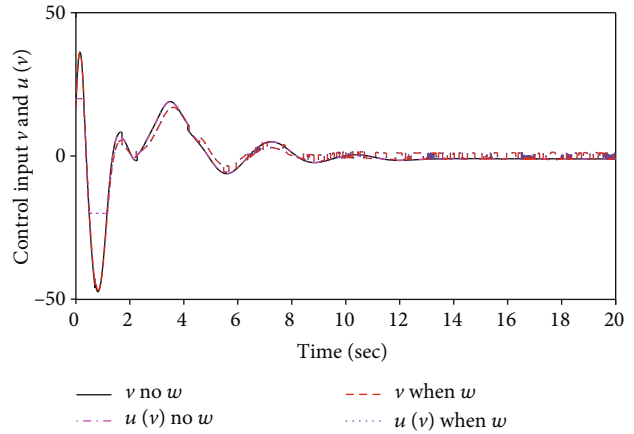


FIGURE 12: Control signal.

chronization were achieved with the proposed adaptive tracking controller and synchronization controller; meanwhile, the uncertain parameters converge to their actual values.

### Data Availability

The simulation of this paper does not require other data.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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