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Research Article

Rational Solutions for the $(2+1)$-Dimensional Modified KdV-CBS Equation

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In this paper, with the help of symbolic computation, three types of rational solutions for the $(2+1)$-dimensional modified KdV-Calogero-Bogoyavlenkskii-Schiff equation are derived. By means of the truncated Painlevé expansion, we show that the $(2+1)$-dimensional modified KdV-Calogero-Bogoyavlenkskii-Schiff equation can be written as a trilinear-linear equation, from which we get explicit representation for rational solutions of the $(2+1)$-dimensional modified KdV-Calogero-Bogoyavlenkskii-Schiff equation.

1. Introduction

It is an important problem to seek exact solutions in the study of the nonlinear evolution equations (NLEEs). With regard to the methods for solving NLEEs, various techniques and approaches have been proposed, for instance, the Darboux transformation [1], the Lie group method [2–4], and the Hirota bilinear method [5]. Recently, based on the Hirota bilinear method, a method for constructing a lump solution by taking the function f in its bilinear form as a positive quadratic function, proposed by Ma [6], has been used to solve many NLEEs. By using this method, the phenomenon of the rogue wave was found in Refs. [7–9]. And then, more and more integrable soliton equations are found to have lump solutions and mixed kink-lump solutions [10–23].

In this paper, we focus on the following $(2+1)$-dimensional modified KdV-Calogero-Bogoyavlenkskii-Schiff (KdV-CBS) equation

$$u_t - 4u^2u_y - 2u_x\partial_x^{-1}\left(u^2\right)_y + 2u_{xxy} - 6u^2u_x + u_{xxx} = 0, \quad (1)$$

which can be derived from the $(2+1)$-dimensional KdV-CBS equation [24, 25]

$$q_t + 4qq_y - 2qx\partial_x^{-1}q_y + q_{xxy} - 6qq_x + q_{xxx} = 0, \quad (2)$$

by means of the Miura transformation $q = u^2 - u_x$ [26]. Here, the subscripts x and y denote the partial derivatives with respect to x and y, respectively. In order to treat the integral appearing in equation (1), so that equation (1) can be better analyzed and solved, let $v_x = 2uu_y$; equation (1) is transformed into the following system:

$$\begin{aligned} u_t - 4u^2u_y - 2u_xv + u_{xxy} - 6u^2ux + u_{xxx} &= 0, \\ 2uu_y &= v_x. \end{aligned} \quad (3)$$

The soliton-cnoidal wave solutions of equation (3) were obtained in Ref. [27]. Interesting studies on the family of KdV-type equations have been carried out in the following papers [28–31]. The aim of this paper is to construct the rational solutions of system (3) by solving a trilinear-linear equation related to its truncated Painlevé expansion.

The outline of this paper is as follows. In Section 2, using the truncated Painlevé expansion, we convert the original modified KdV-CBS equation to a trilinear-linear equation. In Section 3, the first class of rational solutions for the modified KdV-CBS equation is obtained by taking function f as a positive quadratic function. In Section 4, the second class of rational solutions is obtained by taking function f as a positive quadratic and an exponential function. In Section 5, the third class of rational solutions is obtained by taking function f as a positive quadratic and two exponential functions. The conclusion will be given in Section 6.

2. Trilinear-Linear Equation

Based on the Painlevé analysis proposed in Ref. [32], equation (3) possesses a truncated Painlevé expansion as follows:

$$u = u_0 + \frac{u_1}{f}, \qquad v = v_0 + \frac{v_1}{f} + \frac{v_2}{f^2}, \tag{4}$$

with u_0, u_1, v_0, v_1, v_2, and f being the function of x, y, and t. We substitute (4) into (3); the results can be obtained as follows:

$$\begin{aligned} v_1 &= -f_{xy},\\ v_2 &= f_x f_y,\\ u_0 &= -\frac{f_{xx}}{2f_x},\\ u_1 &= f_x,\\ v_0 &= \frac{1}{2}\frac{f_t}{f_x} - \frac{1}{2}\frac{f_{xx}f_{xy}}{f_x^2} + \frac{1}{2}\frac{f_{xxy}}{f_x} + \frac{1}{2}\frac{f_{xxx}}{f_x} - \frac{3}{4}\frac{f_{xx}^2}{f_x^2}, \end{aligned} \tag{5}$$

with f satisfying the following trilinear-linear equation:

$$\begin{aligned} &3f_{xx}f_{yxx}f_x - f_{yxxx}f_x^2 - f_{xt}f_x^2 - f_{xxxx}f_x^2 + f_{xx}f_t f_x \\ &\quad + 4f_{xxx}f_{xx}f_x + f_y f_{xxx}f_x - 3f_{xx}^3 - 3f_{xx}^2 f_{yx} = 0. \end{aligned} \tag{6}$$

By solving the trilinear-linear equation (6), we can get

$$u = -\frac{1}{2}\frac{f_{xx}}{f_x} + \frac{f_x}{f}, \tag{7a}$$

$$v = \frac{1}{2}\frac{f_t}{f_x} - \frac{1}{2}\frac{f_{xx}f_{xy}}{f_x^2} + \frac{1}{2}\frac{f_{yxx}}{f_x} + \frac{1}{2}\frac{f_{xxx}}{f_x} - \frac{3}{4}\frac{f_{xx}^2}{f_x^2} - \frac{f_{yx}^2}{f} + \frac{f_x f_y}{f^2}. \tag{7b}$$

Based on the idea in Refs. [6, 7], we take f as

$$\begin{aligned} f &= (a_1x + a_2y + a_3t + a_4)^2 + (a_5x + a_6y + a_7t + a_8)^2 \\ &\quad + a_9 + \alpha \exp(k_1x + k_2y + k_3t + k_4) \\ &\quad + \beta \exp(-(k_1x + k_2y + k_3t + k_4)), \end{aligned} \tag{8}$$

where $a_i, k, k_j, 1 \le i \le 9$, and $1 \le j \le 4$ are real parameters to be determined. Substituting (8) into trilinear-linear equation (6), we get a set of solutions. In the following, we will give three types of rational solutions of equation (3) via (8).

3. Rational Solutions from the Quadratic Function

Substitution equation (8) into trilinear-linear equation (6), we will get the following set of constraining equations for the parameters

$$\begin{aligned} a_1 &= a_1,\\ a_2 &= -\frac{a_1^2 + a_5^2 + a_5a_6}{a_1},\\ a_3 &= a_3,\\ a_4 &= a_4,\\ a_5 &= a_5,\\ a_6 &= a_6,\\ a_7 &= \frac{a_3a_5}{a_1},\\ a_8 &= a_8,\\ a_9 &= a_9,\\ \alpha &= 0,\\ \beta &= 0,\\ k_1 &= k_1,\\ k_2 &= k_2,\\ k_3 &= k_3,\\ k_4 &= k_4, \end{aligned} \tag{9}$$

where

$$a_1 \ne 0, \qquad a_9 > 0, \tag{10}$$

to guarantee that the corresponding f is positive. By substituting (9) into (8), the function f can be obtained as follows:

$$\begin{aligned} f &= \left(a_1x - \frac{a_1^2 + a_5^2 + a_5a_6}{a_1}y + a_3t + a_4\right)^2 \\ &\quad + \left(a_5x + a_6y + \frac{a_3a_5t}{a_1} + a_8\right)^2 + a_9. \end{aligned} \tag{11}$$

By means of equation (7a) and equation (7b), we get the solutions of (3):

$$u = \frac{4(a_1g + a_5h)^2 - \left(2a_1^2 + 2a_5^2\right)f}{(2ga_1 + 2ha_5)f}, \tag{12a}$$

$$v = \frac{a_3(a_1g - a_5h)}{2a_1(a_1g + a_5h)} - \frac{\left(a_1^2 + a_5^2\right)^2}{4(a_1g + a_5h)^2} + \frac{2f\left(a_1^2 + a_5^2\right)^2 + N}{f^2}, \tag{12b}$$

where

$$
\begin{aligned}
f &= g^2 + h^2 + a_9, \\
g &= a_1 x - \frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} y + a_3 t + a_4, \\
h &= a_5 x + a_6 y + \frac{a_3 a_5}{a_1} t + a_8, \\
N &= -2g\left(\frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} + 2h a_6\right)(a_1 g + a_5 h).
\end{aligned} \tag{13}
$$

The solutions of u via (12a) are singular which can be seen in Figure 1(a); Figure 1(b) is the corresponding density plot. The solutions of v via (12b) can be seen in Figure 1(c) and Figure 1(d) is the corresponding density plot.

4. Rational Solutions Obtained by Adding an Exponential Term to the Quadratic Function

Two types of solutions are obtained in the following.

Case I.

$$
\begin{aligned}
a_1 &= 0, \\
a_2 &= a_2, \\
a_3 &= 0, \\
a_4 &= a_4, \\
a_5 &= a_5, \\
a_6 &= -a_5, \\
a_7 &= -\frac{k_3 a_5}{k_2}, \\
a_8 &= a_8, \\
a_9 &= a_9, \\
\alpha &= 0, \\
\beta &= \beta, \\
k_1 &= -k_2, \\
k_2 &= k_2, \\
k_3 &= k_3, \\
k_4 &= k_4,
\end{aligned} \tag{14}
$$

which should satisfy the constraint conditions

$$
\begin{aligned}
k_2 &\neq 0, \\
a_9 &> 0, \\
\beta &> 0,
\end{aligned} \tag{15}
$$

to ensure that the corresponding f is positive and well defined. By substituting (14) into (8), the function f reads

$$
\begin{aligned}
f = {} & (a_2 y + a_4)^2 + \left(a_5 x - a_6 y - \frac{a_3 a_5 t}{k_2} + a_8\right)^2 \\
& + \beta \exp(k_1 x + k_2 y + k_3 t + k_4) + a_9.
\end{aligned} \tag{16}
$$

Using equation (7a) and equation (7b), the solutions for (3) can be obtained:

$$
u = \frac{2(a_5 h + k_2 \beta \exp(\xi))^2 - (2a_5^2 + k_2^2)\beta \exp(\xi) f}{2(k_2 \beta \exp(\xi) + 2h a_5) f}, \tag{17a}
$$

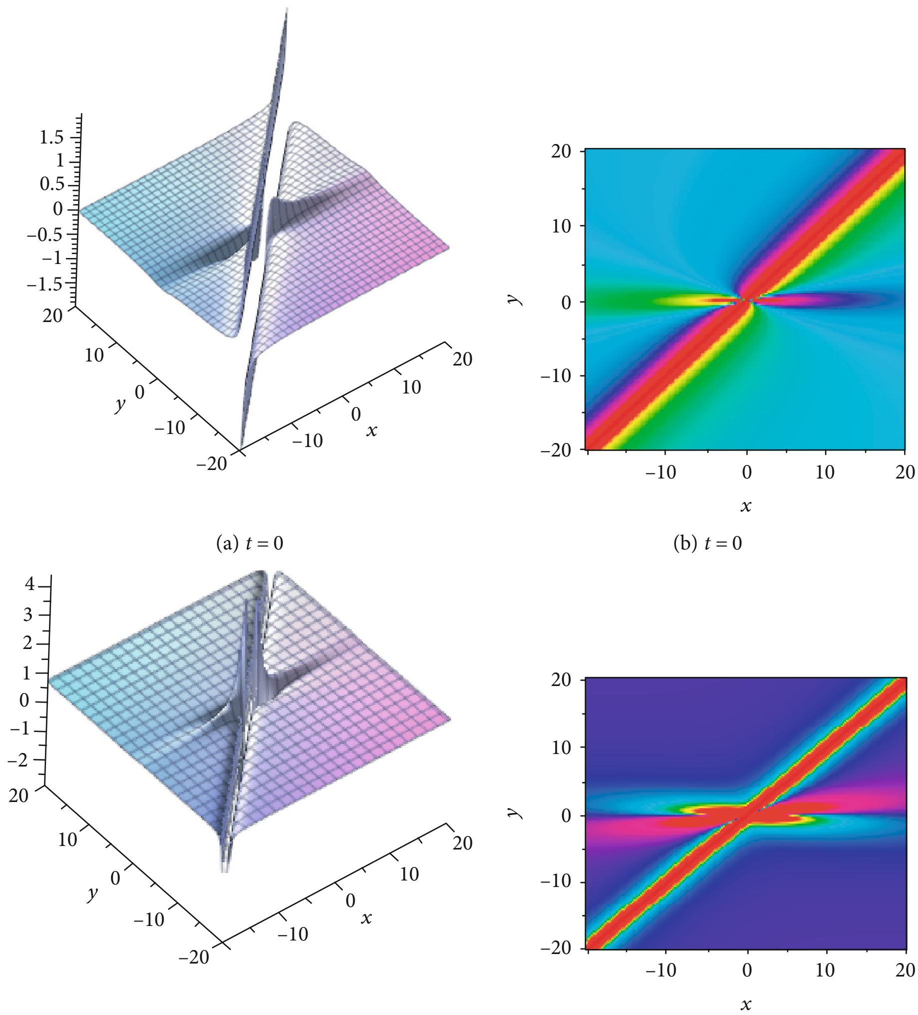
$$
v = \frac{-(k_3(k_2 \beta \exp(\xi) + 2a_5 h))}{k_2(k_2 \beta \exp(\xi) + 4h a_5)} - \frac{B^2}{4A^2} + \frac{A(2g a_2 - A) + Bf}{f^2}, \tag{17b}
$$

where

$$
\begin{aligned}
f &= g^2 + h^2 + \beta \exp(\xi) + a_9, \\
g &= a_2 y + a_4, \\
h &= a_5 x - a_5 y + \frac{k_3 a_5}{k_2} t + a_8, \\
\xi &= k_2 x - k_2 y - k_3 t - k_4, \\
A &= k_2 \beta \exp(\xi) + 2h a_5, \\
B &= k_2^2 \beta \exp(\xi) + 2a_5^2.
\end{aligned} \tag{18}
$$

Case II.

$$
\begin{aligned}
a_1 &= 0, \\
a_2 &= a_2, \\
a_3 &= 0, \\
a_4 &= a_4, \\
a_5 &= a_5, \\
a_6 &= -a_5, \\
a_7 &= -\frac{k_3 a_5}{k_2}, \\
a_8 &= a_8, \\
a_9 &= a_9, \\
\alpha &= \alpha, \\
\beta &= 0, \\
k_1 &= k_1, \\
k_2 &= k_2, \\
k_3 &= k_3, \\
k_4 &= k_4, \\
a_1 &= a_3 = 0, \\
a_6 &= -a_5, \\
a_7 &= -\frac{k_3 a_5}{k_2}, \\
\beta &= 0,
\end{aligned} \tag{19}
$$



(a) $t = 0$ (b) $t = 0$ (c) $t = 0$ (d) $t = 0$

Figure 1: Profiles of the solution u via (12a) and profiles of the solution v via (12b) with $a_1 = 1$, $a_3 = 1.5$, $a_4 = 0$, $a_5 = 2$, $a_6 = 4$, $a_8 = 0$, and $a_9 = 1$. (a) and (c) are the corresponding 3-dimensional plots; (b) and (d) are the corresponding density plots.

which should satisfy the constraint conditions

$$\begin{aligned} &k_2 \neq 0, \\ &a_9 > 0, \\ &\alpha > 0, \end{aligned} \tag{20}$$

to guarantee that the corresponding f is positive and well defined. Substituting (19) into (8), we then have the function f:

$$\begin{aligned} f = {} & (a_2 y + a_4)^2 + \left(a_5 x - a_6 y - \frac{a_3 a_5 t}{k_2} + a_8\right)^2 \\ & + \alpha \exp\left(k_1 x + k_2 y + k_3 t + k_4\right) + a_9. \end{aligned} \tag{21}$$

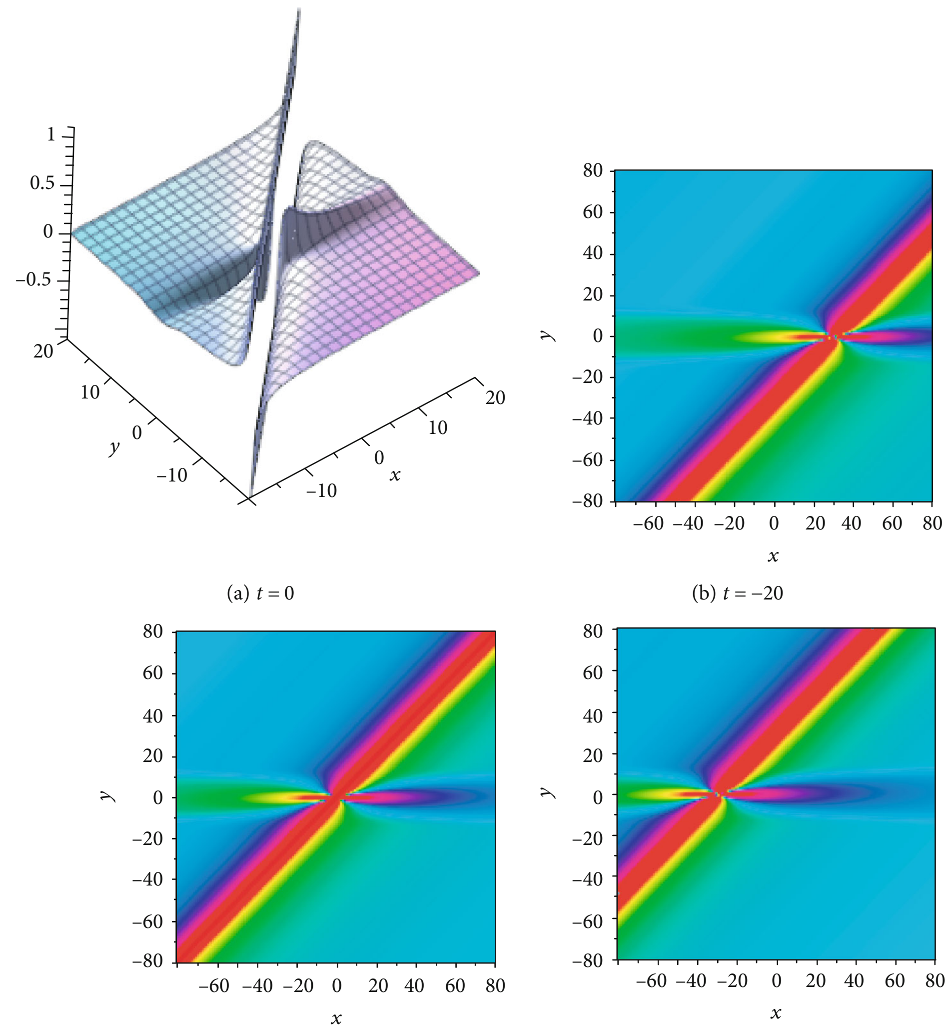
The solutions of this case are similar to the solutions of equation (17a) and equation (17b).

5. Rational Solutions Obtained by Adding Two Exponential Terms to the Quadratic Function

In this section, in order to find interaction solutions of equation (3), we further add two exponential terms to the quadratic function. Three sets of solutions are obtained as follows.

Case I.

$$\begin{aligned} a_1 &= a_1, \\ a_2 &= -\frac{a_1^2 + a_5^2 + a_5 a_6}{a_1}, \\ a_3 &= a_3, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= a_6, \end{aligned}$$



(a) $t = 0$ (b) $t = -20$ (c) $t = 0$ (d) $t = 20$

Figure 2: Profiles of the solution u via (25a) with $a_1 = 1$, $a_3 = 1.5$, $a_4 = 0$, $a_5 = 2$, $a_6 = 4$, $a_8 = 0$, $a_9 = 1$, $\alpha = 1$, $\beta = 1$, $k_2 = 1$, and $k_4 = 0$. (a) 3-dimensional plot; (b–d) the corresponding density plots with different times.

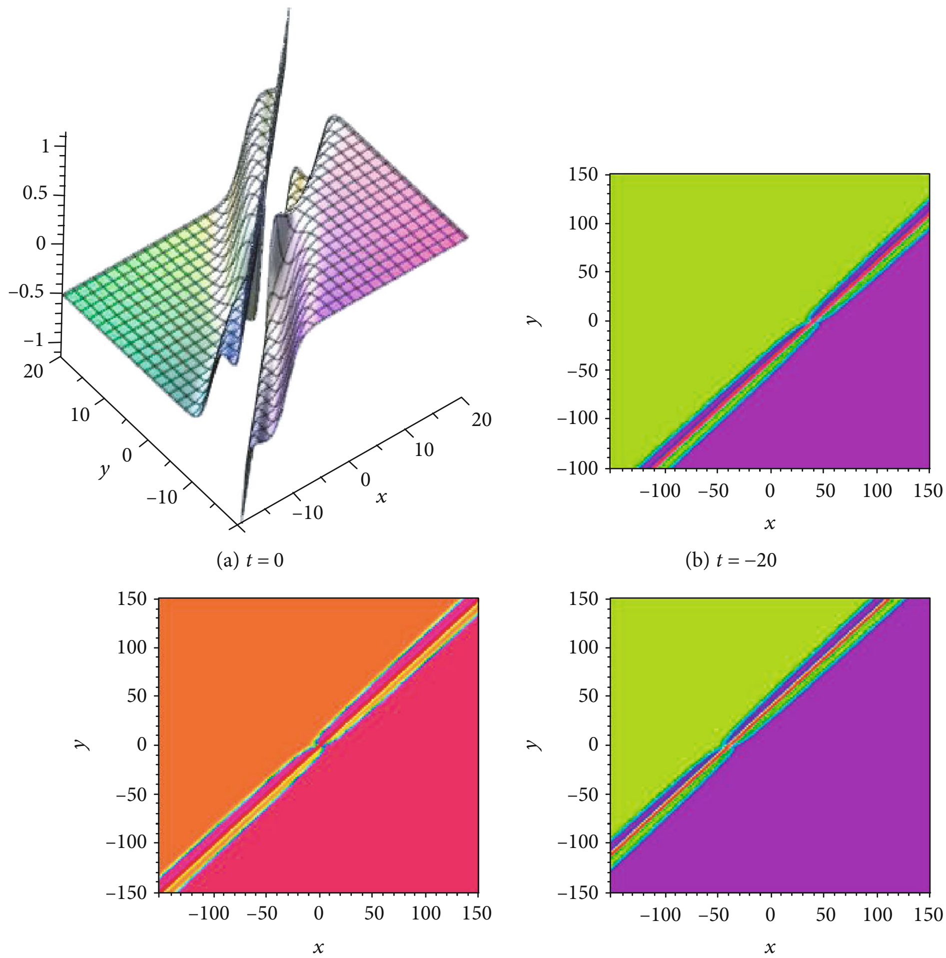
$$\begin{aligned} a_7 &= \frac{a_3 a_5}{a_1}, \\ a_8 &= a_8, \\ a_9 &= a_9, \\ \alpha &= \alpha, \\ \beta &= \beta, \\ k_1 &= 0, \\ k_2 &= k_2, \\ k_3 &= 0, \\ k_4 &= k_4, \end{aligned} \tag{22}$$

where

$$\begin{aligned} a_1 &\neq 0, \\ a_9 &> 0, \\ \alpha &> 0, \\ \beta &> 0, \end{aligned} \tag{23}$$

to ensure that the corresponding f is positive and well defined. By substituting (22) into (8), we can get

$$\begin{aligned} f = {} & \left(a_1 x - \frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} y + a_3 t + a_4 \right)^2 \\ & + \left(a_5 x + a_6 y + \frac{a_3 a_5 t}{a_1} a_8 \right)^2 + a_9, \end{aligned} \tag{24}$$



(a) $t = 0$ (b) $t = -20$

(c) $t = 0$ (d) $t = 20$

FIGURE 3: Profiles of the solution u via (30a) with $a_1 = 1$, $a_4 = 0$, $a_5 = 2$, $a_6 = 4$, $a_8 = 0$, $a_9 = 1$, $\alpha = 1$, $\beta = 1$, $k_1 = 1$, $k_2 = 1.5$, $k_3 = 2$, and $k_4 = 0$. (a) 3-dimensional plot with the time $t = 0$; (b–d) the corresponding density plots with different times.

where a_i, k_j, $1 \le i \le 9$, and $1 \le j \le 4$ are all real parameters to be determined. Then, the solutions of (3) will be obtained:

$$u = -\frac{4(a_1 h + a_5 g)^2 - (a_1^2 + a_5^2) f}{2f(a_1 h + a_5 g)}, \tag{25a}$$

$$v = \frac{a_1 a_3 h + a_3 a_5 g}{2a_1(a_1 h + a_5 g)} - \frac{(a_1 + a_5)^2}{8a_1(a_1 h + a_5 g)} + \frac{D}{f^2}, \tag{25b}$$

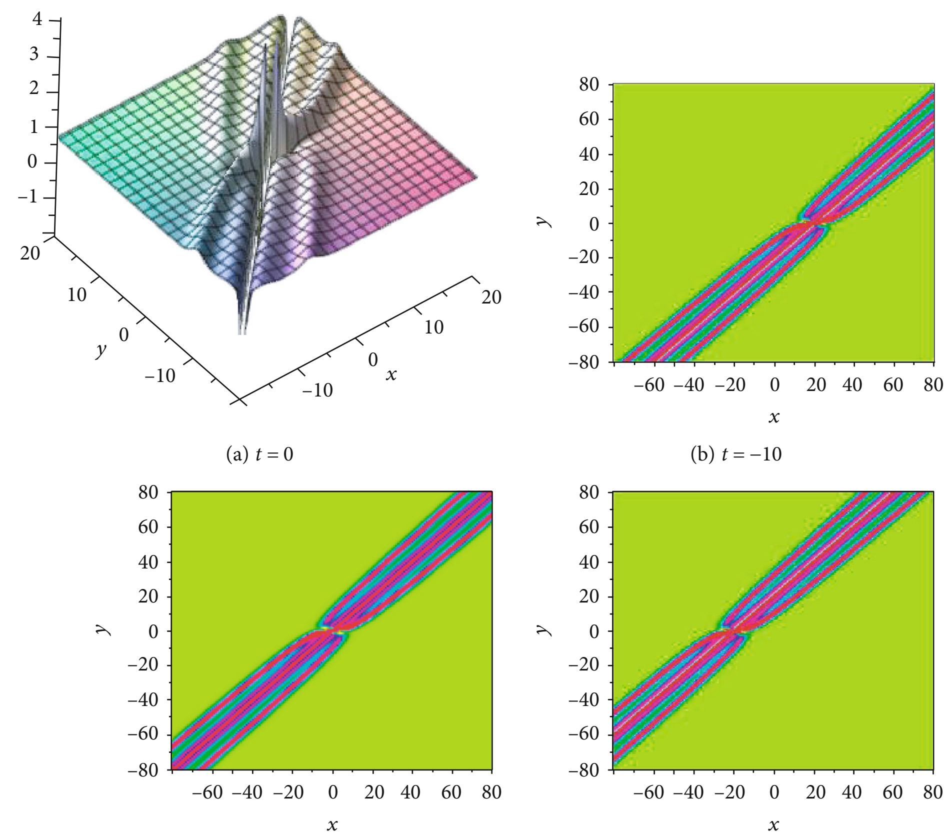
with

$$\begin{aligned} f &= h^2 + g^2 + a_9 + \alpha \exp(\xi) + \beta \exp(-\xi), \\ h &= a_1 x - \frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} y + a_3 t + a_4, \\ g &= a_5 x + a_6 y + \frac{a_3 a_5 t}{a_1} + a_8, \\ \xi &= k_2 y + k_4, \end{aligned} \tag{26}$$

$D = 2a_6 g - 2h((a_1^2 + a_5^2) + a_5 a_6 / a_1) + k_2(\alpha \exp(\xi) - \beta \exp(-\xi) + 2(a_1 h + a_5 g)) + (2f(a_1^2 + a_5^2))$. The solution for (25a) can be seen in Figure 2(a); Figures 2(b)–2(d) are the corresponding density plots with different times.

Case II.

$$\begin{aligned} a_1 &= a_1, \\ a_2 &= -\frac{a_1^2 + a_5^2 + a_5 a_6}{a_1}, \\ a_3 &= \frac{a_1 k_3}{k_1}, \\ a_4 &= a_4, \\ a_5 &= a_5, \\ a_6 &= a_6, \\ a_7 &= \frac{a_5 k_3}{k_1}, \end{aligned}$$



(a) $t = 0$ (b) $t = -10$ (c) $t = 0$ (d) $t = 10$

FIGURE 4: Profiles of the solution v via (30b) with $a_1 = 1, a_4 = 0, a_5 = 2, a_6 = 4, a_8 = 0, a_9 = 1, \alpha = 1, \beta = 1, k_1 = 1, k_2 = 1.5, k_3 = 2$, and $k_4 = 0$. (a) 3-dimensional plot with the time $t = 0$; (b–d) the corresponding density plots with different times.

$$\begin{aligned} a_8 &= a_8, \\ a_9 &= a_9, \\ \alpha &= \alpha, \\ \beta &= \beta, \\ k_1 &= k_1, \\ k_2 &= -k_1, \\ k_3 &= k_3, \\ k_4 &= k_4, \end{aligned} \tag{27}$$

where

$$\begin{aligned} a_1 &\neq 0, \\ k_1 &\neq 0, \\ a_9 &> 0, \\ \alpha &> 0, \\ \beta &> 0, \end{aligned} \tag{28}$$

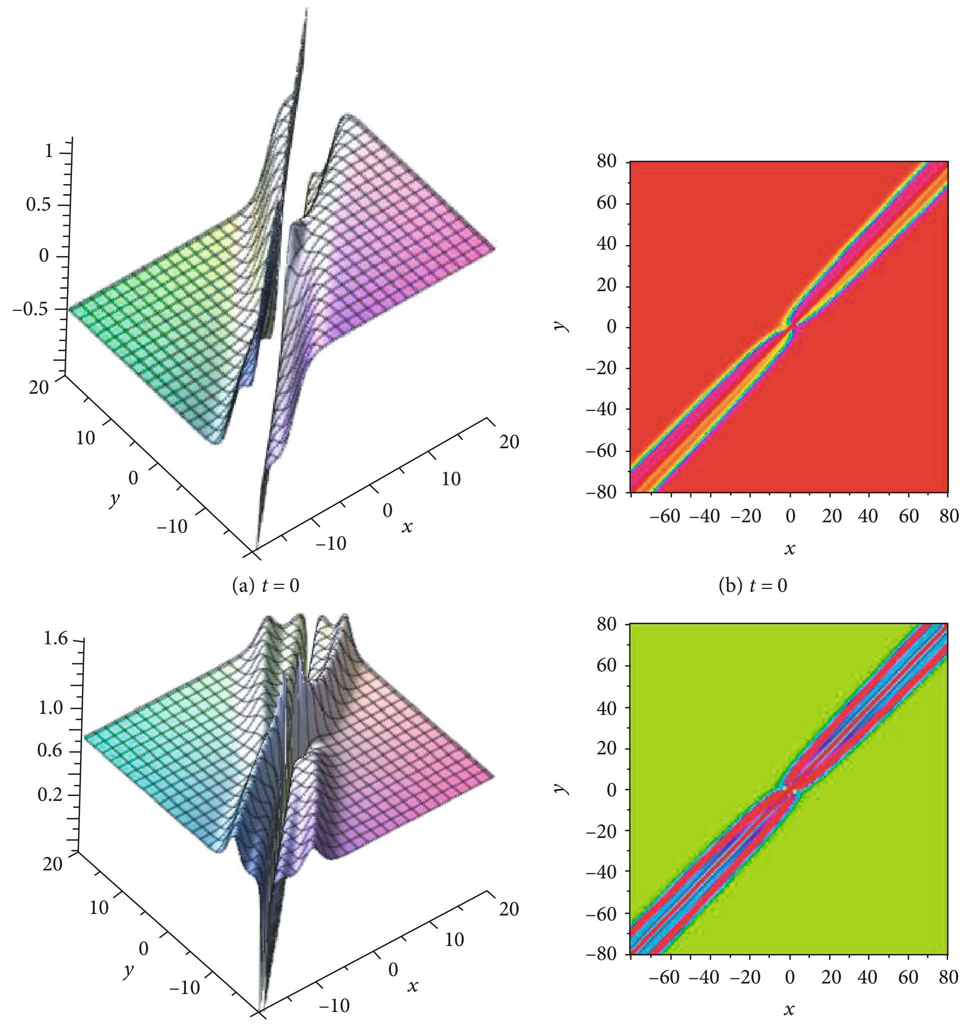
to guarantee that the corresponding f is positive and well defined. We substitute (27) into (8); hence, we can reinstall function f as the following formula:

$$\begin{aligned} f = {} & \left(a_1 x - \frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} y + \frac{a_1 k_3 t}{k_1} + a_4\right)^2 \\ & + \left(a_5 x + a_6 y + \frac{a_5 k_3 t}{a_1} + a_8\right)^2 + a^9, \end{aligned} \tag{29}$$

where $a_i, k_j, 1 \leq i \leq 9$, and $1 \leq j \leq 4$ are all real parameters to be determined. Then, by substituting equation (27) into (29), and with equation (3), we have

$$u = -\frac{2B^2 - Af}{2Bf}, \tag{30a}$$

$$\begin{aligned} v = {} & \frac{k_3}{2k_1} + \frac{A^2}{4B^2} \\ & + \frac{B\left(2ha_6 - 2h\left(\left(a_1^2 + a_5^2 + a_5 a_6\right)/a_1\right) + k_1(\alpha \exp(\xi) - \beta \exp(-\xi)) - Af\right)}{f^2}, \end{aligned} \tag{30b}$$



(a) $t = 0$ (b) $t = 0$

(c) $t = 0$ (d) $t = 0$

Figure 5: Profiles of the solution u via (35a) and profiles of the solution v via (35b) with $a_1 = 1$, $a_4 = 0$, $a_6 = 2$, $a_7 = 1$, $a_8 = 0$, $a_9 = 1$, $\alpha = 1$, $\beta = 1$, $k_1 = 1$, $k_3 = 2$, and $k_4 = 0$. (a) and (c) are the 3-dimensional plots; (b) and (d) are the corresponding density plots.

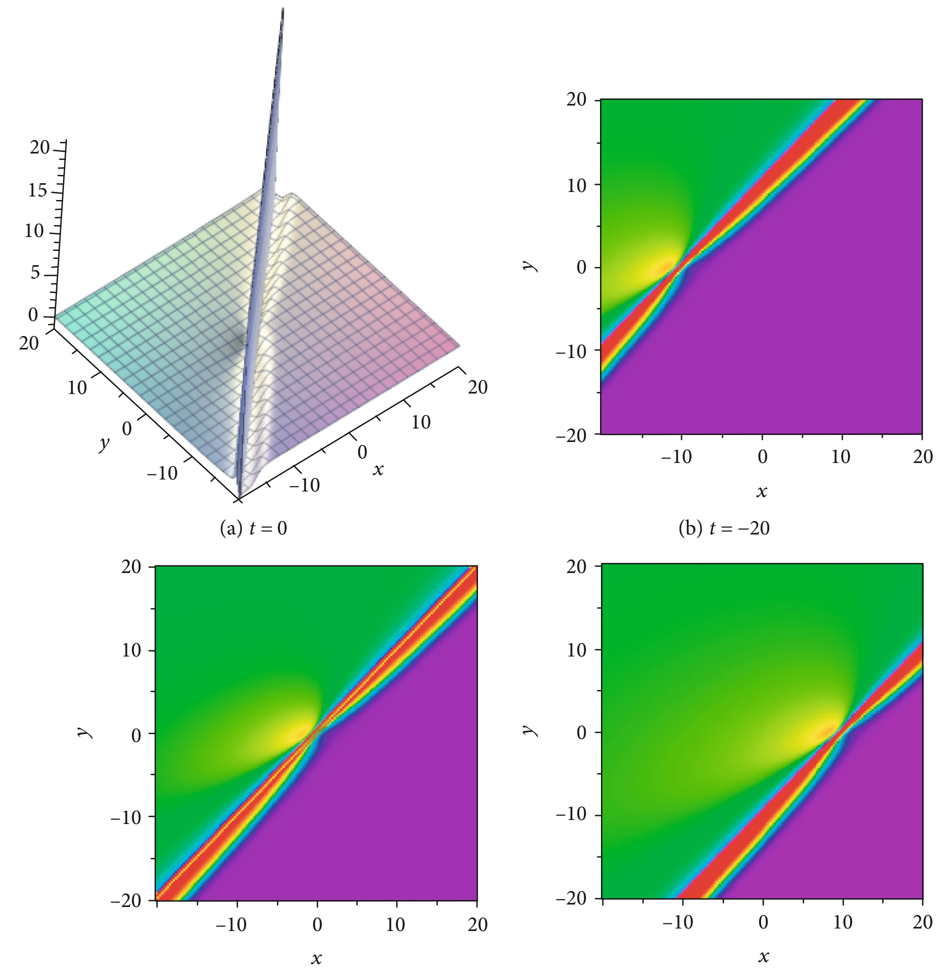
where

$$
\begin{aligned}
f &= h^2 + g^2 + \alpha \exp(\xi) + \beta \exp(-\xi) + a_9, \\
h &= a_1 x - \frac{a_1^2 + a_5^2 + a_5 a_6}{a_1} y + \frac{a_1 k_3 t}{k_1} + a_4, \\
g &= \left(a_5 x + a_6 y + \frac{a_5 k_3 t}{a_1} + a_8 \right)^2 + a_9, \\
\xi &= k_1 x - k_1 y + k_3 t + k_4, \\
A &= 2\left(a_1^2 h + a_5^2 g\right) + k_1^2 (\alpha \exp(\xi) + \beta \exp(-\xi)), \\
B &= 2 a_1 h + a_5 + k_1 (\alpha \exp(\xi) - \beta \exp(-\xi)).
\end{aligned}
\tag{31}
$$

The solution for (30a) as a rational solution can be seen in Figure 3(a) and in Figures 3(b)–3(d) the corresponding density plots with different times. The solution for (30b) as a rational solution which is singular can be seen in Figure 4(a) and in Figures 4(b)–4(d) the corresponding density plots with different times.

Case III.

$$
\begin{aligned}
a_1 &= a_1, \\
a_2 &= -\frac{a_1^2 k_3^2 + a_6 a_7 k_1 k_3 + a_7^2 k_1^2}{a_1 k_3^2}, \\
a_3 &= \frac{a_1 k_3}{k_1}, \\
a_4 &= a_4, \\
a_5 &= \frac{a_7 k_1}{k_3}, \\
a_6 &= a_6,
\end{aligned}
$$



(a) $t = 0$ (b) $t = -20$ (c) $t = 0$ (d) $t = 20$

FIGURE 6: Profiles of the solution u via (17a) with $a_2 = 1$, $a_4 = 0$, $a_5 = 1$, $a_8 = 0$, $a_9 = 1$, beta $= 1$, $k_2 = 2$, $k_3 = 1$, and $k_4 = 0$. (a) 3-dimensional plot with the time $t = 0$; (b–d) the corresponding density plots with different times.

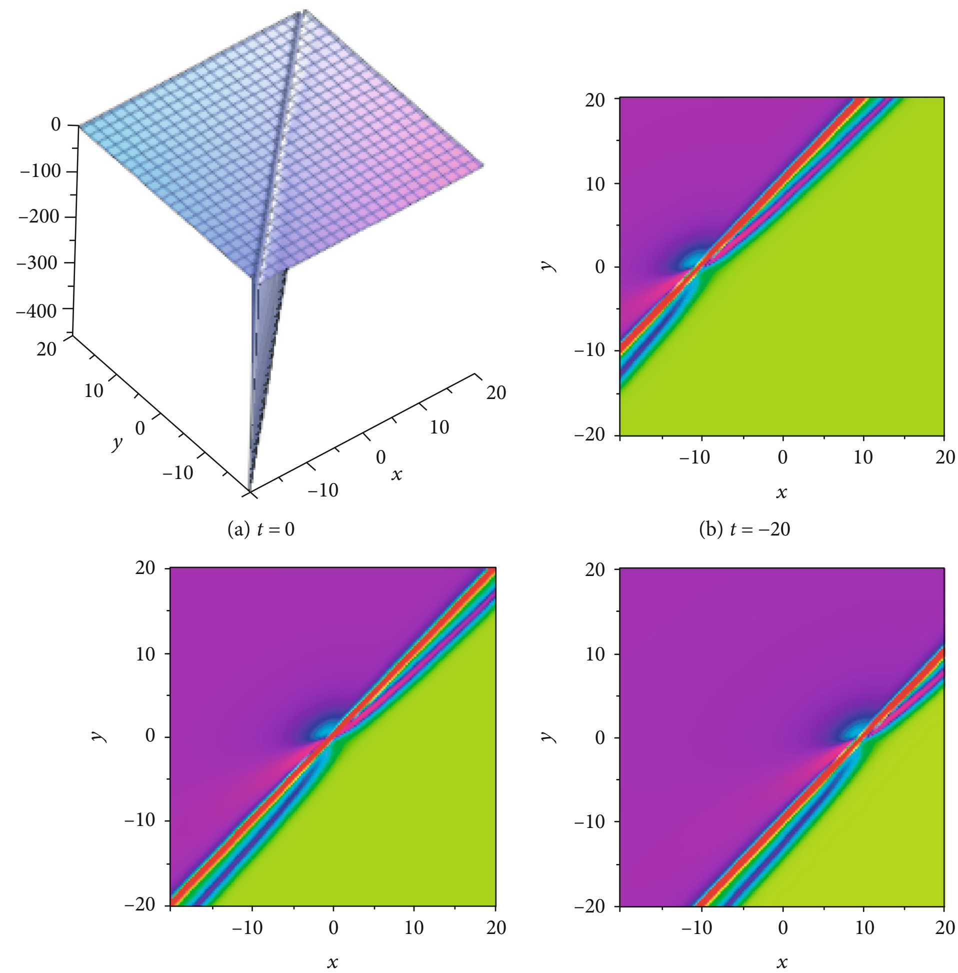
$$\begin{aligned} a_7 &= a_7, \\ a_8 &= \frac{a_4 a_7 k_1}{k_3 a_1}, \\ a_9 &= a_9, \\ \alpha &= \alpha, \\ \beta &= \beta, \\ k_1 &= k_1, \\ k_2 &= -k_1, \\ k_3 &= k_3, \\ k_4 &= k_4, \end{aligned} \tag{32}$$

with

$$\begin{aligned} k_3 a_1 &\neq 0, \\ a_9 &> 0, \\ \alpha &> 0, \\ \beta &> 0, \end{aligned} \tag{33}$$

to guarantee that the corresponding f is positive and well defined. We substitute (32) into (8), then function f reads

$$\begin{aligned} f = {} & \left(a_1 x - \frac{a_1^2 k_3^2 + a_6 a_7 k_1 k_3 + a_7^2 k_1^2}{a_1 k_3^2} y + \frac{a_1 k_3 t}{k_1} + a_4 \right)^2 \\ & + \left(\frac{a_7 k_1}{k_3} x + a_6 y + a_7 t + \frac{a_4 a_7 k_1}{k_3 a_1} \right)^2 \\ & + a_9 + \alpha \exp\left(k_1 x - k_1 y + k_3 t + k_4\right) \\ & + \beta \exp\left(-k_1 x + k_1 y - k_3 t - k_4\right), \end{aligned} \tag{34}$$



(a) $t=0$ (b) $t=-20$

(c) $t=0$ (d) $t=20$

Figure 7: Profiles of the solution v via (17b) with $a_2=1$, $a_4=0$, $a_5=1$, $a_8=0$, $a_9=1$, beta $=1$, $k_2=2$, $k_3=1$, and $k_4=0$. (a) 3-dimensional plot with the time $t=0$; (b–d) the corresponding density plots with different times.

where a_i, k_j, $1\le i\le 9$, and $1\le j\le 4$ are all real parameters to be determined. Then, the rational solution of system (3) can be obtained again:

$$u=-\frac{\left(Jf-2D^2\right)}{2Df}, \tag{35a}$$

$$v=\frac{\left(\left(2a_1k_3h/k_1\right)+2a_7g+N\right)}{2D}+\frac{J^2}{4D^2}+\frac{D(Mh+2a_7g+N)+2a_1\left(M-2\left(a_7k_1/k_3\right)\right)+k_1^2(\alpha\exp(\xi)+\beta\exp(-\xi))}{f^2}, \tag{35b}$$

where

$$f=h^2+g^2+\alpha\exp(\xi)+\beta\exp(-\xi)+a_9,$$

$$h=a_1x-\frac{a_1^2+k_3^2+a_6a_7k_1k_3+a_7^2k_1^2}{a_1k_3^2}y+\frac{a_1k_3t}{k_1}+a_4,$$

$$g=\frac{a_7k_1}{k_3}x+a_6y+a_7t+\frac{a_4a_7k_1}{k_3a_1},$$

$$\xi=k_1x-k_1y+k_3t+k_4,$$

$$D = 2a_1 h + 2\frac{a_7 k_1}{k_3} g + k_1 (\alpha \exp(\xi) - \beta \exp(-\xi)),$$
$$J = 2a_1^2 + 2\left(\frac{a_7 k_1}{k_3}\right)^2 + k_1^2 (\alpha \exp(\xi) + \beta \exp(-\xi)),$$
$$M = \frac{2\left(a_1^2 k_3^2 + a_6 a_7 k_1 k_3 + a_7^2 k_1^2\right)}{a_1 k_3^2},$$
$$N = k_3 (\alpha \exp(\xi) - \beta \exp(-\xi)). \quad (36)$$

The solution for (35a) is rational and can be seen in Figure 5(a) and Figure 5(b) is the corresponding density plot; the solution for (35b) is rational and can be seen in Figure 5(c) and Figure 5(d) is the corresponding density plot.

6. Conclusions

Construction of rational solutions for NLEEs is an important part in nonlinear science. In this study, based on the trilinear-linear equation (6), we obtain three classes of rational solutions ((12a), (12b), (17a), (17b), (25a), (25b), (30a), (30b), (35a), and (35b)) for the $(2+1)$-dimensional modified KdV-CBS equation (3). Three-dimensional plots and the corresponding density plots of the three classes of rational solutions are given, respectively, in Figures 1–7 in this paper.

Data Availability

This article's purpose was not to use data but only to do symbolic calculation with maple, so no data availability statement is required.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

[1] V. B. Matveev and M. A. Salle, *Darboux Transformations and Solitons*, Springer, Berlin, 1991.

[2] G. W. Bluman, A. F. Cheviakov, and S. C. Anco, *Applications of Symmetry Methods to Partial Differential Equations*, Springer, New York, NY, USA, 2010.

[3] M. Ruggieri, "Kink solutions for a class of generalized dissipative equations," *Abstract and Applied Analysis*, vol. 2012, Article ID 237135, 7 pages, 2012.

[4] M. Ruggieri and A. Valenti, "Approximate symmetries in nonlinear viscoelastic media," *Boundary Value Problems*, vol. 2013, no. 1, 2013.

[5] R. Hirota, *The Direct Method in Soliton Theory*, Spring, Berlin, 2004.

[6] W. X. Ma, "Lump solutions to the Kadomtsev-Petviashvili equation," *Physics Letters A*, vol. 379, no. 36, pp. 1975–1978, 2015.

[7] X. Zhang and Y. Chen, "Rogue wave and a pair of resonance stripe solitons to a reduced (3+1)-dimensional Jimbo-Miwa equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 52, pp. 24–31, 2017.

[8] X. Zhang, Y. Chen, and X. Tang, "Rogue wave and a pair of resonance stripe solitons to KP equation," *Computers and Mathematics with Applications*, vol. 76, no. 8, pp. 1938–1949, 2018.

[9] W. X. Ma, "Lump and interaction solutions to linear (4+1)-dimensional PDEs," *Acta Mathematica Scientia*, vol. 39, no. 2, pp. 498–508, 2019.

[10] J. Y. Yang and W. X. Ma, "Abundant lump-type solutions of the Jimbo-Miwa equation in (3+1)-dimensions," *Computers and Mathematics with Applications*, vol. 73, no. 2, pp. 220–225, 2017.

[11] Z. Zhao, Y. Chen, and B. Han, "Lump soliton, mixed lump stripe and periodic lump solutions of a (2+1)-dimensional asymmetrical Nizhnik-Novikov-Veselov equation," *Modern Physics Letters B*, vol. 31, no. 14, article 1750157, 2017.

[12] Y. Yue and Y. Chen, "Dynamics of localized waves in a (3+1)-dimensional nonlinear evolution equation," *Modern Physics Letters B*, vol. 33, no. 9, p. 1950101, 2019.

[13] R. Radha and M. Lakshmanan, "Singularity analysis and localized coherent structures in (2+1) dimensional generalized Korteweg-de Vries equations," *Journal of Mathematical Physics*, vol. 35, no. 9, pp. 4746–4756, 1994.

[14] Y. F. Hua, B. L. Guo, W. X. Ma, and X. Lü, "Interaction behavior associated with a generalized (2+1)-dimensional Hirota bilinear equation for nonlinear waves," *Applied Mathematical Modelling*, vol. 74, pp. 184–198, 2019.

[15] B. Ren, W. X. Ma, and J. Yu, "Characteristics and interactions of solitary and lump waves of a (2+1)-dimensional coupled nonlinear partial differential equation," *Nonlinear Dynamics*, vol. 96, no. 1, pp. 717–727, 2019.

[16] Y. Zhou, S. Manukure, and W. X. Ma, "Lump and lump-soliton solutions to the Hirota-Satsuma-Ito equation," *Communications in Nonlinear Science and Numerical Simulation*, vol. 68, pp. 56–62, 2019.

[17] H. Wang, Y. H. Wang, W. X. Ma, and C. Temuer, "Lump solutions of a new extended (2+1)-dimensional Boussinesq equation," *Modern Physics Letters B*, vol. 32, no. 31, article 1850376, 2018.

[18] W. X. Ma, "Lump and interaction solutions of linear PDEs in (3 + 1)-dimensions," *East Asian Journal on Applied Mathematics*, vol. 9, no. 1, pp. 185–194, 2019.

[19] W. X. Ma, "Abundant lumps and their interaction solutions of (3+1)-dimensional linear PDEs," *Journal of Geometry and Physics*, vol. 133, pp. 10–16, 2018.

[20] S. Batwa and W. X. Ma, "A study of lump-type and interaction solutions to a (3+1)-dimensional Jimbo- Miwa-like equation," *Computers & Mathematics with Applications*, vol. 76, no. 7, pp. 1576–1582, 2018.

[21] L. Huang, Y. Yue, and Y. Chen, "Localized waves and interaction solutions to a (3+1)-dimensional generalized KP equation," *Computers and Mathematics with Applications*, vol. 76, no. 4, pp. 831–844, 2018.

[22] X. Zhang, Y. Chen, and Y. Zhang, "Breather, lump and X soliton solutions to nonlocal KP equation," *Computers & Mathematics with Applications*, vol. 74, no. 10, pp. 2341–2347, 2018.

[23] Y. H. Wang, "Construction of rational solutions for the (2+1)-dimensional Broer–Kaup system," *Modern Physics Letters B*, vol. 33, no. 30, article 1950377, 2019.

[24] Y. Z. Peng, "A new (2+1)-dimensional KdV equation and its localized structures," *Communications in Theoretical Physics*, vol. 54, no. 5, pp. 863–865, 2010.

[25] Y. Wang and Y. Chen, "Binary Bell polynomial manipulations on the integrability of a generalized (2+1) -dimensional Korteweg-de Vries equation," *Journal of Mathematical Analysis and Applications*, vol. 400, no. 2, pp. 624–634, 2013.

[26] R. M. Miura, "Korteweg-de Vries equation and generalizations. I. a remarkable explicit nonlinear transformation," *Journal of Mathematical Physics*, vol. 9, no. 8, pp. 1202–1204, 1968.

[27] Y. H. Wang and H. Wang, "Nonlocal symmetry, CRE solvability and soliton–cnoidal solutions of the (2+1)-dimensional modified KdV-Calogero–Bogoyavlenkskii–Schiff equation," *Nonlinear Dynamics*, vol. 89, no. 1, pp. 235–241, 2017.

[28] J. A. Gear and R. Grimshaw, "Weak and strong interactions between internal solitary waves," *Studies in Applied Mathematics*, vol. 70, no. 3, pp. 235–258, 1984.

[29] M. Ruggieri and M. P. Speciale, "Similarity reduction and closed form solutions for a model derived from two-layer fluids," *Advances in Difference Equations*, vol. 2013, no. 1, 2013.

[30] R. Grimshaw and P. Christodoulides, "Steady gap solitons in a coupled Korteweg-de Vries system: A dynamical systems approach," *Physica D: Nonlinear Phenomena*, vol. 239, no. 10, pp. 635–639, 2010.

[31] M. Ruggieri and M. P. Speciale, "Quasi self-adjoint coupled KdV-like equations," in *AIP Conference Proceedings*, vol. 1558, pp. 1220–1223, 2013.

[32] J. Weiss, M. Tabor, and G. Carnevale, "The Painlevé property for partial differential equations," *Journal of Mathematical Physics*, vol. 24, no. 3, pp. 522–526, 1983.