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Research Article

Multidimensional Schrödinger Equation and Spectral Properties of Heavy-Quarkonium Mesons at Finite Temperature

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The N-radial Schrödinger equation is analytically solved at finite temperature. The analytic exact iteration method (AEIM) is employed to obtain the energy eigenvalues and wave functions for all states n and l. The application of present results to the calculation of charmonium and bottomonium masses at finite temperature is also presented. The behavior of the charmonium and bottomonium masses is in qualitative agreement with other theoretical methods. We conclude that the solution of the Schrödinger equation plays an important role at finite temperature that the analysis of the quarkonium states gives a key input to quark-gluon plasma diagnostics.

1. Introduction

The Schrödinger equation (SE) plays an important role in describing many phenomena related to the vibrations of diatomic molecules and the oscillations of atoms, high energy physics, and quantum chemistry. Thus, the solutions of the SE are important for describing the phenomena in the above-mentioned fields. In [1–9], the authors obtained different solutions of the SE. It is well known that the exact solutions of the SE are found in few cases due to the complexity of the centrifugal potential. Therefore, there are different methods suggested such as those in [10–12], in which the authors solved the SE by using the Nikiforov-Uvarov method. Other authors used asymptotic iteration methods such as in [13–16]. In addition, it is well known that the potential interaction energy of the SE is necessary to obtain the explicit solutions of the SE and the energy eigenvalues, such as the Cornell potential as in [16, 17], the extended Cornell potential [18], and the Hulthén plus ringed-shaped potential as in [19].

At finite temperature, in [3], the authors employed the modified internal potential as a function of temperature to study the quark-gluon plasma using Mayer's expansion and a phenomenological thermodynamic model. In [20], the finite temperature SE is solved by using the Funke-Hecke theorem and is applied to electron and proton systems. In [21], the authors obtained the generalized form of the SE at finite temperature based on the first law of thermodynamics. In [22], the authors numerically solved the SE at finite temperature by employing temperature-dependent effective potential given by a linear combination of color-singlet and internal energies. Matsui and Satz [23] have studied the formation of a hot quark-gluon plasma by studying the effect of the temperature on the J/Ψ radius calculated in the charmonium models. Wong [24] has studied the binding energies and wave functions of heavy quarkonia in quark-gluon plasma by using the color-singlet free energy and total internal energy for a static quark and antiquark in quenched QCD. Thus, the study shows that the model with the new Q-$\overline{Q}$ potential gives dissociation temperatures that agree with the spectral function analyses. Additionally, Wong [25] has investigated the Q-$\overline{Q}$ potential by using the thermodynamic quantities to give spontaneous dissociation temperatures for quarkonium and has also found the quark drip lines which separate the region of bound color-singlet $Q\overline{Q}$ states from the unbound region. Reik and Rapp [26] have studied the evaluation of quarkonium bound-state properties and heavy-quark diffusion. They have applied the thermodynamic T-matrix approach for elastic two-body interactions to obtain the spectral functions of heavy-quark systems in the quark-gluon plasma, in which the spectral functions are used to calculate Euclidean correlators, which are discussed in light of lattice QCD results.

On the other hand, studies of lattice QCD at finite temperature with improved actions have provided consistent estimates of T_c, playing an essential role in investigating the heavy quark. The lattice QCD with two flavors of nonperturbatively improved Wilson fermions at finite temperature is studied to describe the heavy-quark potential [27]. The Debye screening between two opposite color charges is shown in the QCD static potential computed at finite temperature with lattice QCD [28, 29]. Therefore, the heavy-quark bound states may no longer exist well above the deconfinement critical temperature T_c on the order of 200–300 MeV [30].

The aim of the present work is to obtain the solutions of the Schrödinger equation at finite temperature. So far, no attempt has been made to solve the N-radial SE using the AEIM when finite temperature is included. The application is studied on the analysis of the quarkonium states which play an important role in the quark-gluon plasma diagnostics in the heavy-ion collision experiments.

The paper is organized as follows: The N-radial SE is solved by using the AEIM in Section 2. The results are discussed in Section 3. The summary and conclusion are presented in Section 4.

2. The Schrödinger Equation at Finite Temperature

The SE for two particles interacting via a symmetric potential in N-dimensional space takes the form as in [31]:

$$\left[\frac{d^2}{dr^2}+\frac{(N-1)}{r}\frac{d}{dr}-\frac{L(L+N-2)}{r^2}+2\mu(E-V(r))\right]\cdot\Psi(r)=0, \quad (1)$$

where L, N, and μ are the angular momentum quantum number, the dimensionality number, and the reduced mass for the quarkonium particle, respectively. Setting the wave function $\Psi(r) = R(r)/r^{(N-1)/2}$, the following radial Schrödinger equation is obtained:

$$\left[\frac{d^2}{dr^2}+2\mu\left(E-V(r,T)-\frac{(L+(N-2)/2)^2-1/4}{2\mu r^2}\right)\right]\cdot R(r)=0. \quad (2)$$

The $V(r,T)$ can be taken as the internal energy potential [3]:

$$V(r,T)=F_1(r,T)-T\frac{\partial F_1(r,T)}{\partial T}, \quad (3)$$

where

$$F_1(r,T)=\left(cr-\frac{4}{3}\frac{\alpha_s(T)}{r}\right)e^{-m_D(T)r}, \quad (4)$$

where c is a free parameter and $\alpha_s(T)$ is the running coupling constant which is given by

$$\alpha_s(T)=\frac{2\pi}{\left(11-(2/3)n_f\right)\ln\left(T/\beta T_c\right)}, \quad (5)$$

where n_f is the number of quark flavors, T_c is the critical temperature, and $\beta = 0.104 \pm 0.009$. The Debye screening mass $m_D(T)$ is given by

$$m_D(T)=4\pi\eta c_\sigma\alpha_s(T)T, \quad (6)$$

where η and c_σ are parameters of the model. By substituting (3) into (2), we obtain

$$\left[\frac{d^2}{dr^2}+d_1+d_2e^{-m_D(T)r}+\frac{d_3}{r}e^{-m_D(T)r}+d_4re^{-m_D(T)r}+d_5r^2e^{-m_D(T)r}-\frac{(L+(N-2)/2)^2-1/4}{r^2}\right]R(r)=0, \quad (7)$$

where

$$d_1=2\mu E,\qquad d_2=\frac{8\mu T}{3}\alpha_s(T)\frac{d}{dT}m_D(T), \quad (8)$$

$$d_3=\frac{8\mu\alpha_s(T)}{3}+\frac{16\mu\pi\left(11-(2/3)n_f\right)}{3\left[\left(11-(2/3)n_f\right)\ln\left(T/\beta T_c\right)\right]^2}, \quad (9)$$

$$d_4=-2\mu c,\qquad d_5=2\mu cT\frac{d}{dT}m_D(T). \quad (10)$$

By taking the following form of the wave function as in [1]

$$R_{nl}(r)=f_n(r)e^{g_l(r)}, \quad (11)$$

where

$$f_n(r)=\begin{cases}1 & n=0\\ \prod_{i=1}^{n}(r-\alpha_i) & n=1,2,3,\ldots,\end{cases} \quad (12)$$

$$g_l(r)=-\frac{1}{2}\alpha r^2-\beta r+\delta\ln r;\quad \alpha>0,\ \beta>0, \quad (13)$$

$f_n(r)$ represents the Laguerre polynomials. By taking the second derivative of (11), we obtain

$$R''_{nl}(r)=\left(g''_l(r)+g'^2_l(r)+\frac{f''_n(r)+2g'_l(r)f'_n(r)}{f_n(r)}\right)\cdot R_{nl}(r). \quad (14)$$

Comparing (14) and (7),

$$d_1+d_2e^{-m_D(T)r}+\frac{d_3}{r}e^{-m_D(T)r}+d_4re^{-m_D(T)r}+d_5r^2e^{-m_D(T)r}-\frac{(L+(N-2)/2)^2-1/4}{r^2}=-\left(g''_l(r)+g'^2_l(r)+\frac{f''_n(r)+2g'_l(r)f'_n(r)}{f_n(r)}\right). \quad (15)$$

2.1. Calculation of Energy E_{0l}. Calculating the energy E_{0l} at $n = 0$, where $f_0(r) = 1$, using (13), and using the expansion $e^{-m_D(T)r} = \sum_{i=0}^{n}((-m_D(T)r)^i/i!)$, we obtain

$$\begin{aligned}&(d_1 + d_2 - m_D(T)\,d_3) + \frac{1}{r}d_3 \\&\quad + \left(d_4 + d_3 m_D^2 - d_2 m_D\right) r \\&\quad + \left(d_5 + \frac{d_2}{2} m_D^2 - d_4 m_D\right) r^2 \\&\quad + \left(\frac{1}{2} d_4 m_D^2 - d_5 m_D\right) r^3 + \frac{1}{2} d_5 m_D^2 r^4 \\&\quad - \frac{(L + (N-2)/2)^2 - 1/4}{r^2} \\&= -\left(\alpha^2 r^2 + 2\alpha\beta r - (\alpha + 2\delta)\right) + \beta^2 - \frac{2\beta\delta}{r} \\&\quad + \frac{\delta(\delta - 1)}{r^2}.\end{aligned} \tag{16}$$

By comparing the coefficients of the powers of r on both sides, we obtain

$$-d_5 - \frac{d_2}{2} m_D^2 + d_4 m_D = \alpha^2, \tag{17}$$

$$-d_4 - d_3 m_D^2 + d_2 m_D = 2\alpha\beta, \tag{18}$$

$$d_1 + d_2 - m_D d_3 = (\alpha + 2\delta) - \beta^2, \tag{19}$$

$$\begin{aligned}d_3 &= 2\beta\delta, \\ d_4 &= 0, \\ d_5 &= 0,\end{aligned} \tag{20}$$

$$\left(L + \frac{(N-2)}{2}\right)^2 - \frac{1}{4} = \delta(\delta - 1). \tag{21}$$

From (19) and using the formula $d_1 = 2\mu E$ in (8), we obtain

$$E_{0l} = \frac{1}{2\mu}\left[\alpha(1 + 2(\delta + 0)) - \beta^2 - d_2 + m_D(T)\,d_3\right], \tag{22}$$

where the parameters α, β, and δ are obtained from (17), (18), and (21) as follows:

$$\alpha = \sqrt{\left|d_4 m_D - d_5 - \frac{d_2}{2} m_D^2\right|}, \tag{23}$$

$$\beta = \frac{\left|d_2 m_D - d_4 - d_3 m_D^2\right|}{2\sqrt{\left|d_4 m_D - d_5 - (d_2/2)\, m_D^2\right|}}, \tag{24}$$

$$\delta = \frac{1 \pm \sqrt{1 + 4\left((L + (N-2)/2)^2 - 1/4\right)}}{2}. \tag{25}$$

2.2. Calculation of Wave Function at $n = 0$. We can write (11) at $n = 0$:

$$R_{0l}(r) = N_{0l} r^{\delta} e^{-(1/2)\alpha r^2 - \beta r}, \tag{26}$$

where N_{0l} is the normalization constant that can be determined by $\int_0^\infty |R_{0l}(r)|^2 dr = 1$. By using [32], we obtain

$$N_{0l} = \frac{(2\alpha)^{(2\delta+1)/4}\, e^{-\beta^2/4\alpha}}{\sqrt{\Gamma(2\delta + 1)\, D_{-(2\delta+1)}\left(\beta\sqrt{2/\alpha}\right)}}, \tag{27}$$

where $D_\nu(z)$ are the parabolic cylinder functions (see [32] and references therein). The parameters α, β, and δ are calculated in (23), (24), and (25). To satisfy the boundary conditions at $R_{0l}\ (r = 0) = 0$, the parameter δ should have a positive value. Hence, we choose the positive sign in (25). In addition, to satisfy the boundary condition at infinity $R_{0l}\ (r = \infty) = 0$, the parameters α and β should have a positive value. Therefore, the absolute values are taken in (23) and (24).

2.3. Calculation of Energy E_{1l}. To calculate the energy eigenvalue E_{1l}, the index n is taken as $n = 1$. Thus, (11) is written as

$$f_1(r) = (r - \alpha_1). \tag{28}$$

Therefore, we can write (15) as

$$\begin{aligned}&(d_1 + d_2 - m_D(T)\,d_3) + \frac{1}{r}d_3 + \left(d_4 + d_3 m_D^2 - d_2 m_D\right) \\&\cdot r + \left(d_5 + \frac{d_2}{2} m_D^2 - d_4 m_D\right) r^2 + \left(\frac{1}{2} d_4 m_D^2\right. \\&\left. - d_5 m_D\right) r^3 + \frac{1}{2} d_5 m_D^2 r^4 \\&- \frac{(L + (N-2)/2)^2 - 1/4}{r^2} = -\left(\alpha^2 r^2 + 2\alpha\beta r\right. \\&- \alpha(1 + 2(\delta + 1)) + \beta^2 - \frac{2[\beta(\delta + 1) + \alpha\alpha_1]}{r} \\&\left. + \frac{\delta(\delta - 1)}{r^2}\right).\end{aligned} \tag{29}$$

By comparing the coefficients of the powers r on both sides, we obtain

$$-d_5 - \frac{d_2}{2} m_D^2 + d_4 m_D = \alpha^2, \tag{30}$$

$$-d_4 - d_3 m_D^2 + d_2 m_D = 2\alpha\beta, \tag{31}$$

$$d_1 + d_2 - m_D(T)\,d_3 = \alpha(1 + 2(\delta + 1)) - \beta^2, \tag{32}$$

$$\begin{aligned}d_3 &= 2[\beta(\delta + 1) + \alpha\alpha_1], \\ d_4 &= 0, \\ d_5 &= 0,\end{aligned} \tag{33}$$

$$\left(L + \frac{(N-2)}{2}\right)^2 - \frac{1}{4} = \delta(\delta - 1). \tag{34}$$

From (31) and using the formula $d_1 = 2\mu E$ in (8), we obtain E_{1l} as the following form:

$$E_{1l} = \frac{1}{2\mu}\left[\alpha\left(1+2\left(\delta+1\right)\right) - \beta^2 - d_2 + m_D\left(T\right)d_3\right]. \quad (35)$$

2.4. Calculation of Wave Function at $n = 1$. We can write the wave function at $n = 1$:

$$R_{1l}\left(r\right) = N_{1l}\left(r-\alpha_1\right)r^{\delta}e^{-(1/2)\alpha r^2-\beta r}, \quad (36)$$

where the parameter α_1 is obtained from (32):

$$\alpha_1 = \frac{d_3 - 2\beta\left(\delta+1\right)}{2\alpha}, \quad (37)$$

where the parameters α, β, and δ are those given in (23), (24), and (25). N_{1l} is the normalization constant, which can be determined by $\int_0^{\infty}|R_{1l}(r)|^2 dr = 1$. Thus, we obtain

$$N_{1l} = \frac{1}{\sqrt{I_1 - 2\alpha_1 I_2 + \alpha_1^2 I_3}}, \quad (38)$$

where

$$I_1 = \frac{\Gamma\left(2\delta+3\right)D_{-(2\delta+3)}\left(\beta\sqrt{2/\alpha}\right)}{\left(2\alpha\right)^{\delta+3/2}e^{-\beta^2/2\alpha}}$$
$$I_2 = \frac{\Gamma\left(2\delta+2\right)D_{-(2\delta+2)}\left(\beta\sqrt{2/\alpha}\right)}{\left(2\alpha\right)^{\delta+1}e^{-\beta^2/2\alpha}} \quad (39)$$
$$I_3 = \frac{\Gamma\left(2\delta+1\right)D_{-(2\delta+1)}\left(\beta\sqrt{2/\alpha}\right)}{\left(2\alpha\right)^{\delta+1/2}e^{-\beta^2/2\alpha}}.$$

2.5. Calculation of Energy E_{2l}. Following the analytic iteration method for the second node $n = 2$,

$$f_2\left(r\right) = \left(r-\alpha_1\right)\left(r-\alpha_2\right), \quad (40)$$

we can write (15), and using (13), we obtain

$$\begin{aligned}&\left(d_1+d_2-m_D\left(T\right)d_3\right)+\frac{1}{r}d_3+\left(d_4+d_3m_D^2-d_2m_D\right)\\&\cdot r+\left(d_5+\frac{d_2}{2}m_D^2-d_4m_D\right)r^2+\left(\frac{1}{2}d_4m_D^2\right.\\&\left.-d_5m_D\right)r^3+\frac{1}{2}d_5m_D^2r^4-\frac{\left(L+\left(N-2\right)/2\right)^2-1/4}{r^2}\\&=-\left(\alpha^2r^2+2\alpha\beta r-\alpha\left(1+2\left(\delta+2\right)\right)-\beta^2\right.\\&\left.-\frac{2\left[\beta\left(\delta+2\right)+\alpha\sum_{i=1}^{2}\alpha_i\right]}{r}+\frac{\delta\left(\delta-1\right)}{r^2}\right).\end{aligned} \quad (41)$$

The relations between the potential parameters and the coefficients α, β, δ, α_1, and α_2 are as follows:

$$-d_5 - \frac{d_2}{2}m_D^2 + d_4m_D = \alpha^2, \quad (42)$$
$$-d_4 - d_3m_D^2 + d_2m_D = 2\alpha\beta, \quad (43)$$
$$d_1 + d_2 - m_D\left(T\right)d_3 = \alpha\left(1+2\left(\delta+2\right)\right) + \beta^2, \quad (44)$$
$$d_3 = 2\left[\beta\left(\delta+2\right)+\alpha\sum_{i=1}^{2}\alpha_i\right],\quad d_4 = 0,\quad d_5 = 0, \quad (45)$$
$$\left(L+\frac{\left(N-2\right)}{2}\right)^2 - \frac{1}{4} = \delta\left(\delta-1\right). \quad (46)$$

From (44) and using the formula $d_1 = 2\mu E$ in (8), we obtain E_{2l}:

$$E_{2l} = \frac{1}{2\mu}\left[\alpha\left(1+2\left(\delta+2\right)\right) + \beta^2 - d_2 + m_D\left(T\right)d_3\right]. \quad (47)$$

2.6. Calculation of Wave Function at $n = 2$

$$R_{2l}\left(r\right) = N_{2l}\left(r-\alpha_1\right)\left(r-\alpha_2\right)r^{\delta}e^{-(1/2)\alpha r^2-\beta r}, \quad (48)$$

where α_1 and α_2 are obtained from (45) and (37). N_{2l} is the normalization constant which can be obtained as in (38).

2.7. Exact Energy and Wave Function. The iteration method is repeated. Therefore, we obtain the exact energy at finite temperature as the following form:

$$E_{nl} = \frac{1}{2\mu}\left[\alpha\left(1+2\left(\delta+n\right)\right) - \beta^2 - d_2 + m_D\left(T\right)d_3\right], \quad (49)$$

and the wave function is

$$R_{nl}\left(r\right) = N_{nl}\prod_{i=1}^{n}\left(r-\alpha_i\right)r^{\delta}e^{-(1/2)\alpha r^2-\beta r}, \quad (50)$$

where the parameters α, β, and δ are defined in (23), (24), and (25). N_{nl} is the normalization constant that can be obtained as in (27) and (38).

3. Discussion of Results

In the section above, the N-Schrödinger equation is solved at finite temperature by using the AETM as in [1]. The parameters of the present work are shown in Table 1. In this section, we apply the energy eigenvalue that is given in (49). To calculate quarkonium masses at finite temperature, the formula $M = 2m + E_{nl}$ is used when $N = 3$, where m is bare quark mass.

It is important to apply the present results on quarkonium mesons. In [33], the authors have investigated the

Table 1: The parameters of the internal energy.

Parameter	Value [3]
β	0.104
c_σ	0.566
η	2.06
T_c	0.25 GeV
c	0.135 ± 0.015 GeV^2

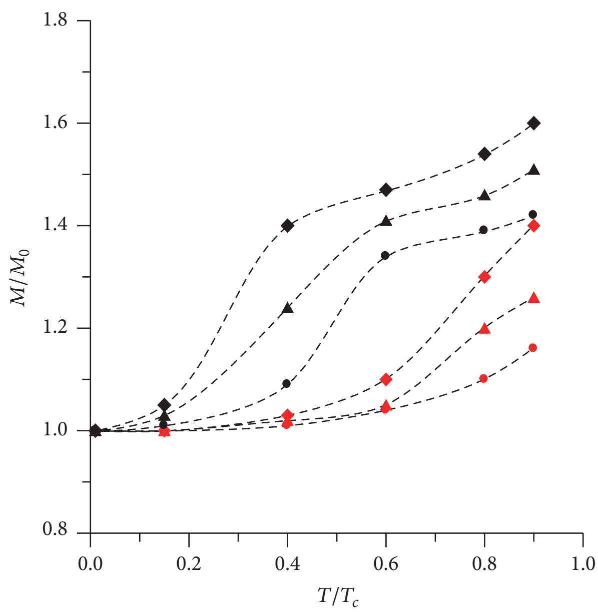

Figure 1: Mass in vacuum mass units of charmonium ground state is a function of temperature in critical temperature units for charm quark mass (m_c) equal to 1200, 1400, and 1640 MeV.

quarkonium spectrum calculations at finite temperature in the framework of QCD sum-rules. They found that the 1S state of quarkonium mass changes little up to $T/T_c = 0.2$ (T_c is critical temperature equal to 250 MeV); then, the behavior of charmonium mass increases with increasing temperature up to $T/T_c = 1$. In addition, the curves shift to lower values by increasing charm mass (m_c) as in Figure 1.

Also in Figure 1, we find that the 1S state of charmonium mass little changes up to $T/T_c = 0.2$ and then the charmonium mass increases with increasing temperature. The curves shift to lower values by increasing m_c in the present study. Additionally, we note that the present results of charmonium mass are shifted to upper values in comparison with [33], since the parameters of each method are changed. So, we note that qualitative agreement between the present results and [33]. The study of Debye mass has much interest in studying quarkonium properties. Unfortunately, there is still quite small diversity of results in this quantity. In Figure 2,

Table 2: The screening mass in the lattice QCD with $N_f = 3.0$ and the present work.

Ratio temperature (T/T_c)	1.5	2
Lattice QCD with $N_f = 3.0$	1.40	2.34
The present work $N_f = 3.0$	1.34	1.76

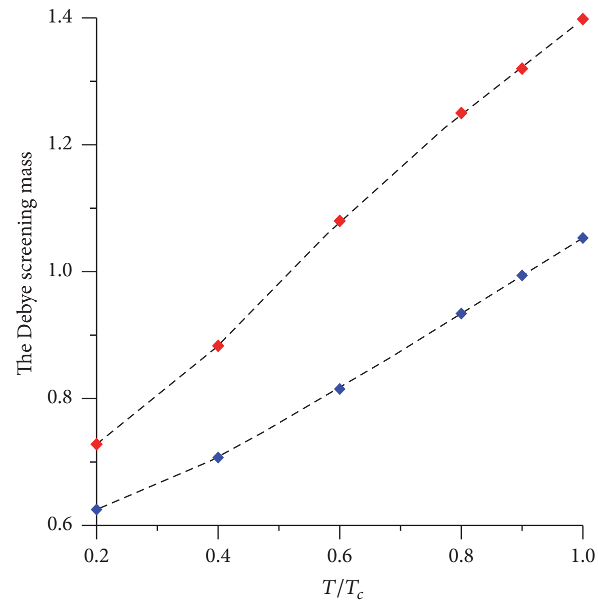

Figure 2: The Debye screening mass is a function of ratio temperature at m_c = 1640 MeV.

the comparison between the Debye screening mass in the present work and its value in [34] is presented. We find that qualitative agreement is noted in comparison with [34]. Additionally, lattice QCD with $N_f = 3$ is studied in this quantity [35]. Table 2 shows the comparison between the present results of screening mass and the lattice QCD. We note that the screening mass is in good agreement with lattice QCD at $T/T_c = 1.5$ and the screening mass is shifted to a higher value at $T/T_c = 2.0$.

4. Summary and Conclusion

In the present work, the solutions of N-radial Schrödinger equation are obtained for all states n and l, where the energy eigenvalue and wave function are obtained at finite temperature. The analytic exact iteration method (AEIM) in [1] is used as the technique for solving the SE. The novelty in this work is that we obtain the analytic solution of the N-radial SE at finite temperature by using the AEIM. In addition, the energy eigenvalues and corresponding wave functions are calculated in the N-dimensional space, in which one obtains the energy eigenvalue and wave function in the 3-dimensional space which are used in the most of the other works.

We apply the theoretical calculations on the quarkonium masses at finite temperature. We find that the behavior of quarkonium masses is in qualitative agreement in comparison with the QCD sum rule and lattice QCD, which are tools for measuring quarkonium masses at quark-gluon plasma. Therefore, the present approach successfully describes the quarkonium states for the given potential, which are a key input to quark-gluon plasma. We hope to extend this work to include electromagnetic forces in future work.

Competing Interests

The author declares no competing interests.

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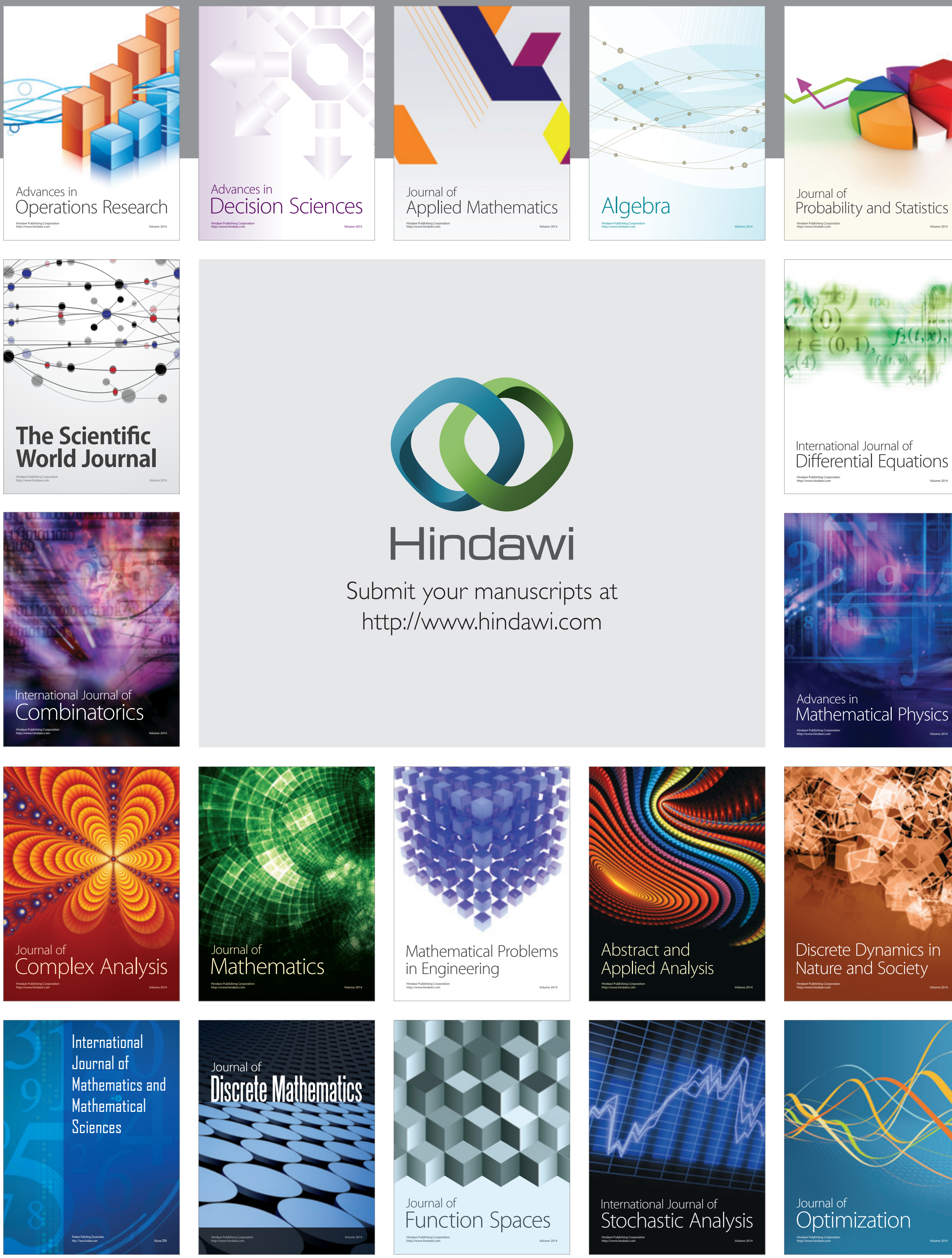
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