Hindawi Publishing Corporation
Advances in High Energy Physics
Volume 2010, Article ID 713659, 18 pages
doi:10.1155/2010/713659

Research Article

Dyon Condensation and Dual Superconductivity in Abelian Higgs Model of QCD

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Received 5 September 2010; Accepted 9 November 2010

Academic Editor: A. Petrov

Constructing the effective action for dyonic field in Abelian projection of QCD, it has been demonstrated that any charge (electrical or magnetic) of dyon screens its own direct potential to which it minimally couples and antiscreens the dual potential leading to dual superconductivity in accordance with generalized Meissner effect. Taking the Abelian projection of QCD, an Abelian Higgs model, incorporating dual superconductivity and confinement, has been constructed and its representation has been obtained in terms of average of Wilson loop.

1. Introduction

Quantum chromodynamics (QCD) is the most favored color gauge theory of strong interaction whereas superconductivity is a remarkable manifestation of quantum mechanics on a truly macroscopic scale. In the process of current understanding of superconductivity, Rajput et al. [1–3] and Kumar [4] have conceived its hopeful analogy with QCD and demonstrated that the essential features of superconductivity, that is, the Meissner effect and flux quantization, provided the vivid models [5–9] for actual confinement mechanism in QCD. Mandelstam [10–12] propounded that the color confinement properties may result from the condensation of magnetic monopoles in QCD vacuum. In a series of papers [13–16], Ezawa and Iwazaki made an attempt to analyze a mechanism of quark confinement by demonstrating that the Yang-Mills vacuum is a magnetic superconductor and such a superconducting state is considered to be a condensed state of magnetic monopole. The condensation of magnetic monopole incorporates the state of magnetic superconductivity [17] and the notion of chromomagnetic superconductor where the Meissner effect confining magnetic field in ordinary superconductivity would be replaced by the chromoelectric

Meissner effect (i.e., the dual Meissner effect), which would confine the color electric flux. As such one conceives the idea of correspondence between quantum chromodynamic situation and chromomagnetic superconductor. However, the crucial ingredient for condensation in a chromomagnetic superconductor would be the non-Abelian force in contrast to the Abelian ones in ordinary superconductivity. Topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles [18]. The method of Abelian projection is one of the popular approaches to confinement problem, together with dual superconductivity [19, 20] picture, in non-Abelian gauge theories. Monopole condensation mechanism of confinement (together with dual superconductivity) implies that long-range physics is dominated by Abelian degrees of freedom [21] (Abelian dominance).

The first model of QCD vacuum, in which the non-Abelian dyons are responsible for confinement, was given by Simonov [19], and almost simultaneously it was demonstrated by Bornyakov and Schierholz [22] that for (anti-) self-dual fields [23, 24] the Abelian monopoles, participating in confinement, become dyons. The non-Abelian dyons give rise to Abelian dyons in the Abelian projection [22]. The mere notions of dyons and of the Kraan-van Baal instantons [25, 26] (i.e., calorons) imply that the Y-M field is periodic in the Euclidean time direction and thus leads to confinement. It has recently been demonstrated by Diakonov and Petrov [27] that the ensemble of dyons can be described mathematically by an exactly solvable field theory in three dimensions and that the resulting vacuum built of dyons has certain features expected for the confining pure Y-M theory. It has also been shown by them that the dyons ensemble induces the area law for spatial Wilson loops which fulfill confinement in three dimensions. To investigate the possible physical implications of the topological structure of non-Abelian dyons in connection with the issue of quark confinement in QCD, extended gauge theory has been formulated [8, 9] in SU(2) and SU(3) groups from the corresponding restricted chromodynamics (RCD), and it has been shown that in this extended QCD the confinement mechanism of the corresponding RCD remains intact, and physical spectrum contains color-singlet generalized electric glue balls made of valence gluon pairs as well as the generalized magnetic glue balls as massive collective modes of condensed vacuum. Recently, it has been shown [3, 4] that a perfect confinement can be achieved with pure dyonic states participating in actual dyonic condensation of RCD vacuum as the result of magnetic symmetry breaking in strong coupling limit.

Evaluating Wilson loops under the influence of the Abelian field due to all monopole currents, monopole dominance has been demonstrated [21, 28]. In the Abelian projection the quarks are the electrically charged particles and, if monopoles are condensed, the dual Abrikosov-string carrying electric flux is formed between quark and antiquark. Due to nonzero tension in this string, the quarks are confined by the linear potential. The conjecture that the dual Meissner effect is the color confinement mechanism is realized if we perform Abelian projection in the maximal gauge where the Abelian component of gluon field and Abelian monopoles are found to be dominant [29, 30]. Then the Abelian electric field is squeezed by solenoidal monopole current [31]. The vacuum of gluodynamics behaves as a dual superconductor, and the key role in dual superconductor model of QCD is played by Abelian monopole. The infrared properties of QCD in the Abelian projection can be described by the Abelian Higgs Model (AHM) in which dyons are condensed. There exists the model [27, 32–34] of QCD vacuum in which the non-Abelian dyons are responsible for the confinement and the non-Abelian dyons give rise to Abelian dyons in the Abelian projection. Therefore, an important problem, before studying the vacuum properties of non-Abelian theories, is to Abelianize them so as to make contribution of the topological magnetic degrees of freedom to the partition function explicit. Such a construction for non-Abelian gauge theories and its relevance to topological magnetic charge and hence to confinement are still lacking in spite of large amount of recent literature [35–40] on the subject.

Starting with generalized field equations and the corresponding Lagrangian of the field associated with Abelian dyons in this paper, it has been demonstrated that topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by dyons which undergo condensation leading to confinement and consequently to superconducting model of QCD vacuum, where the Higgs fields play the role of a regulator only. It has also been demonstrated that for the self-dual fields, the Abelian monopoles become the Abelian dyons, and in low energy QCD the dyon interactions are saturated by duality when Abelian projection is described by the Abelian Higgs model where dyons are condensed leading to confinement and the state of dual superconductivity. Constructing the effective action for dyonic field in Abelian projection of QCD in terms of electric and magnetic constituents, A_μ and B_μ, of the generalized four-potential V_μ, the dyonic current correlators have been derived and it has been demonstrated that the dyonic electric charge produces screening effect for A_μ-propagator and antiscreening effect for B_μ-propagator while the dyonic magnetic charge produces screening effect for B_μ-propagator and antiscreening effect for A_μ-propagator. These antiscreening effects have been shown to lead to dyonic condensation and dual superconductivity and also to maintain the asymptotic freedom of non-Abelian gauge theory (QCD) in its Abelian version. A non-Abelian SU(2) gauge has been obtained in terms of Lagrangian density describing the fields associated with non-Abelian dyons and it has been shown that these non-Abelian dyons give rise to Abelian dyons in the Abelian projection of QCD. The infrared properties of QCD in this Abelian projection have been described by constructing an Abelian Higgs model in which dyons are condensed and the relevant degrees of freedom are two massive gluons, a U(1) gluon and a dyon. This AHM has been shown to incorporate dual superconductivity and confinement as the consequence of dyonic condensation.

The quantum average of Wilson loop has been obtained in the dyonic theory specified by a partition function in terms of dyon Lagrangian in Abelian Higgs model, and the effective electric and magnetic charges and four-currents of dyons have been determined from Wilson loop given in terms of electromagnetic field tensor satisfying field equations identical to those for usual electrodynamics.

2. Electromagnetic Duality and Dyonic Interaction

A gauge invariant and Lorentz covariant quantum field theory of fields associated with dyons has been developed [41–44] in purely group theoretical manner by using two four-potentials and assuming the generalized charge, generalized current, and generalized four-potential as complex quantities with their real and imaginary parts as electric and magnetic constituents, that is,

generalized charge

$$q = e - ig, \tag{2.1a}$$

generalized four-current

$$J_\mu = j_\mu - ik_\mu, \tag{2.1b}$$

and generalized four-potential

$$V_\mu = A_\mu - iB_\mu, \tag{2.1c}$$

where e and g are electric and magnetic charges on dyon, j_μ and k_μ are electric and magnetic four-current densities, and A_μ and B_μ are the electric and magnetic four-potentials associated with dyons. Taking the wave function associated with generalized field as

$$\vec{\varphi} = \vec{E} - i\vec{H}, \tag{2.1d}$$

the generalized field equations of these fields may be written as

$$\begin{aligned} \vec{\nabla} \cdot \vec{\Psi} &= J_0, \\ \vec{\nabla} \times \vec{\Psi} &= -i\vec{J} - i\frac{\partial \vec{\Psi}}{\partial t}, \end{aligned} \tag{2.2}$$

where J_0 and $\vec{J}$ are the temporal and spatial parts of J_μ defined by (2.1b).

In the compact form, these equations may be written as

$$\begin{aligned} G_{\mu\nu,\nu} &= J_\mu, \\ G^d_{\mu\nu,\nu} &= 0, \end{aligned} \tag{2.3}$$

where $G_{\mu\nu}$ the generalized field tensor, is given as

$$G_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu \tag{2.4}$$

and $G^d_{\mu\nu}$ is its dual given as

$$G^d_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\alpha\beta}G_{\alpha\beta}. \tag{2.5}$$

Equation (2.4) may also be written as

$$G_{\mu\nu} = F_{\mu\nu} - iH_{\mu\nu}, \tag{2.6a}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu, \tag{2.6b}$$

$$H_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu. \tag{2.6c}$$

Then (2.3) reduces to the following form:

$$F_{\mu\nu,\nu} = j_\mu, \tag{2.7a}$$

$$H_{\mu\nu,\nu} = k_\mu. \tag{2.7b}$$

These equations are symmetrical under the duality transformations

$$F_{\mu\nu} \longrightarrow H_{\mu\nu}, \qquad H_{\mu\nu} \longrightarrow -F_{\mu\nu}, \qquad j_\mu \longrightarrow k_\mu, \qquad k_\mu \longrightarrow -j_\mu. \tag{2.8}$$

The Lagrangian density for spin-1 generalized charge (i.e., bosonic dyon) of rest mass m_0 may be written as follows [41] in the Abelian theory:

$$\begin{aligned} L &= m_0 - \frac{1}{4}\Big[\alpha\Big\{\left(A_{\nu,\mu} - A_{\mu,\nu}\right)^2 - \left(B_{\nu,\mu} - B_{\mu,\nu}\right)^2\Big\} - 2\beta\big\{\left(A_{\nu,\mu} - A_{\mu,\nu}\right)\left(B_{\nu,\mu} - B_{\mu,\nu}\right)\big\}\Big] \\ &\quad + \left\{\left(\alpha A_\mu - \beta B_\mu\right) j_\mu - \left(\alpha B_\mu + \beta A_\mu\right) k_\mu\right\} \\ &= L_P + L_F + L_I, \end{aligned} \tag{2.9}$$

where α and β are real positive unimodular parameters, that is,

$$|\alpha|^2 + \left|\beta\right|^2 = 1. \tag{2.10}$$

L_P, L_F, and L_I are free particle, field, and interaction Lagrangians, respectively. The action integral may be written as

$$S = \int_{t_1}^{t_2} L\, dt = S_P + S_F + S_I. \tag{2.11}$$

Varying the trajectory of particle without changing the field, we get the following equation of motion:

$$m\ddot{x}_\mu = \operatorname{Re}\left(q * G_{\mu\nu}\right)u^\nu, \tag{2.12}$$

where Re denotes the real part and u^ν is the νth component of four-velocity of dyon.

An Abelian dyon moving in the generalized field of another dyon carries a residual angular momentum [45] (field contribution) besides its orbital and spin-angular momenta. If we consider ith Abelian dyon moving in the field of jth dyon (assumed at rest), its gauge invariant rotationally symmetric orbital angular momentum may be written as [45]

$$\vec{J} = \vec{r} \times \left(\vec{p} - \mu_{ij}\vec{V}^T\right) + \mu_{ij}\frac{\vec{r}}{r}, \tag{2.13}$$

where $\vec{r}$ is the position vector and $\vec{p}$ is the linear momentum of ith dyon, V^T is the transverse generalized vector potential of the field associated with jth dyon, and μ_{ij} is the magnetic coupling parameter defined as

$$\mu_{ij} = e_i g_j - e_j g_i. \tag{2.14}$$

The last term in (2.14) is the residual angular momentum carried by ith dyon besides its usual orbital angular momentum and spin-angular momentum

$$\vec{J}_{\text{res}} = \mu_{ij} \frac{\vec{r}}{r}. \tag{2.15}$$

For each pair of dyons, this residual angular momentum generates a one-dimensional representation of the pair of four-momentum associated with these particles. This is the subgroup of the Lorentz group which leaves both four-momenta invariant. This residual angular momentum leads to chirality-dependent multiplicity in the eigenvalues of angular momentum of an Abelian dyon.

With the development of non-Abelian gauge theories, Dirac monopole has mutated in another way as we have to take into account not only electromagnetic U(1) gauge group but also the color gauge group SU(3)c describing strong interaction. In QCD, because SU(3) is compact, the color electric charges defined with respect to any maximal Abelian subgroup are quantized. It implies that we can write down gauge field configurations that asymptotically look like magnetic monopole of any chosen Abelian direction. The confinement of color electric charge corresponds to the screening of color magnetic charge. There are monopole field configurations in any non-Abelian gauge theory. The phase structure of any such theory can be probed by adding a scalar field (i.e., Higgs field) in the adjoint representation so long as it does not change the nature of flow of the coupling constant with energy. For asymptotically free theories, the low energy behavior is dominated by the Abelian monopoles of almost zero mass which are almost point-like. The interaction of these point-like monopoles with gluons and charged particles can be studied as a dual analogue of point-like charged particle interactions. It leads to condensation of monopole. Thus topologically, a non-Abelian gauge theory is equivalent to a set of Abelian gauge theories supplemented by monopoles which undergo condensation leading to confinement. Thus the non-Abelian confinement of dyonic charge is related to linear Abelian theory in a dyonic superconductor.

Let us first consider the effective action for dyonic field in this Abelian projection of QCD in the following manner [17]:

$$S = -\frac{1}{4} \int G_{\mu\nu}(x) \notin (x-y) G^{\mu\nu} d^4x d^4y + J_\mu V^\mu, \tag{2.16}$$

where $\notin (x-y)$ is the generalized dielectric constant defined as

$$\notin (x-y) = \in (x-y) - i\mu(x-y) \tag{2.17}$$

with $\in (x-y)$ as ordinary dielectric constant and $\mu(x-y)$ as magnetic permeability such that

$$\int \in (x-y)\mu(x-z)d^4y = \delta(x-z), \tag{2.18}$$

where $\delta(x)$ is Dirac-Delta function. The generalized field tensor $G_{\mu\nu}(x)$ of (2.16) satisfies field equations (2.3) or equivalently the field equation (2.7a). The generalized four-current of field equation (2.3) couples to V_μ, with the current-correlators given by

$$\langle J_\mu \rangle = \frac{\delta S}{\delta V_\mu}, \tag{2.19}$$

$$\langle j_\mu(x) J(y) \rangle = \frac{\delta^2 S}{\delta V_\nu(x)\delta V_\mu(y)}. \tag{2.20}$$

Using (2.17) and (2.20), we have

$$\langle J_\mu(x) J_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \left[k^2\delta_{\mu\nu} - k_\mu k_\nu\right] \notin \left(k^2\right), \tag{2.21}$$

where $\notin (k^2)$ is Fourier transform of $\notin (x-y)$. For free fields in vacuum, $\notin (k^2) = 1$. In the perturbation theory, the deviation of $\notin (k^2)$ from 1 can be interpreted as the vacuum polarization due to dyon loops. For perturbatively small $\chi(k^2)$, we have

$$\notin \left(k^2\right) = 1 + \chi\left(k^2\right), \tag{2.22}$$

where

$$\chi\left(k^2\right) = \chi_e\left(k^2\right) - i\chi_g\left(k^2\right) \tag{2.23}$$

with $\chi_e(k^2)$ as perturbation related with electric charge loop and $\chi_g(k^2)$ as the perturbation related with magnetic charge loop.

Let us apply (2.21) to the case of dual superconductivity where $\notin$ includes fully nonperturbative effects. This rigidly excludes generalized electromagnetic field in side dual superconductor in conformity with the generalized Meissner effect with its real and imaginary constituents as the strict Meissner effect and dual Meissner effect, respectively. Then the generalized field V_μ can penetrate into a generalized superconductor up to the generalized London penetration depth

$$\lambda_L = \lambda_e - i\lambda_g, \tag{2.24}$$

where λ_e is strict penetration depth due to Meissner effect and λ_g is the dual penetration depth due to dual Meissner effect. For small values of k^2, we have

$$\notin \left(k^2\right) = \frac{m_L^2}{k^2} - \frac{ik^2}{m_L^2}, \tag{2.25}$$

where

$$m_L = \frac{1}{\lambda_L} = m_{l_e} - i m_{L_g} \tag{2.26a}$$

or

$$m_L = \frac{1}{\lambda_e - i\lambda_g} = \frac{\lambda_e + i\lambda_g}{|\lambda_L|^2}. \tag{2.26b}$$

It gives

$$\begin{aligned} m_{L_e} &= \frac{\lambda_e}{|\lambda_L|^2}, \\ m_{L_g} &= -\frac{\lambda_g}{|\lambda_L|^2}. \end{aligned} \tag{2.27}$$

Equations (2.21) and (2.1b) then give

$$\begin{aligned} \left\langle \left[j_\mu(x) j_\nu(y) + k_\mu(x) k_\nu(y) \right] \right\rangle &= -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \left[k^2 \delta_{\mu\nu} - k_\mu k_\nu \right] \in \left(k^2\right), \\ \left\langle \left[j_\mu(x) k_\mu(y) - j_\nu(x) k_\mu(y) \right] \right\rangle &= -\int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \left[k^4 \delta_{\mu\nu} - k^2 k_\mu k_\nu \right] \mu\left(k^2\right) / m_L^2. \end{aligned} \tag{2.28}$$

These equations give the generalized propagator associated with generalized field V_μ.

3. Dual Superconductivity through Generalized Meissner Effect

Let us consider electric and magnetic charges on different particles (i.e., not dyons). Then field equations (2.3) reduce to the following form:

$$\begin{aligned} F_{\mu\nu,\nu} &= j_\mu, \\ F^d_{\mu\nu,\nu} &= 0, \\ H_{\mu\nu,\nu} &= k_\mu, \\ H^d_{\mu\nu,\nu} &= 0 \end{aligned} \tag{3.1a}$$

or equivalently

$$A_\mu = j_\mu, \qquad B_\mu = k_\mu, \tag{3.1b}$$

and equation of motion (2.12) becomes

$$m\ddot{x}_\mu = \left(eF_{\mu\nu} + gH_{\mu\nu}\right)u^\nu. \tag{3.2}$$

All these equations are dual invariant under the transformations (2.8). The effective action in this Abelian projection of QCD may be written as follows from (2.16):

$$S = -\frac{1}{4}\int F_{\mu\nu}(x) \in (x-y)F^{\mu\nu}(y)d^4xd^4y - \frac{1}{4}\int H_{\mu\nu}(x)\mu(x-y)H^{\mu\nu}(y)d^4xd^4y + j_\mu A^\mu + k_\mu B^\mu. \tag{3.3}$$

The current-correlations (2.20) may then be written as follows:

$$\begin{aligned} \langle j_\mu \rangle &= \frac{\delta S}{\delta A_\mu}, \qquad \langle k_\mu \rangle = \frac{\delta S}{\delta B_\mu}, \\ \langle j_\mu(x) j_\nu(y) \rangle &= \frac{\delta^2 S}{\delta A_\nu(y)\delta A_\mu(x)}, \\ \langle k_\mu(x) k_\nu(y) \rangle &= \frac{\delta^2 S}{\delta B_\nu(y)\delta B_\mu(x)}. \end{aligned} \tag{3.4}$$

For the given action in the present case, these relations lead to

$$\begin{aligned} \langle j_\mu(x) j_\nu(y) \rangle &= -\int \frac{d^4k}{(2\pi)^4}\left[k^2\delta_{\mu\nu} - k_\mu k_\nu\right] \in \left(k^2\right), \\ \langle k_\mu(x) k_\nu(y) \rangle &= -\int \frac{d^4k}{(2\pi)^4}\left[k^2\delta_{\mu\nu} - k_\mu k_\nu\right]\mu\left(k^2\right). \end{aligned} \tag{3.5}$$

For small perturbations, we have

$$\begin{aligned} \in \left(k^2\right) &= 1 \pm \chi_e\left(k^2\right), \\ \mu\left(k^2\right) &= 1 \mp \chi_g\left(k^2\right), \end{aligned} \tag{3.6}$$

where the upper signs in the right-hand sides correspond to vacuum polarization due to charged particle-loops and the lower signs correspond to that due to monopole-loops. Relations (3.5) may also be written as

$$\langle j_\mu(x) j_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] m_{L_e}^2,$$
$$\langle k_\mu(x) k_\nu(y) \rangle = -\int \frac{d^4k}{(2\pi)^4} \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2} \right] m_{L_g}^2. \tag{3.7}$$

These relations show that charged particles $[\chi_e(k^2) \geq 1]$ produce screening effect for the A_μ*-propagator,* with the corresponding photon acquiring the mass m_{L_e} and antiscreening effect for the B_μ*-propagator.* On the other hand, the monopole loops produce screening effect for B_μ*-propagator*, with corresponding photon acquiring the mass m_{L_g} and antiscreening effect for A_μ*-propagator.* Thus any particle (electrically charged or a monopole) screens its own direct potential to which it minimally couples and antiscreens the dual potential (B_μ for electric charge and A_μ for monopole). This dual antiscreening effect leads to dual superconductivity in accordance with generalized Miessner effect.

4. Dyon Condensation and Confinement

The non-Abelian nature of gauge group [SU(3) or SU(2)] is quite crucial to dyon condensation as mechanism of confinement. The method of Abelian projection is one of the popular approaches to the confinement problem in the non-Abelian gauge theories. A general non-Abelian theory of dyons consists of usual four-space (external) and n-dimensional internal group space, where the field associated with dyons has n-fold internal multiplicity and the multiplets of gauge field transform as the basis of adjoint representation of n-dimensional non-Abelian gauge symmetry group. Choosing the internal gauge group as SU(2), the generalized dyonic field tensor may be constructed as

$$\vec{G}_{\mu\nu} = G^a_{\mu\nu} T_a \tag{4.1}$$

with the generalized four-potential defined as

$$\vec{V}_\mu = V^a_{\mu\nu} T_a, \tag{4.2}$$

where repeated indices are summed over 1, 2, and 3 (internal degrees of freedom), vector sign is denoted in the internal group space, and the matrices T_a are infinitesimal generators of group SU(2), satisfying the commutation relation

$$[T_a, T_b] = i\varepsilon_{abc} T_c \tag{4.3}$$

with ε_{abc} as a structure constant of internal group. We may connect $\vec{G}_{\mu\nu}$ and $\vec{V}_{\mu}$ through the following non-Abelian version of (2.4):

$$G^a_{\mu\nu} = \partial_\nu V^a_\mu - \partial_\mu V^a_\nu + |q|\varepsilon^{abc} V_{\mu b} V_{\nu c}, \tag{4.4}$$

where the dyonic generalized charge q is given by (2.1a).

A suitable Lagrangian density of a spontaneously broken non-Abelian gauge theory SU(2), yielding the classical dyonic solutions, may be constructed as

$$\begin{aligned} L &= -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a + \frac{1}{2} (D_\mu \phi)^a (D^\mu \phi)_a - V(\phi) \\ &= L_{\text{dyon}}(A_\mu, B_\mu, \phi), \quad \text{where } D_\mu \phi = \partial_\mu \phi - i\,\text{Re}(q * V_\mu)\phi = (\partial_\mu - ieA_\mu - igB_\mu)\phi, \end{aligned} \tag{4.5}$$

with Re denoting the real part and

$$V(\phi) = \frac{1}{4} (\phi^a \phi_a)^2 - \frac{1}{2} v^2 (\phi^a \phi_a) \quad \text{with } v = \langle \phi \rangle = \langle 0 | \phi | 0 \rangle \tag{4.6}$$

determining the vacuum expectation value of Higgs field. In simplest manner, this equation may be written as

$$V(\phi) = -\eta \left(|\phi|^2 - v^2 \right)^2 \tag{4.7}$$

with η as a constant.

The gauge-dependent part of Lagrangian, that is, first term of rhs in (4.5) is invariant under the following transformations of the fields A_μ and B_μ:

$$V_\mu = \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} \longrightarrow \begin{bmatrix} A'_\mu \\ B'_\mu \end{bmatrix} = V_\mu{}' = R(\delta) \begin{bmatrix} A_\mu \\ B_\mu \end{bmatrix} = R(\delta) V_\mu, \quad \text{where } R(\delta) = \begin{bmatrix} \cos\delta \sin\delta \\ -\sin\delta \cos\delta \end{bmatrix} \tag{4.8}$$

with

$$\delta = \tan^{-1}\left(\frac{g}{e}\right). \tag{4.9}$$

Using the Lagrangian density given by (4.5), the electric and magnetic fields of dyons may be calculated by imposing the following ansatz [46]:

$$V_{ia} = \varepsilon_{aij}(\vec{r})^j \frac{K(r)-1}{|q|r^2},$$
$$V_{0a} = (\vec{r})_a \frac{J(r)}{|q|r^2}, \tag{4.10}$$
$$\phi_a = (\vec{r})_a \frac{H(r)}{|q|r^2},$$

where the functions $K(r)$, $J(r)$, and $H(r)$ satisfy the following equations:

$$r^2 H''(r) = 2HK^2,$$
$$r^2 J''(r) = 2JK^2, \tag{4.11}$$
$$r^2 K''(r) = K\left(K^2-1\right) + K\left(H^2 - J^2\right).$$

A solution of these equations may be written as follows:

$$J(r) = \tilde{\alpha}\phi(r), \quad H(r) = \tilde{\beta}\phi(r), \quad K(r) = \frac{Cr}{\sinh Cr},$$
$$\text{where } \tilde{\beta}^2 - \tilde{\alpha}^2 = 1,\ \phi(r) = C(r)\coth Cr - 1. \tag{4.12}$$

In the Prasad-Sommerfield limit [47],

$$V(\phi) = 0; \quad \text{but } v = \langle \phi \rangle \neq 0. \tag{4.13}$$

In this limit, the dyons have lowest possible energy for given electric and magnetic charges e and g, respectively. Thus we get the following expression for dyonic mass:

$$M = v\left(e^2 + g^2\right)^{1/2} = v|q|, \tag{4.14}$$

where the electric and magnetic fields associated with dyons obey the first-order equations

$$E_i^a = G_{0i}^a = \partial^i V_0^a + |q|\varepsilon^{abc} V_{ib} V_{0c} = \left(D_i\phi\right)^a \sin\alpha,$$
$$B_i^a = \varepsilon_{ijk} G^{jka} = \left(D_i\phi\right)^a \cos\alpha, \qquad \text{where } \alpha = \tan^{-1}\frac{e}{g}. \tag{4.15}$$
$$D_0(\phi)^a = 0,$$

In these equations, i and 0 indicate space and time directions and a is an SU(2) vector index. These electric and magnetic fields associated with dyons are non-Abelian in nature having external as well as internal components. In the Abelian projection, obtained by setting

$$K(r) \to 0, \qquad J(r) \to b + cr, \tag{4.16}$$

where b and c are positive constants having the dimensions of charge and mass, respectively, these fields reduce to the following form in the asymptotic limit:

$$\begin{gathered} E_j^a = -\frac{3b}{|q|r^4}(\vec{r})^a(\vec{r})_j - \frac{2c}{|q|r^3}(\vec{r})^a(\vec{r})_j, \\ B_j^a = -\frac{(\vec{r})_j(\vec{r})^a}{|q|r^4}. \end{gathered} \tag{4.17}$$

For vanishing c (i.e., vanishing mass), these fields correspond to point-like mass-less dyons with electric charge $3b/|q|$ and magnetic charge $1/|q|$. Thus non-Abelian dyons give rise to the Abelian dyons in the Abelian projection. The infrared properties of QCD in the Abelian projection can be described in the Abelian Higgs Model (AHM) in which dyons are condensed. In this model, the relevant degrees of freedom are two massive gluons $W_\mu^\pm$, a U(1) gluon (associated with generalized field V_μ), and a dyon which we take to be scalar represented by complex field ϕ. Then the Lagrangian (4.5) reduces to

$$L_{\text{dyon}}(A_\mu, B_\mu, \phi) = -\frac{1}{4}G_{\mu\nu}G^{\mu\nu} + \frac{1}{2}\left|(\partial_\mu - ieA_\mu - igB_\mu)\phi\right|^2 + \eta\left(|\phi|^2 - v^2\right)^2. \tag{4.18}$$

In terms of this Lagrangian, the partition function in the Euclidean space-time may be written as

$$Z_{\text{dyon}} = \int DA_\mu DB_\mu D\phi \exp\left\{-\int d^4x L_{\text{dyon}}(A_\mu, B_\mu, \phi)\right\}. \tag{4.19}$$

Applying the transformation (4.8) and integrating over the field A'_μ, this partition function reduces to the following form in AHM:

$$\begin{gathered} Z_{\text{dyon}} = \int DB'_\mu D\phi \exp\left\{-\int d^4x L_{\text{AHM}}\left(B'_\mu, \phi\right)\right\}, \\ \text{with } L_{\text{AHM}}\left(B'_\mu, \phi\right) = -\frac{1}{4}H'_{\mu\nu}H'^{\mu\nu} + \frac{1}{2}\left|\left(\partial\mu - i\overline{g}B'_\mu\right)\phi\right|^2 + \eta\left(|\phi|^2 - v^2\right)^2, \end{gathered} \tag{4.20}$$

where the Higgs field ϕ has the magnetic charge

$$\overline{g} = |q|, \qquad H'_{\mu v} = \partial_\mu B'_v - \partial_v B'_\mu. \tag{4.21}$$

This model (AHM) incorporates dual superconductivity and hence confinement as the consequence of dyonic condensation since the Higgs-type mechanism arises here.

5. Dyonic Loop in Abelian Higgs Model

In the dyon theory, specified by partition function (4.19), the quantum average of the Wilson loop is [48]

$$\langle W_l^c \rangle_{\text{dyon}} = \frac{1}{Z_{\text{dyon}}} \int DA_\mu DB_\mu D\phi \exp\left\{ -\int d^4x L_{\text{dyon}}(A_\mu, B_\mu, \phi) \right\} W_l{}^c(A_\mu), \tag{5.1a}$$

where

$$W_l{}^c(A_\mu) = \exp\left\{ ie_0 \int d^4x \eta_\mu A^\mu \right\} \tag{5.1b}$$

with

$$\eta_\mu(x) = \oint_C d\check{x}_\mu \, \delta^{(4)}(x - \check{x}(\tau)), \tag{5.1c}$$

which creates the particle with electric charge e_0 on the world trajectory C.

Let us apply the transformation (4.8) to the quantum average (5.1a) and then integrate over the field A'_μ. Thus we get

$$\langle W_l^c \rangle_{\text{dyon}} = \left\langle K^c_{(q_l, q_m)}\left(B'_\mu\right) \right\rangle_{\text{AHM}} \tag{5.2}$$

with the operator $K^c_{(q_e, q_m)}$ as the product of t′ Hooft loop and the Wilson loop W^c:

$$K^c_{(q_e, q_m)}\left(B'_\mu\right) = H^c_{q_e}\left(B'_\mu\right) \cdot W^c_{q_m}\left(B'_\mu\right), \quad \text{where } q_e = \frac{e_0 g}{|q|}, \; q_m = \frac{e_0 e}{|q|}. \tag{5.3}$$

Then the effective electric and magnetic four-current density may be written as follows:

$$j_\mu = q_e \eta_\mu, \qquad k_\mu = q_m \eta_\mu. \tag{5.4}$$

In (5.3), the operator $H^c_{q_e}(B'_\mu)$ is

$$H^c_{q_e}\left(B'_\mu\right) = \exp\left\{ -\frac{1}{4} \int d^4x \left[\left(H'_{\mu v} - \frac{1}{2} \varepsilon_{\mu v \alpha \beta} F_{\alpha\beta} \right)^2 - H'_{\mu v} H'^{\mu v} \right] \right\}, \quad \text{where } H'_{\mu v} = \partial_\mu B'_v - \partial_v B'_\mu \tag{5.5}$$

and $F_{\alpha\beta}$ is the dual field tensor satisfying

$$F_{\mu v, v} = j_\mu \tag{5.6}$$

which is identical to (2.7a) for the usual electrodynamic field tensor of the field associated with Abelian dyons.

6. Discussion

Equations (2.21), (2.22), and (2.23) for dyonic current correlations show that dyonic electric charge produces the screening effect for A_μ*-propagator* and antiscreening effect for B_μ*-propagator,* while the dyonic magnetic charge produces screening effect for B_μ*-propagator* and antiscreening effect for A_μ*-propagator.* This antiscreening effect maintains the asymptotic freedom of non-Abelian gauge theory (QCD) in its Abelian version. In QCD, because of asymptotic freedom, the Landau singularity (led by charged particles in ordinary electrodynamics) is in the infrared regime and hence the most convenient microscopic theory of low energy QCD is produced by the chromodynamic dyons. The correlations (2.28) give the generalized propagator associated with generalized field V_μ of dyons. In the Abelian projection of QCD with the simultaneous existence of electric charges and monopoles (but not dyons), the effective action is given by (3.3) and the current correlations are given by (3.5), (3.6), and (3.7) which demonstrate that any particle screens its own direct potential to which it minimally couples and antiscreens the dual potential (B_μ for electric charges and A_μ for monopoles). This dual antiscreening effect leads to dual superconductivity in accordance with generalized Miessner effect. This dual superconductivity is the Higgs phase of QCD in its Abelian projection. The antiscreening, described by (3.7), provides the prescription that the magnetic photon (B_μ)-charge particle vertex is identical to the A_μ-charge particle vertex with the constant e replaced by ie. Such prescription of coupling of a gauge particle to its dual charge must be used only when all dual charges appear in loops. The duality prescribed by these equations may be a strong guide to the description of confinement, and interactions of chromomagnetic monopoles should be saturated by this duality, at least for low energy.

The gauge depended part of the Lagrangian density, given by (4.5) for the fields associated with the non-Abelian dyons in the minimal gauge theory, is invariant under the linear transformation (4.8). Equations (4.15) and (4.17) demonstrate that the non-Abelian dyons give rise to Abelian dyons in the Abelian projection obtained by setting up conditions given by (4.16). The infrared properties of QCD in this Abelian projection can be described by the Abelian Higgs model with Lagrangian density given by (4.20) in which dyons are condensed. In this model, the partition function in the Euclidean space-time is given by the first part of (4.20). This model incorporates dual superconductivity and confinement as the consequence of dyonic condensation. In the dyon theory, specified by the partition function given by (4.19) in terms of dyon Lagrangian (4.18), the quantum average of Wilson loop given by (5.1a) corresponds to quark Wilson loop if we consider this partition function as an effective theory of QCD. In (5.2) this average is given in AHM with the effective electric and magnetic charges and the effective electric and magnetic four-current densities given by (5.3) and (5.4), respectively. t′ Hooft loop is precisely given by (5.5) in terms of electromagnetic field tensor $H'_{\mu\nu}$ and the dual field tensor satisfies field equation (5.6) which is identical to (2.7a) for the usual electromagnetic field tensor of field associated with Abelian dyons. It is what we expect in the Abelian projection of QCD in the present Abelian Higgs Model of Abelian dyons in the Abelian version of QCD.

It is generally suspected that the dyonic theory is CP-violating contrary to QCD in the sense that dyon (e, g) and anti-dyon $(-e, -g)$ with the same density, when combined in one vacuum in equal amount, may violate CP-invariance. We have carried out [49] the study of behavior of dyonium in non-Abelian gauge theory and also the study of dyon-dyon bound states [50] and showed that the Bohr radius of dyonium is much smaller than the atomic Bohr radius. The study of bound state of a dyon and an anti-dyon has also been carried out [51], and it has been demonstrated that this state is very short lived and decays in to four or six photons depending on the spin-statistics relationship of the dyons involved. Furthermore, CP-invariance of the vacuum requires that there must be equal number of self-dual and anti-self-dual configurations of dyons up to the thermodynamic fluctuations $\sqrt{V}$ [52–54]. Recently, it has been demonstrated [27] that the integration measure over dyons has the drastic effect on the ensemble of dyons when determinant over nonzero modes is ignored and only the salient features like the renormalization of the coupling constant and the perturbative potential energy [55] are taken into consideration. The question of CP-invariance for dyonic system has also been addressed by Diakonov [56]. It has recently been shown [57] that though the dyon-antidyon pair appears to violate CP-invariance, the CPT-invariance is an exact symmetry for generalized dyon-antidyon system.

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