

Review Article

Lectures on Walking Technicolor, Holography, and Gauge/Gravity Dualities

Maurizio Piai

Physics Department, Swansea University, Singleton Park, Swansea, Wales SA2 8PP, UK

Correspondence should be addressed to Maurizio Piai, m.piai@swansea.ac.uk

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Dynamical electroweak symmetry breaking is an appealing, strongly coupled alternative to the weakly coupled models based on an elementary scalar field developing a vacuum expectation value. In Sections 2 and 3 of this set of lectures, I summarize the arguments, based on low-energy phenomenology, supporting walking technicolor as a realistic realization of this idea. This pedagogical introduction to walking technicolor, and more generally to the physics of extensions of the standard model, makes extensive use of effective field theory arguments, symmetries, and counting rules. The strongly coupled nature of the underlying interactions, and the peculiar quasiconformal behavior of the theory, requires to use nonperturbative methods in order to address many fundamental questions within this framework. The recent development of gauge/gravity dualities provides an ideal set of such nonperturbative instruments. Sections 4 and 5 illustrate the potential of these techniques with two technical examples, one within the bottom-up phenomenological approach to holography in five dimensions, the other within a more systematic top-down construction derived from ten-dimensional type-IIB supergravity.

1. Introduction and Outline

This set of notes on walking technicolor is based in part on sets of lectures delivered to string theory students in the context of their Ph.D. programs at Swansea University and at the University of Barcelona. As such, the content is intended for readers who are familiar with the ideas of gauge/gravity dualities [1–4], but not with technicolor (TC) [5–7], walking technicolor (WTC) [8–10], extended technicolor (ETC) [11, 12], and more in general with the details of the physics of the standard model (SM) and the problematics arising when dealing with its extensions.

This is not a complete and exhaustive review of the subject of technicolor, which can be found elsewhere, but rather a collection of exercises and arguments aimed at singling out a subset of problems that emerge in the context of physics beyond the standard model (BSM), and that are particularly important when, as is the case for WTC, the BSM new physics is

strongly coupled. This set of problems admits a natural formulation (if not yet a natural solution) in the context of gauge/gravity dualities, which is the main message these lectures want to convey. The reader who wants to have a complete overview of the subject of strongly coupled dynamical electroweak symmetry breaking (EWSB) should complement the reading of these notes with other reviews, which develop, in more details, many aspects and ideas that are only glanced at in the present set of lectures [13–18].

The first two parts of these notes are very general, and should be easily accessible to anybody who has a working knowledge of quantum field theory and a basic knowledge of the standard model. For pedagogical reasons, before talking about WTC, I start by reminding the reader about some classical results that are at the core of our modern understanding of the Minimal version of the Standard Model (MSM) (I will use the MSM acronym to mean the version of the standard model with one Higgs scalar doublet, ultimately responsible for EWSB, in such a way as to distinguish it from the acronym SM, by which I mean a generic theory with the fermions and gauge bosons of the Standard Model, without committing to the details of the symmetry-breaking sector itself), and of its weakly coupled extensions. These include the Coleman-Weinberg results for the loop generated effective Higgs potential [19], a discussion of the big and little hierarchy problems, the definition of the electroweak chiral Lagrangian [20–24] and of the oblique precision parameters [25, 26] (and their use in precision electroweak physics), the GIM mechanism and the resulting suppression of flavor-changing neutral current (FCNC) processes [27], the Froggatt-Nielsen mechanism for the generation of fermion mass hierarchy [28], the see-saw mechanism [29–32], and the analysis of perturbative unitarity of longitudinally-polarized gauge-boson elastic-scattering amplitudes [33]. As such, this first half of the paper should also provide a useful introduction for any reader who is interested in BSM physics in general.

Section 2 paper contains some generic introductory material, and part of the discussion is only semiquantitative (more precise statements will be developed in later sections). It starts from a reminder about the basic building blocks of the SM, the introduction of the concept of naturalness, and a discussion of the big hierarchy problem. I also provide an example of technicolor model, for pedagogical reasons chosen to be peculiarly simple [34]. Unconventionally, in comparing three popular scenarios for solving the big hierarchy problem (supersymmetry, technicolor and warped extra-dimensions), rather than the specific differences I highlight the similarities between these three approaches. In particular the fact that at low energies they all yield to the little hierarchy problem. In this way, it should be clear to the reader that the tension between the absence of fine-tuning (for which a new physics scale $\Lambda_1^* \lesssim 1$ TeV would be a natural expectation) and the absence of indirect effects of new physics in precision measurements (which would naturally require a larger new physics scale $\Lambda_2^* \gtrsim 5$ TeV) is a rather general problem of all BSM scenarios, not just a problem of strongly coupled extensions of the SM. After all, the very fact that, thanks to the work of our colleagues in many successful experimental programs, we have now an unprecedented body of high-precision information about nature, per se implies that not every generic new physics model can be allowed by the data. Hence, the presence of what appears to be a $\mathcal{O}(\Lambda_1^{*2}/\Lambda_2^{*2}) \sim 5\%$ fine-tuning in the low energy effective Lagrangian extending the SM is not necessarily a surprising and bad thing, but rather seems to suggest to us that something very special is going to be discovered at the LHC, hence explaining this very special low-energy results. What singles out the strongly coupled scenarios as particularly problematic is the strong coupling itself, which severely limits the possibility of exploring the parameter-space of models, in the search of regions that are compatible with the current data, in the same way as is done in the weakly coupled context, and hence explaining the origin

of the aforementioned 5% tuning in terms of specific, special detail properties of the new physics.

Section 3 contains a more systematic and precise pedagogical discussion, and, besides a preliminary introduction of the idea of walking dynamics, consists of four main subsections. In Section 3.1, precision electroweak physics is discussed, starting from the effective field theory (EFT) treatment, introducing the oblique parameters, and summarizing some classical perturbative results. In comparing to strongly coupled scenarios, I introduce the rules of naive dimensional analysis (NDA) [35] and hidden local symmetry (HLS) [36–43]. Section 3.2 is devoted to the physics of flavor. Starting again from the weakly coupled case, the GIM mechanism is introduced, and the basic logic behind the construction of weakly coupled models of flavor generation explained. ETC and WTC are then introduced and their specific physical implications are explained, and contrasted with what happens in the weakly coupled case. I do not discuss CP violation, the treatment of which to large extent follows the same logic as the physics of FCNC, up to subtleties in dealing with dipole moments that, while important phenomenologically, are not very important for the present purposes. Section 3.3 is a brief reminder about a classical result in the MSM; the scattering amplitude of longitudinally polarized gauge bosons is shown to be manifestly unitary in perturbation theory, at the diagrammatic level, provided all the couplings (including the Higgs self-coupling) are perturbative. This is obviously not the case in TC, where showing explicitly that the scattering amplitudes never grow beyond the unitarity bounds would require a full nonperturbative analysis. Many ideas have been proposed addressing this problem, which is actively studied, but a definite answer does not exist, and hence I will limit the discussion to the well-understood perturbative case. In Section 3.4 I summarize, by means of examples, a few very general expectations about the spectrum of TC, focusing on those bound states (technipions, technirhos and technidilaton) the physics of which is controlled by strong symmetry arguments. I conclude with a brief subsection containing a wish list, to some extent personal and incomplete. This is a list of specific questions about strongly coupled EWSB, all of which emerge from the previous discussion, all of which have crucial phenomenological implications, and answering to any of which requires a very high level of understanding of nonperturbative physics. Gauge/gravity dualities represent a remarkable opportunity for addressing some of these questions in a new context, which is complementary to what is known from traditional techniques.

The second half of this paper contains a set of examples of applications of the ideas and techniques derived from the context of gauge/gravity dualities, in order to address some of the questions in the wish list. This is not done systematically, rather I explain in some details two very special such applications, one in the context of effective five-dimensional theories (bottom-up approach), and one in the context of ten-dimensional supergravity and string theory (top-down approach). This may be understood as a proof of principle, showing that indeed some of the questions in the wish list can be addressed within specific holographic models, and, furthermore, that the technology necessary for this type of studies to be carried out does exist and is well understood. It must be stressed that many more examples exist in the literature, that are not even mentioned in these lectures. Since the early papers on the AdS/CFT correspondence [1–4] and by Randall and Sundrum [44], a huge body of literature on these and related subjects appeared, reviewing which goes very far beyond the aims of these lecture notes.

Section 4 is devoted to one specific five-dimensional effective theory that most resembles what might be the dual of a four-dimensional walking technicolor theory [45–55]. I show how calculability is strongly improved within this approach, in respect to what

happens in four-dimensional EFT and HLS approaches, yielding useful phenomenological correlations between observable quantities such as the oblique precision parameters and the spectrum and couplings of techni-rho mesons. I also show that some fundamental questions cannot be answered within this approach, but require more information about the underlying dynamical properties of the models.

In Section 5, a more ambitious approach is discussed [56–58]. The proposal is to study the properties of walking dynamics in isolation, separating them from the specific embedding of walking into a detailed model of electroweak symmetry breaking and fermion mass generation. This is done by constructing a 10-dimensional string theory model such that, in the supergravity limit, walking, or some feature that resembles it, emerges dynamically, and for which the technology exists allowing to perform a set of calculations that can yield a quantitative answer to some of the questions about walking dynamics that are left open by bottom-up approaches. This is a rather novel research program, and hence only few such calculations exist. I will limit myself to one specific model, which has its own advantages and disadvantages. A few other possible models and other interesting calculations are suggested, but the main aim of this section is to encourage the reader to use his/her own experience, knowledge, and creativity to devise alternative scenarios, and use them to address some of the problems listed earlier on.

2. Electroweak Symmetry Breaking, within and beyond the Standard Model

2.1. The Minimal Standard Model versus Technicolor

The standard model (SM) is a quantum field theory in which weak, strong and electromagnetic interactions are described by a gauge theory with group $G_{\text{SM}} = SU(3)_c \times SU(2)_L \times U(1)_Y$. All the known fundamental matter particles are described by (chiral) fermions transforming in some representation of G_{SM} (see Table 1). If the gauge symmetry was unbroken, all of the resulting particles would be massless. However, the electroweak gauge symmetry is spontaneously broken as $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{e.m.}}$, by some condensate that provides a nontrivial structure for the vacuum of the theory. The unbroken $U(1)_{\text{e.m.}}$ is the gauge symmetry of electromagnetism. Hence all the fermions, together with the W and Z bosons, acquire a mass, proportional to the symmetry-breaking condensate. The proportionality constant is the coupling of the field to the condensate, and depends on the specific field. All of this is well established, and can be found nowadays even in popular science books.

It is also known that none of the SM fermions nor any of the SM gauge interactions are responsible for electroweak symmetry breaking itself. (The chiral condensate of QCD does, in fact, spontaneously break the SM gauge symmetry. But the scale f_π is so small that this fact can be ignored for most phenomenological purposes.) Some new fields and new interactions must be present, in order to implement in the theory a mechanism yielding spontaneous electroweak symmetry breaking. Identifying these new fields and, equally important, new interactions is the main goal of large hadron collider (LHC) program at CERN. In the case of the minimal version of the standard model (MSM), the sector responsible for electroweak symmetry breaking is constituted by one scalar field, transforming as $(1, 2, 1/2)$ under G_{SM} . This is assumed to have a weakly coupled description in terms of a scalar potential with a negative quadratic interaction and a quartic interaction, the latter being the only free parameter to be measured at LHC. The classical minimum of the potential is non-trivial, the Higgs field acquires a vacuum expectation value (VEV), and hence yields electroweak

Table 1: Field content of the Minimal Standard Model. All the fermions are represented by a Left-Handed or a Right-Handed chiral field, with different quantum numbers. Specified is also the number N of different species having the same quantum numbers. The 3 copies of fermions are referred to in the text as *families*. The neutrino singlet ν_R is a hypothetical particle, such as is the Higgs H .

	J	$SU(3)$	$SU(2)_L$	$U(1)_Y$	N
B^μ	1	1	1	0	1
W^μ	1	1	3	0	1
G^μ	1	8	1	0	1
$q_L = \begin{pmatrix} u \\ d \end{pmatrix}_L$	1/2	3	2	1/6	3
u_R	1/2	3	1	2/3	3
d_R	1/2	3	1	-1/3	3
$\ell_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L$	1/2	1	2	-1/2	3
e_R	1/2	1	1	-1	3
ν_R	1/2	1	1	0	3?
H	0	1	2	+1/2	1

symmetry breaking. The complete Lagrangian of the MSM contains five types of terms, which depend on the fields and on their covariant derivatives. We will discuss in more detail some of these terms later on, for the time being it suffices to write them out schematically (suppressing all the indexes).

(i) Gauge-boson Lagrangian

$$\mathcal{L}_1 = -\frac{1}{2} \text{Tr} F_{\mu\nu} F^{\mu\nu} + \dots . \quad (2.1)$$

(ii) Fermion kinetic terms

$$\mathcal{L}_{1/2} = \bar{\psi} i \not{D} \psi + \dots . \quad (2.2)$$

(iii) Scalar kinetic term

$$\mathcal{L}_0 = (D_\mu H)^\dagger D^\mu H. \quad (2.3)$$

(iv) Yukawa couplings

$$\mathcal{L}_y = -y \bar{\psi}_L H \psi_R + \dots . \quad (2.4)$$

(v) Scalar potential

$$\mathcal{L}_V = -\mathcal{V} = -\mu^2 H^\dagger H - \lambda (H^\dagger H)^2. \quad (2.5)$$

The MSM is a very successful model. It agrees to very high accuracy with a number of precision electroweak tests, carried on in particular at SLAC, LEP, and TeVatron. It also successfully reproduces most of the flavor physics studied experimentally in rare processes involving B -mesons, kaons, pions, and muons. One important reason for this success is that the MSM automatically implements the GIM mechanism, so that all sources of flavor changing processes and of CP-violation are encoded in the CKM matrix, hence naturally suppressing flavor changing neutral current (FCNC) processes. It is also a remarkably satisfactory model from a theoretical point of view, in the sense that it automatically yields results that are very non-trivial. For example, thanks to the cancellation of all gauge anomalies, and to the perturbative nature of all the couplings, including the Higgs potential, it is possible to show that unitarity of certain scattering amplitudes is manifest at the diagrammatic level up to very high scales.

Yet, there is no experimental evidence supporting the idea that symmetry breaking be due to the weakly coupled Higgs sector of the MSM, with LEP and TeVatron yielding exclusion regions and bounds on the mass, but no unambiguous positive discovery signal. It is hence important to consider alternatives. In particular, there is no real reason why the sector responsible for electroweak symmetry be weakly coupled. Actually, it is a remarkable fact that in the two most celebrated examples of spontaneous symmetry breaking (the theory of superconductors, and the description of chiral symmetry breaking in QCD) the condensate does not arise from an elementary scalar developing a VEV, but rather a strong interaction produces the formation of a composite condensate.

Besides this simple fact, a set of serious reasons for concern arises when studying the UV behavior of the Higgs sector itself. The Higgs potential does not satisfy the naturalness condition: switching off any of the couplings in the Higgs potential does not result in enhancing the symmetry of the system. We will discuss this in more details later. As a result, both the quadratic and quartic operator receive additive renormalization from (divergent) loop diagrams involving all the couplings of the model. In a sense, the dynamics encoded in the potential is not fundamental, and as a result it is affected by fine-tuning problems. Finally, the beta-function of the quartic coupling of the Higgs and of the Yukawa couplings is not asymptotically free (which is related to the triviality problem). All of this suggests that the Higgs sector might provide a very good description of low-energy physics, but needs to be UV completed.

Technicolor models are a radical alternative to the Higgs of the MSM. There is no weakly coupled scalar, but rather a completely new non-Abelian gauge interaction is assumed to be present, morally analog to QCD. At low-energies (of the order of the electroweak scale), the interaction becomes strong enough to trigger confinement and chiral symmetry breaking, via the formation of a condensate of fermions. As a simple example, consider for instance a $SU(N_T)$ gauge theory, in which fermions Q_L , U_R , D_R , ℓ_L , E_R and N_R with the same quantum numbers as a family of SM fermions transform on the fundamental of $SU(N_T)$ (Here and in several other points of Sections 2 and 3 I chose to refer to this specific model mainly for its striking simplicity and elegance. The reader however should not be misled into thinking that this is always what a TC model has to look like, but should simply think of TC as a strongly coupled theory in which a dynamically generated condensate induces EWSB. For example, lots of attention has been given in recent years to small- N_T models with techniquarks in 2-index representations of the gauge group [59], rather than on the fundamental representation, and nothing prevents from much more exotic scenarios to be considered.) At the electroweak scale, condensates

form

$$\langle \bar{Q}_L U_R \rangle = \langle \bar{D}_R Q_L \rangle = \langle \bar{E}_R L_L \rangle = \langle \bar{L}_L N_R \rangle \sim \mathcal{O}(4\pi v_W^3). \quad (2.6)$$

This condensate breaks electroweak symmetry in exactly the same way as the Higgs of the MSM, because each of the bilinears entering the condensates has the same quantum numbers as H .

The properties of models of this type are the opposite of those of the MSM Higgs sector. All the couplings are natural, there is no fine-tuning nor triviality problem, all the scales are generated dynamically and naturally stabilized. However, calculability (and hence predictivity) becomes problematic, because of the strong coupling. Back-of-the-envelope estimates based on naive dimensional analysis (NDA) suggest that it is difficult to reconcile these models with electroweak precision measurements. Unitarity of scattering amplitudes is not manifest at the diagrammatic level, but relies on uncalculable contributions from the strong dynamics. And, contrary to what happens in the MSM, the generation of the mass of the SM fermions cannot proceed via marginal (Yukawa) couplings, but requires the introduction of a whole new sector of interactions, via what is referred to as extended technicolor (ETC). As a result, there is reason of concern regarding the contribution of this new ETC sector to FCNC processes, because an analog to the GIM mechanism is not automatically present.

One very plausible way to alleviate the shortcomings of technicolor, while preserving its good features, is provided by walking technicolor. The basic idea is that if the new strongly coupled theory is not some variation of QCD, but rather it has very different dynamics, such that it be strongly coupled over a large range of energies above the electroweak scale, then large anomalous dimensions are likely to be generated nonperturbatively. If so, the counting rules of NDA do not hold, and large enough masses for the SM fermions (the top quark is particularly problematic) might be generated dynamically, without necessarily running into troubles with precision electroweak parameters and FCNC processes. The dynamical justification for this scenario is the assumption that an approximate fixed point governs the physics in the IR, just above the electroweak scale, hence slowing down the running of the couplings in the strongly coupled regime, whence the name *walking*. While this adds to the calculability problems of traditional approaches to technicolor, it also provides an ideal arena in which the techniques of the gauge/string duality can be applied. Indeed, in the walking regime the theory is very strongly coupled and approximately conformal.

2.2. Naturalness, Hierarchy Problem(s), and Their Proposed Solutions

All known interactions can be reduced, at the microscopic level, to three fundamental ones: electroweak, strong, and gravitational interactions. The strength of these interactions can be characterized in terms of a typical *scale*, that can be identified, respectively, with three dimensionful coupling constants:

$$v_w \equiv \frac{1}{\sqrt{\sqrt{2}G_F}} \simeq 246 \text{ GeV},$$

$$f_\pi \simeq 93 \text{ MeV},$$

$$M_P \equiv \frac{1}{\sqrt{G_N}} \simeq 1.22 \times 10^{19} \text{ GeV},$$
(2.7)

where G_F is the Fermi constant, f_π is the pion decay constant, and G_N the Newton constant.

At present, the SM does not incorporate a universally accepted theory of quantum gravity. However, it is conceivable that if such a formulation exists, the SM should be thought of as containing the leading-order terms a low-energy effective field theory (EFT) description of all fundamental interactions. In this spirit, quantum gravity should amount to two main effects (at low-energies): the renormalization of the SM Lagrangian \mathcal{L}_{SM} itself, and the generation of higher-order corrections, encoded in higher-dimensional operators to be added to \mathcal{L}_{SM} . Both effects would be controlled by the typical scale of gravitational interactions M_P . Here is one way of seeing the emergence of a technical problem, the *big hierarchy problem*. On the one hand, the fact that

$$\frac{v_w^2}{M_P^2} \simeq 10^{-34}$$
(2.8)

justifies phenomenologically the fact that higher-order terms can be safely neglected, because of being strongly suppressed. On the other hand, the very fact that these two scales be so far apart, in a context in which quantum corrections are present, needs to be explained. In the absence of such an explanation, one would expect either that the electroweak scale and the Planck scale should be of the same order of magnitude, $v_w \sim M_P$, or that their hierarchy is due to accidental cancellations between interactions of very different nature, coming from physics taking place above and below the Planck scale, hence requiring an implausible amount of fine-tuning.

This formulation of the big hierarchy problem shows explicitly that it has a very general origin. The only explicit assumptions used in order to highlight the problem itself being the existence of a quantum theory of gravity (or more general of any new interaction beyond the SM), and of a sensible low-energy EFT description, characterized by the Planck scale M_P (or by a new physics scale $\Lambda \gg v_w$). There is another hidden assumption in all of the above, that the space-time symmetries be described by the four-dimensional Lorentz (or Poincaré) group. A vast literature exists on possible solutions to the big hierarchy problem, that propose to modify the fundamental theory in very different ways. It is remarkable that the three main classes of such solutions (based upon supersymmetry, extra-dimension, and technicolor, resp.) all rely on the idea of extending the group of space-time symmetries.

The reason why this is a good idea has to do with the relation between the concept of *naturalness* and renormalization in a general field theory. Using a definition often attributed to 't Hooft:

A coupling is *natural* if, in the limit in which it vanishes, the theory has an enlarged symmetry.

In field theory, quantum corrections renormalize the bare Lagrangian in two possible ways: some couplings renormalize multiplicatively, while others renormalize additively (due to operator mixing).

Couplings that are not natural may in general renormalize additively. Hence, they may be affected by fine-tuning problems. Setting to a very small value a parameter that is renormalized additively does not ensure that quantum corrections will preserve its smallness. A simple illustration of this is given within the MSM by the Higgs potential \mathcal{U} in (2.5), with neither μ^2 nor λ being a natural parameter. (It should be noticed that setting $\mu^2 = 0$ would render the MSM Lagrangian scale-invariant at the classical level. However, such invariance is not preserved by quantum correction, due to diagrams involving the quartic, gauge, and Yukawa couplings, and even allowing for fine-tuning in the quadratically divergent part of quantum corrections one ends up with the appearance of a scale in the 1-loop generated Higgs potential, which ultimately might lead to electroweak symmetry breaking itself [19].) As a consequence, nothing prevents them from receiving additive renormalization. A way to compute the (divergent) part of the (perturbative) 1-loop correction makes use of the results from Coleman and Weinberg [19] for the quantum effective action

$$\delta\mathcal{U} = \frac{\Lambda^2}{32\pi^2} \mathcal{S}\mathcal{T}r \mathcal{M}^2 + \frac{1}{64\pi^2} \mathcal{S}\mathcal{T}r \left(\mathcal{M}^4 \ln \left(\frac{\mathcal{M}^2}{\Lambda^2} \right) \right), \quad (2.9)$$

where Λ is the UV cut-off, \mathcal{M}^2 is the mass matrix of all fields as a function of the classical external scalar fields, and $\mathcal{S}\mathcal{T}r$ is the supertrace, a trace in which fermion and boson degrees of freedom enter with opposite signs. For instance, in the MSM, the coefficient μ^2 of the quadratic operator receives, among others, (quadratically divergent) positive contributions from loops of gauge bosons and negative from loops of the top as following:

$$\delta\mu^2 \simeq c_g g^2 \Lambda^2 - c_t y_t^2 \Lambda^2 + \dots, \quad (2.10)$$

with c_g and c_t numerical coefficients with mild 1-loop suppressions, g the gauge coupling, and y_t the top Yukawa coupling. If the cut-off is at the Planck scale $\Lambda \sim M_P$, in order for the resulting quadratic coupling to be at the electroweak scale, a very fine-tuned cancellation would be needed between this loop correction and an appropriately chosen counter-term.

Conversely, a natural coupling (barring the possibility of anomalies) renormalizes multiplicatively: in the limit in which the coupling is set to zero, the theory has an extended symmetry, that must be preserved also by quantum corrections. The fact that quantum corrections are themselves proportional to the natural coupling ensures that this symmetry property be automatically satisfied. As such, if one could find a way to render naturally the parameter in the Lagrangian setting the electroweak scale, this would not receive dangerously large additive renormalization via (2.9), and the coexistence of the widely separated electroweak scale v_W and Planck scale M_P would not require fine-tuning, making the EFT useful and predictive. This is what is done in the most popular extensions of the standard model.

A most popular class of solutions proposes to enlarge the symmetry by incorporating some form of supersymmetry, that is, by effectively adding fermionic directions to the space-time (superspace formulation), and assuming that perturbation theory is a good tool up to the Planck scale. The standard example is the Minimal Supersymmetric Standard Model (MSSM) [60, 61]. The basic idea is to enlarge the spectrum by adding a super-partner with different spin and statistics to each of the MSM fields, and to adjust the couplings so that the new Lagrangian is supersymmetric. Details about the construction require to introduce two Higgs doublets instead of one. The technical way in which supersymmetry solves the hierarchy

problem is that, when computing quantum corrections to the relevant operator controlling the electroweak scale, loops involving bosons and fermions in the same supersymmetry multiplets cancel against each other. In other words, in an $\mathcal{N} = 1$ supersymmetric theory in a generic vacuum (and under a few rather soft conditions such as the absence of mixed gauge-gravity anomalies),

$$\mathcal{S}\mathcal{T}r \mathcal{M}^2 = 0. \quad (2.11)$$

As a result, the quadratically divergent corrections to the supersymmetric generalization of (2.5) drop, in favor of a milder logarithmic divergence. In the MSSM, supersymmetry is broken explicitly, by adding to the supersymmetric Lagrangian a restricted set of mass terms and couplings the effect of which are soft, they preserve the logarithmic dependence of the divergence on the cut-off scale. Interestingly, because the Higgs potential, in the supersymmetric limit, does not admit an electroweak symmetry breaking minimum, the latter is controlled by the scale of supersymmetry breaking itself Λ^* , hence linking electroweak scale and supersymmetry-breaking scale.

A second class of solutions to the big hierarchy problem involves the introduction of space-like extra-dimensions. There is a vast number of variations on this idea. One most appealing such realization is the Randall-Sundrum scenario [44], possibly assisted by some implementation of the Goldberger-Wise mechanism [62, 63]. The basic idea is to introduce a fifth dimension z , and assume that the space-time has the geometry of a slice of AdS space,

$$ds^2 = \frac{L^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2), \quad (2.12)$$

with L the curvature scale, and with two 4-dimensional boundaries so that $L_0 < z < L_1$. The radial direction is related to the scale of the 4-dimensional theory living on a probe localized at z , with $z \rightarrow L_0$ corresponding to the UV and $z \rightarrow L_1$ corresponding to the IR. While the 4-dimensional Planck scale M_P is related to the five-dimensional fundamental scale M_5 as

$$M_P^2 \simeq \frac{M_5^3 L^3}{L_0^2}, \quad (2.13)$$

and one chooses (up to $O(1)$ factors) $L_0 \sim L \sim 1/M_5 \sim 1/M_P$, the typical scale of the physics taking place in the IR is suppressed as

$$\Lambda^{*2} \sim \frac{L_0^2}{L_1^2} M_P^2. \quad (2.14)$$

In the Goldberger-Wise mechanism, the hierarchy between the position of the two boundaries is determined dynamically to be

$$\frac{L_0}{L_1} \propto \left(\frac{v_1}{v_0} \right)^{1/\epsilon}, \quad (2.15)$$

where $v_{0,1} = \mathcal{O}(M_P)$ are the boundary values of some dynamical bulk scalar, and ϵ is a natural parameter, related to the coefficient of the only relevant coupling of the five-dimensional scalar (i.e., its five-dimensional mass). If the physics of electroweak symmetry breaking is to take place near the IR boundary, its scale is going to be exponentially suppressed in respect to the Planck scale, hence solving the big hierarchy problem, provided a modest hierarchy (e.g., $v_1/v_0 \sim 1/10$) is chosen, together with a large value of the exponent (e.g., $\epsilon \sim 1/20$). For discussions about the holographic interpretation of this picture see also [64, 65].

A simple way of introducing the idea behind the third class of solutions to the hierarchy problem (technicolor) is to go back to QCD, observing that in what we said up to now, we never mentioned a strong-interaction fine-tuning problem related to the fact that $f_\pi \ll M_P$. For a good reason, there is no such a problem! The Lagrangian of a QCD-like theory with N_f massless fermions transforming on the fundamental representation of $SU(N_c)$ is

$$\mathcal{L} = -\frac{1}{2} \text{Tr} F^2 + i\bar{\psi}\not{D}\psi, \quad (2.16)$$

where the covariant derivative is

$$D\psi = \partial\psi - igA\psi. \quad (2.17)$$

The beta-function computed at 1-loop is

$$\beta(g) = -\frac{g^3}{(4\pi)^2} \left(\frac{11}{3}N_c - \frac{2}{3}N_f \right) = -\frac{g^3}{(4\pi)^2} b_0, \quad (2.18)$$

and accordingly

$$g^2(\mu) = \frac{g^2}{1 + \left(g^2 / (4\pi)^2 \right) b_0 \ln(\mu^2 / \Lambda^2)}, \quad (2.19)$$

where μ is the renormalization scale and g is defined at some reference scale Λ . The expression for $g^2(\mu)$ diverges for $\mu^2 = \Lambda^{*2}$ defined by

$$\Lambda^{*2} \equiv \Lambda^2 e^{-(4\pi)^2 / g^2 b_0}. \quad (2.20)$$

If $\Lambda \sim M_P$, and for choices of N_c and N_f such that $b_0 \sim O(1)$, this means that Λ^* defined in this way is exponentially suppressed, provided that the theory is asymptotically free, and the coupling g measured at the scale Λ is perturbatively small. Because Λ^* is the scale at which the theory becomes strongly coupled, then one has $f_\pi \sim O(\Lambda^*) \ll M_P$.

The symmetry principle behind the mechanism that makes f_π small compared to the Planck scale can be read from (2.16). At the classical level, this Lagrangian is scale invariant because all the operators are exactly marginal. Quantum effects break conformal invariance, the gauge coupling representing a marginally relevant deformation of the massless free theory living at the UV fixed point. However, the breaking is controlled by the coupling

itself, setting the coupling g (as defined at a given scale Λ) to very tiny values, implies that $\Lambda^*/\Lambda \rightarrow 0$, because in the $g = 0$ limit the theory reduces back to a trivial conformal theory of noninteracting, massless gluons and quarks. Again, a space-time symmetry is present, broken only by a natural parameter (the gauge coupling) which renormalizes multiplicatively (all renormalizations of the gauge coupling are proportional to the gauge coupling itself). As a result, no fine-tuning is needed in order to parametrically separate the scale of the strong dynamics from the Planck scale.

The basic mechanism behind technicolor is the same: it relies on removing the Higgs sector from the MSM, and replacing it with a new strong sector with a new gauge symmetry (technicolor) acting on new degrees of freedom (technifermions). The electroweak gauge group is defined by gauging a subgroup of the global symmetry of such fermions (techniflavor). Once the theory becomes strongly coupled at scale Λ^* , a chiral condensate will form dynamically, generating v_W and spontaneously breaking the electroweak symmetry, in analogy to what happens in QCD. Appropriately choosing the new gauge coupling, it is possible to arrange for $\Lambda^* \ll M_P$, without any fine-tuning, because in the $g \rightarrow 0$ limit one recovers a conformal theory, with space-time symmetry described by $SO(4,2)$.

Summarizing, the three examples discussed share the same basic idea of enlarging the space-time symmetry. While the way in which this is done is very different, it yields very similar results, turning the dangerous quadratic divergences relating the electroweak and Planck scales into logarithms, and linking the electroweak scale to a new scale Λ^* characterizing the breaking of the enlarged space-time symmetry itself. Namely, below Λ^* , supersymmetry is lost, the theory is four-dimensional, and scale-invariance is lost, respectively, in the three cases. Provided $\Lambda^* \lesssim 1 \text{ TeV}$, no fine-tuning is present in the electroweak symmetry breaking sector. The scale Λ^* is also the scale at and above which a new physics sector (fields and interactions) appears. It controls the masses of the superpartners of the SM fields, the masses of the KK modes coming from the compact extra-dimension, and the masses of the resonances and composite states in technicolor (technimesons, technibaryons, ...). But Λ^* also controls *indirect* contributions to low-energy precision physics, which we will discuss at length in these lectures.

2.2.1. Little Hierarchy Problem

Whichever your favorite solution to the big hierarchy problem is, provided it is based on the idea of enlarging the space-time symmetries as in the three cases discussed earlier, the net result is that there are now four physical scales, characterizing four distinct interactions: f_π , v_W , M_P , and Λ^* . The more the two scales v_W and Λ^* are close to each other, the more satisfactory is the solution to the hierarchy problem.

Below Λ^* , the process of integrating out all of the sectors responsible for the breaking of the enlarged symmetry (and for the arising of Λ^* itself) means that corrections to the low-energy description of electroweak and strong interactions will be induced, controlled by Λ^* . In other words, the EFT whose leading-order terms are given by the standard model will have corrections suppressed by inverse powers of Λ^* , besides those suppressed by the Planck scale M_P . These corrections are tightly bounded experimentally. The precision electroweak tests can be summarized in the precision parameters of the electroweak chiral Lagrangian (or equivalently in the oblique parameters such as S and T), and they indicate that the physics of the electroweak gauge bosons can receive correction coming from higher-dimensional operators that are at most at the per mille level. Naively, this suggests that $\Lambda^* \gtrsim 5 \text{ TeV}$. The physics of flavor changing neutral currents, tested in great details by studying rare processes

involving kaons, B mesons, charmed mesons, pions, and charged leptons also indicates that, unless Λ^* has nothing to do with flavor, Λ^* must be large.

In the specific case of the MSSM, analogous source of tension can be shown to have a more subtle origin. Some of the scalars from the Higgs multiplets complete, in the supersymmetric limit, the heavy vector multiplets containing the gauge bosons and the gauginos. In this limit, the mass of such scalars is closely linked to the mass of the electroweak gauge bosons. In particular, for the mass of the lightest Higgs scalar in the MSSM the bound $m_h^2 < M_Z^2$ holds in complete generality at the tree-level. (Technically, this originates from the fact that the quartic couplings in the Higgs potential are not free parameters, but actual gauge couplings derived from the D -terms within the supersymmetric gauge theory.). Loop effects involving (finite) diagrams affected by supersymmetry breaking partially remove this bound, and one can show, for example, that the corrections to the physical mass of the lightest scalar particle in the spectrum, very schematically look like [66–69]

$$\delta m_h^2 \simeq c_h v_W^2 \ln \frac{\Lambda^{*2}}{v_W^2}, \quad (2.21)$$

where c_h is again 1-loop suppressed. Because c_h is perturbatively small (loop-suppressed), evading the experimental bounds from direct searches $m_h \gtrsim 114$ GeV requires, again, that Λ^* should be somewhere in the few-TeV range (or at least the mass of one of the scalar partners of the top should). The logarithmic dependence also means that if the lower bound on the Higgs mass raises, evading it would require a very significant amount of fine-tuning.

Concluding, solving the big hierarchy problem requires linking the electroweak scale to a new scale Λ^* which is kept *naturally* smaller than the Planck scale, and at which some new enlarged space-time symmetry be broken. Absence of any fine-tuning would be achieved if $\Lambda^* \lesssim 1$ TeV. However, indirect searches strongly suggest that $\Lambda^* \gtrsim 5$ TeV, both in weakly and strongly coupled scenarios, substantially above the scale dictated by v_W . This tension goes under the name of *little hierarchy problem*. There are several ways of addressing the little hierarchy problem, which may require the introduction of entirely new fields and symmetries (such as in the case of composite Higgs [70–77] models and Little Higgs models [78–82]), hence again naturally separating $v_W \ll \Lambda^*$. Or, one might modify the solution to the big hierarchy problem in such a way as to implement some discrete symmetry, in such a way as to enforce cancellations and suppressing unwanted processes. The literature on the subject is vast, and these proposals are very interesting phenomenologically, both for the LHC and for astrophysics and cosmology (e.g., models with parities might play an important role in the physics of cold dark matter), but are not the subject of these lectures.

It must be stressed that numerically the little hierarchy problem is, indeed, little. In a generic low-energy theory it amounts to a fine-tuning at most $\mathcal{O}(1 \text{ TeV}^2 / (5 \text{ TeV})^2) \sim 5\%$. The arguments presented here are strongly affected by model-dependent coefficients that in this discussion have been taken to be $\mathcal{O}(1)$, but that in special models might just occur to be small enough to evade the experimental bounds. Particularly in the case of technicolor, which naively is the one example in which the little hierarchy problem is most worrisome, the strongly coupled nature of the theory at the Λ^* scale makes the calculation of such coefficients quite delicate. Accurate and efficient ways of computing such observables are needed. Part of these lectures will be devoted to this specific topic, in the context of walking technicolor (which itself involves the introduction of a very special symmetry), and we will see that this possibility is not excluded. While doing so, the arguments presented in this short subsection will be made more rigorous and quantitative, and concrete examples will be developed.

2.2.2. Final Remarks

Before we move away from these general introductory remarks, and start studying more concrete examples and perform more specific calculations, a final set of comments are due to the reader. First of all, one might argue that hierarchy problems as defined in the previous section are not severe failures of the theory that require its complete rewriting, they do not indicate a fatal contradiction between theory and experiment. After all, once properly renormalized, the MSM and its parameters can be used to reproduce correctly a vast amount of experimental data, with no clear indication of a contradiction (barring the fact the the Higgs particle has not been observed yet, and hence might not exist in nature, which is a different problem). In general, the emergence of a hierarchy problem can be looked at as the emergence of an opportunity for research. A fine-tuned parameter is an indication given to us by nature of a possible avenue for discovery of actually new, interesting physical phenomena, by highlighting a sector of the theory which we do not really understand at the fundamental level, and which is probably incomplete. It would be regrettable not to pursue this avenue. This is one reason why so much attention is paid by model-builders to the hierarchy problems. Besides the electroweak hierarchy problem(s) summarized here, similar attitude is applied towards other hierarchy (fine-tuning) problems, such as the strong-CP problem in QCD, and, most important, the problem of the cosmological constant.

Second, it must be stressed that the smallness of a parameter does not necessarily constitute a hierarchy problem. At least, not in the sense of naturalness used here. For instance, one might legitimately ask why there exists such a large hierarchy between the numerical values of the masses of the SM fermions, comparing, for instance, the neutrinos to the electron, and to the top quark. However, all of these masses are controlled by parameters that are *natural*, and hence their values are stable against radiative corrections, without the technical problem of fine-tuning. It would certainly be very interesting and useful to have a theory of flavor, in which these masses and their hierarchical structures are generated dynamically from first principles, and a vast literature exists on the subject, with beautiful and very interesting results having been collected over the years. Yet, the argument in favor of such a research program is somewhat less compelling, at least within weakly coupled theories. However, this kind of research program becomes compelling in strongly coupled extensions of the standard model, for reasons that will be explained later in these notes.

Finally, the ultimate reason why fine-tuned theories are unsatisfactory is deeply connected with the concept of effective field theory. All progress in physics up to recent times can be thought of in terms of a chain of EFTs, and the vast majority of physical phenomena that we believe we understand are actually understood in terms of their EFT description. EFTs, as opposed to would-be fundamental theories, are constructed on the basis of very few and simple general symmetry rules. These rules cannot, by themselves, single out a Lagrangian (or action) which contains only a finite number of local interactions. In general, an EFT contains an infinite number of such interactions, and an infinite number of free parameters. In the absence of anything else, these theories cannot be calculable and predictive, because no matter how many independent experimental quantities are measured and used as input, no unique prediction can be formulated. What makes EFTs work (and they do in real situations) is that one can implement in the EFT also counting rules, ensuring that within the regime of validity of the EFT, and requiring a given accuracy from the calculations to be performed, only a finite number of such parameters are actually important. The (infinite) others are, on the basis of the counting rules, expected to give contributions that,

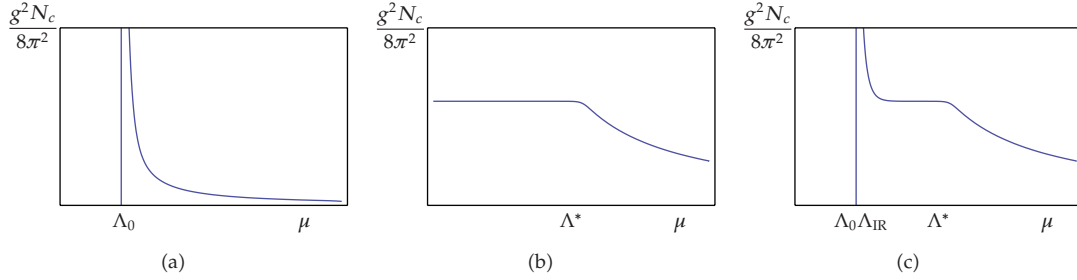


Figure 1: Schematic depiction of the evolution of the gauge coupling as a function of the renormalization scale in three different asymptotically free theories: a QCD-like theory, a theory with an IR fixed point, and a walking theory (left to right).

while uncalculable, are safely smaller than the required accuracy. Hence, the EFT Lagrangian can be truncated to contain only a finite number of terms.

The problem comes from the fact that the counting rules do not yield actual predictions for the values of the (unknown) parameters, but provide only order-of-magnitude estimates. Such estimates are hence useful only provided no fine-tuning turns out to be necessary. In the case where one of the parameters that are kept in the truncated EFT turns out to be severely fine-tuned, such a parameter is explicitly violating the counting rules. It becomes hence impossible to justify the truncation itself, and predictivity is lost. Hence, the hierarchy problem(s) in the standard model highlight a contradiction in the procedure that would allow to use it as (the leading order part of) an EFT description. The history of successes of EFTs itself provides a very compelling reason to look for solutions to the hierarchy problem, beyond the SM.

3. From Weak to Strong Coupling: Walking Technicolor

We have already anticipated that technicolor provides a solution to the big hierarchy problem based on the same mechanism at work in QCD, namely, at the Planck scale the theory is very close to a trivial fixed point of the renormalization group equations. The coupling of a marginally relevant operator (the gauge coupling) drives the theory away from this fixed point towards the IR. The theory becomes strongly coupled at a scale Λ^* . An exponential hierarchy $\Lambda^* \ll M_P$ is natural, because controlled by the (perturbative) value of the gauge coupling in the UV. Finally, the strong coupling triggers the formation of a condensate that induces EWSB, hence linking Λ^* and v_W . We also anticipated that the phenomenological viability of such a scenario is heavily questioned by the fact that the little hierarchy problem is exacerbated by the strong coupling at the scale Λ^* , particularly in a QCD-like theory, and that walking technicolor is one proposal that might soften such a problem. Let us be more specific.

Walking technicolor is based on a gauge theory in which the running of the gauge coupling is completely different from what is expected in a QCD-like theory. Figure 1 illustrates the basic idea. In Figure 1(a) is the QCD-like running as obtained for instance from (2.19). One dynamical scale $\Lambda_0 = \Lambda^*$ exists, identified by the divergence of the running coupling $g^2 N_c / (8\pi^2)$. Figure 1(b) presents a theory that in the IR flows to a non-trivial fixed point. Again, one dynamical scale exists Λ^* , separating the two energy regimes at which the

theory is well approximated by the UV and IR fixed points, respectively. There exists two well-established examples of such behavior. One can be seen to exist by looking at the 2-loop perturbative β -function of a $SU(N_c)$ theory with N_f vectorial fermions on the fundamental representation [83]. If N_f is very close to the limit beyond which asymptotic freedom is lost, the β -function is suppressed by anomalously small coefficients. One then finds that a fixed point exists in the IR, and that provided this is perturbative (so that neglecting higher-loop contributions is justified) the theory flows into a weakly coupled non-Abelian Coulomb phase.

Another example emerges for the supersymmetric version of the same gauge theory, in which case the beta function is [84–86]

$$\beta(g) = -\frac{g^3}{16\pi^2} \left(\frac{3N_c - N_f - N_f \gamma(g^2)}{1 - N_c(g^2/8\pi^2)} \right), \quad (3.1)$$

where the anomalous dimension of the mass is

$$\gamma(g^2) = \frac{g^2}{8\pi^2} \frac{N_c^2 - 1}{N_c}. \quad (3.2)$$

Again, when N_f is very close to $3N_c$, a fixed point in the spirit of [83] exists. However, because of supersymmetry the theory possesses a Seiberg duality [87–89], which in turns means that such a fixed point exists in the whole conformal window,

$$\frac{3}{2}N_c < N_f < 3N_c. \quad (3.3)$$

The interesting fact is that towards the lower end of the conformal window, the fixed point is strongly coupled. Again thanks to supersymmetry, one can compute the anomalous dimension γ at the fixed point as

$$\gamma = -1 + \frac{3N_c}{N_f}, \quad (3.4)$$

which agrees with the intuition by vanishing when $N_f \rightarrow 3N_c$, but towards the other boundary of the conformal window yields

$$\lim_{N_f \rightarrow 3N_c/2} \gamma = 1, \quad (3.5)$$

with the dimension of the (super-)quark bilinear $d(\tilde{Q}Q) = 2 - \gamma$ changing continuously in the range $1 < d(\tilde{Q}Q) < 2$ as a function of N_f , by assuming values that are $\mathcal{O}(1)$ and hence completely nonperturbative.

The dynamics of a walking theory is a combination of these confining and conformal theories. In the IR, below the scale Λ^* , the theory is approaching an approximate fixed point, governed by a strongly coupled conformal field theory, described by operators that have large anomalous dimensions. The fixed point is approximate in the sense that some dynamical

feature prevents the theory from actually reaching such a fixed point. Deep into the IR, the theory actually confines and yields a symmetry breaking condensate at the electroweak scale v_W . The gauge coupling evolves as in Figure 1(c), in a way that is somewhat a hybrid of the previous two cases.

Notice that in the walking case there are at least three dynamical scales. A scale Λ^* analogous to the conformal case, at which the theory enters a quasiconformal phase, a scale Λ_0 deep in the IR in which the gauge coupling diverges in the same sense as in the QCD-like case, and an additional intermediate scale Λ_{IR} , at which the theory stops walking, and enters the fast running leading ultimately to confinement at Λ_0 . The word *walking* refers to the fact that the coupling is approximately constant in the region $\Lambda_{\text{IR}} < \mu < \Lambda^*$, where the beta function is parametrically small in comparison to the coupling itself. Somewhere between the two lower scales (which might coincide) the symmetry breaking scale v_W should be dynamically generated too.

A second feature, of utmost phenomenological relevance, is that because the physics in the walking region is governed by the strongly coupled approximate IR fixed point, its operators acquire in general very large, nonperturbative anomalous dimensions, which drastically change the power-counting rules expected from NDA and implemented in the low-energy effective description of electroweak physics in the standard model.

In principle, one would like to have a nonperturbative, nonsupersymmetric theory with the properties mentioned above. We have no rigorous proof that such a thing exists, but it is usually believed that some critical N_f^c exists above which the theory flows into an IR fixed point, and below which it confines and condensates form. A walking theory is supposed to be given by the limit in which N_f is close to this critical value. In this lecture, we will be more precise about the phenomenological implications of the previous statements, and discuss what are the challenges of this proposal, in direct comparison with what happens in perturbative theories such as the MSM.

3.1. Electroweak Chiral Lagrangian and Oblique Precision Parameters

A convenient way of parameterizing the low-energy effects of the strongly coupled sector responsible for electroweak symmetry breaking is provided by the electroweak chiral Lagrangian [20–24]. The basic idea is that at very low energy the only relevant degrees of freedom are given by the Goldstone bosons and the electroweak gauge bosons. (In these lectures, it is assumed that the SM fermions do not carry quantum numbers of the technicolor sector, and are essentially treated as spectators.) An effective description is then obtained by writing (infinite numbers of) possible local interactions involving pions and gauge bosons, compatibly with the symmetries of the problem, and organizing the expansion as a power-series.

One can think of the resulting effective action as a systematic approximation of the quantum effective action that would be obtained by integrating out all the heavy degrees of freedom in the theory (technimesons, technibaryons, technigluons, ...). In principle, this is a nonlocal function of the momentum q^2 . Provided q^2 is much smaller than a scale Λ^{*2} related to the lightest among the masses of all the degrees of freedom that are integrated out, one can expand in powers of q^2/Λ^{*2} , and truncate at a given order in the expansion. The truncation reduces the number of independent couplings to be finite. At the same time, the truncation determines the level of accuracy of the results of calculations of physical observables, which is itself controlled by a power of q^2/Λ^{*2} .

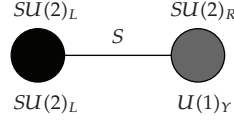


Figure 2: Diagram of the 2-site model, the electroweak chiral Lagrangian.

The electroweak chiral Lagrangian [20–24] is defined from the two-sites model in Figure 2. One defines a $SU(2)_L \times SU(2)_R$ global symmetry, gauges the $SU(2)_L \times U(1)_Y$ subgroup, with $U(1)_Y$ generated by T^3 , and introduces the dimensionless

$$S = e^{2i\pi/f}, \quad (3.6)$$

with $\pi = \pi^a T^a$, $T^a = \tau^a/2$, and τ^a the Pauli matrices, so that S transforms as a bifundamental under $SU(2)_L \times SU(2)_R$. The decay constant is the electroweak VEV, $f = v_W$. The pion fields π^a are the Goldstone bosons of the spontaneous breaking $SU(2)_L \times SU(2)_R \rightarrow SU(2)_V$. In unitary gauge $S = 1$, and the pions become the longitudinal components of the electroweak gauge bosons. The covariant derivative is

$$DS = \partial S + i(gWS - g'SB), \quad (3.7)$$

with $W = W^a T^a$ the weakly coupled gauge bosons of $SU(2)_L$, $B = B^3 T^3$ the gauge boson associated with the diagonal $U(1)_Y \subset SU(2)_R$, and g and g' the (weak) gauge couplings. In terms of the field strength $W_{\mu\nu}$ and $B_{\mu\nu}$, of the covariant derivative $D_\mu S$, of the SM currents $J_{W,B}$ (which contain the SM fermions), and of the custodial-symmetry breaking operator $t = ST^3 S^\dagger$, the Lagrangian reads

$$\mathcal{L}_X = \mathcal{L}_J + \mathcal{L}_g + \mathcal{L}_{2,0} + \mathcal{L}_{2,2} + \mathcal{L}_{4,0} + \mathcal{L}_{4,1} + \mathcal{L}_{4,2} + \mathcal{L}_{4,4} \cdots, \quad (3.8)$$

where

$$\begin{aligned} \mathcal{L}_J &= +2g \operatorname{Tr} W J_W - 2g' \operatorname{Tr} B J_B, \\ \mathcal{L}_g &= -\frac{1}{2} \operatorname{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{2} \operatorname{Tr} B_{\mu\nu} B^{\mu\nu}, \\ \mathcal{L}_{2,0} &= \frac{f^2}{4} \operatorname{Tr} |DS|^2, \\ \mathcal{L}_{2,2} &= \alpha_0 g^2 f^2 \left[\operatorname{Tr} (t D_\mu S S^\dagger) \right]^2, \\ \mathcal{L}_{4,0} &= \alpha_1 g g' \operatorname{Tr} S B_{\mu\nu} S^\dagger W^{\mu\nu} + i\alpha_2 g' \operatorname{Tr} (B_{\mu\nu} [S^\dagger D^\mu S, S^\dagger D^\nu S]) \\ &\quad + i\alpha_3 g \operatorname{Tr} (W_{\mu\nu} [D^\mu S S^\dagger, D^\nu S S^\dagger]) \\ &\quad + \alpha_4 \operatorname{Tr} (D_\mu S S^\dagger D_\nu S S^\dagger) \operatorname{Tr} (D^\mu S S^\dagger D^\nu S S^\dagger) + \alpha_5 \operatorname{Tr} (D_\mu S S^\dagger D^\mu S S^\dagger)^2, \end{aligned}$$

$$\begin{aligned}
\mathcal{L}_{4,1} &= 2\alpha_{11} g e^{\mu\nu\rho\lambda} \text{Tr}(tV_\mu) \text{Tr}(V_\nu W_{\rho\lambda}), \\
\mathcal{L}_{4,2} &= 4\alpha_6 \text{Tr}\left(D_\mu SS^\dagger D_\nu SS^\dagger\right) \text{Tr}\left(tD^\mu SS^\dagger\right) \text{Tr}\left(tD^\nu SS^\dagger\right) \\
&\quad + 4\alpha_7 \text{Tr}\left(D_\mu SS^\dagger D^\mu SS^\dagger\right) \text{Tr}\left(tD_\nu SS^\dagger\right) \text{Tr}\left(tD^\nu SS^\dagger\right) + \alpha_8 g^2 [\text{Tr}(tW_{\mu\nu})]^2 \\
&\quad + 2i\alpha_9 \text{Tr}(tW_{\mu\nu}) \text{Tr}\left(t\left[D^\mu SS^\dagger, D^\nu SS^\dagger\right]\right), \\
\mathcal{L}_{4,4} &= 8\alpha_{10} \left[\text{Tr}\left(tD_\mu SS^\dagger\right) \text{Tr}\left(tD_\nu SS^\dagger\right)\right]^2.
\end{aligned} \tag{3.9}$$

The notation introduced here is somewhat unconventional, and demands for an explanation. In the terms $\mathcal{L}_{i,j}$, the index i indicates the dimension of the operators, obtained by counting $[S] = [E]^0$, $[\partial] = [D] = [E]^1$, and $[W_{\mu\nu}] = [B_{\mu\nu}] = [E]^2$. The index j counts the explicit appearances of t , the operator introducing explicit breaking of custodial symmetry. In practice, there are three expansions being performed, and used in classifying corrections beyond $\mathcal{L}_{2,0}$. One is the aforementioned expansion in derivatives ($q^2/\Lambda^{*2} \ll 1$). But there is also an expansion in a custodial-symmetry breaking parameter δ/Λ^* , implicit within the definition of many of the α_i . And finally we are making use of the perturbative expansion in $g^2/(4\pi^2)$, $g'^2/(4\pi^2) \ll 1$. The way in which the Lagrangian is written is such that the small parameters are all implicit in the definition of the coefficients α_i , which are hence expected to be small.

In order to understand the origin of all these couplings, and how the counting works, let us start by assuming that custodial symmetry be exact. The only terms surviving in this limit are the ones in $\mathcal{L}_{2,0}$ and $\mathcal{L}_{4,0}$. (Some insight might be obtained from the comparison to the chiral Lagrangian, and the classification by Gasser and Leutwyler [90] of the coefficients appearing at the $O(q^4)$. α_1 corresponds to L_{10} , α_2 and α_3 to L_9 , α_4 to L_2 , and α_5 to L_1 .) One could start from $\mathcal{L}_{2,0}$ alone. The fact that the symmetry is nonlinearly realized is reflected in the fact that S contains only the three degrees of freedom of the pion (the constraint $SS^\dagger = 1$ is applied). Expanding in powers of π/f , at the leading order one gets the (canonically normalized) kinetic terms for the pions, while the next-to-leading order yields a (derivative) quartic coupling proportional to $1/f^2$,

$$\mathcal{L}_{2,0} = \text{Tr}(\partial\pi)^2 + \frac{1}{3f^2} \text{Tr}[\pi, \partial\pi], [\pi, \partial\pi] + \dots, \tag{3.10}$$

which, for instance, yields the leading-order contribution to the $\pi\pi \rightarrow \pi\pi$ elastic scattering amplitude. This coupling can also be used to construct 1-loop diagrams. Because the EFT is nonrenormalizable, the divergences of the 1-loop diagrams cannot be easily removed; there are logarithmically divergent amplitudes proportional to q^4 . In order to cure these divergences, one has to introduce new couplings in the Lagrangian, which are those in $\mathcal{L}_{4,0}$. This means that one should think of the expansion in q^2/Λ^{*2} as an expansion in \hbar . Going to higher loops will involve introducing new couplings with higher number of derivatives. Most important, in comparing with experimental data, one has to be careful to use the Lagrangian \mathcal{L}_X appropriately; if one wants to extract a leading-order value for, say α_1 , the comparison should be done by including 1-loop diagrams built with the couplings extracted from $\mathcal{L}_{2,0}$.

Analogously, one can think of custodial-symmetry breaking effects as arising from loop diagrams involving fundamental degrees of freedom (techniquarks) in the underlying theory, whose masses and couplings violate custodial symmetry. And hence we associate also δ^2/Λ^{*2} with $\mathcal{O}(\hbar)$ corrections. The power-counting is hence determined by assuming that

$$\frac{q^2}{\Lambda^{*2}} \sim \frac{\delta^2}{\Lambda^{*2}} \sim \frac{g^2}{4\pi^2} \sim \mathcal{O}(\hbar). \quad (3.11)$$

Looking back at \mathcal{L}_χ one can ask what is the physical meaning of the couplings in terms of the physics of electroweak gauge bosons. One immediately sees that α_0 , α_1 , and α_8 produce corrections to the gauge boson propagators, while α_2 , α_3 , α_{11} , and α_9 correct the cubic couplings and α_4 , α_5 , α_6 , α_7 , and α_{10} the quartic couplings. Precision studies of the electroweak interactions bound mostly the corrections to the vacuum polarizations. Following [26], the oblique parameters can be defined starting from the transverse vacuum polarization amplitudes of the gauge bosons of the standard model, expanding as

$$\Pi(q^2) = \Pi(0) + q^2\Pi'(0) + \frac{1}{2}(q^2)^2\Pi''(0) + \dots, \quad (3.12)$$

and defining, for $\Pi'_{WW}(0) = 1 = \Pi'_{BB}(0)$,

$$\hat{S} = \frac{g}{g'}\Pi'_{W^3B}(0), \quad (3.13)$$

$$\hat{T} = \frac{1}{M_W^2}(\Pi_{W^3W^3}(0) - \Pi_{W^+W^-}(0)), \quad (3.14)$$

$$\hat{U} = (-\Pi'_{W^3W^3}(0) + \Pi'_{W^+W^-}(0)), \quad (3.15)$$

$$W = \frac{M_W^2}{2}\Pi''_{W^3W^3}(0), \quad (3.16)$$

$$X = \frac{M_W^2}{2}\Pi''_{W^3B}(0), \quad (3.17)$$

$$Y = \frac{M_W^2}{2}\Pi''_{BB}(0). \quad (3.18)$$

The parameters α_0 , α_1 and α_8 are hence directly related to S , T , and U of [25], and to the \hat{S} , \hat{T} , and \hat{U} in [26]:

$$\begin{aligned} \hat{S} &= -g^2\alpha_1 = \frac{\alpha}{4\sin^2\theta_W}S, \\ \hat{T} &= 2g^2\alpha_0 = \alpha T, \\ \hat{U} &= g^2\alpha_8 = -\frac{\alpha}{4\sin^2\theta_W}U. \end{aligned} \quad (3.19)$$

(Notice that in the definition of S , T , and U the $\mathcal{O}(q^4)$ corrections are neglected, and hence the identification of \hat{S} and α_1 with S is valid only up to $\mathcal{O}(q^4)$ corrections.) Bounds on the other α_i exist, but are much less stringent, because much less data is available for the cubic and quartic vertexes. The parameters X , W , and Y are related to terms $\mathcal{O}(q^6)$ in \mathcal{L}_χ .

The strategy for extracting bounds on new physics from the experimental data requires computing measurable amplitudes from the MSM at 1-loop level, and include $\mathcal{L}_{2,2}$ and $\mathcal{L}_{4,0}$ (equivalently, introducing the corrections from S and T in the propagators) at the tree level. In this way, all UV and IR divergences cancel, and a consistent set of amplitudes computed at $\mathcal{O}(\hbar)$ can be compared to the data (the reader should remember that at the tree-level the MSM itself does not provide a satisfactory fit of all the electroweak precision observables, for which the inclusion of 1-loop corrections is necessary). One then performs a fit to the data using as free parameters the values of the electroweak MSM parameters g , g' , and v_W , of the masses of top m_t and Higgs m_h , and the precision parameters \hat{S} and \hat{T} . A list of the observables used can be found for instance in [26, 91].

Here comes one subtlety, which is going to be important also later, in computing the MSM 1-loop contributions to the physical observables, an explicit logarithmic dependence on the mass of the Higgs m_h is present. Borrowing the results from [25]:

$$\hat{S} \simeq \frac{\alpha}{48\pi\sin^2\theta_W} \ln \frac{m_h^2}{m_h^2(\text{ref})}, \quad (3.20)$$

with $m_h^2(\text{ref})$ some reference value for the Higgs mass. The result is that the MSM provides a good fit to all the existing data, for values of the Higgs mass $m_h \lesssim 1$ TeV, provided all the precision parameters such as \hat{S} and \hat{T} are at most of order few 10^{-3} . One may arbitrarily decide to do the fit while setting all the oblique parameters to zero. In doing so, one finds a bound on the mass of the Higgs to be $m_h \lesssim 200$ GeV (95% c.l.) [92, 93], because of the logarithmic dependence on m_h of the 1-loop corrections. This bound amounts to assuming that no new physics be present at any scale.

The comparison of the above procedure to a generic extension of the SM can be done rigorously only provided the only light degrees of freedom include all the SM fermions and gauge bosons, together with one Higgs scalar, while all new physics can be integrated out and reabsorbed in the higher-order coefficients in \mathcal{L}_χ . This means that some caution must be used when comparing to models which predict the presence of new light degrees of freedom, for which the 1-loop perturbative part of the calculations should be redone.

But also highlights a difficulty with higgsless models, in which strictly speaking the precision parameters diverge, and are hence not calculable. A physical cut-off should be kept in the calculation, entering logarithmically in place of the logarithmic dependence on the mass of the Higgs. In practice, phenomenological studies usually assume that Higgsless models (including TC) are well described by the limit of the MSM case in which $m_h \simeq 1$ TeV. This is motivated mostly by the pragmatic observation that, because the dependence on the mass of the Higgs is logarithmic, by varying the Higgs mass between $m_h = 115$ GeV and $m_h = 800$ GeV (which covers most of the regime over which the Higgs can be treated perturbatively), the bound on \hat{S} changes only by $\Delta\hat{S} \simeq 10^{-3}$, which is smaller than the error bar on \hat{S} itself. Imposing, for instance, $|\hat{S}| \lesssim 3 \times 10^{-3}$ on the calculable part of the new physics contribution is hence going to yield sensible quantitative constraints. Notice also that the result of the fit of the data yields some ellipsoid in the space of the complete set of precision parameters, and the resulting bounds on \hat{T} and \hat{S} in particular are correlated. Because of the

intrinsic uncertainty in the treatment of strongly coupled systems, in particular due to what we just said about the role of scalars, in the following I will use the bounds as uncorrelated, with the understanding that these are somewhat conservative bounds; models that end up outside the bounds are very unlikely to be fixed by improved calculations, and are hence excluded, while for models that fit into the bounds, detailed strong-coupling calculations would be needed to decide if they can or cannot fit the data.

The nondecoupling of the Higgs degrees of freedom is one of the many subtle, phenomenologically important, aspects of the MSM, which sources theoretical problems in its extensions. In the case of precision electroweak tests, this feature still allows for useful quantitative information to be extracted, thanks to the mild logarithmic dependence on the Higgs mass. In short, generic new physics contributions are larger than the uncertainty introduced by the Higgs-mass dependence, and hence useful (and quite restrictive) constraints can be extracted. We will see that this is not the case for other observables, such as elastic-scattering amplitudes of longitudinally polarized gauge bosons, in which the nondecoupling manifests itself with a strong, quadratic dependence on m_h . In Higgsless theories one is going to face quadratically divergent contributions to these scattering amplitudes, that are removed by completely uncalculable contributions from the scalar sector, hence limiting greatly the amount of information about new physics that can be extracted. We will be more precise on this topic in a later subsection.

Before going any further, the reader should be reminded of the role played by precision studies in the discovery of the top, which is a nice illustration of how powerful simple power-counting is, and how important precision studies are. The electroweak chiral Lagrangian highlights a very special feature: custodial symmetry is broken by an operator of dimension-2, in $\mathcal{L}_{2,2}$. Very interestingly, in the SM there is one very large, explicit source of breaking of $SU(2)_R$, namely, the mass of the top is much larger than the mass of the bottom. Dimensional analysis suggests, hence, that $\hat{T} \propto m_t^2$ (or, equivalently, $\delta \propto m_t$). (We will see later that indeed this is the result of perturbative calculations with a heavy fermion.) Which means that the precision tests of the SM are very sensitive to the precise value of the top mass. And indeed, a clear indication that the top is heavy, in the 170 GeV region, was obtained by analyzing LEP data before the actual TeVatron discovery. As we already saw, similar arguments yield much more limited information about the Higgs mass, that enters only logarithmically in the precision tests.

3.1.1. Perturbative Estimates of \hat{S} and \hat{T}

Following Peskin and Takeuchi, we can start from computing the contribution to the precision parameters from a new family of weakly coupled, heavy fermions. This perturbative calculation will be useful in showing us how the corrections are generated, and we will later discuss what to expect in the nonperturbative case.

Consider a fermion left-handed doublet $(N, E)_L$, with hypercharge Y , and right-handed N_R, L_R such that the physical states have masses, respectively, m and $m+\delta$. Assuming that $m \gg M_Z$, one can compute the correction to the vacuum polarizations induced at 1-loop from diagrams involving loops of such fermions, with SM gauge bosons on the external lines. Assuming also, for simplicity, that $\delta \ll m$, the expressions simplify to

$$\hat{S} \simeq \frac{\alpha}{4 \sin^2 \theta_W} \frac{1}{6\pi},$$

$$\begin{aligned}\hat{T} &\simeq \frac{(g^2 + g'^2)}{48\pi^2} \frac{\delta^2}{M_Z^2}, \\ \hat{U} &\simeq -\frac{\alpha}{4\sin^2\theta_W} \frac{2}{15\pi} \frac{\delta^2}{m^2}.\end{aligned}\tag{3.21}$$

First of all, this confirms that \hat{U} is doubly suppressed, as anticipated in speaking about the electroweak chiral Lagrangian, and hence can be ignored in phenomenological analysis. Second, it explicitly shows that \hat{T} is proportional to the amount of custodial-symmetry breaking in the new physics sector, and is hence negligibly small provided $\delta/M_Z \ll 1$. For these reasons, we focus on the \hat{S} parameter. Finally, if one has N_D different fermions, their contributions sum is

$$\hat{S} \simeq 0.0004N_D,\tag{3.22}$$

which, taking as a reference the bound $\hat{S} \lesssim 0.003$, implies $N_D \lesssim 8$.

If one could borrow this result and use it as an estimate of the TC contribution, this would imply a strong bound on the possible choice of field content of the theory. For example, reconsidering the $SU(N_T)$ vectorial gauge theory introduced earlier on, with N_f families of techniquarks, each family including fermions on the fundamental of $SU(N_T)$ that carry the quantum numbers of a family of SM fermions, one obtains

$$\hat{S}_{\text{perturbative}} \simeq 0.0004 \times (3+1)N_fN_T,\tag{3.23}$$

which effectively restricts to one point of parameter space with $N_T = 2$ and $N_f = 1$. And still, this single point would lie on the 3σ upper bound.

The use of this perturbative estimate is, of course, very hard to motivate: by definition technicolor is strongly coupled. The numerical coefficients have to be taken for what they are. The robust messages that can be extracted from this exercise are that some form of custodial symmetry better be present, to suppress \hat{T} , that \hat{S} is potentially problematic, and that the precision parameters are expected to grow with the number of degrees of freedom N_D coupled to the $SU(2)_L$ gauge bosons.

3.1.2. Nonperturbative Estimates of \hat{S} : NDA and Hidden Local Symmetry

One might want to get a feeling for how big the coefficients of \mathcal{L}_χ are expected to be, in particular α_1 (equivalent to \hat{S}), which is the most severe test of dynamical electroweak symmetry breaking. A set of very simple rules for obtaining such an estimate exists, and goes under the name of Naive Dimensional Analysis (NDA). A summary and discussion of such rules can be found in [35]. Here are the rules, applied to \mathcal{L}_χ , restricted by setting to zero all the custodial-symmetry violating terms. First of all, one has to write out all possible local operators compatible with the symmetries, and include them in the effective Lagrangian.

An estimate of the coefficients is then given by the following three rules:

- (i) include a factor of $f^2 \Lambda^{*2}$ overall,
- (ii) include a factor of $1/f$ for any occurrence of strongly coupled fields (such as the pions π),
- (iii) include appropriate factors of Λ^* to get the dimension to 4.

Hence, the coefficient of the terms in $\mathcal{L}_{2,0}$ is f^2 , and for $\mathcal{L}_{4,0}$ all the coefficients are $\alpha_i = \mathcal{O}(f^2/\Lambda^{*2})$. The scale Λ^* is associated with the mass of the lightest degree of freedom that has been integrated out of the theory, which typically is the mass of the technirho meson M_ρ . Hence, one expects that

$$\hat{S} \sim g^2 \frac{f^2}{M_\rho^2}. \quad (3.24)$$

Identifying f with the electroweak scale, taken literally this result would mean that $M_\rho \gtrsim 3 \text{ TeV}$. We will see that much more sophisticated techniques agree to an impressive degree with this NDA estimate. It must be stressed that additional symmetry arguments provide systematic ways of including exceptions to this rules, as is illustrated by the fact that on the basis of this counting only one would conclude that $\hat{T} = \mathcal{O}(1)$, while promoting $SU(2)_R$ to an approximate symmetry makes $\hat{T} = \mathcal{O}(\delta^2/\Lambda^{*2}) \ll 1$.

The relation between the \hat{S} parameter and the physics of spin-1 resonances in a strongly coupled theory can be made more precise and model-independent with the use of dispersion relations to rewrite (3.13) by saturating on the vector resonances (ρ) and axial resonances (a_1), which results in

$$\hat{S} \simeq \frac{\alpha}{4 \sin^2 \theta_W} 4\pi \sum_n \left(\frac{f_{\rho n}^2}{M_{\rho n}^2} - \frac{f_{a_1 n}^2}{M_{a_1 n}^2} \right), \quad (3.25)$$

where the index $n = 1, 2, \dots$ identifies the tower of resonances with mass M_n^2 and decay constant f_n^2 . In the limit in which $SU(2) \times SU(2)$ were unbroken, axial and vector states would be exactly degenerate and connected by a symmetry, and hence $\hat{S} = 0$. The pion decay constant measures how broken this symmetry is. This expression is useful because it has shown that a suppression of \hat{S} not necessarily implies taking large values of M_ρ , but might be achieved by suppressing the decay constants, or by ensuring a cancellation between vectorial and axial contributions.

One way of drawing a closer relation between \hat{S} and the heavy composite states, in particular the lightest spin-1 meson, the techni-rho, is provided by the inclusion of the latter in the low-energy effective theory. This is done with the help of so-called hidden local symmetry (HLS) [36–43]. The basic trick is to extend the global symmetry of the chiral Lagrangian (truncated to the leading order), by introducing an additional, spontaneously-broken gauged $SU(2)$ as in Figure 3. The $SU(2)_\rho$ in the middle site is gauged with strength g_ρ , and there are now two sigma-model fields, so that in unitary gauge the new gauge bosons are massive.

The important terms in the Lagrangian are, at the leading-order,

$$\mathcal{L}_{3s} = -\frac{1}{2} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{2} \text{Tr} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr} \rho_{\mu\nu} \rho^{\mu\nu} + \frac{f^2}{4} \text{Tr} |DS_1|^2 + \frac{f^2}{4} \text{Tr} |DS_2|^2, \quad (3.26)$$

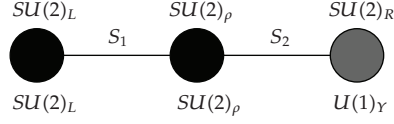


Figure 3: Diagram of the 3-site model.

with

$$S_{1,2} = e^{2i\pi_{1,2}/f}, \quad (3.27)$$

and the covariant derivatives

$$\begin{aligned} DS_1 &= \partial S_1 + i(gWS_1 - g_\rho S_1 \rho), \\ DS_2 &= \partial S_2 + i(g_\rho \rho S_2 - g' S_2 B). \end{aligned} \quad (3.28)$$

In unitary gauge, $S_1 = 1 = S_2$, and the mass of the neutral gauge bosons is, in the (W^3, ρ^3, B) basis,

$$\mathcal{M}^2 = \frac{1}{4} f^2 \begin{pmatrix} g^2 & -g g_\rho & 0 \\ -g g_\rho & 2g_\rho^2 & -g' g_\rho \\ 0 & -g' g_\rho & g'^2 \end{pmatrix}. \quad (3.29)$$

Assuming that $g, g' \ll g_\rho$, one can integrate out the heavy techni-rho, of mass

$$M_\rho^2 = \frac{1}{8} (g^2 + g'^2 + 4g_\rho^2) f^2 \gg M_Z^2 = \frac{1}{8} (g^2 + g'^2) f^2, \quad (3.30)$$

and finally obtain

$$\hat{S} = \frac{1}{4} \frac{g^2}{g_\rho^2} = \frac{1}{4} g^2 \frac{f_\pi^2}{M_\rho^2} = \frac{M_W^2}{M_\rho^2}, \quad (3.31)$$

where $f_\pi^2 = f^2/2 = v_W^2$. This estimate implies that the bound on \hat{S} requires $M_\rho \gtrsim 1.5 \text{ TeV}$, which is a factor of 2 milder than what derived from the NDA estimate (3.24), but consistent with it.

Some lesson and some comment on this little exercise. One might regard the agreement between the equations we derived as a success. They actually indicate a surprising degree of robustness of the NDA estimates. However, one has to use some caution, for a number of reasons. First of all, the bound we just derived implies that $g_\rho \gtrsim 6.5$, which means that the very idea of studying perturbatively the spectrum, couplings, and correlators from \mathcal{L}_{3s} does not work. In practice, this Lagrangian would be useful only in the regime in which g_ρ being perturbative, in which the bounds on \hat{S} are certainly exceeded. Even worse, in writing \mathcal{L}_{3s} we

did not apply the basic rule of EFT, requiring to write, order-by-order, all the possible terms allowed by symmetries, because we omitted

$$\mathcal{L}_\kappa = \kappa \frac{f^2}{4} \text{Tr} |D(S_1 S_2)|^2. \quad (3.32)$$

Aside from the fact that this term is not nearest-neighbour, it is a perfectly legitimate part of the leading-order Lagrangian. Its presence has two main effects, due to the fact that it changes both the kinetic terms of the pions and the mass terms of the gauge bosons. First of all, it splits the pion and techni-rho decay constants

$$f_\rho^2 = \frac{f^2}{2} \neq f_\pi^2 = (1 + 2\kappa) \frac{f^2}{2}. \quad (3.33)$$

Notice that positivity of the kinetic terms requires $\kappa > -1/2$. Furthermore, because the spectrum is given by $M_Z^2 \simeq (1 + 2k)(g^2 + g'^2)f^2/8 = (g^2 + g'^2)f_\pi^2/4$ and $M_\rho^2 \simeq g_\rho^2 f^2/2$, it modifies the \hat{S} parameter,

$$\hat{S} = \frac{1}{4} \frac{g^2}{g_\rho^2} = \frac{1}{8} g^2 \frac{f^2}{M_\rho^2} = \frac{M_W^2}{M_\rho^2} \frac{1}{1 + 2\kappa}. \quad (3.34)$$

Finally, it changes the cubic coupling between ρ and pions:

$$g_{\rho\pi\pi} = \frac{g_\rho}{2 + 4\kappa}. \quad (3.35)$$

Notice that, in real-world QCD, which can be described with the same formalism up to minor modifications, $f_\rho \simeq 150$ MeV and $f_\pi \simeq 93$ MeV, indicating that even in order to describe QCD, κ must be included and is $\mathcal{O}(1)$, not small. In this specific case, the fact that f_π is smaller than f_ρ implies negative values of κ , and hence an enhancement of \hat{S} in comparison with what we said earlier on. This is what actually happens if one estimates the oblique corrections by scaling up the QCD experimental results [25].

Before going any further, another important comment about these nonnearest-neighbour couplings. By looking at (3.34), one might be tempted to conclude that this provides a perfectly natural way out of the little hierarchy problem, just by dialing κ to a somewhat largish, positive coefficient. This train of thought is quite dangerous, and yields to big problems. The reason is that once nonnearest-neighbour interactions are allowed, and particularly if κ is large, nothing prevents from writing much more dangerous terms, one of which is $\text{Tr} S_1 S_2 B_{\mu\nu} S_2^\dagger S_1^\dagger W^{\mu\nu}$, which one recognizes to be a direct contribution to \hat{S} . This is a subleading term to be added to \mathcal{L}_{3s} , and as such one would naively think of it as suppressed, but if κ is taken large, then also the coefficient of this new term should be large, and this would become the dominant (uncalculable) contribution to \hat{S} . In other words, playing the game of dialing up the coefficients of interactions such as (3.32) yields in the end to a useless theory, in which \hat{S} is a free parameter.

In general, terms such as \mathcal{L}_κ are generated when integrating out other heavier states from the theory. The one term in (3.32) is just one example. These terms and their proliferation

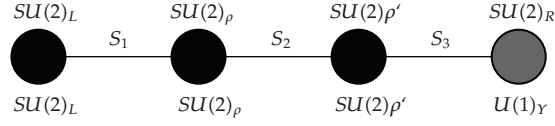


Figure 4: Diagram of the 4-sites model.

with the inclusion of more sites and links represent a very heavy limitation in the predictive power of the hidden local symmetry approach. Also, one must question how appropriate it really is to include in the effective theory the rho mesons, but not other states, such as their axial counterparts, the a_1 . One can use the technique exemplified above to include many spin-1 resonances, along the lines of [36–43]. The resulting model, in the limit of infinite number of sites, is the deconstructed version of a fifth dimension, provided no nonlocal terms are included.

Another simple exercise will help understanding what is the physical meaning of the \hat{S} parameter. In this case, we extend the 3-sites model to a 4-sites model as in Figure 4, hence incorporating in the EFT both the vectorial (techni-rho) meson and its axial partner (techni- a_1). The important terms in the Lagrangian are

$$\begin{aligned} \mathcal{L}_{4s} = & -\frac{1}{2} \text{Tr} W_{\mu\nu} W^{\mu\nu} - \frac{1}{2} \text{Tr} B_{\mu\nu} B^{\mu\nu} - \frac{1}{2} \text{Tr} \rho_{\mu\nu} \rho^{\mu\nu} - \frac{1}{2} \text{Tr} \rho'_{\mu\nu} \rho'^{\mu\nu} \\ & + \frac{F^2}{4} \text{Tr} |DS_1|^2 + \frac{f^2}{4} \text{Tr} |DS_2|^2 + \frac{F^2}{4} \text{Tr} |DS_3|^2, \end{aligned} \quad (3.36)$$

with

$$\begin{aligned} S_{1,3} &= e^{2i\pi_{1,3}/F}, \\ S_2 &= e^{2i\pi_2/f}, \end{aligned} \quad (3.37)$$

and the covariant derivatives

$$\begin{aligned} DS_1 &= \partial S_1 + i(gWS_1 - g_\rho S_1 \rho), \\ DS_2 &= \partial S_2 + i(g_\rho \rho S_2 - g_\rho S_2 \rho'), \\ DS_3 &= \partial S_3 + i(g_\rho \rho' S_3 - g' S_3 B). \end{aligned} \quad (3.38)$$

In unitary gauge, $S_1 = S_2 = S_3 = 1$, and hence all the gauge bosons (aside from the photon) acquire a mass, via the mass matrix

$$\mathcal{M}^2 = \frac{1}{4} \begin{pmatrix} g^2 F^2 & -g g_\rho F^2 & 0 & 0 \\ -g g_\rho F^2 & g_\rho^2 (f^2 + F^2) & -g_\rho^2 f^2 & 0 \\ 0 & -g_\rho^2 f^2 & g_\rho^2 (f^2 + F^2) & -g' g_\rho F^2 \\ 0 & 0 & -g' g_\rho F^2 & g'^2 F^2 \end{pmatrix}. \quad (3.39)$$

In the limit where $f \ll F$, one sees that there is an approximate $SU(2) \times SU(2)$ symmetry. The spectrum has $M_\rho^2 \simeq M_{a_1}^2 \simeq g_\rho^2 F^2/4$, $M_Z^2 = (g^2 + g'^2)f^2/4$, and $\hat{S} \simeq 2g^2 f^2/(g_\rho^2 F^2)$. This explicitly shows the fact that \hat{S} measures the effect of isospin breaking in the resonances. In particular, one can suppress \hat{S} by ensuring that the vectorial and axial-vector resonances are quasidegenerate. However, notice that for this to work one needs a parametric suppression $f_\pi \ll f_\rho$, which is not what expected in a QCD-like theory. Some nontrivial dynamics, in which a scale separation exists suppressing f_π is needed. In particular, notice that models in which this mechanism is at work cannot be described at low energy by a 3-site model, because there is no gap allowing to integrate out the a_1 while keeping the ρ meson. Which is one way of understanding why the small- \hat{S} regime is not accessible within the 3-site model, which becomes uncalculable and fine-tuned in this regime.

Concluding, both perturbative and nonperturbative estimates of the \hat{S} parameter indicate that there is a tension between the expectations in a generic model of technicolor and the experimental data. However, this does not mean that TC is excluded by precision tests. First of all the tension is not dramatic; some models are at the boundary of acceptability. Also, the NDA estimates are just valid up to unknown numerical factors that are very difficult to compute, even in QCD-like theories. In walking technicolor, as we will see in next subsection, NDA expectations are expected to be violated. Large anomalous dimensions can distort the spectrum of masses and decay constants [94], and ultimately a more reliable technique is needed in order to perform actual calculations. This might be provided by the lattice or, as suggested here, by the ideas of gauge/string duality.

3.2. Fermion Masses and FCNC Constraints

Almost by definition, a technicolor model is an incomplete model of electroweak symmetry breaking in the SM. The mechanism illustrated in the previous sections provides a mass for the electroweak gauge bosons, but not for the SM fermions, which are exactly massless. This is because one cannot write the Yukawa couplings, in the absence of an elementary scalar. A simple EFT way of solving this problem is to add to the Lagrangian four-fermion interactions coupling the SM quarks and leptons ψ_{SM} to the techniquarks ψ_{TC} . When techniquarks condense, dimensional transmutation allows to match the theory onto a low-energy EFT in which mass terms are generated for the SM fermions:

$$(\bar{\psi}_{\text{SM}} \gamma_\mu \psi_{\text{TC}})(\bar{\psi}_{\text{TC}} \gamma^\mu \psi_{\text{SM}}) \longrightarrow \langle \bar{\psi}_{\text{TC}} \psi_{\text{TC}} \rangle \bar{\psi}_{\text{SM}} \psi_{\text{SM}}. \quad (3.40)$$

There are three, interrelated problems with this idea. First of all, because these are higher-order operators, adding them by hand to the fundamental Lagrangian would spoil the good UV behavior of the theory that is the original motivation for technicolor itself. It is hence necessary to extend somehow the new physics sector in such a way as to dynamically generate these four-fermion interactions at some intermediate scale $\Lambda_{\text{ETC}} > v_W$. This is the role of Extended Technicolor (ETC). The second problem is that in generating the operators needed in order to yield the masses and mixing among SM fermions, the ETC interactions are expected in general to produce also dangerous higher-order terms, coupling, for instance, four SM fermions, which can easily produce large contributions to FCNC processes, in excess of what allowed by data on flavor physics. Avoiding this requires that $\Lambda_{\text{ETC}} \gg v_W$, but also a challenging model-building exercise, in which the hierarchy of masses is generated

dynamically, and some mechanism resembling the GIM mechanism being at work. This is the role played by tumbling ETC. Finally, some of the SM fermions are quite heavy (the top, e.g., has $m_t \sim v_W$), but NDA suggests that the masses obtained from dimension-6 operators should be $\mathcal{O}(v_W^3/\Lambda_{\text{ETC}}^2) \ll v_W$. Walking dynamics might provide a solution to this problem, by enhancing the fermion masses but not the dangerous operators involved in FCNC processes (and in the custodial-symmetry breaking effects such as \hat{T}), as we are going to see.

In this subsection, I start from recalling the basic mechanisms at work in the SM and its perturbative extension of relevance for flavor physics (the GIM and FN mechanisms), then move to strongly coupled models, and summarize the concepts of tumbling ETC and the role of walking dynamics. Before we embark in this discussion, one should be aware of the fact that, to a large extent because of the uncertainties intrinsic with strongly coupled dynamics, at present there is no satisfactory UV-complete model encompassing all of the above. It would be interesting to build such an explicit UV-complete model of ETC in the context of gauge/string dualities. It must also be mentioned that some very interesting results have been obtained with five-dimensional Higgsless theories that are more closely related to topcolor [13–18], rather than to ETC, in the sense that the SM fermions propagate in the bulk, and are hence composite objects themselves [95–101].

3.2.1. The GIM Mechanism

In the MSM, in the simplifying limit in which no right-handed neutrinos are present, and in which neutrinos are exactly massless, a very rich structure of global family symmetries is present. At the classical level, in the limit in which all the Yukawa couplings in (2.4) vanish, the model has a $U(3)^5$ global symmetry, with each $U(3)$ acting on the three families of matter for each fermion species. The Yukawa couplings break explicitly this symmetry, ultimately generating the hierarchy of masses of the SM. The gauge couplings of $SU(2)_L \times U(1)_Y$ are universal. The phenomenological distinction between families arises only because of the masses induced by the Yukawa couplings, and flavor-changing interactions are controlled by the mixing matrices that diagonalize the mass matrices. For concreteness, restrict attention to the quarks only. The important terms of the Lagrangian after electroweak symmetry breaking read

$$\begin{aligned} \mathcal{L} = & i\bar{u}'_a \not{\partial} u'_a + i\bar{d}'_a \not{\partial} d'_a - \frac{g}{\sqrt{2}} (\bar{u}'_L)_a \gamma^\mu W_\mu^+ (d'_L)_a + \text{h.c.} \\ & - M_{ab} (\bar{u}'_R)_a (u'_L)_b - m_{ab} (\bar{d}'_R)_a (d'_L)_b + \text{h.c.}, \end{aligned} \quad (3.41)$$

where u'_a and d'_a are the up-type and down-type Dirac fermions describing the quarks in what is called the interaction basis, with family index $a = 1, 2, 3$.

The mass matrices can be diagonalized by the biunitary transformations defined as

$$\begin{aligned} M_d &= V_R^\dagger M V_L, \\ m_d &= U_R^\dagger m U_L, \end{aligned} \quad (3.42)$$

where all the rotation matrices are 3×3 unitary matrices $U_L^\dagger U_L = 1 = U_R^\dagger U_R = V_L^\dagger V_L = V_R^\dagger V_R$. These transformations define the mass basis for the quarks

$$\begin{aligned} u'_{L,R} &= V_{L,R} u_{L,R}, \\ d'_{L,R} &= U_{L,R} d_{L,R}. \end{aligned} \quad (3.43)$$

In this new basis, the Lagrangian density can be rewritten as

$$\begin{aligned} \mathcal{L} &= i\bar{u}_a \not{\partial} u_a + i\bar{d}_a \not{\partial} d_a - \frac{g}{\sqrt{2}} U_{ab} (\bar{u}_L)_a \gamma^\mu W_\mu^+ (d_L)_b + \text{h.c.} \\ &\quad - (M_d)_{aa} (\bar{u}_L)_a (u_R)_a - (m_d)_{aa} (\bar{d}_L)_a (d_R)_a + \text{h.c.}, \end{aligned} \quad (3.44)$$

where the matrix $U \equiv V_L^\dagger U_L$ is the CKM mixing matrix, which in general is not diagonal. All flavor-changing processes involve off-diagonal entries of this matrix, which are known to be small [91]. The neutral interactions are diagonal also in the mass basis.

All of this is at the basis of the GIM mechanism for suppressing FCNC transitions. There are three suppression factors. The neutral-current couplings being diagonal, all flavor changing neutral current processes are induced by weak interactions at the loop level, and are hence suppressed by loop factors in the weak couplings. They all involve combinations of the small mixing angles in the CKM matrix, which can yield significant suppression. Because the origin of the mixing is the Yukawa couplings breaking the family symmetry, additional suppression comes from factors involving the masses of fermions in the internal lines of loop diagrams.

An example will help illustrating the point. Consider the semileptonic decay of the K^+ meson, and try to estimate the ratio

$$x = \frac{\Gamma[K^+ \rightarrow \pi^+ \nu \bar{\nu}]}{\Gamma[K^+ \rightarrow \pi^0 e^+ \nu_e]}. \quad (3.45)$$

Naively, the two final states might look very similar. However, a look at the PDG [91] reveals that $x \sim \mathcal{O}(10^{-9})$. A very efficient way to understand this is by integrating out the W boson and hence building an effective theory describing the interaction such as to include the two effective operators

$$\mathcal{L} = c_1 [\bar{s}_L^\alpha \gamma^\mu u_L^\alpha] [\bar{\nu}_{eL} \gamma_\mu e_L] + c_2 [\bar{s}_L^\alpha \gamma^\mu d_L^\alpha] [\bar{\nu}_{eL} \gamma_\mu \nu_L] + \text{h.c.}, \quad (3.46)$$

where c_1 and c_2 are dimensionful. It is clear that the kinematics and the matrix elements of the full calculation are very similar for the two processes, and hence $x \simeq 6|c_2/c_1|^2$, where a factor of 3 comes from the three possible neutrino species in the final state, and a factor of 2 from the matrix elements. The explanation for the smallness of such ratio must come from the weak interactions.

A closer inspection, in the light of the SM, shows that c_1 is generated by the tree-level exchange of a W boson. Hence

$$c_1 \sim \frac{4G_F}{\sqrt{2}} U_{us}^* \quad (3.47)$$

with $U_{us} \simeq 0.22$ the Cabibbo angle. Conversely, c_2 is generated by loop diagrams (boxes and penguins) and is given by

$$c_2 \sim \frac{4G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} \sum_i U_{is}^* U_{id} X \left(\frac{m_i^2}{M_W^2} \right), \quad (3.48)$$

where the function X depends on the ratios of the up-type quarks masses in the loop and M_W . In doing so, contributions that do not depend on the mass of the internal fermions cancel exactly, because of the unitarity of the CKM matrix

$$\sum_i U_{is}^* U_{id} = 0. \quad (3.49)$$

The nonvanishing terms are weighted by functions of the ratio m_i^2/M_W^2 involving the up-type quark masses m_i . The detailed structure of these functions is calculable and well known, and can be expanded in powers and logarithms, such that in the case of degenerate fermion masses the result vanishes identically. In the case of this special decay, top and charm contributions are comparable, because a suppression of the top contribution due to the CKM angles is compensated by the ratio m_t^2/m_c^2 . Hence, one can very roughly estimate $x \sim 10^{-9} - 10^{-8}$, depending on the precise value of the function X evaluated on the charm and top contribution, and to short-distance QCD effects. This estimate agrees with the data, at this level of accuracy. The function X is known, and this statement can be made more precise.

The important thing is that the huge suppression factor has three origins, ultimately all traceable to the GIM mechanism: a (weak-coupling) loop suppression factor α/π , small CKM elements, and ratios of masses of up-type quarks. The charm contribution is not severely Cabibbo-suppressed, but the small charm mass provides another suppression factor, while conversely the top contribution is suppressed by the CKM angles. All of this comes from the fact that ultimately one might rewrite the physical amplitude in terms of the up-type Yukawa $(\mathbf{y}^{(u)} \mathbf{y}^{(u)\dagger})_{12}$ element, which breaks explicitly the aforementioned $U(3)^5$ symmetry.

3.2.2. Froggat-Nielsen and See-Saw Mechanisms

The fact that in the absence of Yukawa couplings the MSM Lagrangian has a very large global family symmetry is at the basis of the GIM mechanism for suppressing FCNC, and it is also at the basis of most attempts to explain the hierarchy of masses and smallness of mixing angles, within models in which the Yukawa couplings are generated dynamically. The basic idea behind this is the generalization of the Froggat-Nielsen (FN) mechanism [28]. A vast literature exists on the subject, all of which is based on the same principles. One starts by taking a subgroup of $G_f \subset U(3)^5$, chosen in such a way that if G_f were unbroken then all the small mixing angles and fermion mass ratios would vanish, because all the Yukawa

couplings would be forbidden. The symmetry is broken by the VEVs $\langle \phi_i \rangle \gg v_W$ of some scalar (flavon) field that transforms under G_f , but is a singlet of the SM gauge group. The breaking is communicated to the SM via the exchange of weakly coupled messenger fields with mass $\Lambda_f \gg v_W$. In doing so, by integrating out the flavons and the messengers one generates the Yukawa couplings of the SM, and depending on the assignment of SM fermions to representations of G_f the resulting coupling is suppressed by powers of the small ratios $\epsilon_i \sim \langle \phi_i \rangle / \Lambda_f$, which are typically in the range $\mathcal{O}(1/5) - \mathcal{O}(1/20)$. Appropriate choices of G_f , of the representations assigned to the fermions, and of the symmetry-breaking pattern allow to produce realistic mass matrices. A number of variations on the theme exist in the literature, in which G_f can be global or local, Abelian or non-Abelian, continuous or discrete, and combinations thereof, within or outside grand unified and/or supersymmetric extensions of the MSM.

For all of this to make sense, the most important ingredient is the fact that everything must be weakly coupled, and a Higgs scalar be present. If so, one can integrate out all the flavor physics involved in the FN mechanism below the very large scale $\Lambda_f \gg v_W$ of the messengers and flavons. As a result, at low energy one generates a series of operators to be added to the SM, in which the only marginal operators are the Yukawa couplings:

- (i) the dimension-4 top Yukawa is often assumed to be unaffected by the FN mechanism, and hence unsuppressed,
- (ii) all other dimension-4 Yukawa couplings are suppressed by powers of ϵ_i , leading to the small mixing angles and small mass ratios,
- (iii) dangerous operators inducing new contributions to FCNC emerge at dimension-5, dimension-6 and higher, and are hence suppressed by powers of v_W / Λ_f ,
- (iv) assuming lepton number be broken in the process, and that no (light) right-handed neutrino singlet be present, a set of dimension-5 operators generates the mass of the SM neutrinos via the see-saw mechanism [29–32].

More on the last point. The operators that would produce the mass for the neutrinos read, schematically,

$$\mathcal{L}_\nu \sim \frac{1}{\Lambda_f} (\text{LH})(\text{LH}). \quad (3.50)$$

When the Higgs develops a VEV, one obtains the mass

$$m_\nu \sim \frac{v_W^2}{\Lambda_f}. \quad (3.51)$$

Taking large values of Λ_f , this results in a suppression of the neutrino masses, that goes under the name of see-saw mechanism. If $m_\nu \sim 10^{-2} - 10^{-1}$ eV, one has to require $\Lambda_f \sim 10^{14} - 10^{16}$ GeV, which is in the range of GUT theories. With such a large scale, all contributions to FCNC are safely negligible.

3.2.3. Extended Technicolor: Tumbling and Walking

In a generic extension of the SM, such as ETC, in which all possible higher-order operators generalizing those in (3.46) are produced at some scale Λ_{ETC} , very strong bounds are inferred from data on rare flavor-changing processes. For instance, requiring that the ETC contribution to c_2 being such as not to spoil the SM agreement with the data implies that $\Lambda_{\text{ETC}} \sim 1/\sqrt{c_2} > 50$ TeV. Much more stringent bounds can be obtained analyzing the effect of such operators as

$$[\bar{s}_L^\alpha \gamma^\mu d_L^\alpha] [\bar{s}_L^\alpha \gamma_\mu d_L^\alpha], \quad (3.52)$$

and similar operators which affect mixing and CP violation in the $K^0 - \bar{K}^0$ system, from which one learns that $\Lambda_{\text{ETC}} \gtrsim 1000$ TeV.

As anticipated, in technicolor there is no simple way to write a set of marginally irrelevant couplings analogous to the Yukawa's of the MSM. The masses for the SM fermions themselves must arise from higher-order operators coupling the SM fermions and the techniquarks. This means that the problems of generating the masses, their hierarchies, the mixing angles, and CP-violation are tangled together with the problem of doing so by suppressing FCNC processes. In other words, the power-counting that made the Froggatt-Nielsen and see-saw mechanisms work in perturbative theories is lost, because there is no simple way to take the limit in which the parameter Λ_f (or its strongly coupled equivalent Λ_{ETC}) to arbitrary large values, while retaining finite fermion masses and CKM mixing angles.

Let us try to explain all of this in more details. Naively, one expects that once some generic mechanism is implement, it will generate operators of the three generic forms

$$\begin{aligned} \mathcal{L}_{\text{ETC}} = & c_{\text{ST}} (\bar{\psi}_{\text{SM}} \gamma_\mu \psi_{\text{TC}}) (\bar{\psi}_{\text{TC}} \gamma^\mu \psi_{\text{SM}}) + c_{\text{TT}} (\bar{\psi}_{\text{TC}} \gamma_\mu \psi_{\text{TC}}) (\bar{\psi}_{\text{TC}} \gamma^\mu \psi_{\text{TC}}) \\ & + c_{\text{SS}} (\bar{\psi}_{\text{SM}} \gamma_\mu \psi_{\text{SM}}) (\bar{\psi}_{\text{SM}} \gamma^\mu \psi_{\text{SM}}), \end{aligned} \quad (3.53)$$

possibly with all different admissible Lorentz and flavor structures. NDA suggests that in the absence of any other dynamical or symmetry argument $c_{\text{SS}} \sim c_{\text{TT}} \sim c_{\text{ST}}$. For the mass of the strange quark, for instance, one finds that

$$m_s \sim c_{\text{ST}} \langle \bar{\psi}_{\text{TC}} \psi_{\text{TC}} \rangle \sim c_{\text{ST}} 4\pi v_W^3, \quad (3.54)$$

so that the coefficient of the operator in (3.52) is expected to be

$$c_{\text{SS}} \sim c_{\text{ST}} \sim \frac{m_s}{4\pi v_W^3} \sim (50 \text{ TeV})^{-2}, \quad (3.55)$$

and the contribution to $K^0 - \bar{K}^0$ mixing results to be somewhere between four and five orders of magnitude too big to be acceptable. This kind of disaster is what people have in mind when they say that TC has a problem with FCNC.

Another related problem emerges when considering the generation of mass for the top. Borrowing the same estimates, including and taking literally the uncertain factor of 4π , the mass of the top would be obtained with $\Lambda_{\text{ETC}} \sim 1/\sqrt{c_{\text{ST}}} \sim 1$ TeV. This optimistic

estimate requires that some strongly coupled sector responsible for generating the mass splitting between the bottom and top quarks be present at a very low scale. Such sector violates custodial symmetry, and hence would generate a contribution to the \hat{T} parameter $\hat{T} \sim c_{\text{ST}} v_W^2 \sim 0.05$, which is largely in excess of the experimental bounds. A more sensible estimate requires the $1/\sqrt{c_{\text{ST}}} \sim \Lambda_{\text{ETC}} \sim 5 \text{ TeV}$, in which case $\hat{T} \sim 0.002$, but then the estimate for the top mass would yield $m_t \sim 7 \text{ GeV}$, which is certainly impossible to reconcile with the data.

All of the above highlights that there is a problem. Clearly, a generic model which is reasonably well approximated by NDA counting arguments is not compatible with the data. However, the estimates we did rely on the assumption that operators of the same kind, but involving completely different fields, have coefficients of the same value because in applying the NDA rules we assumed that all the operators in (3.53) obey the same counting. This needs not be true, provided the underlying dynamics yields a modification of the counting rules. This is based on symmetry considerations within tumbling ETC, and on dynamical considerations within walking technicolor. Effectively, tumbling ETC plays the role of the VEVs and messengers in the FN mechanism in generating different coefficients for operators of the same kind, but involving different SM fermions. Walking dynamics changes the rules of dimensional analysis, by changing the power-counting of operators that involve techniquarks (due to the nonperturbative anomalous dimensions), in such a way as to make the operators generating the masses more relevant (lower-dimensional) than the operators producing dangerous FCNC. The additional suppression factor accounting for the neutrino masses may arise from the fact that one has to break the lepton number, which requires some specific step in the ETC model [102, 103], and ends up making the neutrino masses emerge from very high-dimensional operators.

Let us be more precise about these ideas. In tumbling ETC, one uses a variation of the idea that is at the basis of the FN mechanism, namely, that a subgroup $G_f \subset U(3)^5$ be a gauge symmetry of the underlying dynamics, broken sequentially at several different ETC scales by the dynamical formation of condensates. Technically, one assumes that there is a ETC gauge theory at some very high scale, with $G_f \times G_{\text{TC}} \subset G_{\text{ETC}}$, such that SM fermions and techniquarks transform together in some set of representations of G_{ETC} . The strongly coupled dynamics then breaks sequentially $G_{\text{ETC}} \rightarrow G_1 \rightarrow \dots \rightarrow G_{\text{TC}}$, at scales $\Lambda_1, \Lambda_2, \dots$, in such a way that at lower energy the fermionic field content consists of the techniquarks and the SM singlets, with G_f completely broken. The gauge bosons in the coset $G_{\text{ETC}}/(G_f \times G_{\text{TC}})$ play the role of the messengers in the FN mechanism, and as a result integrating them out yields the four-fermion interactions of the low-energy theory. In this way, one can generate the hierarchy of masses in the SM by exploiting the fact that the c_{ST} coefficients entering the estimate of the mass of different fermions have a different dynamical origin from different scales Λ_i , depending on the SM field content of the operator in consideration. For example, one can embed the by now familiar example of TC model with $SU(N_T)$ gauge symmetry into a $SU(N_T + 3)$ model, the fundamental representation of which contains the techniquarks and the SM fermions. At scales $\Lambda_1 \sim 1000 \text{ TeV}$, $\Lambda_2 \sim 50 \text{ TeV}$, $\Lambda_3 \sim 5 \text{ TeV}$ [104–107], the breaking takes place as

$$SU(N_T + 3) \longrightarrow SU(N_T + 2) \longrightarrow SU(N_T + 1) \longrightarrow SU(N_T), \quad (3.56)$$

so that the $(N_T + 3)$ -dimensional fundamental representation decomposes in one fundamental representation of $SU(N_T)$ and three singlets, identified with the SM fields.

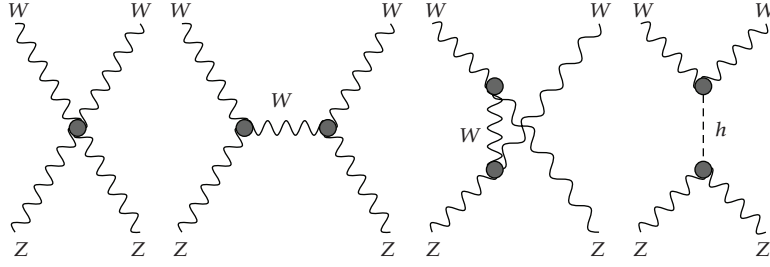


Figure 5: Diagrams contributing to $W_L Z_L$ scattering in the MSM, yielding the amplitudes \mathcal{M}_4 , \mathcal{M}_s , \mathcal{M}_u and \mathcal{M}_h (left to right).

In the estimate of $m_d \ll m_s \ll m_b$ one uses the fact that the important four-fermion operators be generated at scales $\Lambda_1 \gg \Lambda_2 \gg \Lambda_3$, respectively, with larger scales suppressing the mass of the lighter fermions. In doing so, one must also explain and generate the small CKM mixing angles, in terms of combinations of ratios of such scales Λ_i/Λ_j . But then operators such as (3.52), because they violate the strange number by two units instead of one, pick up a suppression factor from the highest scale involved in generating them, times a further suppression in the form of powers of the ratio entering the mass generation, effectively implementing a mild form of GIM mechanism. Examples of this construction have been implemented in many ways, and semirealistic models exist [104–107].

A second crucial ingredient in a realistic model is the fact that the technicolor theory that emerges at the end of the tumbling is assumed to be walking. This further modifies the counting rules, because strong dynamics is assumed to produce anomalous dimensions for operators involving techniquarks, in particular enhancing the c_{ST} coefficients. Assuming the chiral condensate to have anomalous dimension $\gamma \sim \mathcal{O}(1)$, and the walking taking place between the electroweak scale and a scale Λ^* (typically, related to the lowest scale produced by tumbling), the estimate for the mass of the top becomes

$$m_t \sim \left(\frac{\Lambda^*}{v_W} \right)^\gamma c_{ST} 4\pi v_W^3, \quad (3.57)$$

which for $\gamma = 1$, $\Lambda^* = 1/\sqrt{c_{ST}} \sim 5 \text{ TeV}$ yields $m_t \sim 150 \text{ GeV}$, which is acceptably close to the experimental value (this being just an estimate). Most important, the dangerous c_{SS} coefficients are not affected by walking, because they involve fermions that do not directly participate in the strong dynamics. As anticipated, walking changes the counting rules of NDA, making the operators involved in the process of mass generation lower-dimensional than their engineering dimension, and hence less irrelevant, and in this way making ETC more similar to the results of dimensional analysis that we summarized in the FN case.

To conclude, it must be noted that the estimate on the top mass is somewhat optimistic. Nobody really knows for sure of any specific walking theory that reproduces it, particularly because the calculation of the anomalous dimension poses a huge challenge. A specific calculation in a strongly coupled theory with the right properties would be needed in order to put all of the above on firm grounds. The role of walking is certainly that of enhancing the fermion masses, but quantitatively it is difficult to draw a firm conclusion, and it might well be that the top has to be treated separately. Other proposals exist that address the specific problem of the top mass. The basic observation at the basis is that for all practical purposes the

mass of the top is numerically the same as the scale of the condensate breaking electroweak symmetry, and it is hence conceivable that the top plays a special role in the strong dynamics. Models in which the top quark is a composite object emerging from the strong dynamics itself are generically referred to as topcolor models [13–18], with some abuse of language. As anticipated earlier on, in recent years a variation of this idea has been implemented in five-dimensional models, by assuming that the fermions, particularly the top, be allowed to propagate in the bulk of the extra-dimension. The interested reader is referred to [95–101] and references therein.

3.3. Perturbative Unitarity

A very important, non-trivial property of the MSM is related to unitarity. Being a gauge theory, with no anomalies, unphysical degrees of freedom (states with negative-norm or wrong spin-statistics) decouple. But even more is true: scattering amplitudes can be computed diagrammatically, and by looking at their behavior (particularly at large energy), it is possible to ask under what circumstances the theory admits a sensible interpretation, yielding probabilities bounded by 1. In the MSM, this procedure yields a relation between the cut-off Λ , identifying the regime of validity of the theory, and its one undetermined parameter, the mass of the Higgs. In particular, this logic yields the classical result that an absolute upper bound on the mass of the Higgs m_h exists [33].

In this section, I review the classical derivation of this result, in a simplified version of the MSM in which I set the gauge coupling $g' = 0$, so that $M_Z = M_W$, and compute at the tree-level the (elastic) scattering amplitude between longitudinally polarized gauge bosons $W_L Z_L \rightarrow W_L Z_L$. This is a very instructive exercise, which provides deep inside into the functioning of Feynman rules, and of spontaneous symmetry breaking in relation to the Goldstone equivalence theorem. It also teaches us a great lot about the properties of the scalar sector of the spectrum of a strongly coupled model of EWSB.

The scattering amplitude of longitudinally polarized gauge bosons receives four types of contributions at the tree-level in the MSM, depicted in Figure 5. Besides the direct contribution from the 4-gauge boson vertex (\mathcal{M}_4) there are s -channel and u -channel exchanges of charged gauge bosons (\mathcal{M}_s and \mathcal{M}_u) and t -channel exchanges of neutral scalars (\mathcal{M}_h):

$$\begin{aligned}
 -i\mathcal{M}_4 &= \frac{g^2}{M_W^4} \left[(c^2 - 6c - 3)E^4 + (6c + 2)M_W^2 E^2 \right], \\
 -i\mathcal{M}_s &= \frac{1}{M_W^4} \left[c(2E^2 + M_W^2)^2 (4E^2 - 4M_W^2) \right] \frac{g^2}{4E^2 - M_W^2}, \\
 -i\mathcal{M}_u &= \frac{1}{M_W^4} \left[(6 + 10c + 2c^2 - 2c^3)E^6 + (-6 - 26c - 18c^2 + 2c^3)M_W^2 E^4 \right. \\
 &\quad \left. + (2 + 14c + 20c^2)M_W^4 E^2 + (-2 + 2c)M_W^6 \right] \frac{g^2}{2(1+c)(E^2 - M_W^2) + M_W^2}, \\
 -i\mathcal{M}_h &= \frac{1}{M_W^4} \left[M_W^2 \left((c-1)E^2 + M_W^2 \right)^2 \right] \frac{g^2}{2(c-1)(E^2 - M_W^2) - m_h^2}.
 \end{aligned} \tag{3.58}$$

In the above expressions, written in the center of mass frame, g is the gauge coupling of $SU(2)_L$, $c = \cos \theta$ is the scattering angle of the outgoing particles measured in respect to the incoming direction, and E the center of mass energy of the incoming W , related to the Mandelstam variable $s = 4E^2$.

It is customary to discuss the scattering amplitude in terms of partial wave decomposition. In particular, for the $J = 0$ amplitude,

$$t_0(E) = \frac{1}{32\pi} \int_{-1}^{+1} dc \mathcal{M}(E, c). \quad (3.59)$$

Unitarity implies that the total cross-section be limited by the geometric cross-section, and in turns this implies that $|t_0(E)| < 1/2$. The energy at which this relation stops being true gives an estimate of the UV cut-off of the theory Λ , as a function of the parameters in the model.

The expression simplifies considerably in the large momentum limit $p^2 = E^2 - M_W^2 \rightarrow +\infty$:

$$t_0^{\text{SM}} \rightarrow \frac{ig^2}{64\pi M_W^2} \left(m_h^2 - M_W^2 + 2M_W^2 \ln \frac{4p^2}{M_W^2} \right). \quad (3.60)$$

Among the terms in parenthesis, the m_h dependence is by far the most important effect. Neglecting the other two terms gives the anticipated bound $m_h \lesssim 1.2$ TeV. This is a famous result, which puts an upper bound on the MSM Higgs mass. A more complete and rigorous analysis is done, for instance, in [33]. In order to understand what this statement actually means, it is useful to look in details at how it emerges within (3.60), when summing the contributions from the various diagrams.

First, notice how \mathcal{M}_s cannot contribute to this specific channel, because being odd in c , so that it cancels in (3.59). It is useful to consider the behavior at large- p in terms of a series expansion. Truncating at the p^2 term, and hence keeping only terms that asymptotically grow as powers of p :

$$\begin{aligned} t_0^{\mathcal{M}_4} &= \frac{ig^2}{32\pi M_W^4} \left(-\frac{16}{3} p^4 - \frac{20}{3} p^2 M_W^2 \right), \\ t_0^{\mathcal{M}_s} &= 0, \\ t_0^{\mathcal{M}_u} &= \frac{ig^2}{32\pi M_W^4} \left(+\frac{16}{3} p^4 + \frac{23}{3} p^2 M_W^2 \right), \\ t_0^{\mathcal{M}_h} &= \frac{ig^2}{32\pi M_W^4} \left(-p^2 M_W^2 \right), \end{aligned} \quad (3.61)$$

which clearly shows that both most problematic terms are cancelled thanks to gauge symmetry itself. To be more specific, the $\mathcal{O}(p^4)$ terms cancel between diagrams involving the quartic coupling between gauge bosons and diagrams built with two cubic couplings, which is an immediate consequence of the symmetries of the Yang Mills Lagrangian. More subtle is the cancellation of the $\mathcal{O}(p^2)$ term, which involves the Higgs. Notice that for the cancellation

of the coefficient in front of p^2 to be exact, its contribution from the Higgs cannot depend on m_h . Ultimately, gauge invariance ensures that the coupling g of the Higgs to the gauge bosons is the same as the cubic self-coupling. Two terms are left not cancelled at the next order: a term logarithmic in p , arising from \mathcal{M}_u (see the analytical structure of the denominator) and a term proportional to m_h^2 from \mathcal{M}_h . The former can be safely neglected, in the sense that even for p very large it is not growing fast enough to be a source of concern in practical applications. The latter is the origin of the bound.

Let us open here a brief digression. This exercise shows one very typical property of Feynman diagrams: the calculation of single individual diagrams has an inherent tendency to yield nonsensical results. Such are the $\mathcal{O}(p^4)$ and $\mathcal{O}(p^2)$ terms, that would indicate, if really present, an inherent problem of the theory with unitarity. However, once all diagrams are summed, such pathology disappears exactly; the diagrammatic procedure, carried out carefully, contains all the useful information about symmetries and analytical properties of the theory, and is limited only by its perturbative nature. What is even more striking is that the result hence obtained is actually a consequence of a very non-trivial property of spontaneously broken gauge theories, the Goldstone equivalence theorem, which is automatically encoded in the diagrammatic procedure.

To help making the statements in the previous paragraph more clear, let us go back and rewrite the Higgs potential as

$$\mathcal{L}_V = -\mathcal{V} = -\lambda' \left(H^\dagger H - \frac{v_W^2}{2} \right)^2, \quad (3.62)$$

which, provided $\lambda = \lambda'$, differs from (2.5) only by an irrelevant additive constant. The minimum is clearly at

$$H^\dagger H = \frac{v_W^2}{2}, \quad (3.63)$$

and the physical mass of the Higgs particle is

$$m_h^2 = 2\lambda' v_W^2. \quad (3.64)$$

In this way, one important property is highlighted: the physical Higgs particle receives a mass from its coupling to the vacuum, in the same way as the gauge bosons and fermions of the standard model do. Replacing in (3.60) one can rewrite the leading contribution to the partial-wave amplitude as a bound on λ' ,

$$-it_0^{\text{SM}} \longrightarrow \frac{\lambda'}{8\pi} < \frac{1}{2}, \quad (3.65)$$

or $\lambda' < 4\pi$, which is the familiar bound ensuring that the perturbative expansion makes sense.

This yields a more correct interpretation of the result obtained above: it is perturbation theory that breaks down, not unitarity, when the Higgs mass is larger than the bound! The upper bound on the Higgs mass means that either the Higgs is light, and hence all its couplings are perturbative, or otherwise the sector responsible for electroweak symmetry

breaking must be strongly coupled at the TeV scale. This is the basis of the folklore going under the name of *no-lose theorem for LHC*, according to which either the LHC will discover a light Higgs boson, for which the experimental sensitivity stretches up to 800–1000 GeV, or a new strongly coupled sector, with new bound states must be present at the TeV scale, in which case the experimental sensitivity might reach somewhat higher masses, good enough for discovery.

A final parenthetic remark, purely for didactical purposes is presented. Looking at (3.65), one might feel puzzled. We are computing at the tree-level the scattering of gauge bosons, due to a gauge interaction, and the result for the amplitude is, at the leading order, exactly the same as the elastic scattering amplitude of two real scalars in a theory with no gauge interactions at all, in which the only coupling is the quartic interaction among the scalars. The reader who is familiar with the electroweak theory will recognize that this is another indirect manifestation of the Goldstone equivalence theorem: at large energies, processes involving massive gauge bosons are dominated by the contribution coming from the Goldstone bosons that are eaten up by the Higgs mechanism. Another well-known example is, within the MSM, the decay rate of the Higgs onto Z bosons, which at the leading order, and neglecting phase-space corrections, for $m_h > 2M_Z$, reads,

$$\Gamma(h \rightarrow ZZ) \simeq \frac{m_h^3}{32\pi v^2} \left(1 - 4 \frac{M_Z^2}{m_h^2} + 12 \frac{M_Z^4}{m_h^4} \right). \quad (3.66)$$

At large values of m_h , the first term dominates, and the decay rate can be rewritten as

$$\Gamma(h \rightarrow ZZ) \simeq \frac{\lambda'}{16\pi} m_h, \quad (3.67)$$

which again does not depend at all on the gauge coupling, and can be computed directly from the theory of the scalars in the $g \rightarrow 0$ limit, from the decay of the Higgs in two Goldstone bosons.

This being a set of lectures on technicolor, it is time to close these digressions and go back to seeing what all of the above teaches us about dynamical EWSB. First of all, this is another manifestation of the non-decoupling of the scalar sector, as anticipated when talking about precision physics. One might think of building the electroweak chiral Lagrangian by taking the limit $\lambda' \rightarrow +\infty$, while keeping v_W fixed. The Higgs mass grows to infinity in this (classical) limit. However, the Higgs *does not decouple*, because in order to do so one is effectively taking large values of the coupling. In this limit, what goes really wrong is that in the last diagram in Figure 5 one should not, for large- λ , use the perturbative propagator for the Higgs, but should replace it with a nonperturbatively computed scalar correlator, formally written as

$$\frac{1}{p^2 - m_h^2} \longrightarrow \frac{1}{p^2 - \Sigma(p)}. \quad (3.68)$$

The exact form of $\Sigma(p)$ cannot be obtained with simple perturbation theory, and one expects it to have a very non-trivial analytical structure of poles and cuts. In a UV-complete TC model, one might also expect that $\Sigma(p) \rightarrow 0$ at high energies, essentially because all the scales should

be generated dynamically in the IR. And hence, ultimately one expects the $\mathcal{O}(p^2)$ terms to cancel, and the scattering amplitude to be unitary, but uncalculable.

In Higgsless models, one would not include the diagram yielding \mathcal{M}_h , because, by definition, there is no Higgs to start with. As a result, the amplitude under study would grow quadratically with p , in the same way as naively obtained from the chiral Lagrangian. One might be lead to conclude that such models generically violate unitarity already below the TeV scale, unless completed somehow. These statements are misleading, and should be read and used with some caution. The subtle part is that in the calculation we did here, we have been careful to take $p \rightarrow +\infty$ at fixed m_h , before starting to look at large values of m_h . The other way around, in which one first takes $m_h \rightarrow +\infty$, and hence sets $\mathcal{M}_h = 0$, and then looks at large- p , can give sensible answers only provided that one can decouple the Higgs. Which is not the case. We already anticipated this observation earlier: this scattering amplitude is a particularly bad observable quantity, that is quadratically sensitive to the mass of the Higgs, and in general to the structure of the scalar correlators of the symmetry-breaking sector. As such, the amplitude is completely dominated by uncalculable contributions, which cannot be neglected, and hence in general yields very little useful information about new physics. Contrast this with what happens for oblique precision parameters.

One should also explain what is meant by *Higgsless*. The fact that EWSB is not induced by an elementary, weakly coupled scalar sector does not imply that there are no scalars at all. Whatever its origin, it is always possible to excite the vacuum, provided enough energy is at disposal. In doing so, one is probing the structure of the quantum effective potential expanded around the symmetry-breaking vacuum. The fact that we do not know what this potential looks like, does not mean that there are no excited, scalar bound states. The really interesting question has to do with the properties of such excited states: can they be described and detected experimentally as particles? Based on the experience of QCD, one might be tempted to give a firm negative answer. Yet, this is a premature conclusion, answering such a question requires a very detailed knowledge of the strong dynamics itself, and dedicated nonperturbative method should be used, unless symmetry arguments become available, which we will discuss in next subsection.

In conclusion, the existence of a perturbative theory describing the sector responsible for electroweak symmetry breaking, and that this provides a valid description up to high sales, requires the existence of a light scalar degree of freedom with the couplings of the Higgs particle in the MSM, in order for the WW scattering amplitudes to be manifestly unitary. In technicolor, there is no elementary scalar, and hence this statement just amounts to saying that the theory must be strongly coupled and nonperturbative at the TeV scale, which we already knew from the start. One might want to compute the scattering amplitude of longitudinal gauge bosons in presence of such a strongly coupled sector, starting from the underlying dynamics, instead of using EFT, but this is of utmost difficulty. Even in QCD, where data about the analog $\pi\pi$ scattering processes are available, the precise role of the resonances, such as the ρ and the scalars, are still subject of studies. In more modern versions of technicolor, it is believed that the role of techni- ρ be very important in the TeV region, in analogy with QCD, but possible other resonances might contribute. Corrections to the classical SM results encoded in the coefficients of the chiral Lagrangian, in particular α_4 and α_5 , might be measurable, or new resonances detectable at the LHC. In general, this is a very open problem in technicolor, and many interesting and useful studies that can be found in the literature [108–110]. But one has to keep in mind that all of this is very strongly limited by the sensitivity of this particular observable to the uncalculable structure of the correlation functions involving scalar bound states.

3.4. About the Spectrum of WTC

Technicolor being a strongly coupled theory, which confines and produces condensates at low energy, it is expected to have a rich and complicated spectrum of bound states. In this section, I focus on a few very special such bound states, the properties of which are related to the fundamental symmetries of the theory: the techni-rho mesons, the technipions, and the technidilaton.

One would like to know also what the spectrum of anomalous dimensions is, at low energies, where the dynamics is walking. In this energy region, the theory is approximately conformal and the dynamics must be governed by a CFT with some set of operators with non-trivial anomalous dimensions. These anomalous dimensions should be large, in such a way as to modify the NDA counting rules, for the reasons explained earlier. In some classes of supersymmetric theories, supersymmetry itself allows to compute these anomalous dimensions, thanks to the relation between R -symmetry and the superconformal group. But in general, this is a very hard task that requires the use of fully nonperturbative techniques. In principle, within gauge/gravity dualities the program of classifying the relevant operators and their dimensions can be carried out, although it still represents a very non-trivial task.

3.4.1. Techni-Rho Mesons

First of all, the TC sector will have some global symmetry G , a subset of which is identified with the SM gauge group. It is hence expected on general grounds that towers of spin-1 states transforming under this symmetry be present, analogous to the vectorial ρ meson, axial-vectorial a_1 meson, and their heavy excitations. We already saw that these states play a crucial role in the oblique precision observables, particularly in the \hat{S} parameter. They also play a role in the WW scattering amplitudes. And finally, they are the most accessible new states at the LHC. This last statement depends crucially on the couplings (and width) of such states, which is the main topic of this subsection. The discussion will rely almost entirely on the comparison with QCD and with its large- N scaling properties.

Before starting, let us remind the reader about the large- N scaling of the relevant parameters, which can be summarized by

$$\begin{aligned}
 M_\rho &\sim \mathcal{O}(1), \\
 f_\pi^2 &\sim \mathcal{O}(N), \\
 f_\rho^2 &\sim \mathcal{O}(N), \\
 g_{\rho\pi\pi}^2 &\sim \mathcal{O}\left(\frac{1}{N}\right),
 \end{aligned} \tag{3.69}$$

where the $g_{\rho\pi\pi}$ coupling between technirho and technipion mesons in the HLS language is normalized so that

$$\mathcal{L}_{\rho\pi\pi} = 2i g_{\rho\pi\pi} \text{Tr} \rho^\mu [\pi \partial_\mu \pi]. \tag{3.70}$$

The masses of the spin-1 states are going to be controlled by the strong-dynamics scale $\Lambda^* \sim \mathcal{O}(1) \text{ TeV}$, while f_π stands for the electroweak VEV. The $g_{\rho\pi\pi}$ coupling controls the

decay $\rho \rightarrow WW$, which is the analog of the $\pi\pi$ dominant decay of the ρ in QCD, with branching fraction very close to 1. In studying LHC signatures of technicolor, this suggests that a clean signature (at least in a QCD-like technicolor) is the process $pp \rightarrow \rho \rightarrow WW$. On general grounds, the number of events expected at fixed center-of-mass energy is strongly suppressed by the techni-rho mass in the intermediate state. At the same time, the facts that the width being quite large, that the production process might involve vector-fusion or suppressed couplings to the quarks, and that reconstructing the WW final state is not trivial limits the possibility of distinguishing the signal from the SM background to $M_\rho \lesssim 1$ TeV, which is already hard to reconcile with precision electroweak physics.

A different possibility arises at large- N in a non-QCD-like theory. Some basic results are discussed first, exemplified by QCD. The techni-rho, in analogy with its QCD analogue, has two main couplings that determine its decays. There is the cubic coupling to the light W and Z (to the pions), with strength controlled by $g_{\rho\pi\pi}^2 \propto 1/N$. But also, the techni-rho couples to the light SM fermions, with couplings proportional to the decay constant $f_\rho^2 \propto N$. Both yield decays to 2-body final states with effectively massless particles. A look at the PDG reveals that in QCD

$$B_\ell \equiv \frac{\Gamma[\rho \rightarrow \ell^+ \ell^-]}{\Gamma[\rho \rightarrow \pi^+ \pi^-]} \simeq 4.5 \times 10^{-5}, \quad (3.71)$$

both for $\ell = e$ and $\ell = \mu$. Which clearly indicates that the $g_{\rho\pi\pi}$ is the dominant coupling to be considered, as already stated earlier. Indeed, at the tree-level one can show that

$$\Gamma[\rho \rightarrow \ell^+ \ell^-] = \frac{\alpha e^2 f_\rho^2}{3 M_\rho^2} M_\rho, \quad (3.72)$$

$$\Gamma[\rho \rightarrow \pi^+ \pi^-] = \frac{g_{\rho\pi\pi}^2}{48\pi} M_\rho, \quad (3.73)$$

with e the electromagnetic coupling. Some experimental input: $f_\rho \simeq 150$ MeV, $M_\rho \simeq 770$ MeV, $\Gamma_\rho \simeq 150$ MeV, and $\alpha \sim 1/137$. With these one obtains $g_{\rho\pi\pi} \simeq 5.5$, and hence

$$B_\ell \simeq \frac{192\pi^2 \alpha^2 f_\rho^2}{3g_{\rho\pi\pi}^2 M_\rho^2} \simeq 4.2 \times 10^{-5}, \quad (3.74)$$

which agrees quite well with the data.

Having established that these approximations are sensible, at least for QCD, take the result and rescale it to a large- N QCD-like technicolor theory, in which the mass of the ρ is very big (such that all the SM-fermion masses can be neglected). The ratio B_T between partial widths into SM fermions and into W bosons results in

$$B_T = 8 \frac{N_c^2}{3^2} B_\ell, \quad (3.75)$$

where the factor of 8 comes from summing over all the charges of the SM fermions. In order that $B_T \sim \mathcal{O}(1)$, one has to require that $N_c \simeq 12\pi^2$. Hence, for the decay into W bosons (pions)

to be subdominant in respect to the decay into fermions, one needs very large values of N_c . This is in blatant contradiction with the perturbative estimates from the \widehat{S} parameter.

However, remember how we clarified that what enters into \widehat{S} is f_π , while the coupling to the currents of interest for the decays is f_ρ . In a non-QCD-like large- N_c theory, it is possible that the large value of N_c be compensated by a ratio of scales making $f_\pi \ll f_\rho$. Notice also that for such values of $N_c \simeq 12\pi^2$ one expects

$$g_{\rho\pi\pi}(N_c) \sim \frac{\sqrt{3}g_{\rho\pi\pi}(3)}{\sqrt{N_c}} \sim 1, \quad (3.76)$$

which means that this would be precisely the regime in which hidden local symmetry calculations might become reliable.

If $B_T \sim \mathcal{O}(1)$ or larger, then the phenomenology at the LHC of the techni-rho mesons is going to be drastically different from what inferred from QCD; they will be relatively narrow resonances, with couplings to the SM currents comparable to those of the Z and W mesons, and decay predominantly into two fermions. The technirhos will look very similar to weakly coupled Z' (and W') gauge bosons. In particular, the Drell-Yan $pp \rightarrow \rho \rightarrow \ell\ell$ is going to be a very favorable experimental signature of such scenarios. We will come back to this point when discussing the bottom-up approach to holography, but notice how a classical signature of weakly coupled extensions of the SM (a new, narrow spin-1 state) might actually emerge as a low-energy result of a strongly coupled theory, at large- N_c .

3.4.2. Technipion: Composite Higgses and Little Higgses

The second sector of the spectrum that is very important is that of the possible light pseudo-scalars in the theory. In general, the global symmetry G of the technicolor sector may be much larger than the SM gauge group, and the condensates breaking G spontaneously will produce a number n_π of Goldstone bosons. Weakly gauging the $SU(2)_L \times U(1)_Y$ means that 3 of the Goldstone bosons disappear from the spectrum, becoming longitudinal components of W and Z . Also, the gauging of a subgroup of G in general breaks explicitly the symmetry, and hence most of the $n_\pi - 3$ will acquire a mass. The resulting pseudo-Goldstone bosons (PNGB), however, are going to be relatively light, and hence problematic.

As an example, we can perform explicitly the exercise of estimating the masses of the PNGBs in the case of the prototypical example of TC model we have already discussed several times, by using (2.9). In the limit in which electroweak and QCD gauge couplings are turned off there is a $SU(8)_L \times SU(8)_R$ global symmetry, acting on the vectorial techniquarks, which is broken spontaneously to $SU(8)$. Which means there are a total of $n_\pi = 63$ PNGBs. A classification of their properties can be found in [111]. An estimate of their masses can be obtained from the non-linear sigma-model description of the 63 pions π^i :

$$\mathcal{L}_8 = \frac{f^2}{4} \text{Tr} |D\Sigma_8|^2, \quad (3.77)$$

where

$$\Sigma_8 = \exp \frac{2i\pi}{f}, \quad (3.78)$$

the pions describe the $SU(8) \times SU(8)/SU(8)$ coset, and the covariant derivative is

$$D\Sigma = \partial\Sigma + i(g'B + gW + g_s G)\Sigma - i\Sigma(g'B + g_s G), \quad (3.79)$$

with W the matrix of the $SU(2)_L$ gauge bosons, B the matrix of the $U(1)_Y$, and G the gluons. With these normalizations, one deduces masses for the electroweak gauge bosons $M_W^2 = g^2 f^2$ and $M_Z^2 = (g^2 + g'^2)f^2$, implying that $f^2 = v_W^2/4$.

By using (2.9), one sees that the QCD interactions give quadratically divergent masses to the majority of the PNBs. (The weakly coupled effective potential induced by the explicit symmetry-breaking couplings is also important for the study of vacuum alignment [112].) To be more precise, if Λ is the cut-off in (2.9), one finds

$$\begin{aligned} m_{24}^2 &= \frac{g_s^2 \Lambda^2}{4\pi^2}, \\ m_8^2 &= \frac{3g_s^2 \Lambda^2}{8\pi^2}, \\ m_{24}'^2 &= \frac{9g_s^2 \Lambda^2}{16\pi^2}, \end{aligned} \quad (3.80)$$

where the index 24 and 8 indicates the multiplicity, and where corrections due to electroweak couplings have been neglected because the QCD coupling g_s is bigger. These masses are all estimated to be $\mathcal{O}(200)$ GeV (up to very large uncertainties signaled by the quadratic divergence). But notice that all the corresponding states transform as triplets or octets of QCD $SU(3)_c$, and hence carry strong interactions.

There are hence $63 - 24 - 8 - 24 = 7$ massless Goldstone bosons at this stage of the calculation, which do not receive quadratically divergent contributions to their mass from any of the standard-model gauge interactions. Two of them obtain a logarithmically divergent mass from (2.9),

$$m_2^2 = \frac{3g^2 g'^2 f^2}{16\pi^2} c, \quad (3.81)$$

where $c \sim \log \Lambda/f$ is some $\mathcal{O}(1)$ number. These two masses are much lighter, $m_2 \sim \mathcal{O}(10)$ GeV, and the corresponding states electrically are charged and colorless.

Finally, three of the remaining five Goldstone bosons are the longitudinal components of the W and Z bosons. The last two are exact Goldstone bosons, which are electrically neutral and which remain massless as a result of the fact that a global $U(1) \times U(1)$ symmetry is not broken by the gauge interactions. They are sometimes referred to as techni-axions. Their existence is a very major source of concern for technicolor. However, another source of symmetry breaking might be ETC. Among the 4-fermion interactions generated by ETC, some might break the global symmetry G , effectively giving mass to the two undesirable light Goldstone bosons. Walking might also play a role in enhancing dynamically all such masses, because some of the operators produced by ETC, which break explicitly the large global symmetry of the model, will involve techniquarks, and will hence develop large anomalous dimensions, making them more relevant than NDA would indicate.

Notice one curious fact. The mechanisms giving rise to the very existence, and to the small masses, of the technipions also suggests an intriguing alternative possibility. A similar mechanism has been exploited in order to construct variations of strongly coupled theories in which one set of such PNGBs constitutes the Higgs field H of the MSM itself. The symmetry properties of the Goldstone bosons in this case (partially) protect the Higgs potential from dangerous divergences. In these proposals, the strong-coupling scale has nothing to do with electroweak symmetry breaking, which is triggered by the VEV of the Higgs, emerging at a lower scale. These variations go under the name of composite Higgs [70–77], and of little Higgs [78–82] models. The difference between the former and the latter being that in little Higgs models the same mechanism of collective breaking that anomalously suppressed the masses m_2^2 of the example given above (the fact that the quadratically-divergent contribution to the effective potential vanishes at 1-loop) is fully exploited in order to enhance the scale separation between the cut-off Λ and the electroweak scale.

Coming back to our topic, namely, technicolor, this is one particularly simple example, rich enough to yield generic, rather than general, indications of what to expect. The spectrum and interactions of the technipions depend on the global symmetries of the specific realization of technicolor one is interested in, and one should analyze the details of the symmetry structure on a model-by-model basis, but all of the above clearly shows that technipions are a generic source of concern for technicolor, possibly the biggest. On the other hand, the existence of technipions carrying QCD interactions is possibly testable, because of their relatively light mass. The fact that light scalars being present which carry electroweak interactions, possibly very light, is also interesting phenomenologically. And the fact that the number of such states can easily be very large is also interesting in LHC perspective, provided that one identifies models in which all of the technipions could have escaped detection up to now.

3.4.3. Technidilaton

Finally, a very open problem, which is not a general property of dynamical electroweak symmetry breaking but rather specifically related to the nature of walking technicolor. A light scalar particle might be present as a consequence of the spontaneous breaking of dilatation symmetry. Its phenomenological properties are so striking to deserve a systematic discussion.

To understand the implications of this possibility, it is useful to look back at the Lagrangian of the MSM, in which we write the potential in the form of (3.62) rather than (2.5). At the classical level, in the limit in which λ' vanishes, this Lagrangian is exactly scale-invariant, containing only marginal operators. The minimum of the Higgs potential breaks scale invariance spontaneously, at the scale v_W . The coupling λ' , and all the quantum correction giving rise to anomalous dimensions, represent the explicit breaking of scale invariance. As a result, there is a spontaneously broken approximate global symmetry, and hence in the spectrum there must be a dilaton, the PNGB associated with dilatations, whose mass is controlled by the symmetry breaking terms. Indeed, such dilaton is the Higgs particle h , and the expression for its mass

$$m_h^2 = 2\lambda'v_W^2 \quad (3.82)$$

is similar to the relation yielding the mass of the pion in QCD,

$$m_\pi^2 \propto m_q f_\pi. \quad (3.83)$$

In the latter, m_q is the quark mass, that breaks explicitly the chiral symmetry, while f_π breaks it spontaneously. For the Higgs/dilaton, the λ' coupling plays the same role as m_q for the pions. Also, notice that the pion is sensibly described as a PNGB only provided m_q be small. For instance, the mass of the B mesons, which have quantum numbers that make them the analog of the pions when the flavor symmetry of QCD is extended to $SU(5) \times SU(5)$, scales as $m_B^2 \sim m_b^2$. The breaking of chiral symmetry is in this case so large that it does not make sense to speak about $SU(5) \times SU(5)/SU(5)$ Goldstone bosons. In the same sense, only provided λ' be very small, is the Higgs a dilaton.

The striking consequence of thinking about the Higgs as a dilaton, is that one automatically can infer that the couplings of the Higgs to all the SM fields are given by

$$\mathcal{L} = 2\frac{h}{v_W} M_W^2 W_\mu W^\mu + \frac{h}{v_W} M_Z^2 Z_\mu Z^\mu - \frac{h}{v_W} m_\psi \bar{\psi} \psi, \quad (3.84)$$

which again is what expected; each appearance of the Higgs h is accompanied by a $1/v_W$ factor taking into account the scale of spontaneous symmetry breaking (as in the NDA rules), and the coupling is then proportional to the explicit breaking encoded in the masses of the bosons and fermions in the associated dilatation current.

Provided the Higgs be perturbative (i.e., light), most of the main calculations needed to discuss its experimental searches are done at tree-level, while higher-order operators generated by new physics are going to give subleading corrections. For instance, the decay rate into two SM fermions ψ is

$$\Gamma[h \rightarrow \psi\psi] \simeq \frac{\mathcal{N} m_\psi^2 m_h}{8\pi v_W^2}, \quad (3.85)$$

where $\mathcal{N} = 3$ for quarks and $\mathcal{N} = 1$ for leptons. Which means that whatever the theory responsible for electroweak symmetry breaking, if it has a light dilaton in the spectrum, not only its quantum numbers, but also most of its coupling and all of its phenomenology are going to be identical to the higgs h of the MSM. Only at higher energies, and/or measuring very precisely the subleading corrections, one might be able to tell the difference. Which implies that knowing if a walking technicolor has a light dilaton in the spectrum or not is of very direct and immediate relevance for the LHC program. Notice also that such a state would then enter directly into the arguments about perturbative unitarity deduced from the WW scattering, and of course enter logarithmically in the oblique parameters, precisely in the same way as the Higgs field h does. This is another example of the fact that this non-QCD-like technicolor models might be more difficult to distinguish from weakly coupled extensions of the SM than naively expected (see a related comment in Section 3.4.1).

The reason why one might think that a walking technicolor theory might have a light dilaton in the spectrum is not immediately apparent. For instance, in QCD it is known that such a state does not exist. One might associate it with the σ . But then again, the σ is not light, it is certainly not weakly coupled, and it has a possibly complicated internal structure, consisting of an admixture of quark and pion bound states, implying that its couplings are very non-trivial. Also, it is somewhat difficult to argue that there is a sense in which at the Λ_{QCD} scale there is dilatation symmetry, and hence nothing to spontaneously break in order to produce a dilaton.

By contrast, the basic idea of walking technicolor is that the theory being approximately scale invariant in the IR. Barring the problem that a sensible definition of the word *approximate* would be needed, this is clearly suggesting that one cannot exclude the presence of such a light dilaton, provided that the dynamics being very different from QCD-like. Making this statement quantitative is very hard. If the underlying dynamics is some form of gauge theory, the explicit breaking of conformal symmetry is presumably related to the coefficient of the β -function of the gauge coupling being parametrically smaller than the coupling itself. But also this is a confusing and ambiguous statement; at what scale, and in what scheme, should one calculate this coefficient, and hence the resulting mass for the dilaton? One might say that if the gauge coupling runs as illustrated in Figure 1(c), then it is reasonable to believe that conformal symmetry is broken spontaneously at the lower end Λ_{IR} of the plateau region, and that at that scale the β function is indeed small. But then one might ask what is the effect of the lowest scale in the problem, at which the coupling diverges, the theory confines, and the β -function is not small.

An actual calculation is needed in order to answer this question. Examples exist in the literature, done using the most diverse approaches in order to model the strongly coupled sector and gain in calculability, and date back to the early days of walking technicolor [113–119]. Unfortunately, they disagree with each other. This is perhaps the biggest open problem that gauge/gravity dualities might help to solve [57, 120–122], and I will spend more time on it later in these lectures.

3.5. Summary: Questions on Walking TC

Walking technicolor provides a very natural solution to the big hierarchy problem, and is a way of softening the little hierarchy problem of more generic technicolor models. On the basis of what we said so far in this lectures, we can produce a list of open problems that we want to address using holography and gauge/gravity correspondence.

- (i) What is the field content of a model yielding walking dynamics, in particular what are the conditions of the number of colors, type of representation, number of flavor?
- (ii) What are the values of the dynamically generated scales, in particular the value of the chiral condensate, and how are they related to each other and to the quantities in (i)?
- (iii) What are the anomalous dimensions, and in particular how do they depend parametrically on the choice of the theory and on the scale at which they are evaluated?
- (iv) What are the oblique precision parameters, how do they depend on the details in point (i), on the scales at point (ii) and (iii) and on the specific way in which the technicolor sector is coupled to the SM?
- (v) What are the coefficients of the chiral Lagrangian, including those that correct the cubic and quartic interactions among gauge bosons?
- (vi) What does a fully realistic ETC theory look like, what is its field content, what are the ETC scales, what is the pattern of symmetry breaking, and how can one effectively describe the dynamics of tumbling?
- (vii) What are the masses of the fermions, as a function of the points (vi) and (iii)?

- (viii) What are the contributions to FCNC from operators generated by ETC, and how do they depend on the properties of the theory in point (vi)?
- (ix) What is the spectrum of the spin-1 mesons, and what are their couplings and decay constants, how do they depend on (i) and (ii), and what is the precise relation to the oblique parameters in point (iv)?
- (x) What are the global symmetries of the TC theory at the electroweak scale, and what is the spectrum of PNGBs as a function of the operators generated by ETC and of the anomalous dimensions?
- (xi) Is there a light dilaton, and what are its mass and couplings as a function of (i) and (ii)?
- (xii) What is the WW scattering amplitude going to look like at large energies, what are the intermediate states that are relevant from points (ix), (x), and (xi), and what is its analytical structure?

This is an incomplete set of questions. All of them are very difficult, and all of them ultimately need to be answered in order to ask the most fundamental questions:

Are there calculable models of dynamical electroweak symmetry breaking that are compatible with all experimental data? If so, how do we discover them and characterize them at the LHC?

The main aim of these lecture notes is to highlight the fact that gauge/gravity dualities represent one very powerful tool for such a program of investigation of strong dynamics to be carried out, and the next sections will be devoted to this topic. The reader should be aware of the fact that other approaches exist.

For example, in the very recent years a lot of resources have been focused on trying to answer these questions (in particular those at point (i)) with the nonperturbative instruments of lattice field theory. The mere number of papers that appeared recently on the topic gives a measure of how interesting, and difficult, these studies are. An incomplete list includes the studies in [123–151]. A number of collaborations considered several possible four-dimensional models, such as $SU(3)$ and $SU(2)$ gauge theories with fermion field content on the fundamental or symmetric representation of the gauge group. The lattice actions used by the various groups are different (e.g., both Wilson fermions and staggered fermions have been used), and different physical quantities have been analyzed (such as the running of the gauge coupling defined nonperturbatively, the physical spectrum of mesons and of anomalous dimensions, the chiral condensate). The results of several collaborations seem to indicate that some of these models are indeed conformal (and strongly coupled) in the IR, which is the starting point for the construction of a walking technicolor theory.

Much more lattice data is expected to become available in the near future, improving the systematics and possibly yielding robust enough evidence that one or more of these models provides a non-QCD-like dynamical origin for electroweak symmetry breaking. At that point, one might be able to answer quantitatively many of the questions in the list presented here, within those very specific models, and it would be very interesting to know how general these results are, by comparing with other approaches. For example, it would be important to know to what extent the lattice results agree with the extrapolation to $N_c = 2, 3$ of large- N_c studies.

In summary, dynamical electroweak symmetry breaking, and walking technicolor in particular, is a very exciting research field. There are many open questions, related to the

strongly coupled dynamics, that are of direct relevance for the phenomenology of extensions of the standard model, and hence for the experimental program of the LHC. These questions might be answered independently by studies carried on within the lattice community and the string theory community, with completely different and complementary approaches. We will now turn our attention to some example of what can be done along these lines within gauge/gravity dualities.

4. Holographic Technicolor: Bottom-Up Approach

As we have seen, hidden local symmetry is useful, but affected by two related problems: the calculability is limited both by the proliferation of independent parameters when including more states, and by the fact that some couplings take large values, making the perturbative expansion questionable. One basic idea behind the bottom-up holographic approach is to keep the successes of hidden local symmetry (particularly, its simplicity) and overcome these main difficulties by replacing the chain of links and sites with an extra-dimension [45–55]. A background metric inspired by the AdS/CFT correspondence is assumed, which is justified by the assumption of walking, and the effective couplings are kept small by the assumption that the model hence obtained be dual to a large- N_c theory. The gravity formalism can be used because a large value for the large 't Hooft coupling $\lambda = g^2 N_c$ is assumed for the dual field theory.

This idea has been used also in order to model low-energy QCD [152, 153]. The three assumptions about N_c , λ , and the AdS background are hardly justified in this case. Yet, many interesting and useful results have been obtained, and one can argue that they are in acceptable agreement with actual QCD data, at least in a variety of interesting examples.

I will not review all the literature on the subject, which is vast, but focus on one very specific example [45–50]. The main result that I want to convince the reader of is that a sensible description of all the physics of electroweak gauge bosons can be obtained in which only very few parameters with very specific meaning are needed, and that it is possible to choose such parameters in a way that renders \hat{S} compatible with the bounds. Once this is done, this set-up allows to make actual predictions for LHC signatures. The example discussed has the only special feature of being very minimal. In the end, it contains only one more parameter than the MSM, but describes an infinite number of new resonances. Infinite numbers of nonminimal variations of this set up can be formulated, and many have been explored in the literature.

4.1. Holography and AdS/TC: A Simple Model

4.1.1. The Model

A summary of the minimal requirements for a viable model of dynamical EWSB in four dimensions allows to identify the properties of the five-dimensional dual description. Many of these assumptions can in principle be relaxed, but they constitute a simple and natural place to start. There must be a new strong sector possessing the global symmetry $SU(2)_L \times U(1)_Y$ of the standard model. The new interaction must confine, and a symmetry breaking condensate must form. The (weak) gauging of the global symmetry of the strong sector gives the massive SM gauge bosons and the photon. The strong sector has to be close to conformal over the energy range between the electroweak scale and a much larger ETC scale. Large

anomalous dimensions are present so that the chiral condensate has scaling dimension very different from $d = 3$ (for concreteness, here $d = 2$). All the SM fermions are elementary, and do not carry quantum numbers of the new strong interactions, hence ensuring universality of the electroweak gauge coupling.

The (quasiconformal) energy window just above the electroweak scale is described by a slice of AdS_5 , that is, by a five-dimensional space-time containing a warped gravity background given by the metric

$$ds^2 = \left(\frac{L}{z}\right)^2 \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right), \quad (4.1)$$

where x^μ are four-dimensional coordinates, $\eta_{\mu\nu}$ the Minkoski metric with signature $(+, -, -, -)$, and z is the (warped) extra-dimension. The dimensionful parameter L is the AdS_5 curvature, and sets the overall scale of the model. Conformal symmetry is broken by the boundaries

$$L_0 < z < L_1, \quad (4.2)$$

with $L_0 > L$, where L_0 and L_1 correspond to the UV and IR cut-offs of the conformal theory, that is, to the ETC scale and to the confinement scale, respectively.

The field content in the bulk of the five-dimensional model consists of a complex scalar Φ transforming as a $(2, 1/2)$ of the gauged $SU(2)_L \times U(1)_Y$. The generator of $SU(2)_L$ are $T^a = \tau^a/2$ with τ^a the Pauli matrices.

The bulk action for Φ and the gauge bosons $W = W^a T^a$ of $SU(2)_L$ and B of $U(1)_Y$ is

$$\begin{aligned} \mathcal{S}_5 = & \int d^4x \int_{L_0}^{L_1} dz \sqrt{G} \\ & \times \left[\left(G^{MN} (D_M \Phi)^\dagger D_N \Phi - M^2 |\Phi|^2 \right) \left(-\frac{1}{2} \text{Tr}(W_{MN} W_{RS}) - \frac{1}{4} B_{MN} B_{RS} \right) G^{MR} G^{NS} \right], \end{aligned} \quad (4.3)$$

and the boundary terms are

$$\begin{aligned} \mathcal{S}_4 = & \int d^4x \int_{L_0}^{L_1} dz \sqrt{G} \left\{ \delta(z - L_0) D \left[-\frac{1}{2} \text{Tr}[W_{\mu\nu} W_{\rho\sigma}] - \frac{1}{4} B_{\mu\nu} B_{\rho\sigma} \right] G^{\mu\rho} G^{\nu\sigma} \right. \\ & \left. - \delta(z - L_0) 2\lambda_0 \left(|\Phi|^2 - \frac{v_0^2}{2} \right)^2 - \delta(z - L_1) 2\lambda_1 \left(|\Phi|^2 - \frac{v_1^2}{2} \right)^2 \right\}, \end{aligned} \quad (4.4)$$

where the covariant derivative is given by

$$D_M \Phi = \partial_M \Phi + i \left(g W_M \Phi + \frac{1}{2} g' B_M \Phi \right), \quad (4.5)$$

and where the Yang-Mills action is written in terms of the antisymmetric field-strength tensors $W_{\mu\nu}$ and $B_{\mu\nu}$. In the action, M^2 is a bulk mass term for the scalar, and g and g' are the (dimensionful) gauge couplings in five-dimensions. The coefficient D will be fixed later.

Without loss of generality, the VEV of the Φ field can be written as

$$\langle \Phi \rangle = \frac{v(z)}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \quad (4.6)$$

The localized potentials enforce a nonvanishing $v(z)$ that induces electroweak symmetry breaking. For $M^2 = -4/L^2$, in the $\lambda_i \rightarrow +\infty$ limit, (in which the transverse degrees of freedom become infinitely massive and decouple from the spectrum) the bulk equation of the motion

$$\partial_z \left(\frac{L^3}{z^3} \partial_z v \right) - \frac{L^5}{z^5} M^2 v = 0, \quad (4.7)$$

admits the solution

$$v(z) = \frac{v_1}{L_1^2} z^2 = \frac{v_0}{L_0^2} z^2, \quad (4.8)$$

by appropriately choosing $v_0/v_1 = L_0^2/L_1^2$, so as to describe a chiral condensate of dimension $d = 2$.

The localized kinetic terms for the gauge bosons are required by holographic renormalization [154, 155] in order to remove a logarithmic divergence, and retain finite SM gauge couplings in the $L_0 \rightarrow 0$ limit, renormalizing the otherwise divergent kinetic terms of the SM gauge bosons. This procedure ensures that the SM gauge couplings be independent of the strength of the bulk coupling, and that all physical quantities be independent of any UV-sensitive details, such as the precise value of L_0 , or the structure of the UV boundary.

4.1.2. Electroweak Phenomenology

Focusing on the spin-1 sector, and defining

$$\begin{aligned} V^M &\equiv \frac{g' W^{3M} + g B^M}{\sqrt{g^2 + g'^2}}, \\ A^M &\equiv \frac{g W^{3M} - g' B^M}{\sqrt{g^2 + g'^2}}, \end{aligned} \quad (4.9)$$

one obtains the electroweak equivalent of the vector and axial-vector sectors of the chiral Lagrangian. In particular, the massless mode of V^μ is the photon, and the lightest mode of A^μ is the Z boson.

After Fourier transformation in the four-dimensional Minkowski coordinates one can write

$$A^\mu(q, z)^\mu(q) v_Z(z, q), \quad (4.10)$$

and analogous for $W_{1,2}$ and V , where $q = \sqrt{q^2}$ is the four-dimensional momentum. In the limit in which one neglects the five-dimensional gauge coupling, and hence cubic and quartic self-interactions, the bulk equations are

$$\partial_z \frac{L}{z} \partial_z v_i - \mu_i^4 L z v_i = -q^2 \frac{L}{z} v_i, \quad (4.11)$$

where $i = V, Z, W$, with $\mu_V = 0$, $\mu_W^4 = 1/4 g^2 v_0^2 / L^2$, and $\mu_Z^4 = 1/4 (g^2 + g'^2) v_0^2 / L^2$. This approximation and the approximation of restricting all the analysis at the tree-level are valid provided the effective couplings $g/\sqrt{L}, g'/\sqrt{L} \lesssim O(1)$, which corresponds to the large- N limit.

The bulk equations can be solved exactly. Choosing Neumann boundary conditions in the IR (which are compatible with the gauge symmetry) leaves a nonvanishing UV-localized action, that can be interpreted in terms of yielding the vacuum polarizations for the gauge bosons in the form

$$\mathcal{L} = -\frac{1}{2} A_\mu^i \pi_{i,j} P^{\mu\nu} A_\nu^j, \quad (4.12)$$

where $P^{\mu\nu} \equiv \eta^{\mu\nu} - q^\mu q^\nu / q^2$ and $i = B, W^a$. The matrix of the vacuum polarizations $\pi_{i,j}(q^2)$ of the SM gauge bosons results in

$$\begin{aligned} \frac{\pi_+}{\mathcal{N}_W^2} &= Dq^2 + \frac{\partial_z v_W}{v_W} (q^2, L_0), \\ \frac{\pi_{BB}}{\mathcal{N}_B^2} &= Dq^2 + \frac{g^2}{g^2 + g'^2} \frac{\partial_z v_V}{v_V} (q^2, L_0) + \frac{g'^2}{g^2 + g'^2} \frac{\partial_z v_Z}{v_Z} (q^2, L_0), \\ \frac{\pi_{WB}}{\mathcal{N}_W \mathcal{N}_B} &= \frac{g g'}{g^2 + g'^2} \left(\frac{\partial_z v_V}{v_V} (q^2, L_0) - \frac{\partial_z v_Z}{v_Z} (q^2, L_0) \right), \\ \frac{\pi_{WW}}{\mathcal{N}_W^2} &= Dq^2 + \frac{g'^2}{g^2 + g'^2} \frac{\partial_z v_V}{v_V} (q^2, L_0) + \frac{g^2}{g^2 + g'^2} \frac{\partial_z v_Z}{v_Z} (q^2, L_0). \end{aligned} \quad (4.13)$$

The precision electroweak parameters have been defined earlier, in terms of the vacuum polarizations, with the convention of choosing the normalizations \mathcal{N}_i of the fields so that $\pi'_{BB}(0) = \pi'_+(0) = 1$.

By taking for simplicity $L_0 \rightarrow L$, and by expanding for small $L_0 \rightarrow 0$, one has [45–50]

$$\begin{aligned} \frac{\partial_z v_V}{v_V}(q^2, L_0) &= q^2 L_0 \left(\frac{\pi}{2} \frac{Y_0(qL_1)}{J_0(qL_1)} - \left(\gamma_E + \ln \frac{qL_0}{2} \right) \right), \\ \frac{\partial_z v_Z}{v_Z}(q^2, L_0) &= L_0 \left\{ \mu_Z^2 - q^2 \left[\gamma_E + \ln(\mu_Z L_0) + \frac{1}{2} \psi \left(-\frac{q^2}{4\mu_Z^2} \right) - \frac{c_2}{2c_1} \Gamma \left(-\frac{q^2}{4\mu_Z^2} \right) \right] \right\}, \end{aligned} \quad (4.14)$$

where, having imposed Neumann boundary conditions in the IR,

$$\begin{aligned} c_1 &= 2L \left(-1 + \frac{q^2}{4\mu_Z^2}, \mu_Z^2 L_1^2 \right) + L \left(\frac{q^2}{4\mu_Z^2}, -1, \mu_Z^2 L_1^2 \right), \\ c_2 &= -U \left(-\frac{q^2}{4\mu_Z^2}, 0, \mu_Z^2 L_1^2 \right) + \frac{q^2}{2\mu_Z^2} U \left(1 - \frac{q^2}{4\mu_Z^2}, 1, \mu_Z^2 L_1^2 \right), \end{aligned} \quad (4.15)$$

and $\partial_z v_W / v_W = \partial_z v_Z / v_Z (\mu_Z \rightarrow \mu_W)$.

Bounds on oblique precision parameters require that the symmetry-breaking parameter (μ_Z here) be somewhat small compared to the confinement scale (L_1). One can see that this must be the case directly by noticing that

$$\frac{\partial_z v_Z}{v_Z}(0, L_0) = -L_0 \mu_Z^2 \tanh \frac{\mu_Z^2 L_1^2}{2}, \quad (4.16)$$

$\mu_Z^2 \propto g$, and hence one must require $\mu_Z^2 L_1^2 \ll 1$, otherwise the mass of the gauge bosons would be proportional to g , instead of g^2 ! Assuming hence that $\mu_Z^2 L_1^2 \ll 1$, we can expand also in $\mu_Z^2 L_1^2$, and as a result

$$\begin{aligned} \frac{\partial_z v_v}{v_v}(q^2, L_0) &\simeq L_0 \left(q^2 \log \frac{L_1}{L_0} + \mathcal{O}(q^4) \right), \\ \frac{\partial_z v_Z}{v_Z}(q^2, L_0) &\simeq L_0 \left(-\frac{\mu_Z^4 L_1^2}{2} - \frac{3}{16} \mu_Z^4 L_1^4 q^2 + q^2 \log \frac{L_1}{L_0} + \mathcal{O}(q^4) \right). \end{aligned} \quad (4.17)$$

The localized counter-term

$$D = L_0 \left(\ln \frac{L_0}{L_1} + \frac{1}{\varepsilon^2} \right) \quad (4.18)$$

cancels the logarithmic divergences, and choosing a universal normalization $\mathcal{N}^2 = \varepsilon^2 / L_0$ all the dependence on L_0 disappears (at leading order in L_0), the limit $L_0 \rightarrow 0$ can be taken, and

the model is renormalized, with finite (dimensionless) SM couplings $g_4^{(i)2} = \varepsilon^2 g^{(i)2}/L$. The resulting vacuum polarizations (truncating at the order needed to extract \hat{S}) are

$$\begin{aligned}
\pi_+ &= q^2 + \varepsilon^2 g^2 \kappa \left(-\frac{1}{2L_1^2} - \frac{3}{16} q^2 \right), \\
\pi_{BB} &= q^2 + \varepsilon^2 g'^2 \kappa \left(-\frac{1}{2L_1^2} - \frac{3}{16} q^2 \right), \\
\pi_{WB} &= -\varepsilon^2 g g' \kappa \left(-\frac{1}{2L_1^2} - \frac{3}{16} q^2 \right), \\
\pi_{WW} &= q^2 + \varepsilon^2 g^2 \kappa \left(-\frac{1}{2L_1^2} - \frac{3}{16} q^2 \right),
\end{aligned} \tag{4.19}$$

where for notational simplicity the symmetry-breaking parameter $\kappa = \mu_W^4 L_1^4 / g^2$ has been introduced, and only terms at the leading order in κ have been retained. Notice that the normalization of the π' terms are not identically 1, there are small corrections proportional to κ . One should redefine (in a $SU(2) \times U(1)$ invariant way) the normalization of the gauge bosons in such a way to eliminate this. But because κ is very small anyhow, this process just makes the algebra heavier, without changing the final results, and hence I am not going to do so.

From these expressions, one reads that, at the leading order in κ ,

$$M_W^2 \simeq \frac{1}{2} \varepsilon^2 g^2 \kappa \frac{1}{L_1^2}, \tag{4.20}$$

and hence that

$$\hat{S} = \frac{3}{16} \varepsilon^2 g^2 \kappa \simeq \frac{3}{8} M_W^2 L_1^2. \tag{4.21}$$

Notice that because $M_p \propto L_1^{-1}$, the expression for \hat{S} agrees with usual expectations yielding $\hat{S} \propto M_W^2 / M_p^2$. In order to compute \hat{T} one has to carefully include subleading corrections (in κ) in the symmetry-breaking parameters. In particular, the fact derived here that $\pi_+ = \pi_{WW}$ is not an exact result, but holds only at the leading order in the expansion in κ . The approximations made are precise enough only for \hat{S} . A complete treatment of all precision parameters can be found elsewhere [45–50], and goes beyond our present scope.

We can take as indicative of the experimentally allowed range (at the 3σ level), $\hat{S}_{\text{exp}} = (-0.9 \pm 3.9) \times 10^{-3}$ from [26]. These bounds are extrapolated to the case of a Higgs boson with mass of 800 GeV. From the approximate expression for \hat{S} comes the limit on the confinement scale L_1 of the model,

$$L_1^2 \lesssim \frac{1}{(900 \text{ GeV})^2}. \tag{4.22}$$

4.1.3. LHC Phenomenology

In order to talk about phenomenology, one has to talk about the spectrum of spin-1 states first. Because the precision parameters can agree with the data only provided $\kappa \ll 1$, while the SM gauge couplings are $g_4^2 = \varepsilon^2 g^2 / L \sim \mathcal{O}(1)$, this means that the splitting between vector and axial-vector states is going to be very suppressed. Hence, it suffices to consider the spectrum of the excited states of the photon, by reading it from

$$\begin{aligned} \pi_V &= \mathcal{N}^2 \left(Dq^2 + \frac{\partial_z v_V}{v_V} (q^2, L_0) \right) \\ &\simeq q^2 \left[1 + \varepsilon \left(\frac{\pi}{2} \frac{Y_0(qL_1)}{J_0(qL_1)} - \gamma_E - \ln \frac{qL_1}{2} \right) \right], \end{aligned} \quad (4.23)$$

and this will give a good approximation for the spectrum also of the other states (the excitations of the Z and W mesons).

One might be tempted to think that the spectrum of heavy states be given by the zeros of the Bessel $J_0(qL_1)$. This is a good approximation for the zeros of $\pi_V(q^2)$ when $\varepsilon \ll 1$, otherwise it is not. Unfortunately, one cannot make the assumption $\varepsilon \ll 1$. For instance, one sees that (at large- q) $\pi_V \sim \varepsilon q^2 \ln q$ which, if matched brutally to the OPE of large- N_c QCD-like theory, suggests that $\varepsilon \sim N_c / (12\pi^2)$. Most importantly, the large- N_c regime is entered when $\varepsilon \sim \mathcal{O}(1)$ or larger, as seen by the fact that $g_4 = \varepsilon g / \sqrt{L} \sim \varepsilon g_\rho$. Indeed, because $g_4 \sim \mathcal{O}(1)$, and because we are using the tree-level (supergravity) approximation, at most one can have $g_\rho \sim \mathcal{O}(1) \propto 1 / \sqrt{N_c}$.

Finding the actual spectrum requires some numerical work. The result of which is that $M_\rho L_1 \sim 2.4 - 5.5$, where the upper part of the interval is obtained for large values of ε . Hence the bound on M_ρ turns out to be quite stringent, and depending on how small choices of ε we allow for, at best $M_\rho > 2.2$ TeV, possibly another factor of 2 higher for large values of ε . This is a problem. These masses are hard to discover at the LHC. In particular, the second set of resonances will be roughly $M_{\rho'} \simeq M_\rho + \pi / L_1$, which for $L_1^{-1} \sim 1$ TeV means a 3 TeV gap. This almost completely excludes the possibility that LHC may observe several copies, with heavier masses, of this resonance, which would be the unmistakable signature of a strongly coupled model (equivalently, these would be the Kaluza-Klein excitation of a compact extra-dimensions).

There are hence two important open problems: under what circumstances can one see the techni-rho mesons at the LHC? If the techni-rho is discovered, how can one show that it is related to a strongly coupled sector? There are many studies of this type of LHC searches, which depend very strongly on the precise details of the model (in particular, enhancement of the production cross-section is possible if the top quark is playing a role in the new strong dynamics). It goes far beyond the scope of these lectures to give a full account on the topic. I want here just to signal two interesting facts that might happen if the model is actually well described by the large- N_c expectations.

Using the result highlighted earlier on $\varepsilon = g_4 \sqrt{L} / g \simeq N_c / (12\pi^2)$, the comments we made in discussing the phenomenology of techni-rho mesons show that for values of $\varepsilon \gtrsim \mathcal{O}(1)$, the most important coupling of the techni-rho is to the SM fermions, not to the pions. This can be checked numerically to hold by explicitly evaluating the partial width $\Gamma[\rho \rightarrow WW]$ from the five-dimensional set-up [45–50]. We also stressed that only for $\varepsilon \gtrsim \mathcal{O}(1)$ the

calculations performed up to now are trustable. Hence, in the regime of validity of what we did in the previous subsection, the dominant decay mode of the techni-rho is to SM fermions, not to WW . At least as long as we want to trust the calculation of the precision parameter \hat{S} .

Notice that all of the above agrees with the large- N result that the resonances should become narrow in the limit in which N_c is very large, and if the electroweak couplings are neglected the techni-rho becomes effectively stable. In order to study the LHC phenomenology one can use the result on the vacuum polarizations and look at the LHC process $pp \rightarrow \rho \rightarrow f\bar{f}$. In respect to the decay into WW final states, this process is much cleaner at the LHC, because if one focuses attention of the final states involving electrons and muons, this can be reconstructed very precisely, and there is no missing energy.

The very simplest signature of the model is hence the Drell-Yan production of the heavy partners of the electroweak gauge bosons. This should be detectable as a narrow resonance in the few-TeV region of the invariant mass distribution of final states with two fast leptons (muons or electrons). At the partonic level, the dominant tree-level contribution can be very easily computed by simply replacing the complete propagators (inverse of the $\pi(q^2)$ computed earlier) in place of the propagators of the SM gauge bosons. With one caveat: one needs to modify the propagators in order to include the width. Provided one is in the regime in which the decay to SM fermions dominates such width, and the width itself is small, one can pragmatically implement the width by replacing in the Feynman diagrams

$$P_Y(q^2) = \frac{1}{q^2 + i\epsilon} \rightarrow (q^2) = \frac{1}{\pi_V(q^2) + i(8\alpha/3q^2) + i\epsilon}, \quad (4.24)$$

and analog for the Z and W . Notice that this very simple procedure is acceptably accurate only because we assumed that the SM-fermion final state dominates, and that the fermions are localized at the UV boundary. This result is what follows from taking the imaginary part of diagrams in which the five-dimensional photon receives corrections from 1-loop diagrams localized on the four-dimensional boundary. The procedure for including the width due to strongly coupled decays is way more complicated, because not only all the gauge bosons propagate in the bulk of the five-dimensions, so that the effective cubic couplings depend on complicated integrals convoluting the bulk wave functions, but also heavier resonance can decay into combinations of lower modes.

As long as the weak-coupling calculations are acceptably accurate, the partonic cross-section of interest is

$$\hat{\sigma}_{q\bar{q}}(\hat{s}) \equiv \sigma(q\bar{q} \rightarrow \ell^+ \ell^-) = \frac{\hat{s}}{48\pi} e^2 \sum_{A,B} |G_{AB}(\hat{s})|^2, \quad (4.25)$$

where $A, B = L, R$, $\hat{s} = q^2$ is the partonic momentum,

$$\begin{aligned} G_{AB}(\hat{s}) &= Q^{(q)} P_Y(\hat{s}) + \frac{4}{\sin^2 2\theta_W} g_A^{(q)} g_B^{(\ell)} P_Z(\hat{s}), \\ g_L^{(f)} &= T^{3(f)} - Q^{(f)} \sin^2 \theta_W, \\ g_R^{(f)} &= -Q^{(f)} \sin^2 \theta_W, \end{aligned} \quad (4.26)$$

and $T^{3(f)}$ and $Q^{(f)}$ are the isospin and electric charge of the fermion f .

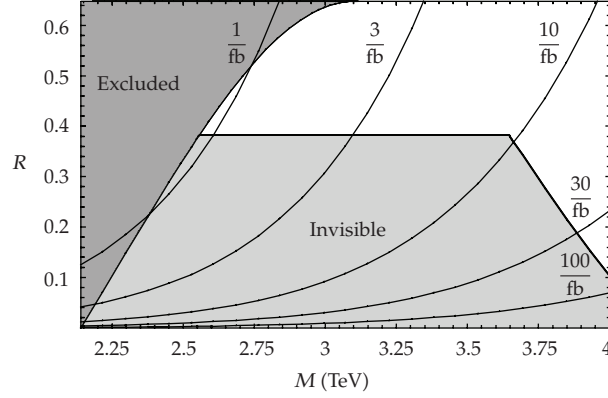


Figure 6: LHC exclusion/discovery reach from $pp \rightarrow \mu^+\mu^- + X$ as a function of $M_\rho = M \simeq M_{\gamma'} \simeq M_{W'} \simeq M_{Z'}$ and of R defined in the text. The curves are obtained by requiring 10 events, for integrated luminosity of 1, 3, 10, 30, 100 fb^{-1} . The darkest region is excluded by indirect limits from $\hat{S} < 3 \times 10^{-3}$. The light-grey-shaded region is allowed by precision data only for $\varepsilon < 0.5$, so that the dominant decay mode of the techni-rho is into longitudinally polarized SM gauge bosons (and hence the signal in SM fermions strongly attenuated, or *invisible*) and the large- N approximations used in the analysis performed here do not hold.

Finally, the differential cross-section for the LHC process as a function of s and \hat{s} is

$$\frac{d\sigma(pp \rightarrow \ell^+\ell^- + X)}{d\hat{s}}(s, \hat{s}) = \frac{1}{3} \sum_q \hat{\sigma}_{q\bar{q}}(\hat{s}) \frac{1}{s} \int_{\hat{s}/s}^1 \frac{d\eta}{\eta} \left[\phi_q(\eta) \phi_{\bar{q}}\left(\frac{\hat{s}}{\eta s}\right) + \phi_{\bar{q}}(\eta) \phi_q\left(\frac{\hat{s}}{\eta s}\right) \right], \quad (4.27)$$

where the functions $\phi_a(\eta)$ are the parton distribution functions yielding the probability of finding a parton of species a with momentum fraction η within the proton, $\sqrt{s} = 14$ TeV is the center of mass energy of the pp collision, and the factor of $1/3$ takes into account the average over color in the partonic initial state.

One can then do the exercise of using the results at the previous section to select the input parameters such as L_1 , and of computing the number of events expected at the LHC as a function of the machine parameters (luminosity and energy) and of the relevant parameters in the model, namely, the mass of the techni-rho M_ρ and its coupling to the SM fermions relative to the coupling of the SM gauge bosons R (which generalizes the factor $e^2 f_\rho^2 / M_\rho^2$ in (3.72)), which is a complicated function of ε .

The results are plotted in Figure 6. A few words of comment are presented. The dark shaded region is excluded by the bound on \hat{S} . Notice that the actual exclusion is a function not only of the mass of the techni-rho mesons, but also of the coupling R , because the latter depends on ε , and we have seen that there is a non-trivial functional dependence between M_ρ , L_1 , and ε . The region at small R is the region in which neglecting the contribution of $\rho \rightarrow WW$ to the width and decay rates of the techni-rho is not justified. In this region, this signal is actually very suppressed in comparison to the final state with two gauge-bosons.

Provided $\varepsilon \sim \mathcal{O}(1)$, the LHC phenomenology is very similar to what one would have in a weakly coupled extension of the SM with an extra $SU(2) \times U(1)$. The four heavy gauge bosons would be almost degenerate in mass. So much so that it would be not possible to distinguish in the invariant-mass distribution of di-lepton final states the peaks due to the

two neutral states. In order to see that the peak is due to more than one resonance, much more sophisticated data analysis would be needed, for example, looking at the forward-backward asymmetry [156].

4.2. Final Remarks

The results obtained for this simple model depend on the specific set-up, and many variations exist of the basic ideas. Before commenting on some very interesting possibilities, it is important to stress that the way in which the bounds from electroweak precision is accommodated relies on the fact that f_ρ and f_π have two different dynamical origins. To be more specific, the former is controlled by the IR boundary L_1 , the latter by the value of the condensate v_0 , which in this model are treated as independent. Both scale as $f_\pi^2 \propto f_\rho^2 \propto N$, but the smallness of the \hat{S} parameter is obtained by assuming the ratio of the two being a somewhat small numerical constant. For this reason, one is allowed to take the large- N limit, and hence treat the spin-1 states as weakly coupled, while at the same time making their masses large so that precision physics bounds do not exclude the model.

There is a number of possible problems arising with this approach. First, in a fully dynamical model, f_π and f_ρ will be related, and there is no known simple example in which their ratio be parametrically small. Second (to be taken with all the caveats we discussed earlier on), if the mass of the techni-rho mesons is in the few-TeV range, it is not possible to show explicitly that the elastic scattering amplitude can be treated perturbatively and stays below the unitarity bounds all the way up to this large mass scale. As such, at least some part of the theory must be strongly coupled, which raises questions about the tree-level calculations being done. But remember how we stressed that the WW scattering amplitude is a particularly bad observable, much more sensitive to the details of the strongly coupled part of the symmetry-breaking sector than the oblique parameters.

Finally, if one takes literally the relation $\varepsilon \sim N_c / (12\pi^2)$, values of $\varepsilon \sim \mathcal{O}(1)$ require very large values of N_c . This means that a large number of fermions in the dual sector is coupled to the electroweak gauge bosons, making the $SU(2)$ gauge coupling blow-up at relatively small scales, not far above the scale set by L_1 . This problem is indirectly related to the perturbative-unitarity bound, and also to the fact that if one looks at the $\mathcal{O}(q^4)$ precision parameter W and Y one finds that they grow with ε . Yet, this model is simple, calculations are doable, the results can be reconciled with precision electroweak tests, and the model is testable at the LHC, which is ultimately the most important thing. If such a signature shows in the data, there will be plenty of time to go back to this model and refine its study, addressing the objections raised here.

Infinite number of variations of this model are possible. Here is a brief and nonexhaustive list of things that can be done (and have been done).

- (i) Custodial symmetry can be implemented, by gauging a complete $SU(2) \times SU(2)$ in the bulk, and then arranging for its breaking via some mechanism localized in the UV, in such a way as to recover a spectrum in which only four gauge bosons are light, but in which \hat{T} is small.
- (ii) The bulk profile of the condensate can be changed. The extreme case is the higgless extra-dimension set-up, in which the field Φ is exactly localized on the IR boundary, effectively modifying the IR boundary conditions [157, 158].

- (iii) More condensates can be present. In particular one may envision the case in which the bulk scalars affect also the dynamics of the vectorial five-dimensional fields, not only the axial-vectorial fields. Also, phenomenologically more realistic models of confinement can be obtained, by introducing modifications of the background AdS metric in the deep IR [51–55, 159].
- (iv) A larger symmetry can be gauged in the bulk, hence allowing the low energy spectrum to contain also pseudo-Goldstone bosons. An extreme variation of this model is dual to composite Higgs models [70–77].
- (v) The fermions can be allowed to propagate in the bulk. In particular, the third-family, such as the top. In this way it is possible to enhance the large mass of the top [95–101].

This huge freedom shows one of the limitations of these models, because they are not fully dynamical, the description they provide has a limited predictive power. However, for many practical purposes they are very effective; they allow to relate in sensible and predictive ways the results from precision studies of the electroweak gauge bosons to the spectrum, couplings and LHC phenomenology of the new particles they predict (new vector states, new axial-vector states, new pseudo-Goldstone bosons, ...). These models are hence very useful, in the dawn of the LHC era, because they give us a plethora of new signatures to look for, within calculable frameworks in which the comparison to the data can be made quantitative, cross-relations between observables exist, and favorable regions of parameter space can be identified and tested experimentally.

5. Holographic Technicolor: Top-Down Approach

5.1. Why a Full String Theory Model?

We have seen that, because of the strong coupling, four-dimensional techniques are limited when applied to technicolor models. We have also seen that five-dimensional models provide a substantial improvement over hidden local symmetry, incorporating some of its ideas and features, but reducing drastically the number of parameters needed to describe the spectrum of composite states, and hence allowing to compute the precision parameters and relate them to measurable quantities, such as masses and decay constants of spin-1 resonances. With the LHC approaching, this framework provides a wonderful opportunity to study the phenomenology of the lightest resonances of a strongly coupled model, relating their masses and couplings to the precision measurements performed in the last two decades. However, the bottom-up approach has itself a limited power; it is based on assuming that the answer to many of the questions highlighted in Section 3.5 is known a priori, and then uses these answers in order to build the five-dimensional action.

To be more specific, let us look back at each of the questions and see what can be said from the bottom-up approach.

- (i) The bottom-up approach describes only bound states, and their low-energy properties, hence it has only indirect information about the fundamental theory (its gauge group and field content are not known).
- (ii) The dynamical scales are put in by hand, choosing the position of the boundaries, the form of the metric, and the presence of condensates.

- (iii) The anomalous dimensions are chosen by hand, they are completely free parameters, bounded only by very general considerations.
- (iv) The precision parameter can be computed. However, they still depend on how the light SM fermions are coupled to the strong sector, and a certain amount of freedom is left.
- (v) Computing the coefficients of the chiral Lagrangian is possible, as a function of what said in (ii). The only limitation being that one is using large- N expansions.
- (vi,vii,viii) This approach has little to say about ETC, the fermion mass hierarchies are controlled by free parameters, not by the underlying dynamics.
- (ix) One can derive relations between the spectrum and couplings of spin-1 states, as a function of the choices in points (ii) and (iii).
- (x) The global symmetries are chosen by hand, and their breaking pattern too, there is no real sense in which symmetry-breaking arises dynamically within this framework.
- (xi) Determining under what conditions a light dilaton is part of the spectrum requires a dynamical study of the underlying theory, with fully back-reacted background geometry.
- (xii) Perturbative unitarity is in part connected with sum rules on the couplings of spin-1 fields, yet the dominant contributions from the nondecoupled scalar sector is missing and uncalculable.

Ultimately, the bottom-up approach yields a very major improvement over hidden local symmetry, and is a very useful and practical way of constructing models that are testable at the LHC. But it cannot answer to those most fundamental questions regarding dynamical scales, spectrum, and symmetries that are intrinsically dependent on the strong dynamics. For this reason, it is important to try and study the extra-dimension dual of the full theory, in a context in which the background and all the condensates can be computed from a fundamental action. This is the topic of this last section.

5.1.1. From $\mathcal{N} = 4$ towards Walking Technicolor

The most celebrated example of gauge/gravity duality is the conjectured correspondence between four-dimensional, superconformal $\mathcal{N} = 4$ Yang-Mills theory with $SU(N_c)$ gauge group and Type IIB string theory on a background with $AdS_5 \times S^5$ geometry [1–4]. The $SO(4,2)$ isometry group of the AdS space is related to the four-dimensional conformal group, while the $SO(6)$ symmetry of the internal S^5 space is related to the $SU(4)_R$ symmetry of $\mathcal{N} = 4$. The value of N_c is related to the flux of the RR F_5 form of type-IIB. One important reason why this relation is very useful is that taking the large- N_c limit, at fixed (but large!) 't Hooft coupling $\lambda = g^2 N_c$ one enters the regime in which the dual description reduces to a weakly coupled supergravity theory in 10 dimensions. Hence, very non-trivial calculations involving strong dynamics in the gauge theory can be rephrased in terms of a more accessible weakly coupled gravitational system.

For phenomenological purposes, the $AdS_5 \times S^5$ background has way too much symmetry; realistic models of nature are neither conformal, nor do they have so much supersymmetry. In the last ten years, the same idea of looking for weakly coupled duals to strongly coupled four-dimensional theories has been pushed in the direction of looking

for sensible supergravity backgrounds which depart from the AdS geometry, and in which a lesser degree of supersymmetry is encoded in a less symmetric structure of the internal five-dimensional manifold. Very interesting models exist, proposing, for instance, backgrounds with $\mathcal{N} = 1$ supersymmetry (such as in [160, 161]), or no supersymmetry at all [162–165], in which the geometry is far away from AdS, and describes the dual of a theory in which all the couplings run. There also exist 10-dimensional models where the geometry is well approximated by an AdS space in the far UV and by a different AdS geometry deep in the IR (at large and small values of the radial coordinate, resp.). These models provide a description of a four-dimensional theory which flows from a UV fixed point to an IR fixed point [166, 167].

Many very interesting aspects of strongly coupled field theories can be studied using the techniques of gauge/gravity dualities. The physics near fixed points (in particular, the calculation of anomalous dimensions) can be studied quantitatively. Correlation functions can be computed. Certain RG flows can be studied in details. New and effective way of describing confinement and the formation of symmetry-breaking condensates exist. A huge amount of formal work has been done in order to put all of these ideas on firm grounds, by producing rigorous prescriptions for performing the calculation of physical quantities of interest in the dual field theory starting from the supergravity (or superstring) framework (e.g., holographic renormalization is a precise prescription for computing correlation functions of local operators [154, 155], while the use of probe-strings provides a tool for the calculation of the Wilson loops, that I will discuss in some detail later).

The reader will immediately realize that many of these observations are (in subtle ways) related to the very large body of very hard field theory questions that emerged in the context of dynamical electroweak symmetry breaking in Sections 2 and 3 of these lecture notes. Hence, it is very natural to try and reformulate some of those questions in terms of dual theories, and look for the answers in this (weakly coupled) context.

In a perfect world, one might even hope to rewrite the whole theory of WTC/ETC in terms of its dual 10-dimensional description. This putative theory should be asymptotically AdS, hence ensuring the UV-completeness of the symmetry-breaking sector of the SM, which would approach a fixed point (not necessarily weakly coupled) in the UV. Relevant deformations would cause the theory to flow away from the UV fixed point, and undergo a set of symmetry-breaking transitions (tumbling), effectively reducing the gauge group of the dual ETC theory down to TC, and in the process producing all the four-fermion operators needed to generate the SM fermion masses. (Something vaguely reminiscent of this scenario exists in the supergravity context [161, 168], although the superficial similarities between tumbling ETC and the cascade are to very large extent overshadowed by the important differences.) At the end of this tumbling process, one should find that the supergravity background has a geometry which is approximately AdS, providing a dual description of walking, and could compute all the anomalous dimensions, and the running of the aforementioned higher-order operators, hopefully showing that enhancement factors are generated along the lines of what we discussed in Section 3. This geometry should also have a global symmetry containing the SM $SU(2)_L \times U(1)_Y$ gauge group. After walking for a while, the dual supergravity geometry should substantially deviate from AdS, to signify the fact that the IR fixed point that the theory was approaching is only approximate. At low energies a set of condensates should form, signaled by the breaking of the internal global symmetry, which corresponds to EWSB in the dual theory. Finally, there should be an end of space in the radial direction in the IR, and the properties of the geometry near this end of space should be healthy enough to allow for a sensible interpretation of it in terms of confinement. The weak

gauging of the $SU(2)_L \times U(1)_Y$ subgroup would produce the massive gauge bosons W and Z , and the presence of higher-order operators enhanced by walking produce the SM fermion masses, but not unacceptably large new sources of FCNC transitions. In principle, given such a complete theory, one would be able to compute all possible observables of relevance for phenomenology; spectra, couplings, scattering amplitudes, production and decay rates of all the possible composite objects in the theory, and their contribution to precision electroweak parameters and FCNC transitions.

Of course, the real world is far from perfect: such a construction does not exist (yet), and much of what I wrote in the previous paragraph is wishful thinking, with limited scientific support. But encouraging steps in this direction have been taken [56–58, 169]. In this final section, I spend some time working out in some details a few examples of these steps. The discussion is far from systematic and pedagogical. The reader is assumed to be familiar with the basic ideas of gauge/gravity dualities (if you are not, a good place to start is the studies in [170, 171]), since part of the discussion will be rather technical, and limited to one very specific system, that exemplifies what can be done with these powerful techniques.

Before beginning along these lines, a clarifying comment is necessary, which anticipates some of the ideas to come. The models I am going to discuss are not, strictly speaking, technicolor models, because they do not implement electroweak symmetry breaking. Yet, these models are used here as an illustration of what walking technicolor might yield, because they are interpreted as the duals to four-dimensional theories that exhibit some of the important features at the basis of walking technicolor. The notion of walking itself is a field-theory property that not necessarily requires coupling to the electroweak sector. Confinement, and the formation of symmetry-breaking condensates, are well-defined concepts even outside the context of electroweak interactions. Many of the questions we are interested in, when talking about strongly coupled EWSB, can be formulated in field-theory terms that do not necessarily require EWSB itself. For these reasons, the exercises we are going to do are going to teach us some very important lessons about walking technicolor, even if none of the models illustrated is the actual dual of a technicolor model.

5.2. Wrapped-D5 System

The set-up we focus on is based on the geometry produced by stacking on top of each other N_c $D5$ -branes that wrap an S^2 inside a CY3-fold and then taking the strongly coupled limit of the gauge theory on this stack, in the (type-IIB) supergravity approximation. We start by recalling the basic definitions that yield the general class of backgrounds obtained from the $D5$ system, which includes the study in [160] as a very special case. Truncating type-IIB supergravity to include only gravity, dilaton Φ and RR 3-form F_3 the 10-dimensional action is

$$S_{\text{IIB}} = \frac{1}{G_{10}} \int d^{10}x \sqrt{-g} \left[R - \frac{1}{2} (\partial\Phi)^2 - \frac{e^\Phi}{12} F_3^2 \right], \quad (5.1)$$

Defining the $SU(2)$ left-invariant one-forms as

$$\tilde{\omega}_1 = \cos \psi d\tilde{\theta} + \sin \psi \sin \tilde{\theta} d\tilde{\phi}, \quad \tilde{\omega}_2 = -\sin \psi d\tilde{\theta} + \cos \psi \sin \tilde{\theta} d\tilde{\phi}, \quad \tilde{\omega}_3 = d\psi + \cos \tilde{\theta} d\tilde{\phi}, \quad (5.2)$$

and using an ansatz which assumes the functions appearing in the background depend only the radial coordinate ρ (but not on the Minkowski coordinates x^μ nor the 5 angles $\theta, \tilde{\theta}, \phi, \tilde{\phi}, \psi$ of the compact internal manifold) allows to write the background (in string frame) as

$$\begin{aligned}
 ds^2 = \alpha' g_s e^{\Phi(\rho)} & \left[\frac{dx_{1,3}^2}{\alpha' g_s} + e^{2k(\rho)} d\rho^2 + e^{2h(\rho)} (d\theta^2 + \sin^2\theta d\phi^2) \right. \\
 & \left. + \frac{e^{2g(\rho)}}{4} \left((\tilde{\omega}_1 + a(\rho) d\theta)^2 + (\tilde{\omega}_2 - a(\rho) \sin\theta d\phi)^2 \right) + \frac{e^{2k(\rho)}}{4} (\tilde{\omega}_3 + \cos\theta d\phi)^2 \right], \\
 F_3 = \frac{N_c}{4} & \left[-(\tilde{\omega}_1 + b(\rho) d\theta) \wedge (\tilde{\omega}_2 - b(\rho) \sin\theta d\phi) \wedge (\tilde{\omega}_3 + \cos\theta d\phi) \right. \\
 & \left. + b' d\rho \wedge (-d\theta \wedge \tilde{\omega}_1 + \sin\theta d\phi \wedge \tilde{\omega}_2) + (1 - b(\rho)^2) \sin\theta d\theta \wedge d\phi \wedge \tilde{\omega}_3 \right].
 \end{aligned} \tag{5.3}$$

The full background is then determined by solving the equations of motion for the functions (a, b, Φ, g, h, k) .

The system of BPS equations derived using this ansatz can be rearranged in a convenient form, by rewriting the functions of the background in terms of a set of functions $P(\rho), Q(\rho), Y(\rho), \tau(\rho)$, and σ as [172–174]

$$4e^{2h} = \frac{P^2 - Q^2}{P \cosh \tau - Q}, \quad e^{2g} = P \cosh \tau - Q, \quad e^{2k} = 4Y, \quad a = \frac{P \sinh \tau}{P \cosh \tau - Q}, \quad N_c b = \sigma. \tag{5.4}$$

Using these new variables, one can manipulate the BPS equations to obtain a single decoupled second-order equation for $P(\rho)$, while all other functions are obtained from $P(\rho)$ as follows:

$$\begin{aligned}
 Q(\rho) &= (Q_0 + N_c) \cosh \tau + N_c (2\rho \cosh \tau - 1), \\
 \sinh \tau(\rho) &= \frac{1}{\sinh(2\rho - 2\hat{\rho}_0)}, \\
 \cosh \tau(\rho) &= \coth(2\rho - 2\hat{\rho}_0), \\
 Y(\rho) &= \frac{P'}{8}, \\
 e^{4\Phi} &= \frac{e^{4\Phi_0} \cosh(2\hat{\rho}_0)^2}{(P^2 - Q^2) Y \sinh^2 \tau}, \\
 \sigma &= \tanh \tau (Q + N_c) = \frac{(2N_c \rho + Q_0 + N_c)}{\sinh(2\rho - 2\hat{\rho}_0)}.
 \end{aligned} \tag{5.5}$$

The second-order equation mentioned above reads

$$P'' + P' \left(\frac{P' + Q'}{P - Q} + \frac{P' - Q'}{P + Q} - 4 \coth(2\rho - 2\hat{\rho}_0) \right) = 0. \quad (5.6)$$

In the following, I will fix the integration constant $Q_0 = -N_c$, so that no singularity appears in the function $Q(\rho)$. When convenient, I will also choose $\hat{\rho}_0 = 0$, together with $\alpha' g_s = 1$, in order to simplify the notation.

5.2.1. Gauge Coupling

As we saw, one clear signal of a walking theory is the existence of an energy range over which the β -function is anomalously small, and hence the gauge coupling does not run. One needs hence to relate the four dimensional gauge coupling of the dual theory to quantities that are well defined in the 10-dimensional geometry. I will keep the factors of α' and g_s explicit, but set $\hat{\rho}_0 = 0$ in this subsection.

The six-dimensional theory on the D5 branes has a 't Hooft coupling given by the dimensionful $\lambda_6 = g_s \alpha' N_c$, and the supergravity limit is taken by keeping this fixed [175]. The branes wrap a small two-cycle Σ_2 , so that at low energies an effectively four-dimensional theory emerges. A natural way of defining its (dimensionless) gauge coupling is given by combining

$$g_4^2 \equiv \frac{g_6^2}{\text{Vol}\Sigma_2}. \quad (5.7)$$

Following [176–178], that consider a five brane (in probe approximation) extended along the Minkowski directions and the two-cycle defined by

$$\Sigma_2 = [\theta = \tilde{\theta}, \phi = 2\pi - \tilde{\phi}, \psi = \pi], \quad (5.8)$$

one arrives to

$$\frac{8\pi^2}{g_4^2} = 2 \left[e^{2h} + \frac{e^{2g}}{4} (a-1)^2 \right] = P e^{-\tau} = \frac{P}{\coth \rho}. \quad (5.9)$$

With this result, together with a particular radius-energy relation, it was shown in [176–178] that one particular solution \hat{P} of (5.6) leads to reproducing the NSVZ beta function. I will sketch in part this result later on.

Before proceeding, four final comments are due, a technical one first. The five-branes on the submanifold $R^{1,3} \times \Sigma_2$ preserve supersymmetry only if the probe brane is at infinite radial distance from the end of the space (i.e., when $\rho \rightarrow \infty$) [179]. This result is valid for the particular solution considered there and does not necessarily extend to the other backgrounds, because a five brane in the configuration described above does not preserve the same spinors as the background itself.

More about the physics. One has to keep in mind that the expansion yielding the supergravity approximation involves the six-dimensional coupling g_6 , and not the four-dimensional coupling defined here. When g_4 becomes small (and it does in the far UV at $\rho \rightarrow +\infty$, as we will see), this does not necessarily mean that the supergravity approximation is breaking down, nor that perturbation theory in the dual gauge theory is useful. At large- ρ , in a putative calculation done in terms of four-dimensional degrees of freedom, one should include a large number of excited modes living on the Σ_2 , and this will compensate for the small coupling of each individual one, ensuring that the four-dimensional dual system is indeed strongly coupled.

There will also be divergences at finite values of ρ . In particular at $\rho = \hat{\rho}_0$. This is in general problematic, because the singularity might mean that unphysical results appear in various amplitudes, and ultimately the supergravity approximation breaks down, spoiling predictivity. However, I will concentrate on backgrounds where the only singularity is at $\rho \rightarrow \hat{\rho}_0$, and for which the Ricci curvature R be finite everywhere. While per se this does not ensure that the background defines a fully sensible theory, a pragmatic way of testing if this is the case is to compute actual physical quantities, and to check how they behave. In particular, we will see that the IR-divergence is associated with confinement, via the study of Wilson loops and the related quark-antiquark potential.

A final, more general comment is given. Given a walking theory, its coupling must be approximately constant over some energy range. However, away from an exact fixed point, the very definition of gauge coupling is ambiguous, and affected by scheme-dependence. One might worry that the presence of a plateau in the gauge coupling defined here, especially in considerations of all the caveats that go into its derivation, might be just a scheme-dependent artifact. These arguments apply to any possible definition of gauge coupling in any theory away from the fixed points, and ultimately derive from the fact that in a generic field theory the gauge coupling per se is not a very well-defined observable quantity. The fact that there are solutions in which the gauge coupling flattens at finite values has to be understood as a first indication of the fact that something very peculiar is happening to the theory. In particular, it indicates the existence of two possibly distinct dynamical scales at which the behavior of the theory changes drastically. In order to understand how physical this is, one has to use the background to compute actual physical quantities, (such as scattering amplitudes or masses) and ask if they show an interesting change in behavior related to the scales indicated by the gauge coupling.

5.2.2. Regular Maldacena-Nunez: Asymptotic behaviors

Solving the equation for P is not easy. As apparent, (5.6) is very nonlinear. And potentially plagued by possible nasty divergences arising from the denominators. One needs to use approximate and/or numerical methods in order to learn about the properties of its solutions. A couple of very general results are useful in doing so.

First, it is known that there exist two different classes of UV-asymptotic solutions for P valid at large- ρ [172–174]:

$$\begin{aligned}
 P &\sim 2N_c\rho + \mathcal{O}(e^{-4\rho}) \quad (\text{class I}), \\
 P &\sim \mathcal{O}(e^{4/3\rho}) + \mathcal{O}(e^{-4/3\rho}) + \mathcal{O}(e^{-8/3\rho}) \quad (\text{class II}).
 \end{aligned}
 \tag{5.10}$$

Notice that both diverge with ρ , implying that the gauge coupling g_4 is becoming small, which we already commented about in the previous subsection.

Second, the simplest and better understood solution of (5.6), which is the only non-trivial exact solution known in closed form, is [160]

$$\hat{P} = 2N_c\rho. \quad (5.11)$$

It belongs to class I, and we already mentioned it earlier on. For this solution, the gauge coupling is

$$\hat{\lambda} = \frac{\hat{g}_4^2 N_c}{8\pi^2} = \frac{N_c \coth \rho}{\hat{P}} = \frac{\coth \rho}{2\rho}. \quad (5.12)$$

Identifying

$$\rho = \frac{3}{2} \ln \mu, \quad (5.13)$$

with μ the renormalization scale in units of a reference scale, and expanding at large ρ yields

$$\hat{\lambda} \simeq \frac{1}{3 \ln \mu}, \quad (5.14)$$

which agrees with (2.19) at asymptotically large μ , provided $b_0 = 3N_c$, and $N_f = 0$ as in Super-Yang-Mills (SYM) theory with $SU(N_c)$ gauge symmetry. This is one simple way to see that this background is related (in a non-trivial way) to the dual to SYM (more precise statement about the dual theory can be found in [180]).

5.2.3. Walking Solutions in Class II

The first class of solutions to (5.6) which yield the running gauge coupling of a walking theory was found in [56], and belongs to class II. This is a 2-parameter family of solutions, the parameters being the integration constants of the second-order differential equation. It has the nice property that it can be obtained starting from an approximate solution, and that all corrections can be in principle computed in a formal power-series, in which the expansion parameter is related to one of the two integration constants. Without loss of generality, I will set $\hat{\rho}_0 = 0$ in the following. One must require that the solution $P(\rho)$ exists and be finite for every $\rho > \hat{\rho}_0$, which imposes some restriction on the integration constants.

Let us see how to construct such solutions. The basic observation is that (5.6) would be exactly solvable if one could set $Q = 0$. It is hence sensible to look for solutions such that $P \gg Q$ for any ρ . Following [172–174], we can write P in a formal expansion in inverse powers of c as [56]

$$P = \sum_{n=0}^{\infty} c^{1-n} P_{1-n}, \quad (5.15)$$

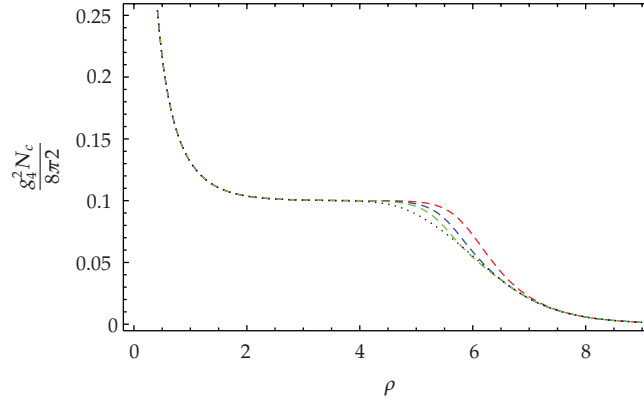


Figure 7: The 't Hooft coupling $g_4^2 N_c / (8\pi^2)$ as a function of ρ for $N_c = 10$, $c = 100$, and $\alpha = 0.0005$ for (iii). The red (long dashes) curve is the $O(c)$ approximation in the expansion of (5.15), the blue (medium dashes) line is the $O(1/c)$ approximation, the green (short dashes) line is the $O(1/c^3)$ approximation, and the black (dotted) line is the numerical solution [56].

where

$$R(\rho) \equiv \left(\cos^3 \alpha + \sin^3 \alpha (\sinh(4\rho) - 4\rho) \right)^{1/3}, \quad (5.16)$$

and where c and α are the two integration constants. Taking c large compared to N_c yields $P \gg Q$, and one obtains the approximate solution

$$P \simeq P_1 = cR(\rho). \quad (5.17)$$

For this to be a well-defined solution, one needs to ensure that $P(\rho) > Q(\rho)$ for all $\rho \geq 0$, since any ρ , where $P = Q$, is a singular point of (5.6). At asymptotically small values of ρ , $Q(\rho) = O(\rho^2)$, while $P(\rho) \approx c \cos \alpha + O(\rho^3)$ and so $P > Q$ is ensured by requiring $\cos \alpha > 0$. For asymptotically large ρ , $Q(\rho) \sim 2N_c \rho$, while $P(\rho) \sim 2^{-1/3} c \sin \alpha e^{4\rho/3}$. So that requiring that $\sin \alpha > 0$ is again sufficient to ensure that $P > Q$ for large ρ . Because P and P' are monotonically increasing functions of ρ , $P \geq c \cos \alpha$ for all ρ . At some special value of $\rho = \rho_*$, $\sinh(4\rho_*) - 4\rho_* \approx \cot^3 \alpha$ and P starts deviating from the constant value $c \cos \alpha$, being approximated by the UV-asymptotic exponential dependence on ρ . In order to allow for a large region of walking behavior, we need to take $\rho_* \gg 1$, in which case

$$\rho_* \approx \frac{1}{4} (\log 2 + 3 \log \cot \alpha) \gg 1. \quad (5.18)$$

In order to ensure that $P > Q$ everywhere, it is hence sufficient to require that $P(\rho_*) > Q(\rho_*)$, which puts an upper bound on $\cot \alpha$, namely,

$$1 \ll \cot \alpha \lesssim \exp\left(\frac{2^{4/3}}{3} \frac{c}{N_c}\right). \quad (5.19)$$

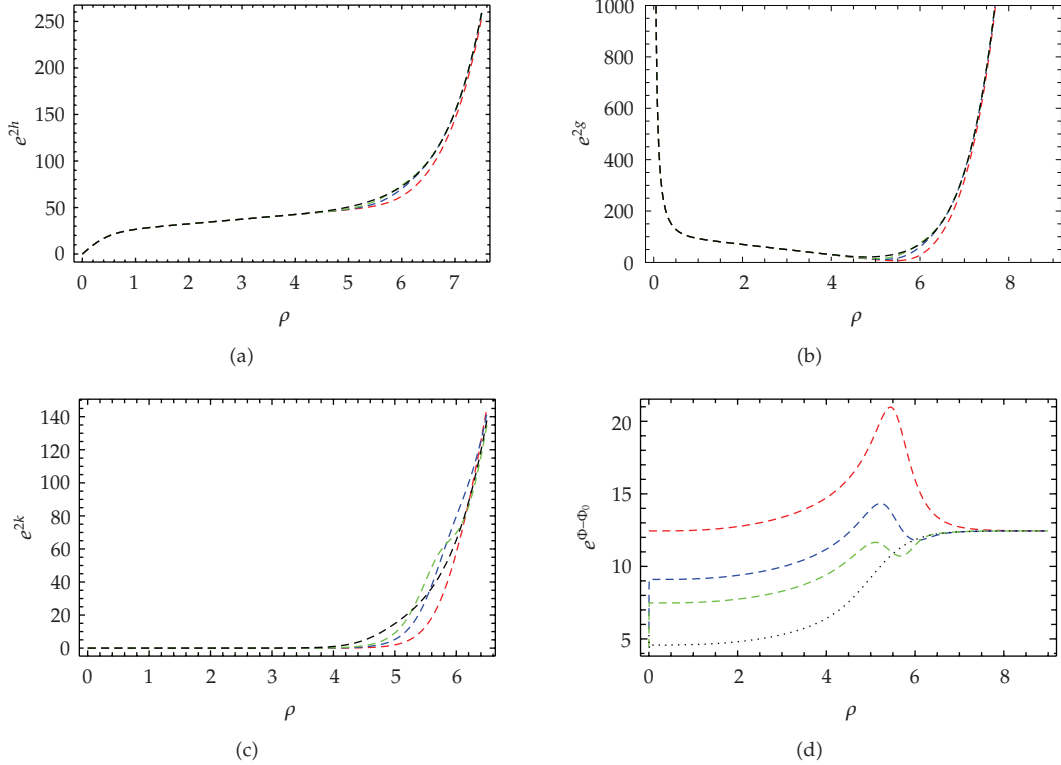


Figure 8: The background functions h , g , k , and Φ as a function of ρ , for the same parameters as in Figure 7, and with the same color-coding. It is clear that the expansion in (5.15) converges sufficiently fast for g and h , but some caution has to be used with Φ and k [56].

In this approximation $P(\rho)$ remains almost constant for $0 \leq \rho \lesssim \rho_*$. The fact that P is almost constant up to very large scales ρ_* produces an intermediate energy region over which the four-dimensional gauge coupling is almost constant, as can be seen from Figure 7. The larger $\cot\alpha$, the wider this region is, with the only limitation provided by the upper bound on the value of $\cot\alpha$, which depends on the ratio c/N_c .

Figure 8 shows the other functions appearing in the background. Notice the divergence of e^{2g} in the IR. Interesting is the behavior of the dilaton; it is practically constant above and below ρ_* , and at ρ_* the IR and UV behaviors are smoothly connected.

This is the first very important result obtained with these methods. We found a background such that the running of the dual gauge coupling has many of the properties of a putative walking theory. There is a region, delimited by two scales ρ_{IR} and ρ_* , where the running almost flattens at a finite value. Below ρ_{IR} the running reappears, and finally there is a scale $\hat{\rho}_o$ at which the space ends, deep in the IR. This is a very encouraging starting point. For all the reasons discussed earlier, in order to understand how physical all of this is, one needs to use this background in order to compute some physical quantity.

5.2.4. Walking Solutions in Class I

The walking behavior described in the previous subsection is appearing in the IR, for $\rho < \rho_*$. It is hence important to understand if it is in any way related to the choice of UV-asymptotic

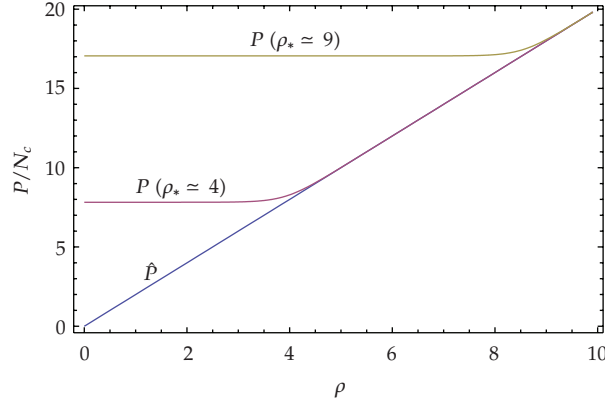


Figure 9: The numerical solutions for $P(\rho)/N_c$ used in the analysis as an example, for $N_c = 100$. The three solutions correspond to the \hat{P} case with $\rho_* = 0$, and to two new numerical solutions with, respectively, $\rho_* \simeq 4$ and $\rho_* \simeq 9$ [58].

behavior of the theory. Also, for several reasons it is more sensible to perform calculations of physical quantities in backgrounds of class I, and it is hence important to know that walking solutions exist also in this class. In principle, one might try and construct numerically such solutions by observing that the walking solutions, for small- ρ , can be written as

$$P(\rho) = P_0 + k_3 P_0 \rho^3 + \frac{4k_3 P_0 \rho^5}{5} - k_3^2 P_0 \rho^6 + \frac{16(2P_0^2 k_3 - 5k_3 N_c^2) \rho^7}{105 P_0} + \dots, \quad (5.20)$$

with P_0 and k_3 being two integration constants. Notice how this expansion does not contain a term linear in ρ . The strategy based on setting up the boundary conditions in the IR in such a way as to reproduce this asymptotic behavior, and then solving numerically towards the UV, is not very efficient, because as we will see the class of walking solutions with UV behavior of class I depends only on one parameter. Hence, one would need to know the precise relation between the integration constants in the IR-expansion, such that asymptotically in the UV one obtains a class-I solution. This is not possible with present knowledge.

In order to build numerically the solution, we start by expanding (5.6), by assuming that the solution can be written as

$$P(\rho) = \hat{P}(\rho) + \varepsilon f(\rho) + \mathcal{O}(\varepsilon^2), \quad (5.21)$$

with $\varepsilon \ll 1$, and expanding at large- ρ by assuming that $\rho \gg 0$. Asymptotically this means that the linearized equations admit solutions that, dropping power-law corrections for notational simplicity, behave as

$$f(\rho) \simeq c_1 e^{-4\rho} + c_2 e^{2\rho}, \quad (5.22)$$

which implies that consistency of the perturbative expansion from (5.21) enforces the choice $c_2 = 0$. Indeed, there are no asymptotic (in the UV) solutions that behave as $e^{2\rho}$.

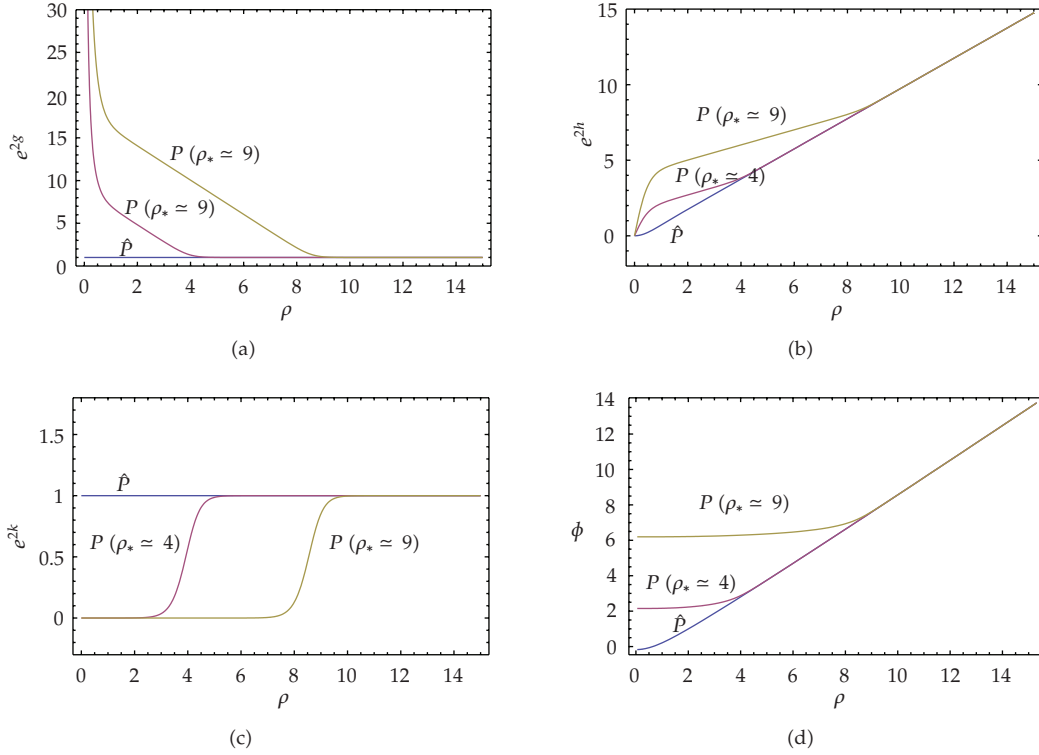


Figure 10: The functions $(e^{2g}, e^{2h}, e^{2k}, \Phi)$ appearing in the metric for the same solutions as in Figure 9, computed rescaling $P \rightarrow P/N_c$ and $Q \rightarrow Q/N_c$ [58].

We have now obtained an important result: at least for large values of ρ , there exists a 1-parameter class of solutions that approach asymptotically the \hat{P} solution. We cannot prove that such solutions are well behaved all the way to $\rho \rightarrow 0$. However, one can use this result in setting up the boundary conditions (at large- ρ) and numerically solve (5.6) towards the IR [57, 58]. By inspection, these solutions turn out to be precisely the ones we are looking for. They start deviating significantly from \hat{P} below some $\rho_* > 0$, below which P is approximately constant. We plot in Figure 9 two such solutions, with $\rho_* \approx 4$ and $\rho_* \approx 9$, together with the \hat{P} solution for the same value of N_c . We also plot in Figure 10 the functions appearing in metric $(e^{2g}, e^{2h}, e^{2k}, \phi)$ for the same solutions. Notice the behavior of e^{2g} for $\rho \rightarrow 0$. Also notice that the dilaton Φ is finite at $\rho \rightarrow 0$.

5.3. Relation to Other Systems

This subsection contains a formal digression that will become useful later on. The Maldacena-Nunez system, and in general the wrapped $D5$ system, can be thought of within a more general class of solutions to type-IIB supergravity that is often referred to as Papadopoulos-Tseytlin ansatz [181]. This ansatz contains as special cases a variety of important models, in particular the Klebanov-Witten (conifold, conformal) [182], the Klebanov-Tseytlin (singular conifold) [183], and the Klebanov-Strassler (resolved conifold) [161] ones. Also, it is possible to show that the 10-dimensional equations yielding the background can be rederived from

a five-dimensional action, in which a sigma-model theory of a set of scalars is coupled to gravity. This section will also introduce a certain amount of notation, which may help the reader in relating the conventions used by several different authors.

Following the notation in [184, 185], we describe the system using an effective five-dimensional action that reads, up to an overall normalization,

$$S = \int d^5y \sqrt{-g} \left[\frac{1}{4} R - \frac{1}{2} G_{ab} g^{MN} \partial_M \Phi^a \partial_N \Phi^b - V(\phi) \right], \quad (5.23)$$

where $\Phi^a = (\tilde{g}, x, p, a, b, \Phi, h_1, h_2, \chi, \mathcal{K})$ and $y^M = (x^\mu, z)$. One uses the two constraints

$$\begin{aligned} \mathcal{K} &= M + 2N(h_1 + bh_2), \\ \partial_M \chi &= \frac{(e^{2\tilde{g}} + 2a^2 + e^{-2}a^4 - e^{-2\tilde{g}}) \partial_M h_1 + 2a(1 - e^{-2\tilde{g}} + a^2 e^{-2\tilde{g}}) \partial_M h_2}{e^{2\tilde{g}} + (1 - a^2)^2 e^{-2\tilde{g}} + 2a^2}, \end{aligned} \quad (5.24)$$

where \mathcal{K} is the normalization of the F_5 form in 10 dimensions, and χ, h_1 , and h_2 appear in the NS B_2 antisymmetric tensor of Type-IIB. N is the normalization of the F_3 form, and essentially counts how many $D5$ branes are present, while M would count the number of $D3$ branes if $N = 0$.

The constraints allow to remove χ and \mathcal{K} from the sigma-model, which is hence defined by

$$\begin{aligned} G_{ab} \partial_M \Phi^a \partial_N \Phi^b &= \frac{1}{2} \partial_M \tilde{g} \partial_N \tilde{g} + \partial_M x \partial_N x + 6 \partial_M p \partial_N p + \frac{1}{4} \partial_M \Phi \partial_N \Phi \\ &+ \frac{1}{2} e^{-2\tilde{g}} \partial_M a \partial_N a + \frac{1}{2} N^2 e^{\Phi-2x} \partial_M b \partial_N b + \frac{e^{-\Phi-2x}}{e^{2\tilde{g}} + 2a^2 + e^{-2\tilde{g}}(1-a^2)^2} \\ &\times \left[(1 + 2e^{-2\tilde{g}}a^2) \partial_M h_1 \partial_N h_1 + \frac{1}{2} (e^{2\tilde{g}} + 2a^2 + e^{-2\tilde{g}}(1+a^2)^2) \partial_M h_2 \partial_N h_2 \right. \\ &\quad \left. + 2a(e^{-2\tilde{g}}(a^2+1) + 1) \partial_M h_1 \partial_N h_2 \right]. \end{aligned} \quad (5.25)$$

The potential is

$$\begin{aligned} V &= -\frac{1}{2} e^{2p-2x} (e^{\tilde{g}} + (1+a^2)e^{-\tilde{g}}) \\ &+ \frac{1}{8} e^{-4p-4x} (e^{2\tilde{g}} + (a^2-1)^2 e^{-2\tilde{g}} + 2a^2) + \frac{1}{4} a^2 e^{-2\tilde{g}+8p} \\ &+ \frac{1}{8} N^2 e^{\Phi-2x+8p} \left[e^{2\tilde{g}} + e^{-2\tilde{g}} (a^2 - 2ab + 1)^2 + 2(a-b)^2 \right] \\ &+ \frac{1}{4} e^{-\Phi-2x+8p} h_2^2 + \frac{1}{8} e^{8p-4x} (M + 2N(h_1 + bh_2))^2. \end{aligned} \quad (5.26)$$

The five-dimensional metric is written as

$$dy^2 = e^{2A} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \quad (5.27)$$

where we use the convention in which the metric is mostly +, and the *warp factor* A is to be determined by the Einstein equations.

The basic idea is that any solution to the equations of motion derived from this 5-dimensional system can be lifted to a full solution of 10-dimensional type-IIB supergravity. The 10-dimensional metric is (in Einstein frame)

$$ds_E^2 = e^{2p-x} dy^2 + \left(e^{x+\tilde{g}} + a^2 e^{x-\tilde{g}} \right) (e_1^2 + e_2^2) + e^{x-\tilde{g}} (e_3^2 + e_4^2 - 2a(e_1 e_3 + e_2 e_4)) + e^{-6p-x} e_5^2, \quad (5.28)$$

where the metric on the internal manifold is written in terms of

$$\begin{aligned} e_1 &= -\sin \theta d\phi, \\ e_2 &= d\theta, \\ e_3 &= \cos \psi \sin \tilde{\theta} d\tilde{\phi} - \sin \psi d\tilde{\theta}, \\ e_4 &= \sin \psi \sin \tilde{\theta} d\tilde{\phi} + \cos \psi d\tilde{\theta}, \\ e_5 &= d\psi + \cos \tilde{\theta} d\tilde{\phi} + \cos \theta d\phi. \end{aligned} \quad (5.29)$$

All the other functions in the type-IIB background (F_3, F_5, H_3, C) can be found explicitly in [184, 185] and references therein.

In looking for solutions to the background, we assume that all the functions have a non-trivial dependence only on the radial direction z . Some very interesting backgrounds can be described by this formalism. When $N = 0$, $\mathcal{K} = M$ is the normalization of F_5 . In this case, there is no F_3 , and hence no b . Further, it is not difficult to show that in this case $H_3 = 0$ satisfies the equations of motion, and hence $h_1 = 0 = h_2 = \chi$. This can be seen directly by minimizing the potential V by brute force. The dilaton Φ disappears from the sigma-model metric and from the potential V , hence its equation of motion is solved by any constant, in particular $\Phi = 0$. Only (\tilde{g}, x, p, a) have to be solved for. There exists in this case a stable minimum for the potential with

$$a = 0, \quad \tilde{g} = 0, \quad x = -\frac{1}{2} \ln \frac{4}{3M}, \quad p = \frac{1}{6} \ln \frac{2}{M}. \quad (5.30)$$

At the fixed points, the Einstein equations imply that

$$(\partial_z A)^2 = -\frac{1}{3} V_0, \quad (5.31)$$

where V_0 is the potential evaluated at the minimum. Replacing in the background one finds

$$\begin{aligned} ds_E^2 &= \frac{2^{4/3}}{\sqrt{3}M^{5/6}} dy^2 + 3\sqrt{3}M \left(\frac{1}{6} (e_1^2 + e_2^2 + e_3^2 + e_4^2) + \frac{1}{9} e_5^2 \right), \\ dy^2 &= e^{(2^{5/3}/3M^{2/3})z} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2, \\ e_1^2 + e_2^2 + e_3^2 + e_4^2 &= \sin^2\theta d\phi^2 + d\theta^2 + \sin^2\tilde{\theta} d\tilde{\phi}^2 + d\tilde{\theta}^2 \end{aligned} \quad (5.32)$$

is the metric on $S^2 \times S^2$. The background has $\text{AdS}_5 \times T^{1,1}$ geometry (the six extra-dimensions describe the conifold). This is the Klebanov-Witten fixed point [182], which describes an $\mathcal{N} = 1$ supersymmetric, conformal theory with $SU(2) \times SU(2) \times U(1)$ global symmetry. Choosing, for instance, $M = 2/\sqrt{27}$, so that the AdS curvature is $L = 1$:

$$ds_E^2 = \sqrt{23}^{3/4} \left(e^{2z} \eta_{\mu\nu} dx^\mu dx^\nu + dz^2 + \frac{1}{6} (e_1^2 + e_2^2 + e_3^2 + e_4^2) + \frac{1}{9} e_5^2 \right). \quad (5.33)$$

One good reason for introducing all of this formalism is that it allows for a very simple exercise to be performed; the calculation of the (quantum) dimensions of the operators of the dual field theory which correspond to the supergravity scalars. By simply expanding the potential around the minimum (and carefully normalizing the kinetic terms), working at unit curvature with $N = 0$ and $M = 2/\sqrt{27}$, one finds that a has mass $m^2 = -3$, \tilde{g} has $m^2 = -4$, h_2 and b mix, the eigenvalues being $m^2 = -3$ and $m^2 = 21$, x and p mix, the eigenvalues of the masses being $m^2 = 12$ and $m^2 = 32$, while h_1 and Φ are massless. Using the celebrated relation $m^2 = (\Delta - 4)\Delta$ yields the scaling dimensions Δ [186, 187]:

$$\begin{aligned} m^2 = -4 &\longrightarrow \Delta = 2, \\ m^2 = -3 &\longrightarrow \Delta = 1, 3, \\ m^2 = 0 &\longrightarrow \Delta = 0, 4, \\ m^2 = 12 &\longrightarrow \Delta = -2, 6, \\ m^2 = 21 &\longrightarrow \Delta = -3, 7, \\ m^2 = 32 &\longrightarrow \Delta = -4, 8. \end{aligned} \quad (5.34)$$

Notice the presence of a set of non-trivial higher-dimensional operators (of dimension 6, 7, and 8). (The reader is assumed to be familiar with the interpretation of m^2 in terms of the dimension of the dual operator and its coupling [188].) A more complete discussion of this model can be found elsewhere, the main lesson we learn from here is that computing the (nonperturbative) dimensions of many important operators in the dual theory is relatively easy (compared, for instance, to the formidable task that doing it in four dimensions represents). Notice also that this exercise clearly shows that this is not simply the dual description of $\mathcal{N} = 1$ super-Yang-Mills; this set-up yields a very rich structure, with many operators being present, all of which in general have very important nonperturbative implications.

An important comment about internal symmetries is presented. Looking back at (5.28), one sees that for $a = 0 = \tilde{g}$ the internal space gains the structure of $T^{1,1}$, and its $SU(2) \times SU(2) \times U(1)$ symmetry. In particular, the functions a and b (the latter appears in the F_3 and B_2 fields), when nonvanishing, induce symmetry-breaking. For instance, in the wrapped $D5$ background defined earlier on, the equations yield a and b that vanish in the UV, but are nonvanishing below ρ_{IR} , signaling that they encode the fact that spontaneous symmetry breaking is taking place at the scale corresponding to ρ_{IR} . The exercise we just did means that the nonvanishing of a and b is related to a dimension-3 condensate in the dual theory, usually identified with the gaugino condensate. (Important subtleties related to the anomalies should be taken into account in discussing the internal symmetries, but for our present purposes they do not change the substance of these results.)

Another important class of solutions can be found with the restriction

$$\begin{aligned} a &= \tanh y, \\ e^{-g} &= \cosh y. \end{aligned} \tag{5.35}$$

With both $M \neq 0 \neq N$, one is left with a system of equations that admits the Klebanov-Strasser background as regular solution. This system is known to provide a description of the duality cascade [168].

Finally, the Maldacena-Nunez system of wrapped $D5$ that is the focus of this section is obtained by setting $M = 0$ and $N = N_c/4$, in which case one can consistently set $h_1 = h_2 = \chi = \mathcal{K} = 0$, reducing to six the number of scalar functions controlling the background. One important thing to stress here is that all of these backgrounds are related to each other, belonging to the same general class [189].

5.4. Glueball Spectrum

By making use of the five-dimensional formalism introduced in the previous subsection, we can compute the spectrum of scalar excitations (glueballs) for the walking solutions with type-I asymptotic behavior in the UV. Following [184, 185], we relate the variables $[A, \tilde{g}, p, x, \Phi, a, b]$ to the functions describing the background in (5.3) as

$$\begin{aligned} \frac{\Phi}{4} &= A + p - \frac{x}{2}, & g &= -A - \frac{\tilde{g}}{2} - p + x + \log 2, \\ h &= -A + \frac{\tilde{g}}{2} - p + x, & k &= -A - 4p + \log 2, \end{aligned} \tag{5.36}$$

and relate the radial coordinates according to $2d\rho e^{-4p} = d\rho e^{A+k} = dz$. The resulting effective 5-dimensional non-linear sigma model with fields $\Phi^a = [\tilde{g}, p, x, \Phi, a, b]$ coupled to gravity can be studied from

$$\mathcal{L}_{5d}^{\text{eff}} = \frac{4(\alpha' g_s N_c)^2 (4\pi)^3}{G_{10}} \sqrt{-g} \left[\frac{R}{4} - \frac{1}{2} G_{ab} \partial \Phi^a \partial \Phi^b - V(\vec{\Phi}) \right], \tag{5.37}$$

where we now made explicit all the constants, and the five-dimensional metric has the form in (5.27).

The sigma-model metric (replacing $N = N_c/4$) is

$$\begin{aligned} 4G_{\phi\phi} = 2G_{gg} = G_{xx} = \frac{G_{pp}}{6} = 1, \\ G_{aa} = \frac{e^{-2\tilde{g}}}{2}, \quad G_{bb} = \frac{N_c^2 e^{\Phi-2x}}{32}, \end{aligned} \quad (5.38)$$

and the potential V is given by (in agreement with [184, 185])

$$\begin{aligned} V = \frac{e^{-2(\tilde{g}+2(p+x))}}{128} \left[16 \left(a^4 + 2 \left((e^{\tilde{g}} - e^{6p+2x})^2 - 1 \right) a^2 + e^{4\tilde{g}} - 4e^{\tilde{g}+6p+2x} (1 + e^{2\tilde{g}}) + 1 \right) \right. \\ \left. + e^{12p+2x+\Phi} \left(2e^{2\tilde{g}}(a-b)^2 + e^{4\tilde{g}} + (a^2 - 2ba + 1)^2 \right) N_c^2 \right]. \end{aligned} \quad (5.39)$$

From here on, we work in units where $\alpha' g_s N_c = 1$. Following [184, 185], after changing coordinates $dz = e^{A+k} d\rho$, writing the fluctuations as $\mathfrak{a}(x, \rho) = e^{iKx} \mathfrak{a}(\rho)$, replacing W by $-K^2 (=M^2)$, and using the equations for the 5d gravity fluctuations we obtain the system of linearized equations for the scalar perturbations [190]

$$\left[D_z^2 + 4A'D_z - e^{-2A} K^2 \right] \mathfrak{a}^a - \left[V_{|c}^a - \mathcal{R}_{bcd}^a \Phi'^b \Phi'^d + \frac{4(\Phi'^a V_c + V^a \Phi'_c)}{3A'} + \frac{16V\Phi'^a \Phi'_c}{9A'^2} \right] \mathfrak{a}^c = 0. \quad (5.40)$$

One important comment is given. The six \mathfrak{a}^c fields solving (5.40) are not simply the fluctuations of the scalars Φ^a in the sigma-model Lagrangian. In fluctuating the geometry, the scalars mix nontrivially with the fluctuations of the metric. The formalism of [184, 185], from which (5.40) is derived, has the advantage of explicitly making use of all the equations of motion (including the Einstein equations) in such a way that the system of six fluctuations written here is already written in the physical basis. In (5.40), primed quantities mean derivatives in respect to z , V is the potential, V_a is its derivative in respect to the field Φ^a , in V^b the index is raised with the inverse sigma-model metric G^{ab} , $V_{|c}^a$ is the (sigma-model) covariant derivative of V^a , \mathcal{R}_{bcd}^a is the Riemann tensor of the sigma-model metric, and D_z is the background covariant derivative defined in [190].

The values of $K^2 = -M^2$ for which the whole system is solved while at the same time satisfying appropriate boundary conditions give us the glueball spectrum of the dual field theory. In order to find them, we employ a numerical method described in [184, 185], that in effect evolves solutions from both the IR and the UV, and then determines whether they match smoothly at a midpoint.

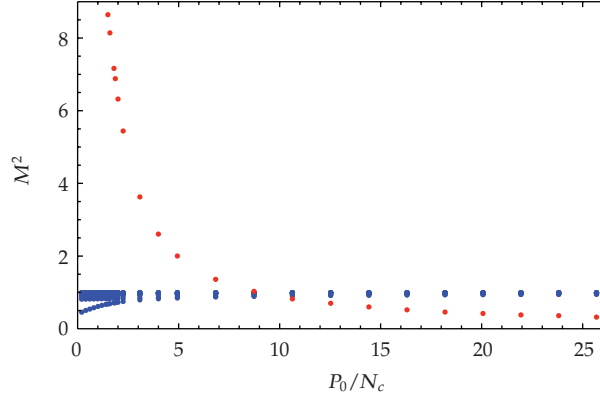


Figure 11: The masses M^2 plotted against $P_0/N_c \simeq 2\rho_*$ [57].

In the UV (for $\rho \rightarrow +\infty$), (5.40) can be diagonalized by a change to a basis in which the fluctuations behave as $\psi^i = e^{C_i\rho} \sum_n a_{i,n} \rho^{b_{i,n}}$, where the exponents $b_{i,n}$ in general are noninteger, while

$$\begin{aligned} C_{1,2,6} &= -1 \pm \sqrt{9 - M^2}, \\ C_{3,4,5} &= -1 \pm \sqrt{1 - M^2}. \end{aligned} \tag{5.41}$$

Notice that this behavior implies the presence of cuts in the two-point functions for $M^2 > 1$ and $M^2 > 9$. The study of the discrete spectrum requires to choose UV boundary conditions such as to select the exponentially suppressed UV-behavior. The IR boundary conditions are more subtle, and the reader who is interested in the details can find them in [57].

The numerical results from the analysis in [57] are plotted in Figure 11. The spectrum for \hat{P} ($P_0 \rightarrow 0$) consists of a series of poles approaching $M^2 = 1$ (in agreement with [184, 185]). For large values of P_0 (equivalently, of ρ_*), all the masses in the series approach the branch points, so that the discrete spectrum effectively disappears into the continuum. With one notable exception: one isolated state becomes lighter, and as ρ_* is increased, its mass is pushed below both the continuum thresholds. For $\rho_* \rightarrow \infty$, this state becomes massless.

The appearance of a single light spin-0 state in the spectrum is typical of systems having some approximate, spontaneously broken symmetry. If this symmetry is some form of scale invariance, the light scalar might be a light dilaton, and hence its couplings should be dictated by this property. With present information, we are not able to reconstruct its composition in terms of the original degrees of freedom in the sigma-model. Notice in particular that the metric is not asymptotically AdS in the UV, hence the rigorous procedure for holographic renormalization is not known, nor is it known how to characterize this state as normalizable or nonnormalizable. Also, the background is not approximately AdS in the walking region, which would be a clear indication of conformal invariance.

The appearance of a very light scalar state, whose mass is parametrically lighter than the dynamical scale controlling the mass of all other composite states, would have profound implications in models of dynamical electroweak symmetry breaking, affecting precision physics (remember that in order to avoid divergences we extracted indicative bounds on \hat{S} from the $m_h \sim 800 \text{ GeV}$ case), and gauge boson scattering amplitudes. It would be the

obvious candidate for a LHC signature. It is remarkable that, if its couplings are dictated by scale invariance, it would be hard to distinguish it from the Higgs of the MSM.

5.5. Wilson Loops

The Wilson loop [191] along the curve \mathcal{C} is defined as

$$W(\mathcal{C}) \equiv \frac{1}{N_c} \text{Tr} P e^{i \oint_{\mathcal{C}} A_\mu dx^\mu}. \quad (5.42)$$

It is one of the most useful gauge-invariant objects, when trying to extract nonperturbative information about a gauge theory. It is computed in the dual string theory by calculating the action of a string bounded by \mathcal{C} at the four-dimensional boundary of the space as follows

$$\langle W(\mathcal{C}) \rangle = \int_{\partial F(\mathcal{C})} \mathfrak{D}F e^{-S_{\text{NG}}[F]}, \quad (5.43)$$

where F denotes all the fields of the string theory and ∂F their boundary values. A good approximation to this path integral is by steepest descent. The Wilson loop is then related to the area of the minimal surface bounded by the curve \mathcal{C} , spanned by classical string configurations (with Nambu-Goto action S_{NG}) that explore the bulk of the space.

All of this was first proposed ten years ago in [192, 193]. In the meantime, this proposal motivated lots of developments, see [194–205] for beautiful papers along this line. See also [206] for a review. In this subsection we are going to compute the Wilson loop using this prescription, for the walking backgrounds of Type I. In doing so, we will be able to show that the scales ρ_0 , ρ_{IR} , and ρ_* correspond to physical scales, at which the functional relation between quark-antiquark potential and quark-antiquark separation changes.

5.5.1. General Treatment

We start by summarizing the general treatment, which uses the ideas of [192, 193] (see also [207]).

We study the action for a string in a background of the generic form

$$ds^2 = -g_{tt} dt^2 + g_{xx} d\vec{x}^2 + g_{\rho\rho} d\rho^2 + g_{ij} d\theta^i d\theta^j. \quad (5.44)$$

We assume that the functions $(g_{tt}, g_{xx}, g_{\rho\rho})$ depend only on the radial coordinate ρ . By contrast, g_{ij} for the internal (typically compact) space can also depend on other coordinates. But we will choose a configuration for a probe string that is not excited on the θ^i directions, and hence we will ignore the internal space.

The configuration we choose is

$$t = \tau, \quad x = x(\sigma), \quad \rho = \rho(\sigma), \quad (5.45)$$

and compute the Nambu-Goto action

$$S = \frac{1}{2\pi\alpha'} \int_{[0,T]} d\tau \int_{[0,2\pi]} d\sigma \sqrt{-\det G_{\alpha\beta}}. \quad (5.46)$$

The induced metric on the 2-d world-volume is $G_{\alpha\beta} = g_{\mu\nu} \partial_\alpha X^\mu \partial_\beta X^\nu$, where

$$G_{\tau\tau} = -g_{tt}, \quad G_{\sigma\sigma} = g_{xx} \left(\frac{dx}{d\sigma} \right)^2 + g_{\rho\rho} \left(\frac{d\rho}{d\sigma} \right)^2. \quad (5.47)$$

Defining for convenience $f(\rho)^2 \equiv g_{tt}g_{xx}$, $g(\rho)^2 \equiv g_{tt}g_{\rho\rho}$, the Nambu-Goto action is

$$S = \frac{T}{2\pi\alpha'} \int_0^{2\pi} d\sigma \sqrt{f^2 x'(\sigma)^2 + g^2 \rho'(\sigma)^2} \equiv \frac{T}{2\pi\alpha'} \int_0^{2\pi} d\sigma L. \quad (5.48)$$

Notice that we consider the situation in which the string does not couple to the NS B_2 -field.

We first compute the Euler-Lagrange equations from (5.46) and then we specify them for the ansatz in (5.45). We also assume a background metric independent of time (we consider the system at the equilibrium). Defining

$$V_{\text{eff}}(\rho) \equiv \frac{f(\rho)}{Cg(\rho)} \sqrt{f^2(\rho) - C^2}, \quad (5.49)$$

where C is an integration constant, we rewrite the equations of motion in terms of only one equation

$$\frac{d\rho}{d\sigma} = \pm \frac{dx}{d\sigma} V_{\text{eff}}(\rho) \longleftrightarrow \frac{d\rho}{dx} = \pm V_{\text{eff}}(\rho). \quad (5.50)$$

The kind of solution we are interested in can be depicted as follows. A string that hangs from infinite radial position at $x = 0$ and drops down towards smaller ρ as x increases. Once it arrives at the smallest ρ compatible with the equations, dubbed ρ_0 , it starts growing in the radial direction up to infinite ρ , where $x = L_{QQ}$; see Figure 12. This means that in the two distinct regions $x < L_{QQ}/2$ and $x > L_{QQ}/2$ the equations of motion will differ only in a sign:

$$\begin{aligned} x < \frac{L_{QQ}}{2}, & \quad \frac{d\rho}{dx} = -V_{\text{eff}}(\rho), \\ x > \frac{L_{QQ}}{2}, & \quad \frac{d\rho}{dx} = V_{\text{eff}}(\rho). \end{aligned} \quad (5.51)$$

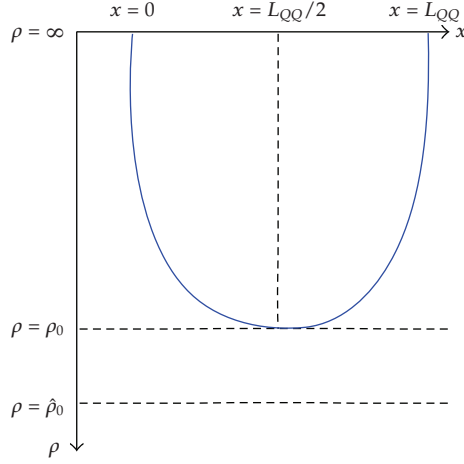


Figure 12: Setting of the string.

We can now formally integrate the equations of motion

$$x(\rho) = \begin{cases} \int_{\rho}^{\infty} \frac{d\rho}{V_{\text{eff}}(r)} & x < \frac{L_{\text{QQ}}}{2}, \\ L_{\text{QQ}} - \int_{\rho}^{\infty} \frac{d\rho}{V_{\text{eff}}(r)} & x > \frac{L_{\text{QQ}}}{2}. \end{cases} \quad (5.52)$$

We need to specify the boundary conditions for the string in (5.45). This is an open string, vibrating in the bulk of a closed string background. Following the ideas of [192, 193], we add a D -brane at a very large radial distance where the open string will end. This string will then satisfy a Dirichlet boundary condition at $\rho \rightarrow \infty$. This means that for large values of the radial coordinate $dx/d\sigma$ must vanish. The only way of satisfying the equation of motion (5.50) for $\rho \rightarrow \infty$, given that the left-hand side has to be nonvanishing, is to have a divergent $V_{\text{eff}}(\rho)$:

$$\lim_{\rho \rightarrow \infty} V_{\text{eff}}(\rho) = \infty. \quad (5.53)$$

This implies that there are restrictions on the asymptotic behavior of the background functions $[f(\rho), g(\rho)]$ in order for the string proposed in (5.45) to exist. This is ultimately the reason why we will perform the calculations in backgrounds in class I in the next subsection.

Provided (5.53) is satisfied, the string moves to smaller values of the radial coordinate down to a turning point ρ_0 , where the quantity $(d\rho/dx)(\rho_0) = 0$, that is, $V_{\text{eff}} = 0$. The turning point can be placed in any possible ρ_0 , with $\hat{\rho}_0 < \rho_0 < \infty$ (where $\hat{\rho}_0$ is the end of the space), where the inversion point is given by imposing $C = f(\rho_0)$. It is clear from (5.50) that $V_{\text{eff}}(\rho)$ controls not only the boundary condition at infinity, but also the possibility for the string to turn around and come back to the brane at infinity.

Given a probe-string hanging from infinity, and that after turning around at a point ρ_0 goes back to the D -brane at infinity, we can compute gauge theory quantities, like the

separation between the two ends of the string, which can be thought of as the separation between a quark-antiquark pair living on the D -brane and coupled to the end-points of the string. And we can compute the Energy of the pair of quarks, that we associate with the length of the string (computed along its path in the bulk). Both of these quantities will be functions of the turning point ρ_0 .

5.5.2. The $D5$ on S^2 System

We now apply all of the above to the walking solutions of type I. For our purposes, it is also convenient to fix $8e^{4\Phi_0} = 1$, and $\hat{\rho}_0 = 0$. The functions we need for the probe string are

$$\begin{aligned} f^2(\rho) &= e^{2\Phi} = \sqrt{\frac{\sinh^2(2\rho)}{(P^2 - Q^2)P'}}, \\ g^2(\rho) &= \frac{1}{2}P'f^2(\rho), \\ V_{\text{eff}}^2(\rho) &= \frac{2}{C^2P'} \left(\sqrt{\frac{\sinh^2(2\rho)}{(P^2 - Q^2)P'}} - C^2 \right), \end{aligned} \quad (5.54)$$

with P and Q defined earlier on, characterizing the background.

We set up the configuration of the string by assuming that its extremes are attached to the brane at $\rho = \rho_1 \gg 1$, and treat this as a UV cut-off. Because the background is known only numerically, and it is not asymptotically-AdS, the calculations are performed at fixed cut-off. The string is stretched in the Minkowski direction $x = x(\rho)$, with $x(\rho_1) = 0$ for convenience. We vary the integration constant $C^2 > f^2(0)$. For each choice of C we define ρ_0 as $V_{\text{eff}}^2(\rho_0) = 0$. In this way, the coordinates of the string are $(x(\rho), \rho)$, where

$$x(\rho) = \int_{\rho}^{\rho_1} \frac{dr}{V_{\text{eff}}(r)}. \quad (5.55)$$

The Minkowski distance between the end-points of the string is hence $L_{QQ} = 2x(\rho_0)$. For the energy, because in the numerical study we do not remove the UV cut-off, we use the unsubtracted action (setting $T/(2\pi\alpha') = 1$) evaluated up to the cut-off:

$$E_{QQ} = 2 \int_{\rho_0}^{\rho_1} dr \sqrt{\frac{f^2(r)g^2(r)}{f^2(r) - C^2}}. \quad (5.56)$$

The numerical results obtained for the three different solutions for P in Figure 9 are shown in Figures 13, 14, and 15.

Let us first focus our attention on the background generated by \hat{P} in (5.11), in which case a great deal of information can be extracted analytically. The deeper the string probes the radial coordinate (smaller values of ρ_0), the longer the separation L_{QQ} between the end-points

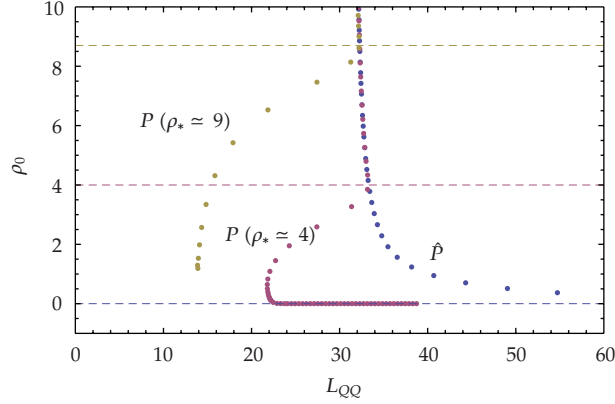


Figure 13: The radial coordinate ρ_0 of the turning point of the string as a function of L_{QQ} , in the three backgrounds from Figure 9, with same color-coding [58].

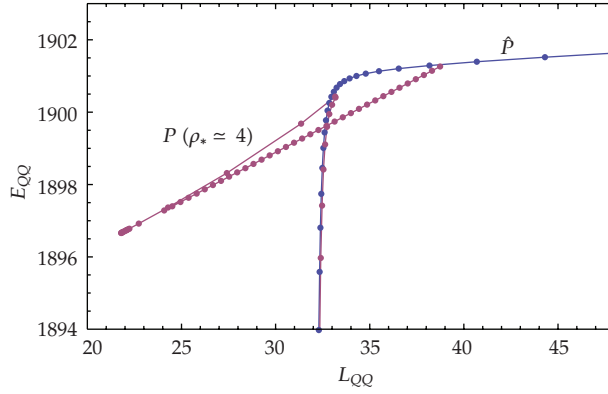


Figure 14: The energy E_{QQ} as a function of the quark-antiquark separation $E_{QQ}(L_{QQ})$. The two solutions in Figure 9 with $\rho_* = 0$ and $\rho_* \approx 4$ are used for the background, and the results are shown using the same color-coding as in Figure 9 [58].

on the UV brane, in agreement with natural expectations. In particular, setting $C^2 = f^2(0)$ (equivalent to $\rho_0 \rightarrow 0$), and expanding around $\rho \sim 0$,

$$V_{\text{eff}}^2(\rho) = \frac{8\rho^2}{9N_c} + \dots, \quad (5.57)$$

which, by means (5.55) and (5.56), implies that both L_{QQ} and E_{QQ} diverge for $C^2 \rightarrow f^2(0)$.

Two regimes can be identified: as long as $\rho_0 > \rho_{\text{IR}}$, then L_{QQ} varies very little with ρ_0 ; for small $\rho_0 < \rho_{\text{IR}}$, further reductions of ρ_0 imply much bigger increase in L_{QQ} . The scale $\rho_{\text{IR}} \sim \mathcal{O}(1)$ is the scale in which the function Q changes from linear to approximately quadratic in ρ , and is also the scale below which the gaugino condensate is appearing (the function $b(\rho)$ in the background is nonzero). This result is better visible in Figure 13. The dependence of L_{QQ} on ρ_0 is monotonic, but shows two very different behaviors for $\rho_0 < \rho_{\text{IR}}$ and $\rho_0 > \rho_{\text{IR}}$, respectively. The transition between the two is completely smooth.

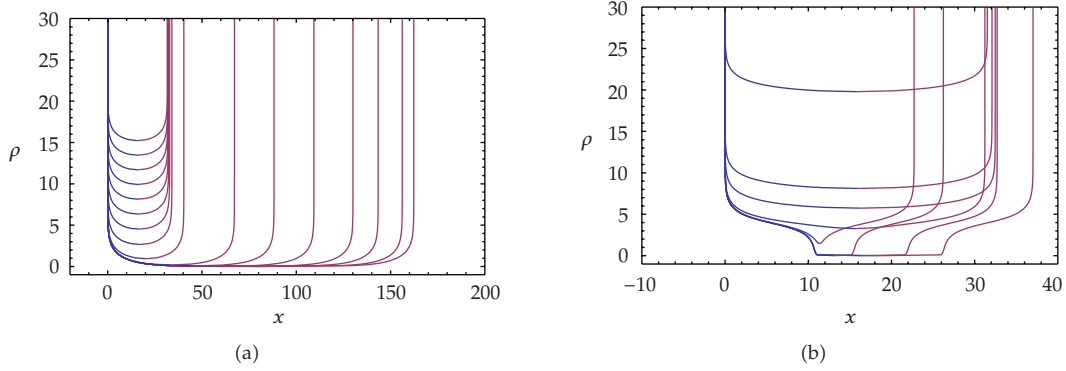


Figure 15: The strings in (x, ρ) -plane, obtained with various choices of $C^2 > f^2(0)$. Left to right, the background is described by the numerical solutions in Figure 9, characterized by increasing values of $\rho_* = 0$ and $\rho_* \simeq 4$, respectively. All calculations performed with $\rho_1 = 30$, and other units set as explained in the text [58].

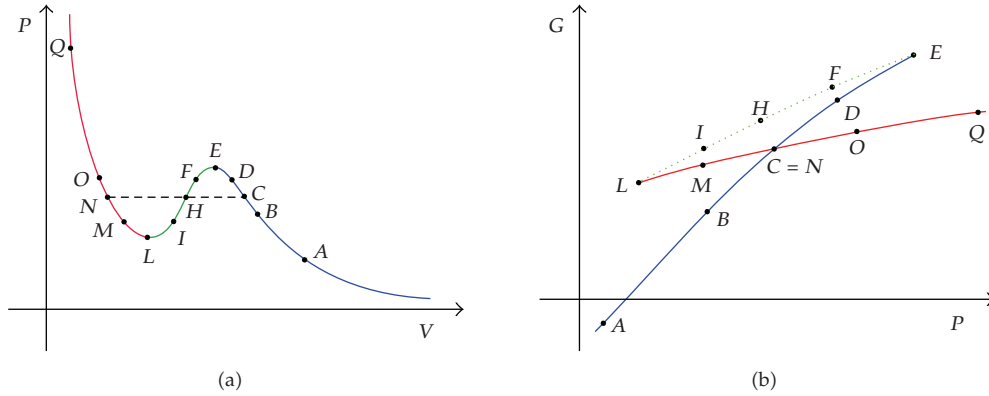


Figure 16: The pressure P as a function of the volume V (a) and the Gibbs free energy G as a function of the pressure P (b) for the same isotherm curve [58].

The physical meaning of this behavior is well illustrated by studying the total energy E_{QQ} of the classical configurations, as a function of L_{QQ} (see Figure 14). For small L_{QQ} , the energy grows very fast with L_{QQ} , until a critical value beyond which the dependence becomes linear, which can be interpreted in terms of the linear behavior of the quark-antiquark potential obtained from the Wilson loop, in agreement with confinement. The important conclusion is that the geometry and the end of the space encodes the information about confinement.

Comparing with the solutions that *walk* in the IR, shows a very different behavior. Starting from Figure 13, one sees that as long as $\rho_0 > \rho_*$, the dependence of L_{QQ} from ρ_0 reproduces the \bar{P} case. Beginning from such large ρ_0 , we start pulling the string down to smaller values of ρ_0 , and follow the classical evolution. Provided we do this adiabatically, we can describe the motion of the string as the set of classical equilibrium solutions. Going to smaller ρ_0 , L_{QQ} increases, and nothing special happens until the tip of the string touches $\rho_0 \simeq \rho_*$. At this point $L_{QQ} = L_{\max}$ has a (local) maximum as a function of ρ_0 . From here

on, the string can keep probing smaller values of ρ_0 only at the price of becoming shorter in the Minkowski direction (smaller L_{QQ}). Another change happens deep in the IR, when $L_{QQ} = L_{\min}$ reaches a local minimum, at which point the tip of the string entered the very bottom section of the space, near its end $\rho_0 < \rho_{\text{IR}}$. From here on, further reducing ρ_0 requires larger values of L_{QQ} . Asymptotically for $\rho_0 \rightarrow 0$, the separation between the end-points of the string is diverging, $L_{QQ} \rightarrow \infty$.

Even more interesting is the behavior of the energy (see Figure 14). For very short L_{QQ} , and again for very large- L_{QQ} , it is just a monotonic function, very similar to the one obtained from \hat{P} . But for a range $L_{\min} < L_{QQ} < L_{\max}$, there are three different configurations allowed by the classical equations for the string we are studying. One of the three solutions (smoothly connected to the small- L_{QQ} configurations) is just the Coulombic potential already seen with \hat{P} . The highest energy one is an unstable configuration, with much higher energy. The third solution (smoothly connected to the unique solution with $L_{QQ} > L_{\max}$) reproduces the linear potential typical of confinement. Notice (from Figure 14) that the solution at large- L_{QQ} is linear, but has a slope much larger than what seen in the \hat{P} case.

This coexistence of several disjoint classical solutions is expected in systems leading to phase transitions. The stability/metastability/instability of the solutions can be illustrated by comparing Figures 13 and 14 with Figure 16 in the Appendix. This is just an analogy, and one should not push it too far. However, identifying the pressure P , volume V and Gibbs free energy G as $L_{QQ} \leftrightarrow P$, $\rho_0 \leftrightarrow V$, and $E_{QQ} \leftrightarrow G$, one sees obvious similarities. In particular, there is a critical distance $L_{\min} < L_c < L_{\max}$ at which the minimum of E_{QQ} is not differentiable, which suggests that this is a first-order (quantum) phase transition.

In order to better understand and characterize the solutions we find, it is useful to look more in details at the shape of the string configurations, focusing in particular on the right panel in Figure 15, in which we plot the string configuration that solves the equations of motion for various values of ρ_0 , on the background with $\rho_* \simeq 4$. Consider those strings that penetrate below ρ_* . Besides having a shorter L_{QQ} , and higher E_{QQ} than those for which $\rho_0 > \rho_*$, these strings show another interesting feature. They start developing a nontrivial structure around their middle point, that becomes progressively more curved the further the string falls at small ρ . Ultimately, this degenerates into a cusp-like configuration, which disappears once ρ_0 approaches the end of the space.

Notice that, as a result, the three different solutions for $L_{\min} < L_{QQ} < L_{\max}$ have three very different geometric configurations. One (stable or metastable) configuration is completely featureless, and practically indistinguishable from the solutions in the background generated by \hat{P} . The second (stable or metastable) configuration shows a funnel-like structure below ρ_* , and most of the string lies very close to the end of the space. The third (unstable) solution presents a highly curved configuration around its middle point. All of this seems to be consistent with the fact that the classical configurations prefer to lie either in the far-UV or deep-IR, outside the $\rho_{\text{IR}} < \rho < \rho_*$ region.

Concluding, we learned two very important things from the study of the Wilson loop. First of all, the end of the space in the IR can indeed be interpreted as the fact that confinement is taking place. The (unphysical) singularity in the running of the gauge coupling yields the expected (physical) linear behavior of the quark-antiquark potential. Second, the existence of a plateau at intermediate scale has a very interesting physical meaning. Because it is ultimately responsible for the arising of the first-order phase transition we found, the hierarchy $\rho_* > \rho_{\text{IR}}$ cannot be undue by simply changing renormalization scheme, but is a physical effect.

5.6. Summary

In this section we focused our attention on one very specific class of models, in which the geometry is obtained by truncating type-IIB supergravity, in the strongly coupled limit of the system of $D5$ -branes wrapping a 2-cycle inside a $CY3$ manifold.

We found new classes of solutions to the BPS equations determining the background that exhibit some very peculiar properties, closely resembling those expected in a walking field theory. Studying these backgrounds, we were able to show that the following holds.

- (i) The (nonperturbative) running of the four-dimensional gauge coupling exhibits a walking behavior, in the sense that there is a finite interval of the radial direction within which the coupling is almost constant, but this behavior disappears both in the UV and in the IR.
- (ii) In the IR, below the walking region, the theory confines, as signaled by the long-distance behavior of the Wilson loop, and condensates emerge, spontaneously breaking the global symmetries of the theory.
- (iii) The existence of the walking region is a physical effect, as signaled by the presence of a discontinuity of the derivative of the quark-antiquark energy as a function of the separation.
- (iv) The spectrum of the scalar glueballs contains a mass gap, and depending on the extension in the radial direction of the walking region one scalar state becomes parametrically lighter than such gap.

These are all very striking and important results, none of which can be obtained analytically by studying a four-dimensional, strongly coupled gauge theory with traditional methods. A long list of other things should be done. I conclude with a sample of possible other open questions.

- (i) The properties of the light scalar (its couplings in particular) are not known. Holographic renormalization, at least in its simplest form, fails to provide a sensible treatment of the correlation functions, because the background is not asymptotically AdS. Some clever strategy must be devised in order to be able to study the possible phenomenology of such a light scalar.
- (ii) The rest of the spectrum (spin-1 fields and fermions in particular) has not been studied yet.
- (iii) The spectrum of anomalous dimensions of the model is not known. This is a peculiarly difficult question, in relation to the fact that the geometry is never really close to AdS, and hence the precise relation between radial direction and renormalization scale is not easily inferred.
- (iv) It would be interesting to couple this model to the standard model, and compute the effect of the walking region on the oblique parameters.
- (v) The field and operator content of the dual field theory is known only in part, and a more systematic exploration would be useful.

Finally, it must be noted that there is no obvious reason why this specific set-up is preferable, in relation to walking technicolor. To large extent, the reason why these studies have been performed in this specific context has to do with the fact that backgrounds based on the conifold and its various deformations provide a very well-studied and understood

place from which to start. But it would be very interesting to know if analogous results hold in completely different models, possibly simpler, and maybe nonsupersymmetric.

6. Conclusions

Dynamical electroweak symmetry breaking relies on the existence of a new strongly coupled sector responsible for the spontaneous symmetry breaking that is at the core of the Standard Model. There are two interconnected reasons why the ideas of gauge/string dualities are useful in constructing and studying these models. First of all, the nonperturbative nature of the underlying strong dynamics calls for a flexible and powerful set of tools which makes calculations doable. But also, the huge amount of experimental data at our disposal indicates quite clearly that a generic model is not going to pass the precision tests. The new (strongly coupled) physics sector has to have very special features in order to be phenomenologically viable. Walking technicolor is one such special possibility, which happens to be a natural candidate for phenomenological applications of gauge/string dualities, because of its quasiconformal behavior, its large anomalous dimensions, and its multiscale nature.

In these notes, besides reviewing the phenomenology of strongly coupled extensions of the standard model (at a very pedagogical level, by comparing to the minimal version of the standard model itself and to its weakly coupled extensions), I provided a few examples illustrating how the ideas taken from gauge/string dualities can indeed yield very useful and interesting results, taking them both from the context of five-dimensional effective theories (bottom-up approach) and from more rigorous ten-dimensional string theory constructions (top-down approach).

Summarizing, the main messages I hope I was able to convey to the reader are the following.

- (i) The unhealthy behavior of the minimal version of the Standard Model, when extrapolated at arbitrary high scale (where Landau poles and fine-tuning appear), requires that it be completed, particularly in the sector responsible for electroweak symmetry breaking, and many proposals exist that do so.
- (ii) At low energy the Standard Model provides such an efficient and precise description of the vast amount of data at our disposal, that only very peculiar, special types of new physics models can be phenomenologically viable.
- (iii) Walking technicolor, supplemented by tumbling ETC, is one such special, phenomenologically viable possibility, in which new strongly coupled interactions and nonperturbative phenomena are present.
- (iv) With the incoming of LHC data, actual calculations of physical observable quantities within this strongly coupled framework are urgently needed.
- (v) Gauge/gravity dualities can be used to perform these calculations, and several examples of useful results exist.
- (vi) The technology of gauge/gravity duality is at such a stage of development that, provided a sensible model is identified, the calculation of actual physical quantities can be done, and is often comparatively simple.
- (vii) While a completely satisfactory proposal for the dual of a walking technicolor theory does not exist at present, yet, circumstantial evidence from the examples we have, together with the existing vast literature on the subject and the number

of open possibilities for model-building in the gauge/gravity context, strongly suggests that there is a lot that can be learned in this direction, which represents a very promising avenue for present and future research.

Appendix

Van Der Waals Gas

Here I summarize some aspect of first-order phase transitions that plays an important conceptual role in the body of the paper. This appendix is taken from [58], and extended discussions can be found in textbooks on statistical mechanics. As an example, I review the classical treatment of the van der Waals gas, in terms of the pressure P , temperature T , and volume V of n moles of particles, by means of the equation of state

$$P = \frac{nRT}{V - bn} - \frac{n^2 a}{V^2}, \quad (\text{A.1})$$

where R , b , and a are constants.

Figure 16 shows one isotherm. The condition for stability of the equilibrium $(\partial^2 F / \partial V^2)_T = -(\partial P / \partial V)_T > 0$ is not satisfied in some region. This implies that a phase transition is taking place.

In order to understand what the physical trajectory followed by the system at equilibrium is, Figure 16 shows the Gibbs free energy $G = G(T, P) = G(P)$ for the same isotherm. From this plot, one sees that the system evolves on the path $ABCOQ$, where $C = N$, in such a way that for every choice of P the Gibbs free energy G is at its minimum. The evolution is smooth along ABC (gas phase: $|\partial P / \partial V|$ is small), but at $C = N$ the free energy is not differentiable, signaling that a first-order phase transition is taking place. In the (P, V) plane the system runs along the horizontal line (constant P) joining C and N . This explains the *Maxwell rule* introducing a curve of constant pressure that separates two regions of equal areas above and below it, delimited by the original isotherm. Afterwards, the evolution follows smoothly the curve NOQ (liquid phase: $|\partial P / \partial V|$ is large).

While the trajectory $ABCNOQ$ follows the stable equilibrium configurations, it is possible to have the system evolving along the path CDE or LMN . Both these paths represent metastable configurations, because $(\partial P / \partial V)_T < 0$. Indeed, these metastable states can be realized in laboratory experiments. For example, in a bubble chamber or in supercooled water, the metastability is exploited as a detector device, because small perturbations induced by passing by charged particles are sufficient to drive the system out of the state and into the stable minimum. The evolution along the path $EFHIL$ is completely unstable and not realized physically (it is a local maximum, as clear from Figure 16(b), and by the fact that $(\partial P / \partial V)_T > 0$).

The analogy with the examples in the main body of the paper is apparent. Notice, for instance, that the pressure P in this system as a function of the volume V behaves as a non monotonic function, hence there are *inversion points* in the curve (the points E, L in Figure 16).

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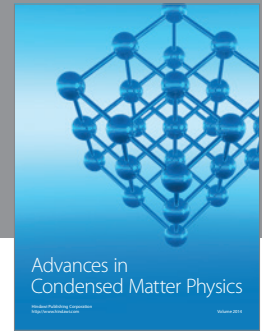
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