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Research Article

Some Modified Fixed Point Results in V-Fuzzy Metric Spaces

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The present research paper focuses on the existence of fixed point in V-fuzzy metric space. The presentation of V-fuzzy metric space in n-tuple encourages us to define different mapping in the symmetric V-fuzzy metric space. Here, the properties of fuzzy metric space are extended to V-fuzzy metric space. The introduction of notion for pair of mappings (f, g) on V-fuzzy metric space called V-weakly commuting of type V_f and $V - R$ weakly commuting of type V_f is given. This proved fixed point theorem in V-fuzzy metric space employing the effectiveness of E.A. property and CLRg property. For the justification of the results, some examples are illustrated.

1. Introduction

Metric space is one of the important basic areas of research for the mathematicians. Many researchers accelerated the concept of metric space either by introducing different contractions in different fields or by extending number of variables in the metric space. Different types of mappings are introduced to facilitate the fixed point in metric spaces such as weakly commuting pair of mappings [1], compatible mappings [2], and weakly compatible mappings [3]. Subsequently, Aamri and Moutawakil [4] introduced the notion of E.A. property. In 2011, Sintunavarat and Kumam[5] stamped the idea of common limit in the range of g (called CLRg property) which relaxes the requirement of completeness (or closedness) of the underlying subspace. Fixed point results are proved through the same concept in fuzzy metric spaces. Many authors [5–15] have given results about the common fixed point results in several spaces. On the basis of number of variables, there are many different generalizations, such as generalized metric space by Mustafa and Sims [16], generalized fuzzy metric spaces by Sun and Yang [17], new generalized metric space called S-metric space by Sedghi [18], and A-metric spaces by Abbas et al. [19], which is generalization of S-metric spaces. Also, V-fuzzy metric spaces were introduced by Gupta and Kanwar [20], which are based on fuzzy metric for n-tuples.

The above mentioned generalizations of metric spaces are described below.

Definition 1 ([16]). Let X be a nonempty set and let $G : X^3 \longrightarrow [0, +\infty)$ be a function satisfying the following conditions for all $x, y, z, a \in X$:

(G-1) $G(x, y, z) = 0$ if $x = y = z$,

(G-2) $0 \leq G(x, y, z)$ with $x \neq y$,

(G-3) $G(x, x, y) \leq G(x, y, z)$ with $y \neq z$,

(G-4) $G(x, y, z) = G(x, z, y) = G(y, x, z) = G(z, x, y) = G(z, y, x) = G(y, z, x)$,

(G-5) $G(x, y, z) \leq G(x, a, a) + G(a, y, z)$.

The function G is called a generalized metric on X and the pair (X, G) is called a G-metric space.

In 2012, Sedghi et al. [18] introduced a new generalized metric space called S-metric space.

Definition 2 ([18]). Let X be a nonempty set. Suppose a function $S : X^3 \longrightarrow [0, +\infty)$ satisfies the following conditions:

(S-1) $S(x, y, z) \geq 0$,

(S-2) $S(x, y, z) = 0$ if and only if $x = y = z = 0$, (S-3) $S(x, y, z) \leq S(x, x, a) + S(y, y, a) + S(z, z, a)$ for any $x, y, z, a \in X$.

Then the ordered pair (X, S) is called S-metric space.

Abbas et al. [19] established the notion of A-metric spaces, which is considered as generalizations of S-metric space.

Definition 3 ([19]). Let X be a nonempty set. A function $A : X^n \longrightarrow [0, +\infty)$ is called an A-metric on X, if for any $x_i, a \in X, i = 1, 2, 3, \ldots, n$, the following conditions hold:

(A-1) $A(x_1, x_2, x_3, \ldots, x_n) \geq 0$,

(A-2) $A(x_1, x_2, x_3, \ldots, x_n) = 0$ if and only if $x_1 = x_2 = x_3 == \cdots = x_n = 0$,

(A-3) $A(x_1, x_2, x_3, \ldots, x_n) \leq A(x_1, x_1, x_1, \ldots, (x_1)_{n-1}, a) + A(x_2, x_2, x_2, \ldots, (x_2)_{n-1}, a) + \ldots + A(x_n, x_n, x_n, \ldots, (x_n)_{n-1}, a)$.

The pair (X, A) is called A-metric space.

Fuzzy sets introduced by Zadeh [21] are the engender for all the research in different fields. Kramosil and Michalek [22] introduced the concept of fuzzy metric spaces.

Definition 4 ([23]). A binary operation $* : [0.1] \times [0, 1] \longrightarrow [0, 1]$ is called continuous t-norms; it satisfies following conditions:

(T-1) $*$ is commutative and associative,

(T-2) $*$ is continuous,

(T-3) $a * 1 = a, \forall a \in [0, 1]$,

(T-4) $a*b \leq c*d$ whenever $a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0, 1]$.

Definition 5 ([22]). The 3-tuple $(X, M, *)$ is called a fuzzy metric space if X is an arbitrary set, $*$ is continuous t-norm, and M is a fuzzy set in $X \times X \times [0, \infty)$ satisfying the following conditions:

for all $x, y, z \in X$ and $s, t > 0$,

(FM-1) $M(x, y, 0) = 0$,

(FM-2) $M(x, y, t) = 1, \forall t > 0$ if and only if $x = y$,

(FM-3) $M(x, y, t) = M(y, x, t)$,

(FM-4) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$,

(FM-5) $M(x, y, \cdot) : [0, \infty) \longrightarrow [0, 1]$ is left continuous.

Note that $M(x, y, t)$ can be thought of as the degree of nearness between x and y with respect to t.

Example 6. Let (X, d) be a metric space. Define t-norm $a*b = ab$ or $a * b = \min\{a, b\}$. For all $x, y \in X, t > 0$.

$$M(x, y, t) = \frac{t}{t + d(x, y)}. \tag{1}$$

Then $(X, M, *)$ is a fuzzy metric space.

Lemma 7 ([24]). *Let $(X, M, *)$ be a fuzzy metric space. If there exists $k \in (0, 1)$ for all $x, y \in X$, $x, t > 0$ such that*

$$M(x, y, kt) \geq M(x, y, t) \tag{2}$$

for all $x, y \in X$, $t > 0$, then $x = y$.

In the process of generalization of fuzzy metric space, Sun and Yang [17] presented the notion of G-fuzzy metric space as follows.

Definition 8 ([17]). A 3-tuple $(X, G, *)$ is said to be G-fuzzy metric space (denoted by GF space) if X is an arbitrary nonempty set, $*$ is continuous t-norm, and G is a fuzzy set on $X \times X \times X \times [0, \infty)$ satisfying the following conditions:

for each $x, y, z \in X$ and $s, t > 0$,

(GF-1) $G(x, x, y, t) > 0$ with $x \neq y$,

(GF-2) $G(x, x, y, t) \geq G(x, y, z, t)$ with $y \neq z$,

(GF-3) $G(x, y, z, t) = 1$ if and only if $x = y = z$,

(GF-4) $G(x, y, z, t) = G(x, z, y, t) = G(y, x, z, t) = G(z, x, y, t) = G(y, z, x, t) = G(z, y, x, t)$,

(GF-5) $G(x, y, z, t + s) \geq G(x, a, a, t) * G(a, y, z, s)$.

(GF-6) $G(x, y, z, \cdot) : (0, \infty) \longrightarrow [0, 1]$ is left continuous.

Lemma 9 ([17]). *Let $(X, G, *)$ be a GF space. Then $G(x, y, z, t)$ is nondecreasing with respect to t for all $x, y, z \in X$.*

2. V-Fuzzy Metric Space

These all generalizations advocate V-fuzzy metric spaces. In 2016, Gupta and Kanwar [20] stamped the move of these generalization to n-tuples as discussed below.

Definition 10 ([20]). Let X be nonempty set. A 3-tuple $(X, V, *)$ is said to be a V-fuzzy metric space (denoted by VF-space), where $*$ is a continuous t-norm and V is a fuzzy set on $X^n \times (0, \infty)$ satisfying the following conditions for each $t, s > 0$:

(VF-1) $V(x, x, x, \ldots, x, y, t) > 0$ for all $x, y \in X$ with $x \neq y$;

(VF-2) $V(x_1, x_1, x_1, \ldots, x_1, x_2, t) \geq V(x_1, x_2, x_3, \ldots, x_n, t)$ for all $x_1, x_2, x_3, \ldots, x_n \in X$ with $x_2 \neq x_3 \neq \cdots \neq x_n$;

(VF-3) $V(x_1, x_2, x_3, \ldots, x_n, t) = 1$ if $x_1 = x_2 = x_3 = \ldots = x_n$;

(VF-4) $V(x_1, x_2, x_3, \ldots, x_n, t) = V(p(x_1, x_2, x_3, \ldots x_n), t)$, where p is a permutation function;

(VF-5) $V(x_1, x_2, x_3, \ldots, x_{n-1}, t + s) \geq V(x_1, x_2, x_3, \ldots, x_{n-1}, l, t) * V(l, l, l, \ldots, l, x_n, s)$;

(VF-6) $\lim_{t \longrightarrow \infty} V(x_1, x_2, x_3, \ldots, x_n, t) = 1$;

(VF-7) $V(x_1, x_2, x_3, \ldots, x_n) : (0, \infty) \longrightarrow [0, 1]$ is continuous.

Example 11 ([20]). Let $(X, V, *)$ be a V-metric space. Define t-norm $a*b = ab$ or $a*b = \min\{a, b\}$. For all $x_1, x_2, x_3, \ldots, x_n \in X, t > 0$,

$$V(x_1, x_2, x_3, \ldots, x_n, t) = \frac{t}{t + A(x_1, x_2, x_3, \ldots, x_n)} \tag{3}$$

then $(X, V, *)$ is a V-fuzzy metric space.

Lemma 12 ([20]). *Let $(X, V, *)$ be a V-fuzzy metric space; then $V(x_1, x_2, x_3, \ldots, x_n, t)$ is nondecreasing with respect to t.*

Lemma 13 ([20]). *Let $(X, V, *)$ be a V-fuzzy metric space such that*

$$V(x_1, x_2, x_3, \ldots, x_n, kt) \geq V(x_1, x_2, x_3, \ldots, x_n, t), \quad (4)$$

with $k \in (0, 1)$; then $x_1 = x_2 = x_3 = \ldots = x_n$.

Definition 14 ([20]). Let $(X, V, *)$ be a V-fuzzy metric space. A sequence $\{x_r\}$ is said to converge to a point $x \in X$ if $V(x_r, x_r, x_r, \ldots, x_r, x, t) \longrightarrow 1$ as $r \longrightarrow \infty$ for all $t > 0$; that is, for each $\epsilon > 0$, there exists $n \in N$ such that for all $r \geq N$ we have $V(x_r, x_r, x_r, \ldots, x_r, x, t) > 1 - \epsilon$ and we write $\lim_{t \longrightarrow \infty} x_r = r$.

Definition 15 ([20]). Let $(X, V, *)$ be a V-fuzzy metric space. A sequence $\{x_r\}$ is said to be a Cauchy sequence if $V(x_r, x_r, x_r, \ldots, x_r, x_q, t) \longrightarrow 1$ as $r, q \longrightarrow \infty$ for all $t > 0$; that is, for each $\epsilon > 0$ there exists $n_0 \in N$ such that for all $r, q \geq n_0$, we have $V(x_r, x_r, x_r, \ldots, x_r, x_q, t) > 1 - \epsilon$.

Definition 16 ([20]). A V-fuzzy metric space $(X, V, *)$ is said to be complete if every Cauchy sequence in X is convergent.

In the present research paper, topology is induced by V-fuzzy metric spaces. The introduction of concepts of V-weakly commuting of type V_f and V-R weakly commuting of type V_f in V-fuzzy metric spaces is given which helps in determining the fixed point theorem for symmetric V-fuzzy metric spaces.

3. Topology Induced by V-Fuzzy Metric Space

Definition 17. Let $(X, V, *)$ be a V-fuzzy metric space. For $t > 0$, the open ball $B_V(x, r, t)$ with center $x \in X$ and radius $0 < r < 1$ is defined as

$$B_V(x, r, t) = \{y \in X : V(x, y, \ldots, y, t) > 1 - r\}. \quad (5)$$

Result 1. Every open ball is an open set.

Consider an open ball $B_V(x, r, t)$. Now

$$y \in B_V(x, r, t) \Longrightarrow V(x, y, \ldots, y, t) > 1 - r. \quad (6)$$

Since $V(x, y, \ldots, y, t) > 1 - r$, we can find t_0, $0 < t_0 < t$, such that $V(x, y, \ldots, y, t_0) > 1 - r$.

Let $r_0 = V(x, y, y, \ldots, y, t_0) > 1 - r$.

Since $r_0 > 1 - r$, we can find s, $0 < s < 1$, such that $r_0 > 1 - s > 1 - r$.

Further for a given r_0 and s such that $r_0 > 1 - s$ we can find r_1, $0 < r_1 < 1$ such that $r_0 * r_1 \geq 1 - s$.

Consider the Ball

$$B_V(y, 1 - r_1, t - t_0). \quad (7)$$

We will show that

$$B_V(y, 1 - r_1, t - t_0) \subset B_V(x, r, t). \quad (8)$$

Now $z \in B_V(y, 1 - r_1, t - t_0)$ implies $V(y, z, \ldots, z, t - t_0) > r_1$.

Therefore

$$V(x, z, \ldots, z, t) \geq V(x, y, \ldots, y, t_0) * V(y, z, \ldots, z, t - t_0) \geq r_0 * r_1 \geq 1 - s. \quad (9)$$

Therefore $z \in B_V(x, r, t)$ and hence

$$B_V(y, 1 - r, t - t_0) \subset B_V(x, r, t). \quad (10)$$

Result 2. Let $(X, V, *)$ be a V-fuzzy metric space. Define

$$\tau = \{A \subset X : x \in A \text{ if and only if there exist } t > 0 \text{ and } r,\ 0 < r < 1 \text{ such that } B_V(x, r, t) \subset A\}. \quad (11)$$

Then τ is a topology on X.

Definition 18. Let $(X, V, *)$ be a V-fuzzy metric space. The following condition is satisfied:

$$\lim_{n_i \longrightarrow \infty} V(x_{n_1}, x_{n_2}, \ldots, x_n, t_{n_0}) = V(x_1, x_2, \ldots, x_N, t), \quad i = 0, 1, 2, \ldots, N, \quad (12)$$

whenever $\lim_{n_1 \longrightarrow \infty} x_{n_1} = x_1$, $\lim_{n_2 \longrightarrow \infty} x_{n_2} = x_2$, ..., $\lim_{n_N \longrightarrow \infty} x_{n_N} = x_N$, and

$$\lim_{n_0 \longrightarrow \infty} V(x_1, x_2, \ldots, x_N, t_{n_0}) = V(x_1, x_2, \ldots, x_N, t); \quad (13)$$

then V is called continuous function on $X^N \times (0, \infty)$.

Lemma 19. *Let $(X, V, *)$ be a V-fuzzy metric space. Then V is a continuous function $X^N \times (0, \infty)$.*

Proof. Since

$$\lim_{n_1 \longrightarrow \infty} x_{n_1} = x_1, \quad \lim_{n_2 \longrightarrow \infty} x_{n_2} = x_2, \quad \ldots, \quad \lim_{n_N \longrightarrow \infty} x_{n_N} = x_N \quad (14)$$

and

$$\lim_{n_0 \longrightarrow \infty} V(x_1, x_2, \ldots, x_N, t_{n_0}) = V(x_1, x_2, \ldots, x_N, t), \quad (15)$$

then there exists $n_p \in N$ such that

$$|t - t_{n_0}| < \delta \quad \text{for } n_0 \geq n_p \text{ and } \delta < \frac{t}{2}. \quad (16)$$

As $V(x_1, x_2, \ldots, x_n, t)$ is nondecreasing with respect to t, we have

$$V(x_{n_1}, x_{n_2}, \ldots, x_{n_N}, t_n) \geq V(x_{n_1}, x_{n_2}, \ldots, x_{n_N}, t - \delta) \geq V\left(x_{n_1}, x_1, \ldots, x_1, \frac{\delta}{N}\right)$$

$$
\begin{aligned}
&* V\left(x_1, x_{n_2}, \ldots, x_{n_N}, t - \frac{N+1}{N}\delta\right) \\
&\geq V\left(x_{n_1}, x_1, \ldots, x_1, \frac{\delta}{N}\right) * V\left(x_{n_2}, x_2 \ldots x_2, \frac{\delta}{N}\right) \\
&* \ldots \\
&* V\left(x_{N-1}, x_{N-2}, \ldots, x_1, x_{n_N}, t - \frac{N+(N-1)}{N}\delta\right) \\
&\geq V\left(x_{n_1}, x_1, \ldots, x_1, \frac{\delta}{N}\right) * V\left(x_{n_2}, x_2 \ldots x_2, \frac{\delta}{N}\right) \\
&* \ldots * V\left(x_{n_N}, x_N, \ldots, x_1, x_N, \frac{\delta}{N}\right) \\
&* V\left(x_N, x_{N-1}, \ldots, x_1, t - (N-1)\delta\right)
\end{aligned} \tag{17}
$$

and

$$
\begin{aligned}
&V\left(x_1, x_2, \ldots, x_N, t + (N-1)\delta\right) \\
&\geq V\left(x_1, x_2, \ldots, x_N, t_n + \delta\right) \\
&\geq V\left(x_1, x_{n_1}, \ldots, x_{n_1}, \frac{(N-1)}{N}\delta\right) \\
&* V\left(x_{n_1}, x_2, x_3, \ldots, x_N, t_n + \frac{1}{N}\delta\right) \\
&\geq V\left(x_1, x_{n_1}, \ldots, x_{n_1}, \frac{\delta}{N}\right) \\
&* V\left(x_2, x_{n_2}, x_{n_2}, \ldots, x_{n_2}, \frac{\delta}{N}\right) * \ldots \\
&* V\left(x_N, x_{n_N}, x_{n_N}, \ldots, x_{n_N}, t_n + \frac{\delta}{N}\right) \\
&\geq V\left(x_1, x_{n_1}, x_{n_1}, \ldots, x_{n_1}, \frac{\delta}{N}\right) \\
&* V\left(x_2, x_{n_2}, x_{n_2}, \ldots, x_{n_2}, \frac{\delta}{N}\right) * \ldots \\
&* V\left(x_N, x_{n_N}, x_{n_N}, \ldots, x_{n_N}, \frac{\delta}{N}\right) \\
&* V\left(x_N, x_{N-1}, \ldots, x_2, x_1, t_n\right).
\end{aligned} \tag{18}
$$

Considering continuity of the function V with respect to t and letting $n \longrightarrow \infty$, we have

$$
\begin{aligned}
&V\left(x_1, x_2, \ldots, x_N, t + (N-1)\delta\right) \\
&\geq V\left(x_N, x_{N-1}, \ldots, x_1, t\right) \\
&\geq V\left(V\left(x_N, x_{N-1}, \ldots, x_1, t - (N-1)\delta\right)\right).
\end{aligned} \tag{19}
$$

Therefore, V is continuous function on $X^N \times [0, \infty)$. □

Remark 20. In the present paper, $(X, V, *)$ will denote an *VF*-space with a continuous t-norm $*$ defined as $a * b = \min\{a, b\}$ for all $a, b \in [0, 1]$ and we assume that

$$
\lim_{t \longrightarrow \infty} V\left(x_1, x_2, \ldots, x_N, t\right) = 1 \quad \text{for all } x_i \in X,\ i = 1, 2, \ldots, N. \tag{20}
$$

Define $\phi = \{\phi : R^+ \longrightarrow R^+\}$, where $R^+ = [0, \infty]$ and each $\phi \in \Phi$ satisfying the following conditions:

(ϕ-1) ϕ is strict increasing,

(ϕ-2) ϕ is upper semicontinuous from the right,

(ϕ-3) $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $t > 0$.

Lemma 21. *Let $(X, V, *)$ be an VF-space. If there exists $\phi \in \Phi$ such that*

$$
V\left(x_1, x_2, x_3, \ldots, x_n, \phi(t)\right) \geq V\left(x_1, x_2, x_3, \ldots, x_n, t\right) \quad \text{for all } t > 0, \tag{21}
$$

then $x_1 = x_2 = x_3 = \ldots = x_n$.

Proof. Since

$$
V\left(x_1, x_2, x_3, \ldots, x_n, \phi(t)\right) \geq V\left(x_1, x_2, x_3, \ldots, x_n, t\right) \tag{22}
$$

and also $\phi(t) < t$, by using Lemma 12, we have

$$
V\left(x_1, x_2, x_3, \ldots, x_n, \phi(t)\right) \leq V\left(x_1, x_2, x_3, \ldots, x_n, t\right). \tag{23}
$$

From (22) and (23) and definition of V-fuzzy metric space, one can get $x_1 = x_2 = x_3 = \cdots = x_n$. □

Remark 22. Let $x_1 = w$, $x_2 = x_3 = \ldots = x_{n-1} = \mu$, in (VF-5); we have

$$
\begin{aligned}
&V\left(w, u, u, \ldots, u, t + s\right) \\
&\geq V\left(w, v, v, \ldots, v, t\right) * V\left(v, u, u, \ldots, u, s\right),
\end{aligned} \tag{24}
$$

which implies that

$$
\begin{aligned}
V\left(u, u, \ldots, w, s + t\right) \geq\ & V\left(v, v, \ldots, v, w, t\right) \\
&* V(v\left(u, u, \ldots, u, v, s\right)
\end{aligned} \tag{25}
$$

for all $u, v, w \in X$ and $s, t > 0$.

An *VF*-space is said to be symmetric if $V(x, x, \ldots, x, y, t) = V(x, y, y, \ldots, y, t)$ for all $x, y \in X$ and for each $t > 0$.

Lemma 23. *Let $(X, V, *)$ be an VF-metric space; if we define $E_\lambda : X \times X \times X \times \cdots \times X \longrightarrow [0, \infty)$ by*

$$
\begin{aligned}
&E_\lambda\left(x_1, x_2, \ldots, x_n\right) \\
&\quad = \inf\left\{t > 0, V\left(x_1, x_2, \ldots, x_n, t\right) > 1 - \lambda\right\}
\end{aligned} \tag{26}
$$

for all $\lambda \in (0,1]$ *and* $x_1, x_2, \ldots, x_n \in X$, *then we have*

(i) *for each* $\lambda \in (0,1]$, *there exists* $\mu \in (0,1]$ *such that*

$$E_\lambda(x_1, x_2, \ldots, x_n) \le \sum_{i=1}^{n-1} E_\mu(x_i, x_i, \ldots, x_{i+1}) \quad \text{for all } x_1, x_2, \ldots, x_n \in X \tag{27}$$

(ii) *the sequence* $\{x_n\}_{n\in N}$ *in* X *is convergent if and only if* $E_\lambda(x_n, x_n, \ldots, x_n, x) \longrightarrow 0$ *as* $n \longrightarrow \infty$ *for all* $\lambda \in (0,1]$.

Proof. (i) For any $\lambda \in (0,1]$, let $\mu \in (0,1]$ and $\mu < \lambda$. Therefore, by the triangular inequality (VF-5) and Remark 22

$$\begin{aligned} V\Bigg(& x_1, x_1, \ldots, x_n, \sum_{i=1}^{n-1} E_\mu(x_i, x_i, \ldots, x_{i+1}) + (n-1)\delta\Bigg) \\ &\ge V\left(x_1, x_1, \ldots, x_2, E_\mu(x_1, x_1, \ldots, x_2) + \delta\right) \\ &* V\left(x_2, x_2, \ldots, x_3, E_\mu(x_2, x_2, \ldots, x_3) + \delta\right) \\ &+ \cdots V\left(x_{n-1}, x_{n-1}, \ldots, x_n, E_\mu(x_{n-1}, x_{n-1}, \ldots, x_n) + \delta\right) \ge \min\{(1-\mu), (1-\mu)\ldots(1-\mu)\} \ge 1-\lambda, \end{aligned} \tag{28}$$

which gives, using (26),

$$\begin{aligned} E_\lambda(x_1, x_1, \ldots, x_n) \le\ & E_\mu(x_1, x_1, \ldots, x_2) \\ &+ E_\mu(x_2, x_2, \ldots, x_3) \\ &+ E_\mu(x_{n-1}, x_{n-1} \ldots, x_n) \\ &+ (n-1)\delta. \end{aligned} \tag{29}$$

Since $\delta > 0$ is arbitrary, we have

$$\begin{aligned} E_\lambda&(x_1, x_1, x_1, \ldots, x_n) \\ &\le E_\mu(x_1, x_1, \ldots, x_2) + E_\mu(x_2, x_2, \ldots, x_3) + \ldots \\ &+ E_\mu(x_{n-1}, x_{n-1}, \ldots, x_n). \end{aligned} \tag{30}$$

(ii) Since V is continuous in its $(n+1)$th argument (by (26)), we have

$$V(x_n, x_n, \ldots, x_n, x, \eta) > 1 - \lambda \quad \text{for all } \eta > 0. \tag{31}$$

This proves the lemma. □

Lemma 24. *Let* $(X, V, *)$ *be a* V*-Fuzzy metric space and* $\{y_n\}$ *be a sequence in* X. *If there exists* $\phi \in \Phi$ *such that*

$$\begin{aligned} V(y_n, y_n, y_n, \ldots, y_n, y_{n+1}, \phi(t)) &\ge V(y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n, t) \\ &* V(y_n, y_n, \ldots, y_n, y_{n+1}, t) \end{aligned} \tag{32}$$

for all $t > 0$ *and* $n = 1, 2, 3, \ldots$, *then* $\{y_n\}$ *is a Cauchy sequence in* X.

Proof. Let $\{E_\lambda(x, y, z)\}_{\lambda\in(0,1]}$ be defined by (26).

For each $\lambda \in (0,1]$ and $n \in N$, put

$$a_n = E_\lambda(y_{n-1}, y_{n-1}, \ldots, y_{n-1}, y_n). \tag{33}$$

We will prove that

$$a_{n+1} \le \phi(a_n) \quad \text{for all } n \in N. \tag{34}$$

Since ϕ is upper semicontinuous from right, for given $\epsilon > 0$ and each a_n, there exists $p_n > a_n$ such that $\phi(p_n) < \phi(a_n) + \epsilon$. From (26), it follows from $P_n > a_n = E_\lambda(y_{n-1}, y_{n-1}, \ldots, y_n)$ that $V(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, P_n) > 1 - \lambda$ for all $n \in N$.

Thus, by (32), (34), and Lemma 12, we get

$$\begin{aligned} V&(y_n, y_n, y_n, \ldots, y_{n+1}, \phi(\max\{p_n, p_{n+1}\})) \\ &\ge V(y_{n-1}, y_{n-1}, \ldots, y_n, \max\{p_n, p_{n+1}\} \\ &\quad * V(y_n, y_n, \ldots, y_{n+1}, \max\{p_n, p_{n+1}\}) \\ &\ge V(y_{n-1}, y_{n-1}, y_{n-1}, \ldots, y_n, p_n) \\ &\quad * V(y_n, y_n, \ldots, y_1, y_{n+1}, p_{n+1}) > 1 - \lambda. \end{aligned} \tag{35}$$

Again, by (26), we get

$$\begin{aligned} E_\lambda(y_n, y_n, \ldots, y_n, y_{n+1}) &\le \phi(\max\{p_n, p_{n+1}\}) \\ &= \max\{\phi(p_n), \phi(p_{n+1})\} \\ &\le \max\{\phi(a_n), \phi(a_{n+1})\} + \epsilon. \end{aligned} \tag{36}$$

By the arbitrariness of ϵ, we have

$$\begin{aligned} a_{n+1} &= E_\lambda(y_n, y_n, \ldots, y_{n+1}) \\ &\le \max\{\phi(a_n), \phi(a_{n-1})\}. \end{aligned} \tag{37}$$

So, we can interpret that $a_{n+1} \le \phi(a_n)$.

If not, then by (37), we have $a_{n+1} \le \phi(a_{n+1}) < a_{n+1}$; this is a contradiction. Hence (37) implies that $a_{n+1} \le \phi(a_n)$, and (34) is proved.

By repeated application of (34), we get

$$\begin{aligned} E_\lambda(y_n, y_n, \ldots, y_{n+1}) &\le \phi(E_\lambda(y_{n-1}, y_{n-1}, \ldots, y_n) \\ &\le \cdots \le \phi^n(E_\lambda(y_0, y_0, \ldots, y_1) \quad \text{forall } n \in N. \end{aligned} \tag{38}$$

By Lemma 23, for each $\lambda \in (0,1]$, there exists $\mu \in (0,\lambda]$ such that

$$E_\lambda(y_n, y_n, \ldots, y_n, y_n) \le \sum_{i=n}^{m-1} E_\mu(y_i, y_i, \ldots, y_i, y_{i+1}), \quad m, n \in N \text{ with } m > n. \tag{39}$$

Since $\phi \in \Phi$, by condition (ϕ-3), we have

$$\sum_{n=0}^{\infty} \phi^n \left(E_\mu \left(y_0, y_0, \ldots, y_1 \right) \right) < +\infty. \tag{40}$$

So, for given $\epsilon > 0$, there exists $n_0 \in N$ such that $\sum_{i=n_0}^{\infty} \phi^i(E_\mu(y_0, y_0, \ldots, y_1)) < \epsilon$. Thus, it follows from (39) that

$$\begin{aligned} E_\lambda \left(y_n, y_n, \ldots, y_m \right) &\leq \sum \phi^i \left(E_\mu \left(y_0, y_0, y_0, \ldots, y_1 \right) \right) \\ &< \epsilon \quad \text{for all } n \geq n_0, \end{aligned} \tag{41}$$

which implies that $V(y_n, y_n, y_n, \ldots, y_n, \epsilon) > 1-\lambda$ for all $m, n \in N$ with $m > n \geq n_0$. Therefore $\{y_n\}$ is a Cauchy sequence in X. □

Definition 25. A pair of self-mappings (f, g) of V-fuzzy metric space $(X, V, *)$ is said to be V-weakly commuting of type V_f if

$$\begin{aligned} &V \left(fgx, gfx, fgx, \ldots, ffx, t \right) \\ &\quad \geq V \left(fx, gx, fx, fx, \ldots, \ldots, fx, t \right) \end{aligned} \tag{42}$$

for all $x \in X$ and $t > 0$.

Definition 26. A pair of self-mappings (f, g) of a V-fuzzy metric space $(X, V, *)$ is said to be V-R weakly commuting of type V_f if there exists some positive real number R such that

$$\begin{aligned} &V \left(fgx, gfx, fgx, \ldots, ff(x) t \right) \\ &\quad \geq V \left(fx, gx, fx, fx, \ldots, fx, \frac{t}{r} \right) \end{aligned} \tag{43}$$

for all $x \in X$ and $t > 0$.

Remark 27. If we interchange f and g in above definitions, then the pair of self-mappings (f, g) of V-fuzzy metric space $(X, V, *)$ is said to be V-weakly commuting of type V_g and V-R weakly commuting of type V_g, respectively.

For proving our main results, we use the following relation.

The following example shows that a pair of mapping (f, g) that is V-weakly commuting of type V_f does not need to be V-weakly commuting of type V_g.

Example 28. Let $x = (0, 1]$ be the V-fuzzy metric space with

$$\begin{aligned} &A \left(x_1, x_2, \ldots, x_n \right) \\ &\quad = \max \left\{ \left| x_1 - x_2 \right|, \left| x_2 - x_3 \right| \ldots, \left| x_{n-1} - x_n \right| \right\} \end{aligned} \tag{44}$$

for all $x_1, x_2, \ldots, x_n \in X$.

Define $fx = x^2/4$, $g(x) = x^2$.

Then we find

$$A \left(fgx, gfx, \ldots, ffx \right) = \frac{15}{64} x^4 \tag{45}$$

and

$$A \left(fx, gx, gx, \ldots, fx \right) = \frac{3}{4} x^2. \tag{46}$$

Then, one can get

$$\begin{aligned} A \left(gfx, fgx, fgx, \ldots, gg(x) \right) &= \frac{15}{16} \\ &\nleq A \left(gx, fx, fx, \ldots gx \right) = \frac{3}{4}, \end{aligned} \tag{47}$$

which implies

$$V \left(fgx, gfx, \ldots, ffx, t \right) \geq V \left(fx, gx, \ldots, fx, t \right) \tag{48}$$

and

$$V \left(gfx, fgx, \ldots, ggx, t \right) \ngeq V \left(gx, fx, \ldots, gx, t \right). \tag{49}$$

Hence the pair (f, g) is not V-weakly commuting of type V_g, but it is V-weakly commuting of type V_f.

Lemma 29. *If f and g are V-weakly commuting of type V_f or V-R-weakly commuting of type V_f, then f and g are weakly compatible.*

Proof. Let x be a coincidence point of f and g; i.e., $f(x) = g(x)$; then if pair (f, g) is V-weakly commuting of type V_f, we have

$$\begin{aligned} V \left(fgx, gfx, \ldots, fgx, t \right) &= V \left(fgx, gfx, \ldots, ffx, t \right) \\ &\geq V \left(fx, gx, \ldots, fx, t \right) = 1. \end{aligned} \tag{50}$$

It follows that $fxg = gfx$. Hence f and g commute at their coincidence point.

Similarly, if pair (f, g) is V-R weakly commuting of type V_f, we have

$$\begin{aligned} V \left(fgx, gfx, \ldots, fgx, t \right) &= V \left(fgx, gfx, \ldots, ffx, t \right) \\ &\geq V \left(fx, gx, \ldots, fx, \frac{t}{R} \right) = 1, \end{aligned} \tag{51}$$

and thus $fgx = gfx$; then the pair (f, g) is weakly compatible.

The converse of the lemma need not be true. □

Example 30. $X = [0, 1]$ and $A(x_1, x_2, \ldots, x_n) = |x_1 - x_2| + |x_2 - x_3|$.

Define $f, g : X \longrightarrow X$ by $f(x) = 4x - 1$ and $g(x) = 3x^2$, $x \in X$; we see that $x = 1/3$ is the only coincidence point and $f(g(1/3)) = 1/3$ and $g(f(1/3)) = g(1/3)$, so f and g are weakly compatible.

But by easy calculation, for $x = 1$, one can have

$$A \left(fgx, gfx, \ldots, ffx \right) = 36 \tag{52}$$

and

$$A \left(fx, gx, \ldots, fx \right) = 0, \tag{53}$$

i.e.,

$$\begin{aligned} V \left(fgx, gfx, \ldots, ffx, t \right) &= 36 \nleq 0 \\ &= \left(fx, gx, \ldots, fx, t \right). \end{aligned} \tag{54}$$

Therefore, f and g are not V-weakly commuting of type V_f.

Definition 31 ([4]). A pair of self-mappings (f, g) on X is said to satisfy the property E.A. if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = z \quad \text{for all } z \in X. \tag{55}$$

Definition 32 ([25]). A pair of self-mappings (f, g) on X is said to satisfy the CLRg property if there exists a sequence $\{x_n\}$ such that

$$\lim_{n \to \infty} fx_n = \lim_{n \to \infty} gx_n = gz \quad \text{for all } z \in X. \tag{56}$$

Now, we are ready to prove our results for symmetric V-fuzzy metric spaces.

Theorem 33. *Let $(X, V, *)$ be a symmetric VF-space and mappings $f, g : X \longrightarrow X$ satisfying the following conditions:*

(i) *f and g are V-weakly commuting of type V_f;*

(ii) *$f(X) \subseteq g(X)$;*

(iii) *$g(X)$ is a V-complete subspace of X;*

(iv) *these exists $\phi \in \Phi$ such that for all $x_i \in X$ and $i = 1, 2, \ldots, n$, $t > 0$*

$$\begin{aligned} &V(fx_1, fx_2, fx_3, \ldots, fx_n, \phi(t)) \\ &\quad \geq V(gx_1, gx_1, \ldots, fx_1, t) \\ &\qquad * V(gx_2, gx_2, \ldots, fx_2, t) \\ &\qquad * V(gx_3, gx_3, \ldots, fx_3, t) * \ldots \\ &\qquad * V(gx_n, gx_n, \ldots, fx_n, t). \end{aligned} \tag{57}$$

Then f and g have common fixed point.

Proof. Let $z_1 \in X$ such that $fz_0 = gz_1$ and $z_2 \in X$, where $fz_1 = gz_2$, and then by induction we can define a sequence $\{y_n\} \in X$ as follows

$$y_n = fz_n = gz_{n+1}, \quad n \in N. \tag{58}$$

We will prove that $\{y_n\}$ is a Cauchy sequence in X.

$$\begin{aligned} &V(y_n, y_n, \ldots, y_n, y_{n+1}, \phi(t)) \\ &\quad = V(fz_n, fz_n, \ldots, fz_{n+1}, \phi(t)) \\ &\quad \geq V(gz_n, gz_n, \ldots, fz_n, t) \\ &\qquad * V(gz_n, gz_n, \ldots, fz_n, t) * \cdots \\ &\qquad * V(gz_{n+1}, gz_{n+1}, \ldots, fz_{n+1}, t) \\ &\quad \geq V(gz_n, gz_n, \ldots, gz_{n+1}, t) \\ &\qquad * V(gz_n, gz_n, \ldots, gz_{n+1}, t) * \cdots \\ &\qquad * V(gz_{n+1}, gz_{n+1}, \ldots, gz_{n+2}, t), \end{aligned} \tag{59}$$

which gives

$$\begin{aligned} &V(y_n, y_n, \ldots, y_{n+1}, \phi(t)) \\ &\quad \geq V(y_{n-1}, y_{n-1}, \ldots, y_n, t) * \cdots \\ &\qquad * V(y_n, y_n, \ldots, y_{n+1}, t). \end{aligned} \tag{60}$$

By Lemma 24, the sequence $\{y_n\}$ is a V-Cauchy sequence. Since $y_n = gz_{n+1}$, $\{gz_{n+1}\}$ is a V-Cauchy sequence in $g(x)$.

By hypothesis (iii), we know that $g(X)$ is V-complete; then there exists $u \in g(X)$ such that

$$\lim_{n \to \infty} gz_n = u = \lim_{n \to \infty} fz_n. \tag{61}$$

Now $u \in g(x)$, so there exists $p \in X$ such that $u = gp$. Therefore

$$\lim_{n \to \infty} gz_n = gp = \lim_{n \to \infty} fz_n. \tag{62}$$

We will prove that $fp = gp$:

$$\begin{aligned} &V(fp, fp, \ldots, fz_n, \phi(t)) \\ &\quad \geq V(gp, gp, \ldots, fp, t) * V(gp, gp, \ldots, fp, t) \\ &\qquad * V(gp, gp, \ldots, fp, t) * \ldots \\ &\qquad * V(gz_n, gz_n, \ldots, fz_n, t), \end{aligned} \tag{63}$$

taking limit as $n \longrightarrow \infty$,

$$\begin{aligned} &V(fp, fp, \ldots, gp, \phi(t)) \\ &\quad \geq V(gp, gp, \ldots, fp, t) * V(gp, gp, \ldots, fp, t) \\ &\qquad * V(gp, gp, \ldots, gp, t), \end{aligned} \tag{64}$$

which implies,

$$V(fp, fp, \ldots, gp, \phi(t)) \geq V(gp, gp, \ldots, fp, t). \tag{65}$$

Since V-fuzzy metric space is symmetric, we have

$$\begin{aligned} &V(fp, fp, \ldots, gp, \phi(t)) \geq V(gp, gp, \ldots, fp, t) \\ &\quad = V(fp, fp, \ldots, gp, t), \end{aligned} \tag{66}$$

which implies $fp = gp$ (by Lemma 21).

Since pair (f, g) is V-weakly commuting of type V_f, then

$$\begin{aligned} &V(fgp, gfp, fgp, \ldots, ffp, \phi(t)) \\ &\quad \geq V(fp, gp, fp, gp, \ldots, fp, t) = 1, \end{aligned} \tag{67}$$

which implies

$$ffp = fgp = gfp = ggp. \tag{68}$$

Hence $fu = fgp = fgp = gu$.

Eventually, we show that $u = gp$ is common fixed point of f and g. Suppose $fu \neq u$; then

$$\begin{aligned} V(fu, fp, fp, \ldots, fp, \phi(t)) &\geq V(gu, gu, \ldots, fu, t) * V(gp, gp, \ldots, fp, t) \\ &\quad * \ldots * V(gp, gp, \ldots, fp, t) \\ &\geq V(fu, fu, \ldots, fu, t) * V(fp, fp, \ldots, fp, t) \\ &\quad * \ldots * V(fp, fp, \ldots, fp, t). \end{aligned} \tag{69}$$

$V(fu, u, \ldots u, \phi(t)) \geq 1 * 1 * \ldots * 1 = 1$,
which is the contradiction. Hence, $fu = gu = u$.

To prove the uniqueness, suppose u and v are such that $u \neq v$, $fv = gv = v$ and $fu = gu = u$; then again using condition (iv), we have

$$\begin{aligned} V(u, v, \ldots, v, \phi(t)) = V(fu, fv, \ldots, fv, \phi(t)) &\geq V(gu, gu, \ldots, fu, t)) \\ &\quad * V(gv, gv, \ldots, fv, t) * \ldots \\ &\quad * V(gv, gv, \ldots, fv, t) \\ &= 1 * 1 * \ldots * 1 = 1. \end{aligned} \tag{70}$$

Hence, $V(u, v, v, \ldots, v, \phi(t)) \geq 1$, which gives a contradiction. Hence $u = v$. Therefore u is a unique common fixed point of f and g. □

Example 34. Let $X = [0, 1]$ be a standard V-fuzzy metric space.

Let $\phi(t) = t/2$ and define $f, g : X \longrightarrow X$ by $f(x) = x/6$, $g(x) = (x/2)(x + 1)$, $x \in X$.

We see that $x = 0$ is the only coincidence point and f and g are weakly compatible.

Let $x_n = 1/n$ be a sequence such that

$$V(fp, fp, \ldots, fx_n, \phi(t)) \geq V(fp, fp, \ldots, gp, t), \tag{71}$$

where p is a coincidence point.

Then the pair (f, g) is V-weakly commuting of type V_f. Further f and g have a unique common fixed point of f and g.

Corollary 35. *Theorem 33 remains true if we replace V-weakly commuting and $V - R$-weakly commuting of type V_f by V-weakly commuting and $V - R$-weakly commuting of type V_g (considering the other conditions are the same).*

Theorem 36. *Let $(X, V, *)$ be a symmetric V-fuzzy metric space and suppose mappings $f, g : X \longrightarrow X$ are V-weakly commuting of type V_f satisfying the following conditions:*

(i) *f and g satisfy the E.A property;*

(ii) *$g(X)$ is a closed subspace of X;*

(iii) *there exists a $\phi \in \Phi$ such that for all $x_i \in X$, $i = 1, 2, \ldots, n$ and $t > 0$,*

$$\begin{aligned} V(fx_1, fx_2, \ldots, fx_n, \phi(t)) &\geq V(gx_1, gx_1, \ldots, fx_1, t) \\ &\quad * V(gx_2, gx_2, \ldots, fx_2, t) * \ldots \\ &\quad * V(gx_n, gx_n, \ldots, fx_n, t). \end{aligned} \tag{72}$$

Then f and g have a unique common fixed point.

Proof. Since, the mappings f and g satisfy the E.A. property, then there exists a sequence $\{z_n\}$ in X satisfying

$$\lim_{n\longrightarrow\infty} gz_n = u = \lim_{n\longrightarrow\infty} fz_n \quad \text{for some } u \in X. \tag{73}$$

Since $g(X)$ is closed subspace of X and $\lim_{n\longrightarrow\infty} gz_n = u$, then there exists $p \in X$ such that $gp = u$.

Also, $\lim_{n\longrightarrow\infty} gz_n = gp = \lim_{n\longrightarrow\infty} fz_n$.

We will prove, $fp = gp$:

$$\begin{aligned} V(fp, fp, \ldots, fz_n, \phi(t)) &\geq V(gp, gp, \ldots, fp, t) * V(gp, gp, \ldots, fp, t) \\ &\quad * \ldots * V(gz_n, gz_n, \ldots, fz_n, t), \end{aligned} \tag{74}$$

and, taking limit as $n \longrightarrow \infty$, we have

$$\begin{aligned} V(fp, fp \ldots, gp, \phi(t)) &\geq V(gp, gp, \ldots, fp, t) * \cdots \\ &\quad * V(gp, gp, \ldots, gp, t) V(fp, fp \ldots, gp, \phi(t)) \\ &\geq V(gp, gp, \ldots, fp, t). \end{aligned} \tag{75}$$

Since V-fuzzy metric space is symmetric, we have

$$V(fp, fp, \ldots, gp, \phi(t)) \geq V(fp, fp, \ldots, gp, t), \tag{76}$$

which implies $fp = gp = u$ (by Lemma 21).

Since pair (f, g) is V-weakly commuting of type V_f, then

$$\begin{aligned} V(fgp, gfp, \ldots, ffp, \phi(t)) &\geq V(fp, gp, fp, \ldots, fp, t) = 1, \end{aligned} \tag{77}$$

which implies $ffp = fgp = gfp = ggp$.

Hence $fu = fgp = gfp = gu$.

Finally, we show that $u = gp$ is a common fixed point of f and g. Suppose $fu \neq u$, then

$$\begin{aligned} V(fu, fp, \ldots, fp, \phi(t)) &\geq V(gu, gu, \ldots, fu, t) * V(gp, gp, \ldots, fp, t) \\ &\quad * \ldots * V(gp, gp, \ldots, fp, t) \\ &\geq V(fu, fu, \ldots, fu, t) * V(fp, fp, \ldots, fp, t) \\ &\quad * \ldots * V(fp, fp, fp, \ldots, fp, t) \\ &= 1 * 1 * 1 * \ldots * 1 = 1. \end{aligned} \tag{78}$$

Thus, $V(fu, u, u \ldots u, \phi(t)) \geq 1$

which is a contradiction. Hence $fu = gu = u$.

To prove the uniqueness, suppose that u and v are such that $u \neq v$, $fu = gu = u$, and $fv = gv = v$; then again using condition (iii), we have

$$\begin{aligned} V(u, v, \ldots, v, \phi(t)) &= V(fu, fv, \ldots, fv, \phi(t)) \\ &\geq V(gu, gu, \ldots, fu, t) \\ &\quad * V(gv, gv, \ldots, fv, t) \ldots \\ &\quad * V(gv, gv, \ldots, fv, t) \\ &= 1 * 1 * 1 * \ldots * 1 = 1. \end{aligned} \tag{79}$$

Hence $V(u, v, \ldots, v, \phi(t)) \geq 1$, which gives a contradiction. Hence $u = v$. Therefore 'u' is a unique common fixed point of f and g. □

Theorem 37. *Let $(X, V, *)$ be a symmetric V-fuzzy metric space and suppose mappings $f, g : X \longrightarrow X$ are V-weakly commuting of type V_f satisfying the following conditions:*

(i) *f and g satisfy the CLRg property;*

(ii) *$g(X)$ is a closed subspace of X;*

(iii) *there exists a $\phi \in \Phi$ such that for all $x_i \in X$, $i = 1, 2, \ldots, n$ and $t > 0$,*

$$\begin{aligned} &V(fx_1, fx_2, \ldots, fx_n, \phi(t)) \\ &\quad \geq V(gx_1, gx_1, \ldots, fx_1, t) \\ &\qquad * V(gx_2, gx_2, \ldots, fx_2, t) * \ldots \\ &\qquad * V(gx_n, gx_n, \ldots, fx_n, t). \end{aligned} \tag{80}$$

Then f and g have a unique common fixed point.

Proof. Proof follows on the same lines of Theorem 33 and by definition of CLRg property. □

Data Availability

The related results, applications, definitions, and all other data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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