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Research Article

Memory State Feedback Control for Time-Varying Delay Switched Fuzzy Systems

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Consider the problem of memoryless state feedback controller for time-delay system, which cannot consider both the memoryless and the memory items in the system. Therefore, the memoryless state feedback controller has certain limitations and is more conservative. This paper addresses the memory state feedback control for the time-varying delay switched fuzzy systems based on T-S fuzzy model to overcome the problem discussed above. The state vector and input of the time-varying delay systems contain unknown time-varying delay with known bounds. The designed controller whose parameters are solvable can introduce past state information and reduce the system conservativeness. The more general Lyapunov-Krasovskii functional is selected and the switching law is designed in order to analyze the open-loop system stability, and the memory state feedback controller is designed for the closed-loop system and the criterion for its asymptotic stability. Discuss the solvability of the above two criteria. Finally, a numerical example is given. The simulation results show that the proposed method is more feasible and effective.

1. Introduction

Hybrid systems are more common in real physical systems. Complex systems in which discrete event subsystems interact with continuous variable subsystems are called hybrid dynamic systems [1–5]. Switched system, as classical hybrid dynamic system, has attracted much attention. And the model of switched system can be interpreted as a group of subsystems and switching laws (logical rules) describing the relationship between the subsystems [6–10]. In practice, many engineering systems, such as network control systems [11, 12], robot systems [13, 14], and electromechanical systems [15], can be described by the switched system models. The T-S fuzzy model provided new method for the nonlinear system problems [16–21]. The fuzzy rules describing the input-output relationship of each subsystem of nonlinear system are important components of the T-S fuzzy model. Each sublinear fuzzy system is connected by membership function. Therefore, the mature linear system theory can be used to solve the problems of nonlinear system based on T-S fuzzy model [22–25].

Switched fuzzy systems whose subsystems are composed of T-S fuzzy systems, Yang [26] et al. proposed in 2008, further optimize the switching fuzzy systems. The switching law is used to switch the subfuzzy system controlled by the designed controller. Compared with the switching fuzzy system, this system does not rely on the regional rules and optimizes the secondary fuzzy rules into first-order fuzzy rules. The switched fuzzy system model consisting of "switching" and "fuzzy" ideas does not have the restraint like traditional control system, broadening the performance requirements for control. It is suitable for more complex practical systems. Some researches have made results regarding the proposed switched fuzzy systems [27–34]. The network systems were transformed into switched fuzzy model. The free weighting matrix was used to deal with the delay and packet loss caused by the network in [35]. Authors in [36, 37] focused on stability analysis for the switched fuzzy systems; however, the designed controllers all are memoryless state feedback controllers. This kind of controller cannot have effect in the time-delay systems because it does not introduce the past state information of the systems. That is why memory state feedback controller has attracted much attention [38–44]. For example, in [45], a memory state feedback controller was given to control the affine linear parameter variable systems, which verified that the designed H_∞ memory state feedback controller had better immunity and effectively reduced the influence of time delay on the system.

Delays have been encountered in almost all real systems, which can be caused in many conditions. Generally, the transmission of information in a system often leads to the occurrence of time delay, such as communication systems and power systems. [46–54]. Scholars are keen to study stability for delay systems because delays in systems often lead to system performance degradation and instability. Reference [55] introduced the nonfragile control of memory state feedback for stochastic systems with time delays and presented the sufficient conditions for the asymptotic stability of closed-loop systems by combining Lyapunov function with linear matrix inequality (LMI). The delay-dependent switched fuzzy system was asymptotically stable by the observer switching method, when the actuator was seriously invalid in [56]. The stability conditions of time-delay-independent system are more conservative than delay-dependent system. It is worth noting that, although there are many studies on memory state feedback control, the control problem of memory state feedback has not been involved in time-varying delay (delay-dependent) switched fuzzy systems.

Therefore, this paper uses the Lyapunov function method to study the memory state feedback control problem for switched fuzzy time-varying delay switched fuzzy systems. The stability problem of the open-loop system is discussed, and the criterion for making the open-loop system asymptotically stable is given. The suitable switching law and the memory state feedback controller which has less conservativeness are designed and the more general Lyapunov functionals are selected in order to obtain the sufficient condition for the asymptotic stability of the closed-loop system. This sufficient condition is compared with that under the control of the memoryless state feedback controller for the closed-loop systems to explore the solvability problems. Simulation results testify that the proposed method is better than memoryless state feedback control under the same initial condition.

2. Analysis of Time-Varying Delay Switched Fuzzy Systems

Time-varying delay switched fuzzy model based on T-S fuzzy model is proposed. That is, each time-varying delay switched subsystem is composed of T-S fuzzy system. The model is different from the switching fuzzy model, and its switching rule is a first-level switching rule. Consider time-varying delay switched fuzzy systems consisting of $N_{\sigma(t)}$ rules as follows:

$$\begin{aligned}
R^j_{\sigma(t)}\colon \mathit{if}\quad & z_{\sigma(t)1}(t) \ \mathit{is}\ M^j_{\sigma(t)1} \cdots \ \mathit{and}\ z_{\sigma(t)p}(t)\ \mathit{is}\ M^j_{\sigma(t)p},\\
\mathit{then}\quad & \dot{x}(t)\\
&= A_{\sigma(t)j}x(t) + A_{q\sigma(t)j}x\left(t-h_{\sigma(t)}(t)\right)\\
&\quad + B_{\sigma(t)j}u_{\sigma(t)}(t),\\
& x(t) = \varphi(t), \quad t\in[-d,0],\\
& y(t) = C_{\sigma(t)j}x(t), \quad j = 1,2,\cdots,N_{\sigma(t)}
\end{aligned} \tag{1}$$

where $R^j_{\sigma(t)}$ indicates the jth fuzzy rule of the switched fuzzy model; $\sigma(t) : \overline{M} = \{1,2,\cdots,m\}$ is the switching signal to be designed; $z(t) = \{z_{\sigma(t)1}, z_{\sigma(t)2},\ldots,z_{\sigma(t)p}\}$ is the premise variable; $N_{\sigma(t)}$ is the number of fuzzy rules; $x(t) \in \mathbb{R}^n$ is the state variable vector of the systems; $y(t) \in \mathbb{R}^s$ is the output variable vector of the systems; $\varphi(t)$ is a continuously differentiable vector-valued initial function which satisfies $t \in [-d,0]$ and $d > 0$; $M^j_{\sigma(t)p}$ is fuzzy set; $u_{\sigma(t)j}(t) \in \mathbb{R}^q$ is the input variable of the jth switched subsystem; $A_{\sigma(t)j}$, $A_{q\sigma(t)j}$, $B_{\sigma(t)j}$, $C_{\sigma(t)h}$ are constant matrices of known appropriate dimensions; and $h_{\sigma(t)}(t)$ is time-varying delay function of the jth switched subsystem which meets the following inequalities called slow time-varying delay.

Assumption 1.

$$\begin{gathered}
0 \le h_{\sigma(t)}(t) \le d,\\
\dot{h}_{\sigma(t)}(t) \le h < 1
\end{gathered} \tag{2}$$

where h is a known constant.

2.1. Switching Signal Design. Switching signal $\sigma(x(t)) = \{1,\ x(t)\in\overline{\Omega}_i;\ 0,\ x(t)\notin\overline{\Omega}_i\}$ divides the entire space into m subregions, $\overline{\Omega}_1,\overline{\Omega}_2,\ldots,\overline{\Omega}_m$, if and only if $x(t)\in\overline{\Omega}_i$, $\sigma(x(t)) = 1$.

Switching law

$$\sigma = \sigma(x(t)) = i\colon\ [0,+\infty) \longrightarrow \overline{M} = \{1,2,\cdots,m\} \tag{3}$$

The jth subsystem of time-varying delay switched fuzzy systems is as follows:

$$\begin{aligned}
R^j_i\colon \mathit{if}\quad & z_{i1}(t)\ \mathit{is}\ M^j_{i1}\cdots \mathit{and}\ z_{ip}(t)\ \mathit{is}\ M^j_{ip},\\
\mathit{then}\quad & \dot{x}(t) = A_{ij}x(t) + A_{qij}x\left(t-h_i(t)\right) + B_{ij}u_i(t),\\
& x(t) = \varphi(t), \quad t\in[-d,0],\\
& y(t) = C_{ij}x(t),\\
&\qquad j = 1,2,\cdots,N_i,\ i = 1,2,\cdots,m
\end{aligned} \tag{4}$$

The membership function of the proposed time-varying delay switched fuzzy systems is

$$\eta_{ij}(z(t)) \triangleq \frac{\prod^p_{\rho=1} M^j_{i\rho}\left(z_\rho(t)\right)}{\sum^{N_i}_{j=1}\prod^p_{\rho=1} M^j_{i\rho}\left(z_\rho(t)\right)}, \tag{5}$$

$M^j_{i\rho}(z_\rho(t))$ denotes the membership function in which $z_\rho(t)$ belongs to the fuzzy set $M^j_{i\rho}$. And the membership function $\eta_{ij}(z(t))$ satisfies the following conditions. The membership function $\eta_{ij}(z(t))$ is expressed as η_{ij} in order to simplify the expression:

$$\begin{gathered}
0 \le \eta_{ij}(z(t)) \le 1,\\
\sum^{N_i}_{h=1}\eta_{ij}(z(t)) = 1
\end{gathered} \tag{6}$$

Therefore, the global model for the jth time-varying delay switched fuzzy systems based on T-S model is obtained:

$$\begin{aligned}
\dot{x}(t) &= \sum_{j=1}^{N_i} \eta_{ij}\left[A_{ij}x(t) + A_{qij}x(t-h_i(t)) + B_{ij}u_i(t)\right], \\
x(t) &= \varphi(t), \quad t \in [-d, 0], \\
y(t) &= \sum_{j=1}^{N_i} \eta_{ij} C_{ij} x(t), \quad i = 1, 2, \cdots, m
\end{aligned} \tag{7}$$

The open-loop system of the proposed systems based on T-S model is given:

$$\begin{aligned}
\dot{x}(t) &= \overline{A}(t)x(t) + \overline{A}_q(t)x(t-h_i(t)) \\
x(t) &= \varphi(t), \quad t \in [-d, 0], \\
y(t) &= \overline{C}(t)x(t), \quad j = 1, 2, \cdots, N_i, \; i = 1, 2, \cdots, m
\end{aligned} \tag{8}$$

along with

$$\begin{aligned}
\overline{A}(t) &\triangleq \sum_{i=1}^{m}\sum_{j=1}^{N_i} \eta_{ij} A_{ij}, \\
\overline{A}_q(t) &\triangleq \sum_{i=1}^{m}\sum_{j=1}^{N_i} \eta_{ij} A_{qij} \\
\overline{C}(t) &\triangleq \sum_{i=1}^{m}\sum_{j=1}^{N_i} \eta_{ij} C_{ij},
\end{aligned} \tag{9}$$

2.2. The Controller Form. Memoryless state feedback controller exists in most literature, which does not introduce past state information. Therefore, consider the memory state feedback controller form of the jth subsystem of the proposed time-varying delay switched fuzzy system as follows:

$$\begin{aligned}
R_i^j\text{: } &\textit{if} \quad z_{i1}(t) \textit{ is } M_{i1}^j \cdots \textit{and } z_{ip}(t) \textit{ is } M_{ip}^j, \\
&\textit{then} \quad u_i = K_{ij}x(t) + K_{1ij}x(t-h_i(t))
\end{aligned} \tag{10}$$

where K_{ij} and K_{1ij} are the two gain matrices of the memory state feedback controller to be sought. The global model of the memory state feedback controller can be described as follows:

$$u_i = \sum_{j=1}^{N_i} \eta_{ij}\left[K_{ij}x(t) + K_{1ij}x(t-h_i(t))\right] \tag{11}$$

According to the given switching law (3) and the memory state feedback controller (11), closed-loop system (12) is obtained:

$$\begin{aligned}
&\dot{x}(t) \\
&\quad = \sum_{i=1}^{m}\sum_{j=1}^{N_i}\eta_{ij}\sum_{r=1}^{N_i}\eta_{ir}\left[A_{ij}x(t)\right. \\
&\quad + A_{qij}x(t-h_i(t)) + B_{ij}K_{ir}x(t) \\
&\quad \left. + B_{ij}K_{1ir}x(t-h_i(t))\right]
\end{aligned} \tag{12}$$

Remark 2. In order to better study the proposed time-varying delay switched fuzzy systems, the stability of the open-loop system (8) is considered at first, that is, the characteristics of the system itself.

3. Main Results

In this section, the stability of the open-loop system and the control of the closed-loop system with feedback are mainly studied.

Lemma 3. *Given the constant matrices X and Y of the appropriate dimension, for any constant $\varepsilon > 0$, the following inequality holds:*

$$X^TY + Y^TX \le \varepsilon X^TX + \varepsilon^{-1}Y^TY \tag{13}$$

3.1. Stability Analysis for Open-Loop System.

Theorem 4. *Consider switched fuzzy systems (8); suppose there exist constants $\beta_{i\lambda}$ $(i, \lambda \in \overline{M})$ (either all nonnegative or all nonpositive); $h_{\sigma(t)}(t)$ satisfies Equation (2); there exist positive definite symmetric matrices P_{Li}, W_{Li}, Q_{Li}, and positive scalars ε_i, ε_{1i}, d, such that the matrix inequality (14) is satisfied; then system (8) is symptomatically stable:*

$$\begin{bmatrix}
\left\{A_{ij}^TP_{Li} + P_{Li}A_{ij} + \varepsilon_i^{-1}P_{Li}P_{Li} + A_{ij}^T\left(W_{Li} + \varepsilon_{1i}I\right)A_{ij}\right] + Q_{Li} + \sum_{\lambda=1}^{m}\beta_{i\lambda}\left(P_{Li} - P_{L\lambda}\right)\right\} & 0 \\
0 & \left\{\varepsilon_i A_{qij}^TA_{qij} + \varepsilon_{1i}^{-1}A_{qij}^TW_{Li}W_{Li}A_{qij} + A_{qij}^TW_{Li}A_{qij} - \left(1 - h_i(t)\right)Q_{Li}\right\}
\end{bmatrix} < 0 \quad i = 1, 2, \cdots, m, \; j, r = 1, 2, \cdots, N_i. \tag{14}$$

Proof. Without loss of generality, suppose $\beta_{i\lambda} \ge 0$. For any $i \in \overline{M}$, if $x^T(t)(P_{Li} - P_{L\lambda})x(t) \ge 0$, $\forall\lambda \in \overline{M}$. From (14), the inequality (15) is given:

$$\begin{bmatrix} \left\{A_{ij}^T P_{Li} + P_{Li} A_{ij} + \varepsilon_i^{-1} P_{Li} P_{Li} + A_{ij}^T \left(W_{Li} + \varepsilon_{1i} I\right) A_{ij}\right] + Q_{Li}\right\} & 0 \\ 0 & \left\{\varepsilon_i A_{qij}^T A_{qij} + \varepsilon_{1i}^{-1} A_{qij}^T W_{Li} W_{Li} A_{qij} + A_{qij}^T W_{Li} A_{qij} - \left(1 - h_i(t)\right) Q_{Li}\right\} \end{bmatrix} < 0 \tag{15}$$

Obviously, for $\forall x(t) \in \mathbb{R}^n \setminus \{0\}$, there certainly is an $i \in \overline{M}$ such that $x^T(t)(P_{Li} - P_{L\lambda})x(t) \geq 0$, $\forall \lambda \in \overline{M}$, For arbitrary $i \in \overline{M}$, let $\Omega_i = \{x(t) \in R^n \setminus \{0\} \mid x^T(t)(P_{Li} - P_{L\lambda})x(t) \geq 0, \forall \lambda \in \overline{M}\}$. Then $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$. Further, construct the sets $\overline{\Omega}_1 = \Omega_1, \cdots, \overline{\Omega}_i = \Omega_i - \bigcup_{\lambda=1}^{i-1} \overline{\Omega}_\lambda, \cdots, \overline{\Omega}_m = \Omega_m - \bigcup_{\lambda=1}^{m-1} \overline{\Omega}_\lambda$. Obviously, we have $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$, and $\overline{\Omega}_i \cap \overline{\Omega}_\lambda = \Phi$, $i \neq \lambda$.

Consider the following Lyapunov functional:

$$V_{Li}(x(t)) = V_{L1}(x(t)) + V_{L2}(x(t)) + V_{L3}(x(t)) \tag{16}$$

where

$$\begin{aligned} V_{L1}(x(t)) &= x^T(t) P_{Li} x(t) \\ V_{L2}(x(t)) &= d \int_{-d}^{0} \int_{t+\theta}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds\, d\theta \\ V_{L3}(x(t)) &= \int_{t-h_i(t)}^{t} x^T(s) Q_{Li} x(s)\, ds \end{aligned} \tag{17}$$

P_{Li}, W_{Li}, Q_{Li} are all positive definite symmetric matrices, which satisfy matrix inequality (14). When $x(t) \in \overline{\Omega}_i$, $i \in \overline{M}$, then

$$\dot{V}_{Li}(x(t)) = \dot{V}_{L1}(x(t)) + \dot{V}_{L2}(x(t)) + \dot{V}_{L3}(x(t)) \tag{18}$$

Assuming that inequality (2) is established, and according to Lemma 3, the following formulas are obtained:

$$\begin{aligned} \dot{V}_{L1}(x(t)) &= \dot{x}^T(t) P_{Li} x(t) + x^T(t) P_{Li} \dot{x}(t) \\ &= \sum_{j=1}^{N_i} \eta_{ij} \left\{\left[A_{ij} x(t) + A_{qij} x(t - h_i(t))\right]^T P_{Li} x(t) \right. \\ &\quad \left. + x^T(t) P_{Li} \left[A_{ij} x(t) + A_{qij} x(t - h_i(t))\right]\right\} \leq \sum_{j=1}^{N_i} \eta_{ij} \\ &\quad \cdot \left\{x^T(t) \left[A_{ij}^T P_{Li} + P_{Li} A_{ij} + \varepsilon_i^{-1} P_{Li} P_{Li}\right] x(t) + x(t \right. \\ &\quad \left. - h_i(t))^T \varepsilon_i A_{qij}^T A_{qij} x(t - h_i(t))\right\} \\ \dot{V}_{L2}(x(t)) &= d^2 \dot{x}^T(t) W_{Li} \dot{x}(t) - d \int_{t-d}^{t} \dot{x}^T(s) \\ &\quad \cdot W_{Li} \dot{x}(s)\, ds \leq d^2 \dot{x}^T(t) W_{Li} \dot{x}(t) \end{aligned} \tag{19}$$

$$\begin{aligned} &- d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds \\ &= \sum_{j=1}^{N_i} \eta_{ij} \left\{d^2 \left[A_{ij} x(t) + A_{qij} x(t - h_i(t))\right]^T \right. \\ &\quad \left. \cdot W_{Li} \left[A_{ij} x(t) + A_{qij} x(t - h_i(t))\right]\right\} \\ &- d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds \leq d^2 \sum_{j=1}^{N_i} \eta_{ij} \left\{x^T(t) \right. \\ &\quad \cdot A_{ij}^T W_{Li} A_{ij} x(t) + \varepsilon_{1i} x^T(t) A_{ij}^T A_{ij} x(t) \\ &\quad + \varepsilon_{1i}^{-1} x(t - h_i(t))^T A_{qij}^T W_{Li} W_{Li} A_{qij} x(t - h_i(t)) \\ &\quad \left. + x(t - h_i(t))^T A_{qij}^T W_{Li} A_{qij} x(t - h_i(t))\right\} \\ &- d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds = d^2 \sum_{j=1}^{N_i} \eta_{ij} \left\{x^T(t) \right. \\ &\quad \cdot \left[A_{ij}^T \left(W_{Li} + \varepsilon_{1i} I\right) A_{ij}\right] \times x(t) + x(t - h_i(t))^T \\ &\quad \cdot \left[\varepsilon_{1i}^{-1} A_{qij}^T W_{Li} W_{Li} A_{qij} + A_{qij}^T W_{Li} A_{qij}\right] \\ &\quad \left. \cdot x(t - h_i(t))\right\} - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds \end{aligned} \tag{20}$$

According to Assumption 1, $(1 - \dot{h}_i(t)) e^{-\alpha h_i(t)} \geq (1 - h_i) e^{-\alpha d} > 0$; then

$$\begin{aligned} \dot{V}_{L3}(x(t)) &= x^T(t) Q_{Li} x(t) - \left(1 - \dot{h}_i(t)\right) x^T(t \\ &\quad - h_i(t)) Q_{Li} x(t - h_i(t)) \leq \sum_{j=1}^{N_i} \eta_{ij} \left[x^T(t) Q_{Li} x(t) \right. \\ &\quad \left. - (1 - h_i(t)) x^T(t - h_i(t)) Q_{Li} x(t - h_i(t))\right] \end{aligned} \tag{21}$$

According to formula (19), formula (20), and formula (21), $\dot{V}_i(x(t))$ can be written in the following form

$$\begin{aligned}\dot{V}_{\mathrm{L}i}(x(t)) &= \dot{V}_{\mathrm{L}1}(x(t)) + \dot{V}_{\mathrm{L}2}(x(t)) + \dot{V}_{\mathrm{L}3}(x(t)) \le \sum_{j=1}^{N_i}\eta_{ij}\left\{x^T(t)\left[A_{ij}^T P_{\mathrm{L}i} + P_{\mathrm{L}i}A_{ij} + \varepsilon_i^{-1}P_{\mathrm{L}i}P_{\mathrm{L}i} + A_{ij}^T\left(W_{\mathrm{L}i} + \varepsilon_{1i}I\right)A_{ij}\right] + Q_{\mathrm{L}i}\right]x(t) \\ &+ x\left(t-h_i(t)\right)^T\left[\varepsilon_i A_{\mathrm{q}ij}^T A_{\mathrm{q}ij} + \varepsilon_{1i}^{-1}A_{\mathrm{q}ij}^T W_{\mathrm{L}i}W_{\mathrm{L}i}A_{\mathrm{q}ij} + A_{\mathrm{q}ij}^T W_{\mathrm{L}i}A_{\mathrm{q}ij} - \left(1-h_i(t)\right)Q_{\mathrm{L}i}\right] \times x\left(t-h_i(t)\right)\Big\} - d\int_{t-\tau}^{t}\dot{x}^T(s)W_{\mathrm{L}i}\dot{x}(s)\,ds \\ &= \sum_{j=1}^{N_i}\eta_{ij}\left\{\left[x^T(t)\,x\left(t-h_i(t)\right)^T\right]\right. \\ &\times \begin{bmatrix} A_{ij}^T P_{\mathrm{L}i} + P_{\mathrm{L}i}A_{ij} + \varepsilon_i^{-1}P_{\mathrm{L}i}P_{\mathrm{L}i} + A_{ij}^T\left(W_{\mathrm{L}i} + \varepsilon_{1i}I\right)A_{ij}] + Q_{\mathrm{L}i}\} & 0 \\ 0 & \left\{\varepsilon_i A_{\mathrm{q}ij}^T A_{\mathrm{q}ij} + \varepsilon_{1i}^{-1}A_{\mathrm{q}ij}^T W_{\mathrm{L}i}W_{\mathrm{L}i}A_{\mathrm{q}ij} + A_{\mathrm{q}ij}^T W_{\mathrm{L}i}A_{\mathrm{q}ij} - \left(1-h_i(t)\right)Q_{\mathrm{L}i}\right\}\end{bmatrix} \\ &\times \begin{bmatrix} x(t) \\ x\left(t-h_i(t)\right)\end{bmatrix} < 0\end{aligned} \tag{22}$$

We know inequality (22) is established from inequality (15). Therefore, $V_{\mathrm{L}i}(x(t))$ is the Lyapunov functional and the open-loop system (8) is asymptotically stable. □

Remark 5. The Lyapunov functions selected in this paper are composed of basic independent matrices, and the constant coefficient d is added to the $V_{\mathrm{L}2}(x(t))$ functions. So, the study of nonlinear systems (8) is more general.

3.2. Memory State Feedback Controller Design. The sufficient conditions for the stability of the open-loop system (8) have been obtained by the Lyapunov functional method. Suppose that the state of the proposed time-varying time-delay switched fuzzy system is measurable, suitable controller and switching law can be designed to make the closed-loop system (12) asymptotically stable.

Theorem 6. *Suppose there exist constants $\beta_{i\lambda}$ $(i,\lambda \in \overline{M})$ (either all nonnegative or all nonpositive); there exist positive scalars $\varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}, d$, and positive definite symmetric matrices P_i, W_i, Q_i, $h_{\sigma(t)}(t)$ satisfy Equation (2), such that the matrix inequalities (23) are satisfied; then there are memory state feedback controller (11) and switching law $\sigma(t)$: $\overline{M} = \{1, 2, \cdots, m\}$ to make system (12) symptomatically stable:*

$$\begin{bmatrix}\prod_{11} & 0 \\ 0 & \prod_{22}\end{bmatrix} < 0 \tag{23}$$

$$i = 1, 2, \cdots, m, \quad j, r = 1, 2, \cdots, N_i$$

along with

$$\begin{aligned}\prod_{11} &= A_{ij}^T P_i + P_i A_{ij} + K_{ir}^T B_{ij}^T P_i + P_i B_{ij}K_{ir} + \varepsilon_{2i}^{-1}P_i P_i \\ &+ \varepsilon_{3i}^{-1}P_i P_i + d^2\left(A_{ij}^T + K_{ir}^T B_{ij}^T\right)\left(W_i + \varepsilon_{4i}I\right)\left(A_{ij}\right. \\ &\left.+ B_{ij}K_{ir}\right) + Q_i + \sum_{\lambda=1}^{m}\beta_{i\lambda}\left(P_i - P_\lambda\right) \\ \prod_{22} &= \varepsilon_{2i}A_{\mathrm{q}ij}^T A_{\mathrm{q}ij} + \varepsilon_{3i}K_{1ir}^T B_{ij}^T B_{ij}K_{1ir} - \left(1-h_i(t)\right) \\ &\cdot Q_i + d^2\left[\left(A_{\mathrm{q}ij}^T + K_{1ir}^T B_{ij}^T\right)W_i\left(A_{\mathrm{q}ij} + B_{ij}K_{1ir}\right)\right. \\ &\left.+ \varepsilon_{4i}^{-1}\left(A_{\mathrm{q}ij}^T + K_{1ir}^T B_{ij}^T\right)W_i W_i\left(A_{\mathrm{q}ij} + B_{ij}K_{1ir}\right)\right]\end{aligned} \tag{24}$$

Proof. Without loss of generality, suppose $\beta_{i\lambda} \ge 0$. For any $i \in \overline{M}$, if $x^T(t)(P_i - P_\lambda)x(t) \ge 0, \forall\lambda \in \overline{M}$. From (23), we have

$$\begin{bmatrix}\Gamma_{11} & 0 \\ 0 & \Gamma_{22}\end{bmatrix} < 0 \tag{25}$$

where

$$\begin{aligned}\Gamma_{11} &= A_{ij}^T P_i + P_i A_{ij} + K_{ir}^T B_{ij}^T P_i + P_i B_{ij}K_{ir} + \varepsilon_{2i}^{-1}P_i P_i \\ &+ \varepsilon_{3i}^{-1}P_i P_i \\ &+ d^2\left(A_{ij}^T + K_{ir}^T B_{ij}^T\right)\left(W_i + \varepsilon_{4i}I\right)\left(A_{ij} + B_{ij}K_{ir}\right)] \\ &+ Q_i \\ \Gamma_{22} &= \prod_{22}\end{aligned} \tag{26}$$

Obviously, for $\forall x(t) \in \mathbb{R}^n \setminus \{0\}$, there certainly is an $i \in \overline{M}$ such that $x^T(t)(P_i - P_\lambda)x(t) \ge 0, \forall\lambda \in \overline{M}$, For arbitrary $i \in \overline{M}$, let $\Omega_i = \{x(t) \in R^n \setminus \{0\} | x^T(t)(P_i - P_\lambda)x(t) \ge 0, \forall\lambda \in \overline{M}\}$. Then $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$. Further, construct the sets $\overline{\Omega}_1 = \Omega_1, \cdots, \overline{\Omega}_i = \Omega_i - \bigcup_{\lambda=1}^{i-1}\overline{\Omega}_\lambda, \cdots, \overline{\Omega}_m = \Omega_m - \bigcup_{\lambda=1}^{m-1}\overline{\Omega}_\lambda$. Obviously, we have $\bigcup_{i=1}^m \Omega_i = \mathbb{R}^n \setminus \{0\}$, and $\overline{\Omega}_i \cap \overline{\Omega}_\lambda = \Phi, i \ne \lambda$.

Consider more general Lyapunov functional:

$$V_i(x(t)) = V_1(x(t)) + V_2(x(t)) + V_3(x(t)) \tag{27}$$

where

$$\begin{aligned}V_1(x(t)) &= x^T(t)P_i x(t) \\ V_2(x(t)) &= d\int_{-d}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)W_i\dot{x}(s)\,ds\,d\theta \\ V_3(x(t)) &= \int_{t-h_i(t)}^{t} x^T(s)Q_i x(s)\,ds\end{aligned} \tag{28}$$

P_i, W_i, Q_i are positive definite symmetric matrices satisfying the matrix inequality (23). When $x(t) \in \overline{\Omega}_i$, $i \in \overline{M}$, there is

$$\dot{V}_i(x(t)) = \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) \tag{29}$$

Assuming that inequality (2) is established, and according to Lemma 3, the following formulas are obtained:

$$\begin{aligned}
\dot{V}_1(x(t)) &= \dot{x}^T(t) P_i x(t) + x^T(t) P_i \dot{x}(t) \\
&= \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ \left[A_{ij} x(t) \right.\right. \\
&\quad + A_{qij} x(t - h_i(t)) + B_{ij} K_{ir} x(t) \\
&\quad \left. + B_{ij} K_{1ir} x(t - h_i(t)) \right]^T P_i x(t) + x^T(t) P_i \left[A_{ij} x(t) \right. \\
&\quad + A_{qij} x(t - h_i(t)) + B_{ij} K_{ir} x(t) \\
&\quad \left.\left. + B_{ij} K_{1ir} x(t - h_i(t)) \right] \right\} \\
&\le \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ x^T(t) \right. \\
&\quad \cdot \left[A_{ij}^T P_{Li} + P_{Li} A_{ij} + P_i B_{ij} K_{ir} + \varepsilon_{2i}^{-1} P_i P_i + \varepsilon_{3i}^{-1} P_i P_i \right. \\
&\quad \left. + K_{ir}^T B_{ij}^T P_i \right] x(t) + x(t - h_i(t))^T \left[\varepsilon_{3i} K_{1ir}^T B_{ij}^T B_{ij} K_{1ir} \right. \\
&\quad \left.\left. + \varepsilon_{2i} A_{qij}^T A_{qij} \right] x(t - h_i(t)) \right\}
\end{aligned} \tag{30}$$

$$\begin{aligned}
\dot{V}_2(x(t)) &= d^2 \dot{x}^T(t) W_i \dot{x}(t) - d \int_{t-d}^{t} \dot{x}^T(s) W_i \dot{x}(s)\, ds \\
&\le d^2 \dot{x}^T(t) W_i \dot{x}(t) \\
&\quad - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) \\
&\quad \cdot W_i \dot{x}(s)\, ds \\
&= \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ d^2 \left[A_{ij} x(t) \right.\right. \\
&\quad + A_{qij} x(t - h_i(t)) + B_{ij} K_{ir} x(t) \\
&\quad \left. + B_{ij} K_{1ir} x(t - h_i(t)) \right]^T W_i \left[A_{ij} x(t) \right. \\
&\quad + A_{qij} x(t - h_i(t)) + B_{ij} K_{ir} x(t) \\
&\quad \left.\left. + B_{ij} K_{1ir} x(t - h_i(t)) \right] \right\} \\
&\quad - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) \\
&\quad \cdot W_i \dot{x}(s)\, ds \\
&= d^2 \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ x^T(t) \right. \\
&\quad \cdot \left[\left(A_{ij}^T + K_{ir}^T B_{ij}^T \right) W_i \times \left(A_{ij} + B_{ij} K_{ir} \right) \right] x(t) + x^T(t) \\
&\quad \cdot \left[\left(A_{ij}^T + K_{ir}^T B_{ij}^T \right) W_i \times \left(A_{qij} + B_{ij} K_{1ir} \right) \right] x(t \\
&\quad - h_i(t)) + x(t - h_i(t))^T \left[\left(A_{qij}^T + K_{1ir}^T B_{ij}^T \right) W_i \right. \\
&\quad \left. \times \left(A_{ij} + B_{ij} K_{ir} \right) \right] x(t) + x(t - h_i(t))^T \\
&\quad \cdot \left[\left(A_{qij}^T + K_{1ir}^T B_{ij}^T \right) W_i \times \left(A_{qij} + B_{ij} K_{1ir} \right) \right] x(t \\
&\quad \left. - h_i(t)) \right\} - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_i \dot{x}(s)\, ds \\
&\le d^2 \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ x^T(t) \right. \\
&\quad \cdot \left[\left(A_{ij}^T + K_{ir}^T B_{ij}^T \right) \times (W_i + \varepsilon_{4i} I) \times \left(A_{ij} + B_{ij} K_{ir} \right) \right] \\
&\quad \cdot x(t) + x(t - h_i(t))^T \left[\left(A_{qij}^T + K_{1ir}^T B_{ij}^T \right) W_i \right. \\
&\quad \times \left(A_{qij} + B_{ij} K_{1ir} \right) + \varepsilon_{4i}^{-1} \left(A_{qij}^T + K_{1ir}^T B_{ij}^T \right) \\
&\quad \left.\left. \times W_i W_i \left(A_{qij} + B_{ij} K_{1ir} \right) \right] x(t - h_i(t)) \right\} \\
&\quad - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) \\
&\quad \cdot W_i \dot{x}(s)\, ds
\end{aligned} \tag{31}$$

$$\begin{aligned}
\dot{V}_3(x(t)) &= x^T(t) Q_i x(t) - (1 \\
&\quad - \dot{h}_i(t)) x^T(t - h_i(t)) Q_i x(t - h_i(t)) \\
&\le \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left[x^T(t) \right. \\
&\quad \cdot Q_i x(t) - (1 - h_i(t)) x^T(t - h_i(t)) Q_i x(t \\
&\quad \left. - h_i(t)) \right]
\end{aligned} \tag{32}$$

From formulas (30)–(32), we get formula (33):

$$\begin{aligned}
\dot{V}_i(x(t)) &= \dot{V}_1(x(t)) + \dot{V}_2(x(t)) + \dot{V}_3(x(t)) \\
&\le \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \left\{ x^T(t) \right. \\
&\quad \cdot \left[A_{ij}^T P_i + P_i A_{ij} + K_{ir}^T B_{ij}^T P_i + P_i B_{ij} K_{ir} + \varepsilon_{3i}^{-1} P_i P_i \right. \\
&\quad + \varepsilon_{2i}^{-1} P_i P + Q_i \\
&\quad \left. + d^2 \left(A_{ij}^T + K_{ir}^T B_{ij}^T \right) (W_i + \varepsilon_{4i} I) \left(A_{ij} + B_{ij} K_{ir} \right) \right] \\
&\quad \cdot x(t) + x(t - h_i(t))^T \left[\varepsilon_{2i} A_{qij}^T A_{qij} \right.
\end{aligned}$$

$$
\begin{aligned}
&+ \varepsilon_{3i} K_{1ir}^T B_{ij}^T B_{ij} K_{1ir} - \left(1 - h_i(t)\right) Q_i \\
&+ d^2 \left[\left(A_{qij}^T + K_{1ir}^T B_{ij}^T\right) W_i \left(A_{qij} + B_{ij} K_{1ir}\right) \right. \\
&\left. + \varepsilon_{4i}^{-1} \left(A_{qij}^T + K_{1ir}^T B_{ij}^T\right) W_i W_i \left(A_{qij} + B_{ij} K_{1ir}\right) \right] x(t \\
&- h_i(t)) \Big\} - d \int_{t-h_i(t)}^{t} \dot{x}^T(s) W_{Li} \dot{x}(s)\, ds
\end{aligned}
\tag{33}
$$

Equation (33) can be rewritten as

$$
\dot{V}_i(t) \le \sum_{j=1}^{N_i} \eta_{ij} \sum_{r=1}^{N_i} \eta_{ir} \begin{bmatrix} x(t) \\ x(t - h_i(t)) \end{bmatrix}^T \Theta_{ijr} \begin{bmatrix} x(t) \\ x(t - h_i(t)) \end{bmatrix} \tag{34}
$$

in which

$$
\Theta_{ijr} = \begin{bmatrix} \begin{matrix} A_{ij}^T P_i + P_i A_{ij} + K_{ir}^T B_{ij}^T P_i + P_i B_{ij} K_{ir} + \varepsilon_{2i}^{-1} P_i P_i + \varepsilon_{3i}^{-1} P_i P_i \\ + Q_i + d^2 \left(A_{ij}^T + K_{ir}^T B_{ij}^T\right) \times \left(W_i + \varepsilon_{4i} I\right) \left(A_{ij} + B_{ij} K_{ir}\right) \end{matrix} & 0 \\ 0 & \begin{matrix} \varepsilon_{2i} A_{qij}^T A_{qij} + \varepsilon_{3i} K_{1ir}^T B_{ij}^T B_{ij} K_{1ir} - \left(1 - h_i(t)\right) Q_i \\ + d^2 \left[\left(A_{qij}^T + K_{1ir}^T B_{ij}^T\right) \times W_i \left(A_{qij} + B_{ij} K_{1ir}\right) + \varepsilon_{4i}^{-1} \left(A_{qij}^T + K_{1ir}^T B_{ij}^T\right) W_i \times W_i \left(A_{qij} + B_{ij} K_{1ir}\right) \right] \end{matrix} \end{bmatrix} \tag{35}
$$

When Equation $\Theta_{ijr} < 0$, $\dot{V}(x(t)) < 0$ can be obtained. Therefore, the controller (11) can ensure that system (12) is asymptotically stable under the switching law $\sigma(x(t)) = i$. □

Remark 7. The integral term $-d \int_{t-\tau}^{t} \dot{x}^T(s) W_i \dot{x}(s) ds < 0$ is obtained, because $0 < d < 1$, W_i is a positive definite symmetry matrix and the physical meaning of the integral term is the area.

There are Lyapunov functional matrices P_i, W_i and closed-loop system product terms in Equation (23),which can be seen from Theorem 6. In order to solve this problem, an additional matrix is introduced to decouple, and feasible solution in which Equation (23) depends on linear matrix inequalities is obtained.

Theorem 8. *Suppose there exist constants $\beta_{i\lambda}$ $(i, \lambda \in \overline{M})$ (either all nonnegative or all nonpositive); there exist positive scalars $\varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}, d$ and positive definite symmetric matrices P_i, W_i, Q_i, S_i, continuously differentiable symmetry matrices Y_{ir}, Y_{1ir}, $h_{\sigma(t)}(t)$ satisfy Equation (2), such that the matrix inequalities (36) are satisfied; then there are memory state feedback controller (42) and switching law $\sigma(t)$: $\overline{M} = \{1, 2, \cdots, m\}$ to make system (12) symptomatically stable:*

$$
\begin{bmatrix}
\gamma_{11} & \gamma_{12} & \gamma_{13} & 0 & 0 & 0 & 0 & 0 \\
* & \gamma_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\
* & 0 & \gamma_{33} & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \gamma_{44} & \gamma_{45} & \gamma_{46} & \gamma_{47} & \gamma_{48} \\
0 & 0 & 0 & * & \gamma_{55} & 0 & 0 & 0 \\
0 & 0 & 0 & * & 0 & \gamma_{66} & 0 & 0 \\
0 & 0 & 0 & * & 0 & 0 & \gamma_{77} & 0 \\
0 & 0 & 0 & * & 0 & 0 & 0 & \gamma_{88}
\end{bmatrix} < 0
$$

$$
i = 1, 2, \cdots, m, \quad j, r = 1, 2, \cdots, N_i \tag{36}
$$

along with

$$
\begin{aligned}
\gamma_{11} &= X_i A_{ij}^T + A_{ij} X_i + Y_{ir}^T B_{ij}^T + B_{ij} Y_{ir} + \left(\varepsilon_{2i}^{-1} + \varepsilon_{3i}^{-1}\right) I \\
&\quad + S_i + \sum_{\lambda=1}^{m} \beta_{i\lambda} \left(P_i - P_\lambda\right) \\
\gamma_{12} &= X_i A_{ij}^T + Y_{ir}^T B_{ij}^T \\
\gamma_{13} &= X_i A_{ij}^T + Y_{ir}^T B_{ij}^T \\
\gamma_{22} &= -\left(d^2 W_i\right)^{-1} \\
\gamma_{33} &= -\left(\varepsilon_{4i} d^2 I\right)^{-1} \\
\gamma_{44} &= -\left(1 - h_i(t)\right) S_i \\
\gamma_{45} &= X_i A_{qij}^T \\
\gamma_{46} &= Y_{1ir}^T B_{ij}^T \\
\gamma_{47} &= X_i A_{qij}^T + Y_{1ir}^T B_{ij}^T \\
\gamma_{48} &= X_i A_{qij}^T + Y_{1ir}^T B_{ij}^T \\
\gamma_{55} &= -\left(\varepsilon_{2i} I\right)^{-1} \\
\gamma_{66} &= -\left(\varepsilon_{3i} I\right)^{-1} \\
\gamma_{77} &= -\left(d^2 W_i\right)^{-1} \\
\gamma_{88} &= -\left(d^2\right)^{-1} \varepsilon_{4i} I
\end{aligned}
\tag{37}
$$

Proof. Using the Schur method for the matrix inequality (35)

$$\begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & 0 & 0 & 0 & 0 & 0 \\ * & \gamma_{22} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & 0 & \gamma_{33} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \psi_{44} & \psi_{45} & \psi_{46} & \psi_{47} & \psi_{48} \\ 0 & 0 & 0 & * & \gamma_{55} & 0 & 0 & 0 \\ 0 & 0 & 0 & * & 0 & \gamma_{66} & 0 & 0 \\ 0 & 0 & 0 & * & 0 & 0 & \gamma_{77} & 0 \\ 0 & 0 & 0 & * & 0 & 0 & 0 & \gamma_{88} \end{bmatrix} < 0 \qquad (38)$$

in which

$$\begin{aligned} \psi_{11} &= A_{ij}^T P_i + P_i A_{ij} + K_{ir}^T B_{ij}^T P_i + P_i B_{ij} K_{ir} + \varepsilon_{2i}^{-1} P_i P_i \\ &\quad + \varepsilon_{3i}^{-1} P_i P_i + Q_i \\ \psi_{12} &= A_{ij}^T + K_{ir}^T B_{ij}^T \\ \psi_{13} &= A_{ij}^T + K_{ir}^T B_{ij}^T \\ \psi_{44} &= -\left(1 - h_i(t)\right) Q_i \\ \psi_{45} &= A_{qij}^T \\ \psi_{46} &= K_{1ir}^T B_{ij}^T \\ \psi_{47} &= A_{\mathrm{q}ij}^T + K_{1ir}^T B_{ij}^T \\ \psi_{48} &= A_{\mathrm{q}ij}^T + K_{1ir}^T B_{ij}^T \end{aligned} \qquad (39)$$

Multiply matrix $\operatorname{diag}\{P_i^{-1}, I, I, P_i^{-1}, I, I, I, W_i^{-1}\}$ by Equation (38) and define it:

$$\begin{aligned} X_i &= P_i^{-1}, \\ S_i &= X_i Q_i X_i, \\ Y_{ir} &= K_{ir} X_i, \\ Y_{1ir} &= K_{1ir} X_i \end{aligned} \qquad (40)$$

Then, formula (23) is equivalent to formula (36). Proof is to be true. □

The gain matrices of the memory state feedback controller are obtained by solving LMI (36):

$$\begin{aligned} K_{ir} &= Y_{ir} X_i^{-1}, \\ K_{1ir} &= Y_{1ir} X_i^{-1} \end{aligned} \qquad (41)$$

The global model (11) of the memory state feedback controller can be written in the following form:

$$u_i = \sum_{j=1}^{N_i} \eta_{ij} \left[Y_{ij} X_i^{-1} x(t) + Y_{1ij} X_i^{-1} x\left(t - h_i(t)\right) \right] \qquad (42)$$

Inference 1. Suppose there exist constants $\beta_{i\lambda}$ $(i, \lambda \in \overline{M})$ (either all nonnegative or all nonpositive); there exist positive scalars $\varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}, d$ and positive definite symmetric matrices P_i, W_i, Q_i, S_i; continuously differentiable symmetry matrices Y_{ir}, Y_{1ir}, $h_{\sigma(t)}(t)$ satisfy Equation (2) and then the matrix inequalities

$$\begin{bmatrix} \mathrm{E}_{11} & \gamma_{12} & \gamma_{13} & 0 & 0 & 0 & 0 \\ * & \gamma_{22} & 0 & 0 & 0 & 0 & 0 \\ * & 0 & \gamma_{33} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \gamma_{44} & \gamma_{45} & \mathrm{E}_{46} & \mathrm{E}_{47} \\ 0 & 0 & 0 & * & \gamma_{55} & 0 & 0 \\ 0 & 0 & 0 & * & 0 & \mathrm{E}_{66} & 0 \\ 0 & 0 & 0 & * & 0 & 0 & \mathrm{E}_{77} \end{bmatrix} < 0 \qquad (43)$$

where

$$\begin{aligned} \mathrm{E}_{11} &= X_i A_{ij}^T + A_{ij} X_i + Y_{ir}^T B_{ij}^T + B_{ij} Y_{ir} + \varepsilon_{2i}^{-1} I + S_i \\ &\quad + \sum_{\lambda=1}^{m} \beta_{i\lambda} \left(P_i - P_\lambda\right) \\ \gamma_{12} &= X_i A_{ij}^T + Y_{ir}^T B_{ij}^T \\ \gamma_{13} &= X_i A_{ij}^T + Y_{ir}^T B_{ij}^T \\ \gamma_{22} &= -\left(d^2 W_i\right)^{-1} \\ \gamma_{33} &= -\left(\varepsilon_{4i} d^2 I\right)^{-1} \\ \gamma_{44} &= -\left(1 - h_i(t)\right) S_i \\ \gamma_{45} &= X_i A_{\mathrm{q}ij}^T \\ \mathrm{E}_{46} &= X_i A_{\mathrm{q}ij}^T \\ \mathrm{E}_{47} &= X_i A_{\mathrm{q}ij}^T \\ \gamma_{55} &= -\left(\varepsilon_{2i} I\right)^{-1} \\ \mathrm{E}_{66} &= \gamma_{77} = -\left(d^2 W_i\right)^{-1} \\ \mathrm{E}_{77} &= \gamma_{88} = -\left(d^2\right)^{-1} \varepsilon_{4i} I \end{aligned} \qquad (44)$$

The closed-loop system is asymptotically stable, when the memoryless state feedback controller is $u_i = K_{\mathrm{q}ij} X_i^{-1} x(t)$, $K_{\mathrm{q}ir} = Y_{ir} X_i^{-1}$ and the switching law is $\sigma(t) : \overline{M} = \{1, 2, \cdots, m\}$.

Remark 9. This paper discusses the situation that the designed controller can stabilize the original system regardless of time-varying delay of the systems. Comparing the linear matrix inequalities (36) with (43), it can be seen that when the inequality (36) has a solution, Equation (43)˜must have a solution (take Y_{1ir} to 0); on the contrary, it is not necessarily true. Formula (36) has better solvability than Formula (43).

Therefore, the memory state feedback controller can control the system in which the memoryless state feedback controller is not controllable.

Remark 10. Equation (23) is transformed into a strict linear matrix inequality (36) by Schur theorem and the matrix inequality method. The transformed LMI matrix increases the conservativeness of the result but makes the final result MATLAB solvable.

4. Simulation Results

Consider the switched fuzzy model with time-varying delay as follows in order to prove the effectiveness of the proposed control method:

$$\begin{aligned}
R_1^1&: if \quad \xi \ is\ M_{11}^1,\\
then &\quad \dot{x}(t)\\
&= A_{11}x(t) + A_{q11}x(t-h_1(t)) + B_{11}u_1(t)\\
&y(t) = C_{11}x(t)\\
R_1^2&: if \quad \xi \ is\ M_{11}^2,\\
then &\quad \dot{x}(t)\\
&= A_{12}x(t) + A_{q12}x(t-h_1(t)) + B_{12}u_1(t)\\
&y(t) = C_{12}x(t)\\
R_2^1&: if \quad \delta \ is\ M_{21}^1,\\
then &\quad \dot{x}(t)\\
&= A_{21}x(t) + A_{q21}x(t-h_2(t)) + B_{21}u_2(t)\\
&y(t) = C_{21}x(t)\\
R_2^2&: if \quad \delta \ is\ M_{21}^2,\\
then &\quad \dot{x}(t)\\
&= A_{22}x(t) + A_{q22}x(t-h_2(t)) + B_{22}u_2(t)\\
&y(t) = C_{22}x(t)
\end{aligned} \tag{45}$$

where

$$\begin{aligned}
A_{11} &= \begin{bmatrix} -11 & 1 \\ 12 & -18 \end{bmatrix},\\
A_{12} &= \begin{bmatrix} -21 & 6 \\ 19 & 0 \end{bmatrix},\\
A_{21} &= \begin{bmatrix} -11.3 & 3 \\ 14 & -17 \end{bmatrix}\\
A_{22} &= \begin{bmatrix} -11 & 6 \\ 18 & 0 \end{bmatrix},\\
B_{11} &= \begin{bmatrix} -17 & 3 \\ 2 & 28 \end{bmatrix},\\
B_{12} &= \begin{bmatrix} -18 & -2 \\ -22 & 38 \end{bmatrix}\\
B_{21} &= \begin{bmatrix} -15.23 & 4 \\ 3.01 & 27 \end{bmatrix},\\
B_{22} &= \begin{bmatrix} -19 & -1 \\ -21 & 37 \end{bmatrix},\\
A_{q11} &= \begin{bmatrix} -35 & 41 \\ 31 & 13 \end{bmatrix},\\
A_{q12} &= \begin{bmatrix} -37 & 0.3 \\ 0.1 & 14 \end{bmatrix},\\
A_{q21} &= \begin{bmatrix} -27 & 40 \\ 29 & 15 \end{bmatrix}\\
A_{q22} &= \begin{bmatrix} -36 & 0.2 \\ 0.3 & 16 \end{bmatrix}
\end{aligned} \tag{46}$$

Select the time-varying delay functions depending on [48] as follows:

$$h_1(t) = 0.003 - 0.003\sin(t) \tag{47a}$$

$$h_2(t) = 0.01 - 0.01\sin(t) \tag{47b}$$

Then, $h_1 \le 0.003$, $h_2 \le 0.01$, and $d \le 0.02$

$M_{11}^1, M_{11}^2, M_{21}^1, M_{21}^2$ are represented by the following membership functions:

$$\begin{aligned}
M_{11}^1(x_2(t)) &= \eta_{11}(z(t)) = 1 - \frac{1}{1+e^{-2z(t)}},\\
M_{11}^2(z(t)) &= \eta_{12}(z(t)) = \frac{1}{1+e^{-2z(t)}},\\
M_{21}^1(\delta(t)) &= \eta_{21}(\delta(t)) = 1 - \frac{1}{1+e^{(-2(\sigma(t)-0.3))}},\\
M_{21}^2(\delta(t)) &= \eta_{22}(\delta(t)) = \frac{1}{1+e^{(-2(\delta(t)-0.3))}}
\end{aligned} \tag{48}$$

The parameters in the stability criterion are as follows:

$\varepsilon_{2i} = \varepsilon_{3i} = \varepsilon_{4i} = 0.5$, $\beta_{i\lambda} = 1$, $r, j, i = 1, 2$. Using MATLAB to solve the linear matrix inequality (36), we get

$$\begin{aligned}
P_1 &= \begin{bmatrix} 0.1014 & 0.0411 \\ 0.0411 & 0.0531 \end{bmatrix},\\
P_2 &= \begin{bmatrix} 0.0987 & 0.0280 \\ 0.0280 & 0.0411 \end{bmatrix},\\
K_{11} &= \begin{bmatrix} 9.9351 & 4.8792 \\ -1.0104 & -2.7214 \end{bmatrix},\\
K_{12} &= \begin{bmatrix} 9.9351 & 4.8792 \\ -1.0104 & -2.7214 \end{bmatrix}\\
K_{21} &= \begin{bmatrix} 9.4119 & 3.3242 \\ -0.6760 & -2.4021 \end{bmatrix},
\end{aligned}$$

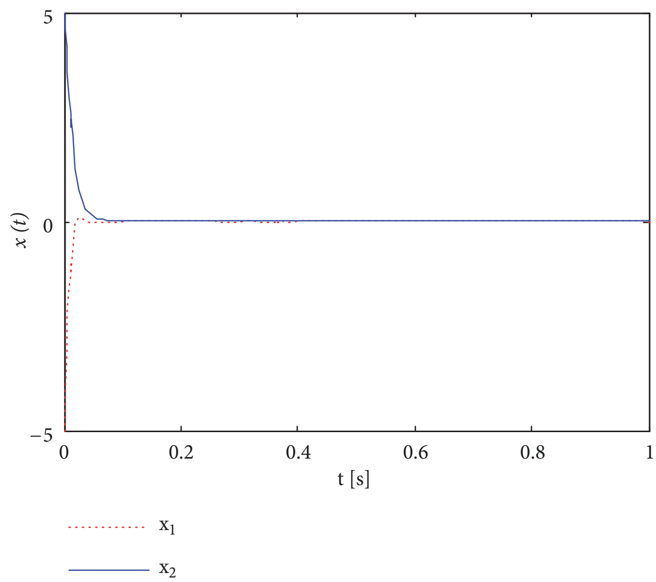

FIGURE 1: State response curve of closed-loop system controlled by memory state feedback controller.

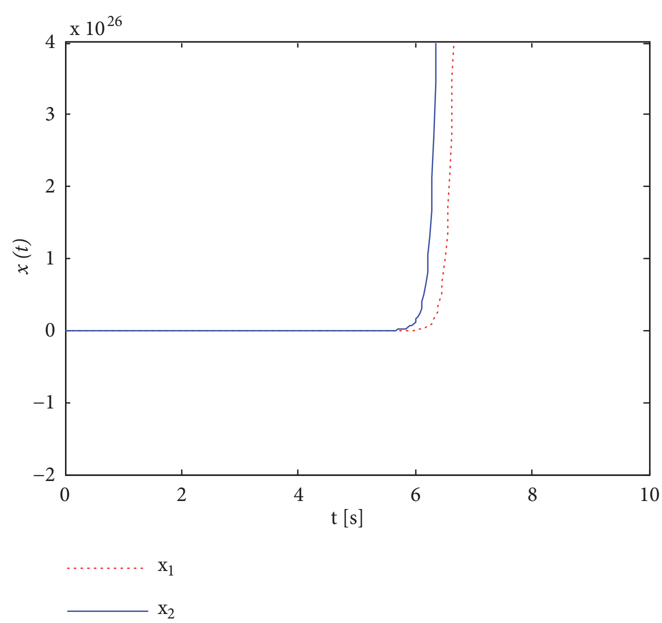

FIGURE 2: State response curve of closed-loop system under control of memoryless feedback controller.

$$K_{22} = \begin{bmatrix} 9.4119 & 3.3242 \\ -0.6760 & -2.4021 \end{bmatrix}$$
$$K_{111} = \begin{bmatrix} 0.0253 & 0.0152 \\ 0.0039 & 0.0041 \end{bmatrix}, \quad K_{112} = \begin{bmatrix} 0.0253 & 0.0152 \\ 0.0039 & 0.0041 \end{bmatrix}$$
$$K_{121} = \begin{bmatrix} 0.0193 & 0.0090 \\ 0.0005 & 0.0020 \end{bmatrix}, \quad K_{122} = \begin{bmatrix} 0.0193 & 0.0090 \\ 0.0005 & 0.0020 \end{bmatrix} \tag{49}$$

And the gain of the memoryless feedback controller:

$$K_{q11} = \begin{bmatrix} 0.8958 & 0.2771 \\ -0.0165 & -0.0911 \end{bmatrix}, \quad K_{q12} = \begin{bmatrix} 0.8958 & 0.2771 \\ -0.0165 & -0.0911 \end{bmatrix}$$
$$K_{q21} = \begin{bmatrix} 0.7328 & 0.1181 \\ 0.0246 & -0.0647 \end{bmatrix}, \quad K_{q22} = \begin{bmatrix} 0.7328 & 0.1181 \\ 0.0246 & -0.0647 \end{bmatrix} \tag{50}$$

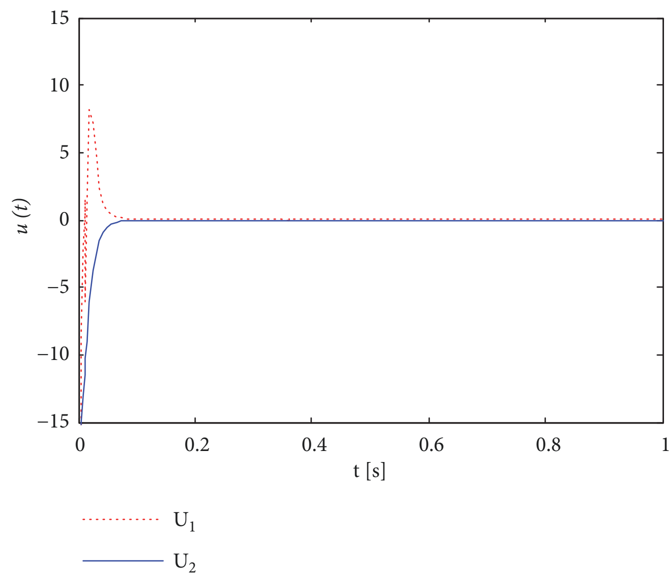

Figure 3: Control curve of memory state feedback controller.

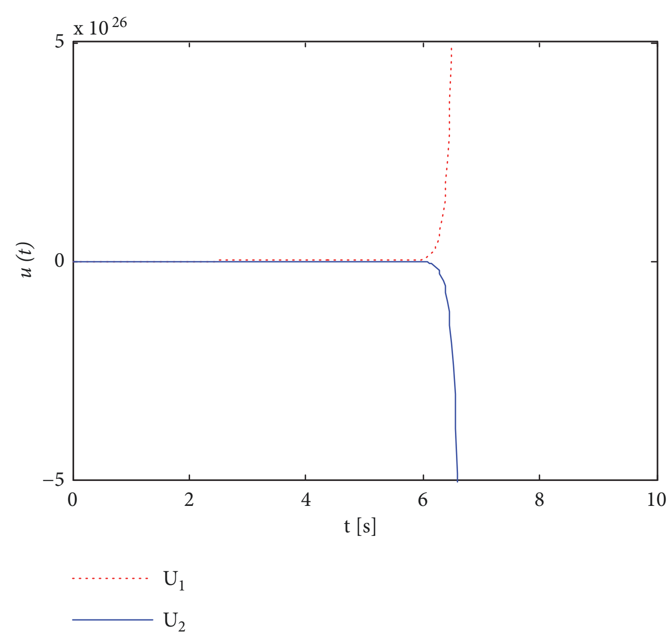

Figure 4: Control curve of memoryless state feedback controller.

Figures 1 and 2 show the state response curves of the closed-loop system (12) controlled by the memory state feedback controller and the memoryless state feedback controller, with the same initial conditions and the switching laws, under the simulink simulation of MATLAB. Figures 3 and 4 show the control curves of the two controllers, where $x(0) = [5, -5]$.

The state response curve in Figure 1 quickly converges to 0, and the closed-loop system is asymptotically stable; the state response curve in Figure 2 is divergent and the closed-loop system is unstable, which are shown from Figures 1 and 2 under the same initial conditions. The control curve of the memory state feedback controller converges rapidly and the control curve of the memoryless state feedback controller diverges in Figures 3 and 4 under the same initial conditions. Therefore, the control effect with the memory state feedback controller is better than the memoryless state feedback controller for the time-varying delay switched fuzzy system under the given initial conditions.

The initial value is changed to 100 times; that is, $x(0) = [500, -500]$; we get the state response curve and control curve of the time-varying delay switched fuzzy system in Figures 5 and 6 under the memory state feedback controller and the switching law in order to further verify the large-scale

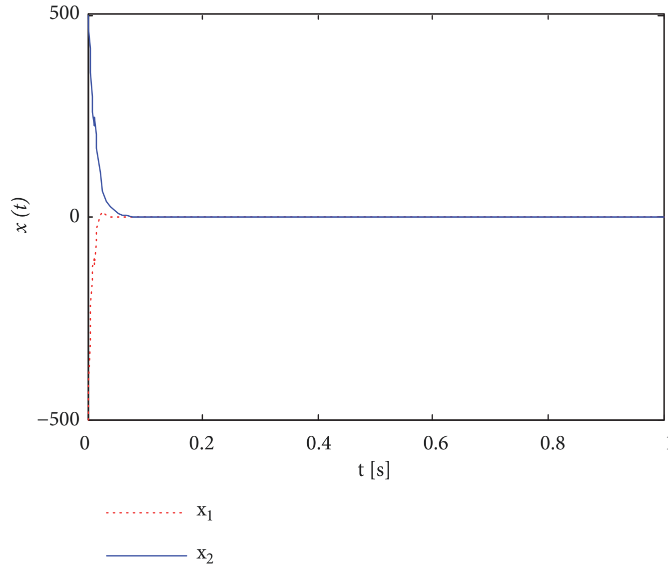

FIGURE 5: The state response curve of the closed-loop system under the control of the memory state feedback controller when the initial value is expanded.

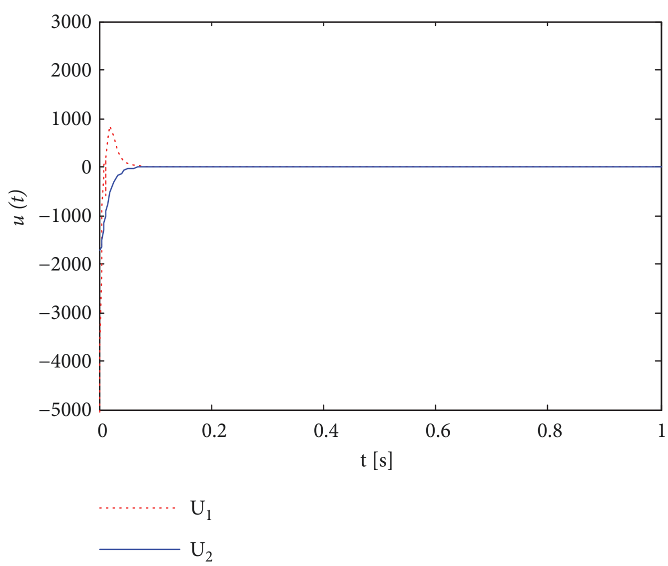

FIGURE 6: The state control curve of the closed-loop system under the control of the memory state feedback controller when the initial value is expanded.

stability of the system. The results show that the changing initial value has little effect on the convergence speed of the system.

5. Conclusion

In this paper, memory state feedback controller and switching law are designed for a class of time-varying delay switched fuzzy systems. The stability problem of closed-loop systems based on memory state feedback control is studied. According to the definition of asymptotic stability of the system, the Lyapunov functional is constructed, and the LMI solvable criterion for asymptotic stability of the closed-loop system is obtained. Finally, numerical examples and comparative analysis experiments are carried out to verify the effectiveness of the proposed method for the proposed control of time-varying switched fuzzy systems.

Data Availability

This paper does not use the data of other papers; all the data is chosen by us.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this article.

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