Hindawi Publishing Corporation
Advances in Fuzzy Systems
Volume 2015, Article ID 251646, 12 pages
http://dx.doi.org/10.1155/2015/251646

Research Article

New Closeness Coefficients for Fuzzy Similarity Based Fuzzy TOPSIS: An Approach Combining Fuzzy Entropy and Multidistance

Mikael Collan,[1] Mario Fedrizzi,[2] and Pasi Luukka[1]

[1] *School of Business and Management, Lappeenranta University of Technology, P.O. Box 20, 53851 Lappeenranta, Finland*
[2] *Department of Industrial Engineering, University of Trento, Via Mesiano 77, 38123 Trento, Italy*

Correspondence should be addressed to Pasi Luukka; pasi.luukka@lut.fi

Received 18 March 2015; Revised 4 June 2015; Accepted 8 June 2015

Academic Editor: Francisco Herrera

This paper introduces new closeness coefficients for fuzzy similarity based TOPSIS. The new closeness coefficients are based on multidistance or fuzzy entropy, are able to take into consideration the level of similarity between analysed criteria, and can be used to account for the consistency or homogeneity of, for example, performance measuring criteria. The commonly known OWA operator is used in the aggregation process over the fuzzy similarity values. A range of orness values is considered in creating a fuzzy overall ranking for each object, after which the fuzzy rankings are ordered to find a final linear ranking. The presented method is numerically applied to a research and development project selection problem and the effect of using two new closeness coefficients based on multidistance and fuzzy entropy is numerically illustrated.

1. Introduction

This paper investigates new closeness coefficients that are based on multidistance and on fuzzy entropy and that are usable with new variants of the well known Technique for Order Performance by Similarity to Ideal Solution (TOPSIS), such as fuzzy TOPSIS and fuzzy similarity based fuzzy TOPSIS. Fuzzy TOPSIS was originally introduced by Chen in 2000 [1] and Chen et al. [2] later extended it to include trapezoidal fuzzy numbers. In these contributions a vertex based fuzzy distance method was used as a measure of distance from ("similarity to") the ideal solutions. A similarity measure based version of fuzzy TOPSIS was introduced by Luukka in 2011 [3], where the similarity to the ideal solutions is calculated by using a fuzzy similarity measure. This strain of research was continued by Niyigena et al. in 2012 [4], where two different fuzzy similarity measures were considered, and by Collan and Luukka in 2014 [5], where four fuzzy similarity measure based fuzzy TOPSIS variants and a way of holistic overall ranking of projects were presented. This research continues on this same strain and explores further the possibilities to extend fuzzy similarity based fuzzy TOPSIS by introducing new closeness coefficients.

Fuzzy TOPSIS uses fuzzy numbers as inputs and is thus able to incorporate inaccurate and imprecise information in the analysis (there is no need to simplify reality by using crisp numbers). The main difference in and the intuition behind introducing fuzzy similarity measures in the place of (crisp) distance measures in the TOPSIS environment with fuzzy numbers is that fuzzy similarity measures can take into consideration more of the information that is stored in fuzzy numbers, for example, with regard to the perimeter and the area of a fuzzy number. When crisp distance measures are used what is done is essentially defuzzification of the imprecise information, that is, the fuzzy number, in order to calculate a distance between the resulting crisp number and an ideal solution. In some cases, using a defuzzified crisp distance based measure may cause a loss of relevant information. The fuzzy similarity measure used here was introduced by Wen et al. [6] and can take into account

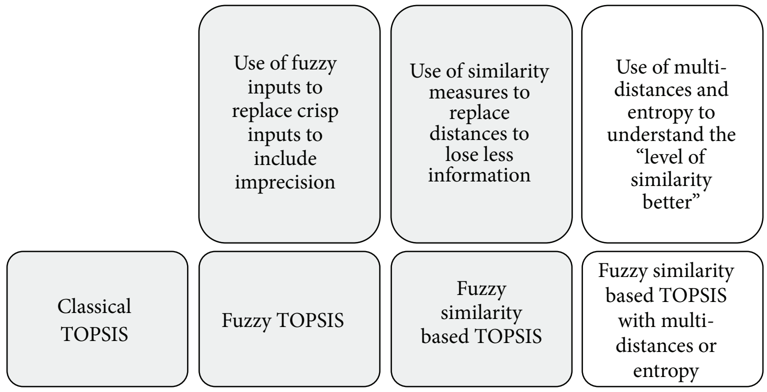

Figure 1: Evolution of TOPSIS variants with the contribution area of this research on light background.

the center of gravity, the perimeter, and the area of a fuzzy number. Similarity measures were previously studied in the context of fuzzy similarity based TOPSIS method [5].

The new contribution of this paper concentrates, in addition to presenting a new "system construct," on the application of multidistance and fuzzy entropy in creating additional information for project ranking by similarity coefficients, after they have been analyzed with fuzzy similarity measure based fuzzy TOPSIS. Multidistances are used in analyzing the "level" of similarity to the ideal solution between the analyzed criteria. High level of similarity means a low multidistance and can be interpreted as homogeneity or consistency of, for example, performance or expectations. Multidistance depends on the order, in which the partial distances are calculated; therefore, multidistance can be used, when order of measurement is defined and has a significance. Using fuzzy entropy is independent of measurement order and is in that sense a more flexible measure of consistency between criteria, but, on the other hand, fuzzy entropy cannot differentiate between specific order in which measurement is made between the criteria like multidistance can. Therefore, we observe that the two approaches are complementary. The abovementioned type of information may be valuable in the analysis and offers an additional differentiator between objects. Multidistances were examined by Martín and Mayor [7] and presented as a generalization of the notion of distance. Martín and Mayor [8] proposed also the construction of multidistances by means of OWA functions. The OWA based multidistances functions, used here, combine the distance values of all pairs of elements in the collection into OWA based multidistances [9]. Using the multidistance in the aggregation will add a step of pairwise distance measurement of similarities between criteria (values) in the procedure. Use of multidistances and fuzzy entropies with fuzzy TOPSIS is, to the best of our knowledge, a new approach. Figure 1 illustrates one direction of evolution from the classical TOPSIS to the new variants presented here, with the contribution of this research being highlighted.

The remainder of the paper is organized as follows. In Section 2, the fuzzy similarity relations between fuzzy numbers, fuzzy entropy measures, the OWA operator, multidistances, and total ordering of fuzzy numbers are introduced. Section 3 is devoted to the description of the new approach to fuzzy TOPSIS based on fuzzy similarity, multidistances, and fuzzy entropy measures. Numerical examples are introduced in Section 4 and some conclusions in Section 5 close the paper.

2. Preliminaries

In this section some preliminary mathematical background, used in this paper and on which the method discussed relies, is shortly introduced. The issues reviewed here include fuzzy similarity measures, fuzzy entropy measures, the OWA operator, and an often used method to generate the weights for the OWA operator, the O'Hagan's method. Two fuzzy entropy measures, by De Luca and Termini [10] and by Parkash et al. [11], are presented. Multidistances are defined following the work of Martín and Mayor [7] and the relationship between the OWA operator and multidistances is presented. Additionally, a way to find total ordering for fuzzy numbers is shortly revisited.

2.1. Fuzzy Similarity of Fuzzy Numbers. By focusing on uncertain objects like fuzzy sets or fuzzy numbers, the notion of a fuzzy subset generalizes that of the classical subset, where the concept of similarity can be considered as a many valued generalization of the classical notion of equivalence as stated by Zadeh [12]. As an equivalence relation is a familiar way to classify similar objects, fuzzy similarity is an equivalence relation that can be used to classify multivalued objects [4]. The concept of a similarity measure is given as follows.

For any fuzzy subset $F \neq \emptyset$ of $\mathbb{R}^n$ and for any elements $A, B \in F$ the function of a similarity measure [13] is a mapping:

$$s(A, B) : F \times F \longrightarrow [0, 1]. \tag{1}$$

Satisfying the following properties for any $x, y, z \in F$:

(i) $s(x, x) = 1, \forall x \in F$ (Reflexivity).

(ii) $s(x, y) = s(y, x)$ (Symmetry).

(iii) $\forall x, y, z \in F, s(x, z) \geq \max_y(\min(s(x, y), s(y, z)))$ (Transitivity).

Since fuzzy numbers can be considered as a type of restricted fuzzy sets, it is natural that the similarity measures used for generalized fuzzy numbers come from the family of similarity measures created and available for fuzzy sets.

Represented by Chen [14], a generalized trapezoidal fuzzy number's notation is $\widetilde{A} = (a, b, c, d; w)$, where a, b, c, and d are real values and $0 < w \leq 1$. The membership function $\mu_{\widetilde{A}}$ satisfies the following conditions:

(1) $\mu_{\widetilde{A}}(x)$ is a continuous mapping from the universe of discourse X to the closed interval in $[0, 1]$.

(2) $\mu_{\widetilde{A}}(x) = 0$, where $-\infty < x \leq a$.

(3) $\mu_{\widetilde{A}}$ is monotonically increasing in $[a, b]$.

(4) $\mu_{\widetilde{A}}(x) = w$, where $b \leq x \leq c$.

(5) $\mu_{\widetilde{A}}(x)$ is monotonically decreasing in $[c, d]$.

(6) $\mu_{\widetilde{A}}(x) = 0$, where $d \leq x < \infty$.

Due to the fit and the applicability of similarity measures in the context of decision making, various similarity measures have been proposed for the calculation the degree of similarity between fuzzy numbers. In this work, a recently introduced similarity measure by Wen et al. [6] is used. The similarity measure takes into consideration the center of gravity, the perimeter, and the area of fuzzy numbers. The similarity measure is denoted by $s(M, N)$ and involves fuzzy numbers $M = (m_1, m_2, m_3, m_4; \omega_m)$ and $N = (n_1, n_2, n_3, n_4; \omega_n)$ with $0 \leq m_1 \leq m_2 \leq m_3 \leq m_4 \leq 1$, $0 \leq n_1 \leq n_2 \leq n_3 \leq n_4 \leq 1$, and $M(x_i)$ and $N(x_i)$, their corresponding membership functions with $i \in \{1, 2, 3, 4\}$ for generalized trapezoidal fuzzy numbers, where ω_m and ω_n are their corresponding heights. The similarity is given as follows:

$$s(M, N) = (1 - |X_m - X_n|)(1 - |\omega_m - \omega_n|) \cdot \frac{\min(p(m), p(n)) + \min(a(m), a(n))}{\max(p(m), p(n)) + \max(a(m), a(n))}, \quad (2)$$

where X_m and X_n are center of gravity of the generalized trapezoidal fuzzy numbers, calculated as follows:

$$X_m = \frac{Y_m(m_3 + m_2) + (m_4 - m_1)(\omega_m - Y_m)}{2\omega_m},$$
$$Y_m = \begin{cases} \dfrac{\omega_m((m_3 - m_2)/(m_4 - m_1) + 2)}{6} & \text{if } m_1 \neq m_4,\ 0 < \omega_m \leq 1 \\ \dfrac{\omega_m}{2} & \text{if } m_1 = m_4,\ 0 < \omega_m \leq 1. \end{cases} \quad (3)$$

The values $p(m)$ and $p(n)$ represent the perimeters of the trapezoidal fuzzy numbers M and N and are defined as

$$p(m) = \sqrt{(m_1 - m_2)^2 + \omega_m^2} + \sqrt{(m_3 - m_4)^2 + \omega_m^2} + (m_3 - m_2) + (m_4 - m_1),$$
$$p(n) = \sqrt{(n_1 - n_2)^2 + \omega_n^2} + \sqrt{(n_3 - n_4)^2 + \omega_n^2} + (n_3 - n_2) + (n_4 - n_1). \quad (4)$$

The values $a(m)$ and $a(n)$ represent the areas of the trapezoidal fuzzy numbers M and N, and they are defined as

$$a(m) = \frac{1}{2}\omega_m(m_3 - m_2 + m_4 - m_1),$$
$$a(m) = \frac{1}{2}\omega_n(n_3 - n_2 + n_4 - n_1). \quad (5)$$

Notice that the result of the above similarity measure $s(M, N)$ belongs to the unit interval $[0, 1]$ and the larger the value of the similarity measure is, the stronger the similarity between the fuzzy numbers M and N is.

2.2. Fuzzy Entropy Measures. In many cases, it is of interest to have a suitable measure of the level of imprecision and vagueness, a so-called fuzziness measure, which gives an answer to the question: "How far is a given fuzzy set from a well-defined crisp reference set?" [15]. The specific contribution of fuzzy sets [16] is to capture the idea of partial membership, which creates this difference between the crisp and the fuzzy sets. Taking into consideration the concept of fuzzy sets, De Luca and Termini [10] suggested that, corresponding to Shannon's [17] probabilistic entropy, the measure of fuzzy entropy can be given as

$$H_1(A) = -\sum_{j=1}^{n}\left(\mu_A(x_j)\log\mu_A(x_j) + (1 - \mu_A(x_j))\log(1 - \mu_A(x_j))\right), \quad (6)$$

where $\mu_A(x_j)$ are the fuzzy values. This fuzzy entropy measure is considered to be a fuzziness measure [15] and it evaluates global deviations from the type of ordinary sets; that is, from any crisp set A_0 leads to $H_1(A_0) = 0$. Note that the fuzzy set A with $\mu_A(x) = 0.5$ plays the role of maximum element of the ordering defined by H.

Newer fuzzy entropy measures were introduced by Parkash et al. [11]:

$$H_2(A; w) = \sum_{j=1}^{n} w_j \left(\sin\frac{\pi\mu_A(x_j)}{2} + \sin\frac{\pi(1 - \mu_A(x_j))}{2} - 1\right) \quad (7)$$

or equivalently as

$$H_3(A; w) = \sum_{j=1}^{n} w_j \left(\cos\frac{\pi\mu_A(x_j)}{2} + \cos\frac{\pi(1 - \mu_A(x_j))}{2} - 1\right). \quad (8)$$

Besides applying fuzzy entropy measures to measure the entropy of fuzzy sets they can also be applied to fuzzy similarity values as is done, for example, by Luukka [18] in connection with a feature selection problem. Information about entropy is relevant in decision making as the ability to distinguish between the entropy of two fuzzy sets gives information about the informational value contained in the sets.

2.3. The OWA Operator. In 1988 Yager [19] introduced an aggregation operator, called ordered weighted averaging (OWA) operator, and formalized it as follows.

An ordered weighted averaging (OWA) operator of dimension m is a mapping $R^m \rightarrow R$ that has associated weighting vector $W = [w_1, w_2, \ldots, w_m]$ of dimension m with

$$\sum_{i=1}^{m} w_i = 1, \quad w_i \in [0,1], \ 1 \le i \le m \tag{9}$$

such that

$$\text{OWA}(a_1, a_2, \ldots, a_m) = \sum_{i=1}^{m} w_i b_i, \tag{10}$$

where b_i is the ith largest element of the collection of objects $a_1, a_2, \ldots, a_m$. One of the measures related to the OWA is the so-called "orness" measure. For a given weighting vector $W = [w_1, w_2, \ldots, w_m]^T$ the measure of orness of the OWA operator for W is given as

$$\text{orness}(W) = \frac{1}{m-1} \sum_{i=1}^{m} (m-i) w_i. \tag{11}$$

The weighting vector has an important role in the operation of the OWA operator: it determines how large a weight that each aggregated object receives is. The distribution of weights depends on the selected value of orness that can be selected from [0, 1]. If orness is 0, then the first ordered object gets all weight and the rest of the objects get a weight of zero. If the orness value is 1, then the weight is evenly distributed among all objects and the weighting is actually the same as a normal nonweighted average. In 1988 O'Hagan [20] introduced a technique for (optimal) computation of the weights used with the OWA. The procedure assumes a predefined degree of orness; the weights are obtained by maximizing the entropy $-\sum_{i=1}^{m} w_i \ln(w_i)$. The solution is based on the constrained optimization problem

$$\begin{aligned} \text{maximize} \quad & -\sum_{i=1}^{m} w_i \ln(w_i) \\ \text{subject to} \quad & \alpha = \frac{1}{m-1} \sum_{i=1}^{m} (m-1) w_i \\ & \sum_{i=1}^{m} w_i = 1 \\ & w_i \ge 0. \end{aligned} \tag{12}$$

The above constrained optimization problem can be solved by using different methods. Here an analytical solution introduced by Fullér and Majlender [21] is used. Below, this weighting scheme is presented:

(a) If $m = 2$, it implies that $w_1 = \alpha$ and $w_2 = 1 - \alpha$.

(b) If $\alpha = 0$ or $\alpha = 1$, it implies that the corresponding weighting vectors are $w = (0, \ldots, 0, 1)$ or $w = (1, 0, \ldots, 0)$ respectively.

(c) If $m \ge 3$ and $0 \le \alpha \le 1$, then we have

$$\begin{aligned} & w_i = \left(w_1^{m-i} \cdot w_m^{i-1}\right)^{1/(m-1)}, \\ & w_m = \frac{((m-1) \cdot \alpha - m) \cdot w_1 + 1}{(m-1) \cdot \alpha + 1 - m \cdot w_1}, \\ & w_1 \left[(m-1) \cdot \alpha + 1 - m \cdot w_1\right]^m \\ & \quad = ((m-1) \cdot \alpha)^{m-1} \cdot \left[((m-1) \cdot \alpha - m) \cdot w_1 + 1\right]. \end{aligned} \tag{13}$$

For $m \ge 3$, the weights are computed by initially obtaining the first weight, followed by the last weight, before other weights are computed.

2.4. Multidistances. A multidistance is a representation of the notion of multiargument distances. The set X is a union of all m-dimensional lists of elements of X; multidistance is defined as a function $D : X \rightarrow [0, \infty)$ on a nonempty set X provided that the following properties are satisfied for all m and $x_1, x_2, \ldots, x_m, y \in X$:

(c1) $D(x_1, x_2, \ldots, x_m) = 0$ if and only if $x_i = x_j$ for all $i, j = 1, 2, \ldots, m$.

(c2) $D(x_1, x_2, \ldots, x_m) = D(x_{\sigma(1)}, x_{\sigma(2)}, \ldots, x_{\sigma(m)})$ for any permutation σ of $i, j = 1, 2, \ldots, m$.

(c3) $D(x_1, x_2, \ldots, x_m) \le D(x_1, y) + D(x_2, y) + \cdots + D(x_m, y)$.

We say that D is a strong multidistance if it satisfies $c1$, $c2$, and

(c3*) $D(\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_m) \le D(\vec{x}_1, \vec{y}) + D(\vec{x}_2, \vec{y}) + \cdots + D(\vec{x}_m, \vec{y})$, for all $\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_m, \vec{y} \in X$.

In application contexts, the estimation of distances between more than two elements of the set X can be constructed using multidistances by means of the OWA operator as suggested by Martín and Mayor [7]:

$$\begin{aligned} & D_w(x_1, x_2, \ldots, x_m) \\ & \quad = \text{OWA}_w\left(d(x_1, x_2), d(x_2, x_3), \ldots, d(x_{m-1}, x_m)\right). \end{aligned} \tag{14}$$

In this case, elements $x_1, x_2, \ldots, x_m$ are obtained from the similarity measure (2), and the distance applied is $d(x, y) = |x - y|$.

2.5. Total Ordering of Fuzzy Numbers. There are several ways to rank fuzzy numbers. Since ranking of fuzzy number is a much more complex problem than ranking ordinary numbers, often only partial ordering is found. Kaufmann and Gupta [22] propose a method to try to find *total order* or *linear order* for fuzzy numbers, where all fuzzy numbers and fuzzy intervals are comparable. The method is based on using three properties of fuzzy sets as criteria to separate fuzzy numbers into classes. If using the first criterion does not give a unique linear order, that is, each class includes only one number, then one moves on to separate the numbers in a multinumber class with the next criterion. The description of the three different criteria used in the Kaufmann and Gupta ordering method is given below.

(1) The Removal. Let us consider an ordinary number $k \in \mathbb{R}$ and a fuzzy number A. The left side removal of A with respect to k, denoted by $R_l(A,k)$, is defined as the area bounded by k and the left side of the fuzzy number A. Similarly, the right side removal $R_r(A,k)$ is defined. The removal of the fuzzy number A with respect to k is defined as the mean of $R_l(A,k)$ and $R_r(A,k)$. Thus,

$$R(A,k) = \frac{1}{2}\left(R_l(A,k) + R_r(A,k)\right). \quad (15)$$

The position of k can be located anywhere on the x-axis including $k = 0$. By definition, the areas are positive quantities, but here they are evaluated by integration taking into account the position (negative, zero, or positive) of the variable x; therefore, $R(A,k)$ can be positive, negative, or null.

The first criterion, used in ordering, is the removal with respect to k. Two different fuzzy numbers can have the same removal with respect to the same k. The criterion decomposes a set of fuzzy numbers into classes having the same removal number. The classes can be ordered according to the removal number; if there is only one fuzzy number per each class, then we have linear ordering of the fuzzy numbers.

The removal number $R(A,k)$ defined in this criterion, relocated to $k = 0$, is equivalent to an "ordinary representative" of the fuzzy number. In the case of a triangular fuzzy number this ordinary representative is given by

$$\widehat{A} = \frac{a_1 + 2a_2 + a_3}{4}, \quad (16)$$

where $A = (a_1, a_2, a_3)$.

If after using the removal criteria there are classes with multiple fuzzy numbers, one has to go forward and use the second criteria for ordering the fuzzy numbers within the "multiple number" classes.

(2) The Mode. In each class of (multiple) fuzzy numbers, one should look for the mode of each fuzzy number in the class; these modes will generate subclasses. If the fuzzy numbers under consideration have a nonunique mode, one takes the mean position of the modal values. It must be noted that this is only one way of obtaining subclasses, and one may need the following third divergence criterion for further subclassification:

$$\text{Mode}(A) = \{x \in U \mid A(x) = 1\}. \quad (17)$$

If there are still classes (or rather subclasses) with multiple fuzzy numbers, one will then resort to the third ordering criterion.

(3) The Divergence. The consideration of the divergence around the mode in each subclass leads to sub-subclasses, and this criterion may be sufficient to obtain the final linear ordering of fuzzy numbers (we do not know of a situation where using the three criteria has not been able to create a linear order):

$$\text{Divergence}(A) = \sup\left(\text{supp}(A)\right) - \inf\left(\text{supp}(A)\right). \quad (18)$$

Summarizing the Method. When one orders fuzzy numbers to size order, one proceeds as follows. Apply the above presented three criteria in the exact given order, such that if the unique linear order is not obtained with a criterion, then move to the next criterion. Let us recall that this is one of the many methods available in the literature; a good survey has been most recently proposed by Brunelli and Mezei [23].

3. Proposed New Model That Uses Fuzzy Similarity Based Fuzzy TOPSIS with Fuzzy Entropy and Multidistances

TOPSIS was originally introduced by Hwang and Yoon [24] (see also work in Lai et al. [25]). We start with a short introduction of the original TOPSIS method and then present the new proposed extension. The idea of evaluation that TOPSIS uses is to simultaneously consider the distance between an analyzed alternative and a positive and a negative ideal solution. The best alternative is the closest to the positive ideal solution and the furthest away from the negative ideal solution. The procedure of TOPSIS starts from the construction of an evaluation matrix $X = [x_{ij}]$, where x_{ij} denotes the valuation of the ith alternative with respect to jth criterion. It can be summarized as follows.

Step 1. Calculation of normalized decision matrix $Z = [z_{ij}]$

$$z_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} x_{ij}^2}}, \quad j = 1,\ldots,m,\ i = 1,\ldots,n. \quad (19)$$

Step 2. Calculation of the weighted normalized decision matrix $V = [v_{ij}]$

$$v_{ij} = z_{ij}(\cdot)\, w_j, \quad j = 1,\ldots,m,\ i = 1,\ldots,n. \quad (20)$$

Step 3. Determination of the positive and negative ideal solution A^+ and A^-

$$\begin{aligned} A^+ &= \{v_1^+,\ldots,v_m^+\} \\ &= \left\{\left(\max_i v_{ij} \mid j \in B\right), \left(\min_i v_{ij} \mid j \in C\right)\right\}, \\ A^- &= \{v_1^-,\ldots,v_m^-\} \\ &= \left\{\left(\min_i v_{ij} \mid j \in B\right), \left(\max_i v_{ij} \mid j \in C\right)\right\}, \end{aligned} \quad (21)$$

where B is for benefit criteria and C is for cost criteria.

Step 4. Calculation of the distance of each alternative from the positive ideal solution and negative ideal solution

$$\begin{aligned} d_i^+ &= \sqrt{\sum_{j=1_m} \left(v_{ij} - v_j^+\right)^2}, \quad i = 1,\ldots,n, \\ d_i^- &= \sqrt{\sum_{j=1_m} \left(v_{ij} - v_j^-\right)^2}, \quad i = 1,\ldots,n. \end{aligned} \quad (22)$$

Step 5. Calculation of the relative closeness to the ideal solutions

$$\mathrm{CC}_i = \frac{d_i^-}{d_i^+ + d_i^-}, \quad i = 1, \ldots, n. \tag{23}$$

Step 6 (ranking of alternatives). The closer the CC_i is to one the higher the priority of the ith alternative is.

A fuzzy extension to the Technique for Order Performance by Similarity to Ideal Solution (TOPSIS) was presented by Chen [1] and it has been extended to solve problems involving trapezoidal fuzzy numbers and applied, that is, to solving supplier selection problems as done by Chen et al. [2] (see also work of Jahanshahloo et al. [26]). Fuzzy TOPSIS is a multiple criteria decision making (MCDM) [2, 27] method useful in ranking objects, based on the similarity of the object characteristics to the characteristics of an ideal object (ideal solution). The method is based on the idea that the higher the objects are ranked, the shorter their distance is from the *Fuzzy Positive Ideal Solution* (FPIS) and the further away the objects simultaneously are from the *Fuzzy Negative Ideal Solution* (FNIS). One advantage of having extended TOPSIS method to the fuzzy environment is that linguistic assessment can be used in describing the (properties of) alternatives, instead of being constrained to using only numerical values; linguistic variables can be mapped to fuzzy numbers.

To present the new proposed approach we first introduce it shortly as a stepwise algorithm and then we go more into details of the steps presented; the steps of the new proposed TOPSIS approach are as follows.

Step 1. Form a committee of decision makers and identify the evaluation criteria.

Step 2. Choose appropriate linguistic variables for the importance of weight of the criteria and the linguistic ratings for alternatives.

Step 3. Aggregate the weight of criteria to get the aggregated fuzzy weight $\widehat{w}_j$ of the criterion C_j, and join decision makers' ratings to get an aggregated fuzzy rating x_{ij} of the project P_i in consideration of the criterion C_j.

Step 4. Construct a fuzzy decision matrix and a normalized fuzzy decision matrix.

Step 5. Construct a weighted normalized fuzzy decision matrix.

Step 6. Determine a fuzzy positive (and negative) ideal solution FPIS (and FNIS).

Step 7. Construct a similarity matrix by calculating the similarity of each alternative to the FPIS (and to the FNIS).

Step 8. Calculate aggregated similarity values for each alternative, with regard to the FPIS and the FNIS by using OWA.

Step 9. Calculate a multidistance value or fuzzy entropy value for each alternative with regard to the FPIS.

Step 10. Calculate a closeness coefficient for each alternative, in order to determine the alternatives' ranking within the set of alternatives.

For Steps 8–10, use multiple orness values for each alternative to get multiple ranking results.

Step 11. Rank the set of alternatives for each orness value and calculate the minimum, the mean, and the maximum ranking of each alternative, to form a triangular fuzzy ranking score for each alternative by using the three values.

Step 12. Make a final "overall" ranking of the alternatives by forming fuzzy triangular numbers from the ranks by using minimum, mean, and maximum ranks. The resulting final fuzzy numbers are put in the final order by using the Kaufmann and Gupta [22] method.

Next, the details of the new proposed procedure are presented. Solution to the selection problem, when using our new fuzzy TOPSIS approach, can be presented by considering a situation of a finite set of alternatives $A = \{A_i \mid i = 1, 2, \ldots, m\}$, which need to be evaluated by a committee of decision makers $D = \{D_l \mid l = 1, 2, \ldots, k\}$, by considering a finite set of given criteria $C = \{C_j \mid j = 1, 2, \ldots, n\}$.

Let us consider a decision matrix [28] representing a set of performance ratings of each alternative A_i, $i = 1, 2, \ldots, m$, with respect to each criterion C_j, $j = 1, 2, \ldots, n$, as follows:

$$\mathbf{X} = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1n} \\ x_{21} & x_{22} & \cdots & x_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ x_{m1} & x_{m2} & \cdots & x_{mn} \end{bmatrix}. \tag{24}$$

Let us also assume the weight w_j of the jth criterion C_j, such that the weight vector is represented as follows:

$$\mathbf{W} = [w_1, w_2, \ldots, w_n], \tag{25}$$

where m rows represent m possible alternatives, n columns represent n relevant criteria, and x_{ij} represent the performance rating of the ith alternative A_i with respect to the jth criterion C_j. The above fuzzy ratings for each decision maker D_l, $l = 1, 2, \ldots, k$, are represented by positive trapezoidal fuzzy numbers $\widehat{R}_l = (a_l, b_l, c_l, d_l)$, $l = 1, 2, \ldots, k$, with the respective membership function $\mu_{\widehat{R}_l}(x)$. As the rating $\widehat{R}_l = (a_l, b_l, c_l, d_l)$ is for the lth decision maker, the aggregated fuzzy number that can stand for all decision makers' rating is

$$\widehat{R} = (a, b, c, d) \tag{26}$$

with $a = \min_l\{a_l\}$, $b = (1/k)\sum_{l=1}^{k} b_l$, $c = (1/k)\sum_{l=1}^{k} c_l$, and $d = \max_l\{d_l\}$. The fuzzy rating and importance of weight of the lth decision maker can, respectively, be represented by $x_{ijl} = (a_{ijl}, b_{ijl}, c_{ijl}, d_{ijl})$ and $\widehat{w} = (w_{jl1}, w_{jl2}, w_{jl3}, w_{jl4})$ with $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots, n$. Then, the aggregated fuzzy ratings x_{ij} of alternatives, with respect to each criterion, are

$$x_{ij} = \left(a_{ij}, b_{ij}, c_{ij}, d_{ij}\right), \tag{27}$$

calculated as $a_{ij} = \min_l\{a_{ijl}\}$, $b_{ij} = (1/k)\sum_{l=1}^{k} b_{ijl}$, $c_{ij} = (1/k)\sum_{l=1}^{k} c_{ijl}$, and $d_{ij} = \max_l\{d_{ijl}\}$. The aggregated fuzzy weight $\widehat{w}_j$ of each criterion can be calculated as

$$\widehat{w}_j = \left(w_{j1}, w_{j2}, w_{j3}, w_{j4}\right) \tag{28}$$

with $w_{j1} = \min_l\{w_{jl1}\}$, $w_{j2} = (1/k)\sum_{l=1}^{k} w_{jl2}$, $w_{j3} = (1/k)\sum_{l=1}^{k} w_{jl3}$, and $w_{j4} = \max_l\{w_{jl4}\}$. After aggregation the decision matrix and the weight vector are of the following forms: $X = \{x_{ij}\}_{m\times n}$ and $W = \{w_j\}_{1\times n}$, where $i = 1,2,\ldots,m$ and $j = 1,2,\ldots,n$. These matrices' elements are given by positive trapezoidal fuzzy numbers as $x_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij})$ and $w_j = (w_{j1}, w_{j2}, w_{j3}, w_{j4})$. A linear scale transformation is used to transform the various criteria scales into comparable scales, in order to avoid overly complex mathematical operations in a decision process. The set of criteria can be divided into *benefit criteria B*, where the larger the rating, the greater the preference, and to *cost criteria C*, where the smaller the rating, the greater the preference. A normalization method designed to preserve the property, in which the elements are normalized trapezoidal fuzzy numbers, is used. The normalized value of x_{ij} is r_{ij}, and the normalized fuzzy decision matrix is then represented as

$$R = \left[r_{ij}\right]_{m\times n} \tag{29}$$

with

$$\begin{aligned} r_{ij} &= \left(\frac{a_{ij}}{d_j^+}, \frac{b_{ij}}{d_j^+}, \frac{c_{ij}}{d_j^+}, \frac{d_{ij}}{d_j^+}\right), \quad j \in B, \\ r_{ij} &= \left(\frac{a_j^-}{d_{ij}}, \frac{a_j^-}{c_{ij}}, \frac{a_j^-}{b_{ij}}, \frac{a_j^-}{a_{ij}}\right), \quad j \in C, \end{aligned} \tag{30}$$

where $d_j^+ = \max_i\{d_{ij}\}$, $j \in B$, and $a_j^- = \min_i\{a_{ij}\}$, $j \in C$. The weighted normalized value of r_{ij} is called v_{ij}, and, by considering the importance of each criterion, the weighted normalized fuzzy decision matrix is represented as

$$V = \left[v_{ij}\right]_{m\times n}, \tag{31}$$

where $v_{ij} = r_{ij} \cdot w_j$. For all i, j, the elements v_{ij} are now normalized positive trapezoidal fuzzy numbers.

Next, the ideal solutions must be determined and taken from the given criteria, which are linguistically expressed; they are commonly referred to as *Fuzzy Positive Ideal Solution* (FPIS) and *Fuzzy Negative Ideal Solution* (FNIS). By considering a finite set of given criteria $C = \{C_j \mid j = 1,2,\ldots,n\}$, the ways to select the FPIS(A^+) and the FNIS(A^-) come from the weighted normalized decision matrix $V = (v_{ij})_{m\times n}$, where the obtained weighted normalized values v_{ij} are fuzzy numbers expressed as

$$v_{ij} = \left(v_{ij1}, v_{ij2}, v_{ij3}, v_{ij4}\right). \tag{32}$$

The fuzzy positive ideal solution A^+ and the fuzzy negative ideal solution A^-, respectively, are

$$A^+ = \left[v_1^+, v_2^+, \ldots, v_n^+\right], \tag{33}$$

$$A^- = \left[v_1^-, v_2^-, \ldots, v_n^-\right]. \tag{34}$$

Several ways for choosing the FPIS(A^+) and the FNIS(A^-) have been presented by Luukka [3]. Next, we shortly review the one used here. Every element of A^+ is the maximum for all i weighted normalized value v_{ij} and every element of A^- is the minimum for all i weighted normalized value v_{ij} as

$$\begin{aligned} v_j^+ &= \left(\max_i v_{ij1}, \max_i v_{ij2}, \max_i v_{ij3}, \max_i v_{ij4}\right), \\ v_j^- &= \left(\min_i v_{ij1}, \min_i v_{ij2}, \min_i v_{ij3}, \min_i v_{ij4}\right). \end{aligned} \tag{35}$$

The similarity measure between each project and the ideal solutions A^+ and A^- is needed in calculating the closeness coefficients to determine the ranking order of all possible alternative projects. The similarities s_i^+ from the positive and negative ideal solution are calculated as

$$\begin{aligned} s_i^+ &= \left\{s_{i1}\left(v_{i1}, v_1^+\right), s_{i2}\left(v_{i2}, v_2^+\right), \ldots, s_{in}\left(v_{in}, v_n^+\right)\right\}, \\ s_i^- &= \left\{s_{i1}\left(v_{i1}, v_1^-\right), s_{i2}\left(v_{i2}, v_2^-\right), \ldots, s_{in}\left(v_{in}, v_n^+\right)\right\}, \end{aligned} \tag{36}$$

where for similarity we used the similarity measure from (2). These similarity vectors are then aggregated using OWA, as follows:

$$\begin{aligned} S_{iw}^+ &= \mathrm{OWA}_w\left(s_{i1}^+, s_{i2}^+, \ldots, s_{in}^+\right), \\ S_{iw}^- &= \mathrm{OWA}_w\left(s_{i1}^-, s_{i2}^-, \ldots, s_{in}^-\right). \end{aligned} \tag{37}$$

In addition to the similarity measure we also aggregate s_i^+ vector by using multidistance as

$$\begin{aligned} &D_{iw}^+\left(s_{i1}^+, s_{i2}^+, \ldots, s_{in}^+\right) \\ &\quad = \mathrm{OWA}_w\left(d\left(s_{i1}^+, s_{i2}^+\right), d\left(s_{i2}^+, s_{i3}^+\right), \ldots, d\left(s_{i(n-1)}^+, s_{in}^+\right)\right). \end{aligned} \tag{38}$$

The first fuzzy entropy measure of the similarity vector here introduced is calculated by using De Luca and Termini entropy as

$$H_{Di}^+\left(s_i^+\right) = -\sum_{j=1}^{n} s_{ij}^+ \ln\left(s_{ij}^+\right) + \left(1 - s_{ij}^+\right)\ln\left(1 - s_{ij}^+\right). \tag{39}$$

The second fuzzy entropy measure of the similarity vector we use is calculated with Parkash entropy and is

$$\begin{aligned} &H_{Pi}^+\left(s_i^+\right) \\ &\quad = -\sum_{j=1}^{n} w_j\left(\sin\left(\frac{\pi s_{ij}^+}{2}\right) + \sin\left(\frac{\pi\left(1 - s_{ij}^+\right)}{2}\right) - 1\right). \end{aligned} \tag{40}$$

We now want to extend the closeness coefficient to consider not only the similarity of the objects from the positive and the negative ideal solution but also the information about the "consistency" of the similarity of different criteria carried by using the multidistance of fuzzy entropy.

Closeness coefficients of the alternative A_i with respect to the positive ideal solution by using the closeness coefficient (CC_i^D) that uses the multidistance are defined as

$$\mathrm{CC}_i^D = \frac{S_{iw}^- + D_{iw}^+}{D_{iw}^+ + S_{iw}^+ + S_{iw}^-}, \quad i = 1,2,\ldots,m. \tag{41}$$

An extension that applies a De Luca and Termini fuzzy entropy measure in the closeness coefficient to account for criteria consistency can be constructed in the following way:

$$CC_i^H = \frac{S_{iw}^- + H_{Di}^+/H^*}{S_{iw}^+ + S_{iw}^- + H_{Di}^+/H^*}, \quad i = 1, 2, \ldots, m, \tag{42}$$

where H^* is used as a scaling factor and it is the maximum entropy value gained for the particular problem, when similarity vector would consist of values $s_{ij} = 0.5 \; \forall j$ (yielding the maximum entropy).

Another closeness coefficient extension with a fuzzy entropy measure, with Parkash entropy, can be constructed as

$$CC_i^H = \frac{S_{iw}^- + H_{Pi}^+/H^*}{S_{iw}^+ + S_{iw}^- + H_{Pi}^+/H^*}, \quad i = 1, 2, \ldots, m. \tag{43}$$

The philosophy of entropy measures is based on the idea that one of the tasks is to discover patterns or regularities in the data. Regularities and structure are characterized by low entropy (value), whereas a high level of randomness is associated with high entropy (value) [29]. Entropy value can tell us whether one distribution contains more information than another. Low entropy indicates the existence of information, while high entropy indicates greater uncertainty about the information content. Minimum entropy gives minimum uncertainty, which is the limit of our knowledge about the structure of the system [30]. With these measures added to the closeness coefficient we can take also this kind of information into account. A similar rationale is also found behind the concept of multidistance.

To finish the ranking of the alternatives the closeness coefficients are ordered in an ascending order. After this we then repeat this process by using several different orness values. This way we do not just get one ranking for each alternative, but we get a set of rankings. From these sets we then form approximation by creating a triangular fuzzy number by using the minimum, the mean, and the maximum rankings. After the triangular fuzzy numbers are created, they are ranked by using the method summarized in Section 2.5, and the total order is found and used in forming a final ranking. This approach to get an "overall ranking" in the presence of parameter uncertainty, or parameter value ranges, has earlier been presented in [5].

4. Numerical Example

This numerical example is based on the data used previously in Hassanzadeh et al. [31] and Luukka et al. [32]. A pharmaceutical company can select a certain number of projects for investment from among twenty R&D projects. Criteria in the example come from costs, revenues, budget constraints, and the real option value (ROV) of the projects, calculated for each project by using the payoff method [33] for real option valuation; the values of these four criteria are represented by trapezoidal fuzzy numbers. The first and the third criteria are cost criteria and the second and the fourth ones are benefit criteria.

In Table 1 one can see evaluations of the different criteria by using trapezoidal fuzzy numbers. The fourth (ROV) criterion is carried out in computations as a fuzzy number of form $A = (a_1, a_2, a_3, a_4)$, where $a_1 = a_2 = a_3 = a_4$. The four criteria are used in the TOPSIS analysis according to the procedure outlined above. The most important contribution here, in addition to and as a part of the proposed extended procedure, has to do with the extension of the closeness coefficients; therefore, we concentrate on discussing them in more detail. The ranking of the projects depends on the choice of the orness value associated to OWA operator's weights and thus on the risk profile of the decision makers. Risk-averse behavior would demand that most criteria be satisfied and this implies a conjunctive behavior, that is, orness < 0.5. Risk-seeking behavior of decision makers would mean that they are willing to accept that only some criteria are satisfied, that is, orness > 0.5. Since the managerial problem here addressed asked for a prudent approach to the holding of risky prospects we have assumed that the decision makers are risk-averse and opted to compute the ranking of the projects with multiple orness values, chosen in the open interval $(0, 0.5)$. For this reason we compute the ranking of each project with multiple α; in fact we have used several values, starting from 0.005 and running up to 0.495 with the interval of 0.005, that is, $\alpha = 0.005 : 0.005 : 0.495$. Table 2 summarizes the experimental results with the minimum, the mean, and the maximum rankings for the tested orness values. These three values are used in the formation of a triangular fuzzy number ranking for each project.

Total ordering is found for the fuzzy numbers presented in Table 2 by using the method introduced by Kaufmann and Gupta [22]. For this purpose the removal number, dispersion, and modal values are calculated in the way already presented in Table 2. Results reporting also removal number, dispersion, and mode values from these computations can be found in Tables 4–6. In Table 3 we simply present the ranking results from these three methods.

According to Table 2 results we can see that using the two new closeness coefficients (CCs) has a different effect on the resulting overall rankings. Both entropy based CCs are giving quite similar results, whereas the new multidistance based CC is clearly different. Top four projects according to the two entropy based measures are P_{13}, P_{16}, P_9, and P_4, whereas the multidistance based CC ends up with top four of P_{16}, P_{13}, P_{17}, and P_9. The top two projects were always P_{13} and P_{16}. The difference between the results highlights the fact that when measurement order matters the results are and should be different. This is not trivial, as there may be cases where the measurement order is highly significant, such as cases where "the act of measurement" changes the state of the variable.

5. Summary and Conclusions

A new multiple criteria decision making approach was presented that is a new extension of the fuzzy similarity based fuzzy TOPSIS. Aggregation of similarity to fuzzy negative and positive ideal solutions for each criterion was done by using ordered weighted averaging (OWA) and multidistance and two fuzzy entropy measures were introduced for collecting

Table 1: Evaluation of R&D projects.

Project	C_1	C_2	C_3	C_4
P_1	(53, 62, 68, 78)	(43, 50, 55, 63)	(115, 128, 128, 141)	0.06
P_2	(83, 98, 108, 123)	(85, 100, 110, 125)	(126, 140, 140, 154)	0.0594
P_3	(157, 185, 204, 231)	(170, 200, 220, 250)	(170, 189, 189, 208)	18
P_4	(204, 240, 268, 300)	(170, 200, 220, 250)	(164, 182, 182, 200)	0.54
P_5	(259, 305, 336, 381)	(510, 600, 660, 750)	(209, 232, 232, 255)	3.10
P_6	(85, 100, 110, 125)	(85, 100, 110, 125)	(185, 206, 206, 227)	5
P_7	(259, 305, 336, 381)	(510, 600, 660, 750)	(209, 232, 232, 255)	3.10
P_8	(94, 110, 121, 138)	(85, 100, 110, 125)	(177, 197, 197, 217)	1.58
P_9	(140, 165, 182, 206)	(153, 180, 198, 225)	(238, 264, 264, 290)	17.15
P_{10}	(190, 223, 245, 279)	(323, 380, 418, 475)	(257, 285, 285, 314)	1.65
P_{11}	(60, 70, 77, 88)	(68, 80, 88, 100)	(148, 164, 164, 180)	10.03
P_{12}	(91, 107, 118, 134)	(85, 100, 110, 125)	(144, 160, 160, 176)	2.39
P_{13}	(247, 290, 319, 363)	(34, 40, 44, 50)	(297, 330, 330, 363)	0
P_{14}	(370, 435, 479, 544)	(595, 700, 770, 875)	(338, 375, 375, 413)	278.25
P_{15}	(166, 195, 215, 244)	(425, 500, 550, 625)	(279, 310, 310, 341)	320.25
P_{16}	(221, 260, 286, 325)	(255, 300, 330, 375)	(315, 350, 350, 385)	39.66
P_{17}	(235, 277, 305, 346)	(298, 350, 385, 438)	(311, 346, 346, 381)	72.48
P_{18}	(281, 330, 363, 413)	(468, 550, 605, 688)	(331, 368, 368, 405)	231
P_{19}	(344, 405, 446, 506)	(680, 800, 880, 1000)	(365, 406, 406, 447)	414.75
P_{20}	(451, 530, 583, 663)	(978, 1150, 1265, 1438)	(394, 438, 438, 482)	651

Table 2: The minimum, mean, and the maximum rankings with closeness coefficients created with (a) multidistance measure, (b) De Luca and Termini entropy measure, and (c) Parkash entropy measure.

Project	Min	Mean	Max	Min	Mean	Max	Min	Mean	Max
P_1	19	19	19	2	5.54	9	2	4.55	9
P_2	15	15.7	16	11	11.15	12	10	10.97	11
P_3	6	8.0	11	6	6.61	7	5	6.28	7
P_4	3	6.0	8	4	4.13	5	3	3.85	4
P_5	9	9.9	11	16	17.79	20	16	17.80	20
P_6	13	13.1	14	9	9.46	10	9	9.91	11
P_7	17	18.0	18	13	13.08	14	12	12.48	13
P_8	12	12	12	7	7.82	8	7	7.91	8
P_9	4	5.1	6	2	3.30	5	2	3.71	5
P_{10}	4	5.0	7	11	12.57	14	12	13.56	14
P_{11}	17	17.0	18	12	13.29	15	12	12.96	14
P_{12}	13	14.1	15	8	9.46	10	8	9.05	10
P_{13}	2	2.1	3	1	1	1	1	1	1
P_{14}	9	10.3	11	16	16.27	18	16	16.22	17
P_{15}	7	9.0	11	17	17.80	19	17	18.16	19
P_{16}	1	1	1	2	2.20	3	2	2.56	3
P_{17}	2	3.0	4	5	5.48	6	6	6.22	7
P_{18}	5	6.6	8	14	14.91	15	15	15	15
P_{19}	14	15.1	16	18	19.07	20	18	18.76	20
P_{20}	20	20	20	17	19.08	20	17	19.06	20

Table 3: Overall rankings of the R&D projects with multidistance, De Luca and Termini entropy measure and with Parkash entropy measure.

Project	Multidistance	De Luca and Termini	Parkash
P_1	19	6	5
P_2	16	11	11
P_3	8	7	6
P_4	6	4	4
P_5	10	17	17
P_6	13	10	10
P_7	18	13	12
P_8	12	8	8
P_9	4	3	3
P_{10}	5	12	14
P_{11}	17	14	13
P_{12}	14	9	9
P_{13}	2	1	1
P_{14}	11	16	16
P_{15}	9	18	18
P_{16}	1	2	2
P_{17}	3	5	7
P_{18}	7	15	15
P_{19}	15	20	20
P_{20}	20	19	19

information about the "similarity of these similarities" that can be understood as a measure of homogeneity or consistency of a given project. This has allowed the inclusion of more relevant information in the analysis than is possible, when adopting a simple defuzzification procedure alone. The method was applied to an R&D project selection problem.

Table 4: Overall rankings of the R&D projects with multidistance using removal number, dispersion, and modal value.

Project	Rank	Removal number	Div.	Mode
P_{16}	1	1	0	1
P_{13}	2	2.29	1	2.07
P_{17}	3	2.99	2	2.99
P_9	4	5.03	2	5.05
P_{10}	5	5.27	3	5.03
P_4	6	5.76	5	6.02
P_{18}	7	6.57	3	6.63
P_3	8	8.26	5	8.01
P_{15}	9	8.99	4	8.98
P_5	10	9.93	2	9.87
P_{14}	11	10.17	2	10.34
P_8	12	12	0	12
P_6	13	13.31	1	13.12
P_{12}	14	14.03	2	14.06
P_{19}	15	15.04	2	15.08
P_2	16	15.62	1	15.74
P_{11}	17	17.26	1	17.01
P_7	18	17.74	1	17.99
P_1	19	19	0	19
P_{20}	20	20	0	20

Table 5: Overall rankings of the R&D projects with De Luca and Termini entropy measure using removal number, dispersion, and modal value.

Project	Rank	Removal number	Div.	Mode
P_{13}	1	1	0	1
P_{16}	2	2.35	1	2.20
P_9	3	3.40	3	3.30
P_4	4	4.32	1	4.13
P_{17}	5	5.49	1	5.48
P_1	6	5.52	7	5.54
P_3	7	6.55	1	6.61
P_8	8	7.66	1	7.82
P_{12}	9	9.23	2	9.46
P_6	10	9.48	1	9.45
P_2	11	11.33	1	11.15
P_{10}	12	12.53	3	12.57
P_7	13	13.29	1	13.08
P_{11}	14	13.40	3	13.29
P_{18}	15	14.70	1	14.91
P_{14}	16	16.64	2	16.27
P_5	17	17.89	4	17.79
P_{15}	18	17.90	2	17.80
P_{20}	19	18.79	3	19.07
P_{19}	20	19.04	2	19.08

It was observed that analysis results depend on the proper selection of the orness parameter, α, when the weights are generated for the OWA operator. This weight generation was based on O'Hagan's method that finds the weights as an optimal solution for a predefined (given) orness value (α). The effect of the choice of the orness value to the resulting ranking was tested and "cancelled" by using multiple orness values to form triangular fuzzy numbers from three descriptive numbers (min, max, and the mean) from several rankings generated for each project by using different orness values.

Table 6: Overall rankings of the R&D projects with Parkash entropy measure using removal number, dispersion, and modal value.

Project	Rank	Removal number	Div.	Mode
P_{13}	1	1	0	1
P_{16}	2	2.53	1	2.56
P_9	3	3.60	3	3.71
P_4	4	3.67	1	3.85
P_1	5	5.02	7	4.55
P_3	6	6.14	2	6.28
P_{17}	7	6.36	1	6.22
P_8	8	7.70	1	7.91
P_{12}	9	9.03	2	9.05
P_6	10	9.95	2	9.91
P_2	11	10.73	1	10.97
P_7	12	12.49	1	12.48
P_{11}	13	12.98	2	12.96
P_{10}	14	13.28	2	13.56
P_{18}	15	15	0	15
P_{14}	16	16.36	1	16.22
P_5	17	17.90	4	17.80
P_{15}	18	18.08	2	18.16
P_{20}	19	18.78	3	19.06
P_{19}	20	18.88	2	18.76

A measure of homogeneity of similarity of the different criteria of each project to the fuzzy positive ideal solution was calculated introducing multidistances and two fuzzy entropy measures. This was done in order to include information about the consistency of the level of goodness of projects (by the selected criteria). This information was included in the closeness coefficient that is used in the ranking of the projects. The final ranking thus includes information about the goodness of each project (as ranked by TOPSIS) and about the "stability" of the level of goodness of each of the criteria of each project. The two different types of CCs complement each other, as the multidistance based CC is usable, when the measurement order is significant, and entropy based CCs are usable also in more general cases, while they cannot consider measurement order.

Forming a fuzzy number from different rankings allows one to include different points of view and create an intelligent overall ranking. Using multiple orness values in forming the final ranking is relevant in situations, where there is uncertainty or imprecision with regard to the correct orness parameter selection. It is clear that if there is absolutely no uncertainty involved in the orness parameter selection one should use the certain parameter alone in creating the ranking. Furthermore, an increased amount of relevant information is carried along in the analysis until the ranking stage, enabling the ranking to take more things into consideration than in the classical TOPSIS method. This allows a more holistic analysis of ranking and selection problems. By measuring entropy and including it in CC the new proposed variant is able to consider also uncertainty related to the evaluations which has not been possible with the previous TOPSIS variants.

Interesting future research directions include the introduction of consensual dynamics in the aggregation of individual TOPSIS scores, using Choquet aggregation in the place of the OWA operator aggregation and using the histogram method together with OWA operator aggregation to make the aggregation orness parameter selection independent.

Conflict of Interests

The authors declare that there is no conflict of interests regarding the publication of this paper.

References

[1] C.-T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets and Systems*, vol. 114, no. 1, pp. 1–9, 2000.

[2] C.-T. Chen, C.-T. Lin, and S.-F. Huang, "A fuzzy approach for supplier evaluation and selection in supply chain management," *International Journal of Production Economics*, vol. 102, no. 2, pp. 289–301, 2006.

[3] P. Luukka, "Fuzzy similarity in multicriteria decision-making problem applied to supplier evaluation and selection in supply chain management," *Advances in Artificial Intelligence*, vol. 2011, Article ID 353509, 9 pages, 2011.

[4] L. Niyigena, P. Luukka, and M. Collan, "Supplier evaluation with fuzzy similarity based fuzzy TOPSIS with new fuzzy similarity measure," in *Proceedings of the 13th IEEE International Symposium on Computational Intelligence and Informatics (CINTI '12)*, pp. 237–244, IEEE, Budapest, Hungary, November 2012.

[5] M. Collan and P. Luukka, "Evaluating R & D projects as investments by using an overall ranking from four new fuzzy similarity measure based TOPSIS variants," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 3, pp. 505–515, 2014.

[6] J. Wen, X. Fan, D. Duanmu, and D. Yong, "A modified similarity measure of generalized fuzzy numbers," *Procedia Engineering*, vol. 15, pp. 2773–2777, 2011.

[7] J. Martín and G. Mayor, "Some properties of multi-argument distances and Fermat multidistance," in *Proceedings of the 13th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems*, pp. 703–711, 2010.

[8] J. Martín and G. Mayor, "Multi-argument distances," *Fuzzy Sets and Systems*, vol. 167, pp. 92–100, 2011.

[9] M. Brunelli, M. Fedrizzi, M. Fedrizzi, and F. Molinari, "On some connections between multidistances and valued m-ary adjancency relations, advances on computational intelligence," in *Proceedings of the 14th International Conference on Information Processing and Management of Uncertainty in Knowledge-Based Systems (IPMU '12)*, vol. 297, pp. 201–207, Catania, Italy, July 2012.

[10] A. de Luca and S. Termini, "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory," *Information and Computation*, vol. 20, pp. 301–312, 1972.

[11] O. Parkash, P. K. Sharma, and R. Mahajan, "New measures of weighted fuzzy entropy and their applications for the study of maximum weighted fuzzy entropy principle," *Information Sciences*, vol. 178, no. 11, pp. 2389–2395, 2008.

[12] L. A. Zadeh, "Similarity relations and fuzzy orderings," *Information Sciences*, vol. 3, pp. 177–200, 1971.

[13] R. N. Shepard, "Toward a universal law of generalization for psychological science," *Science*, vol. 237, no. 4820, pp. 1317–1323, 1987.

[14] S. H. Chen, "Operations on fuzzy numbers with function principle," *Tamkang Journal of Management Sciences*, vol. 6, no. 1, pp. 13–25, 1985.

[15] H. Bandemer and W. Näther, *Fuzzy Data Analysis*, Kluwer Academic Publishers, 1992.

[16] L. A. Zadeh, "Fuzzy sets," *Information and Computation*, vol. 8, pp. 338–353, 1965.

[17] C. E. Shannon, "A mathematical theory of communication," *The Bell System Technical Journal*, vol. 27, pp. 379–423, 1948.

[18] P. Luukka, "Feature selection using fuzzy entropy measures with similarity classifier," *Expert Systems with Applications*, vol. 38, no. 4, pp. 4600–4607, 2011.

[19] R. R. Yager, "On ordered weighted averaging aggregation operators in multicriteria decisionmaking," *IEEE Transactions on Systems, Man and Cybernetics*, vol. 18, no. 1, pp. 183–190, 1988.

[20] M. O'Hagan, "Aggregating template or rule antecedents in real-time expert systems with fuzzy set logic," in *Proceedings of the 22nd Asilomar Conference on Signals, Systems and Computers*, vol. 2, pp. 681–689, IEEE, Pacific Grove, Calif, USA, November 1988.

[21] R. Fullér and P. Majlender, "An analytic approach for obtaining maximal entropy OWA operator weights," *Fuzzy Sets and Systems*, vol. 124, no. 1, pp. 53–57, 2001.

[22] M. Kaufmann and M. Gupta, *Fuzzy Mathematical Models in Engineering and Management Science*, Elsevier Science, 1988.

[23] M. Brunelli and J. Mezei, "How different are ranking methods for fuzzy numbers? A numerical study," *International Journal of Approximate Reasoning*, vol. 54, no. 5, pp. 627–639, 2013.

[24] C. L. Hwang and K. P. Yoon, *Multiple Attribute Decision Maing, Methods and Applications*, Springer, Berlin, Germany, 1981.

[25] Y.-J. Lai, T.-Y. Liu, and C.-L. Hwang, "TOPSIS for MODM," *European Journal of Operational Research*, vol. 76, no. 3, pp. 486–500, 1994.

[26] G. R. Jahanshahloo, F. H. Lotfi, and M. Izadikhah, "Extension of the TOPSIS method for decision-making problems with fuzzy data," *Applied Mathematics and Computation*, vol. 181, no. 2, pp. 1544–1551, 2006.

[27] M. S. García-Cascales and M. T. Lamata, "Solving a decision problem with linguistic information," *Pattern Recognition Letters*, vol. 28, no. 16, pp. 2284–2294, 2007.

[28] Z. X. Cui, H. K. Yoo, J. Y. Choi, and H. Y. Youn, "Multi-criteria group decision making with fuzzy logic and entropy based weighting," in *Proceedings of the 5th International Conference on Ubiquitous Information Management and Communication (ICUIMC '11)*, pp. 1–7, February 2011.

[29] Y. Yao, S. Wong, and C. Butz, "On information-theoretic measures of attribute importance," in *Proceedings of the Pacific-Asia Conference on Knowledge Discovery and Data Mining*, 1999.

[30] L. Yuan and H. K. Kesavan, "Minimum entropy and information measure," *IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews*, vol. 28, no. 3, pp. 488–491, 1998.

[31] F. Hassanzadeh, M. Collan, and M. Modarres, "A practical approach to R&D portfolio selection using the fuzzy pay-off method," *IEEE Transactions on Fuzzy Systems*, vol. 20, no. 4, pp. 615–622, 2012.

[32] P. Luukka, M. Fedrizzi, L. Niyigena, and M. Collan, "Fuzzy similarity based fuzzy TOPSIS with multi-distances," in *Proceedings of the 5th International Joint Conference on Computational Intelligence (FCTA '13)*, pp. 193–200, Algarve, Portugal, September 2013.

[33] M. Collan, R. Fullér, and J. Mezei, "A fuzzy pay-off method for real option valuation," *Journal of Applied Mathematics and Decision Sciences*, vol. 2009, Article ID 238196, 14 pages, 2009.